<u>TTMath School</u>

Level 4: Class 20

1. Monotonic Functions: Strictly increasing or strictly decreasing
   Rigorous definition for strictly increasing: $f(x+c) - f(x) > 0$ for all $x \in \mathbb{R}$ and $c > 0$ and in the domain of $f(x)$.
   (a) Properties:
      i. The sum of 2 increasing functions is increasing.
      ii. The product of 2 increasing functions is not necessarily increasing.
      iii. Monotonic functions are invertible.
2. Piece-wise Functions
3. Absolute Value
4. Floor and Ceiling Function
   (a) Properties:
      i. $\lfloor x+n \rfloor = \lfloor x \rfloor + n$ for all real numbers x and integers n.
      ii. $\{x+n\} = \{x\}$ for all real numbers x and integers n.
      iii. $\lfloor x+y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$ for all real x, y.

Homework

Complete the following problems. Remember to simplify your answer. Do not use a calculator.

1. Textbook reference and additional practice: Intermediate Algebra: Chapter 15.4, 16
2. Prove that $x(15-x)$ is a decreasing function for $x > 7.5$.
3. Find the area of the region described by $|5x| + |6y| \leq 30$.

4. Find all values of x that satisfy $\lfloor x \rfloor + \lceil x \rceil = 9$.

5. Solve the equation $7t + \lfloor 2t \rfloor = 52$.

6. Find all real numbers k satisfying $\lfloor k \rfloor = 5k - 14$.

7. Find the inverse of f if $f(x) = \begin{cases} \sqrt{2-x} & \text{if } x < 0, \\ 1 - x^2 & \text{if } x \geq 0. \end{cases}$

8. Find all x such that $8^x(3x+1) = 4$, and prove that you have found all values of x that satisfy this equation.

9. Let $\{x\} = x - \lfloor x \rfloor$. Find $\dfrac{\{\sqrt{3}\}^2 - 2\{\sqrt{2}\}^2}{\{\sqrt{3}\} - 2\{\sqrt{2}\}}$ without a calculator.

10. For how many positive integers n is it true that $n < 1000$ and that $\lfloor \log_2 n \rfloor$ is a positive even integer?

11. Let
$$S_n = \lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{n}\rfloor.$$
Compute the largest value of $k < 1997$ such that $S_{1997} - S_k$ is a perfect square.

12. Suppose that $f(x)$ and $g(x)$ are positive for all x. If $f(x)$ and $g(x)$ are both monotonically increasing, then must $f(x) \cdot g(x)$ be monotonically increasing?