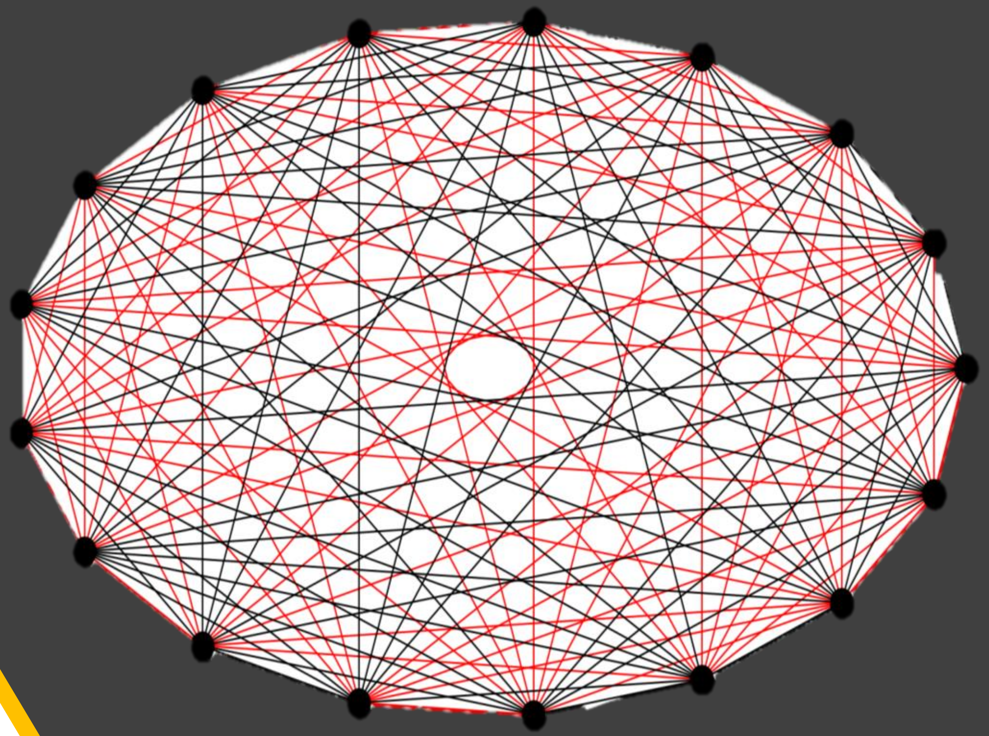
James Marshall Reber

Methods for Improving the Lower Bound of R(5,5)



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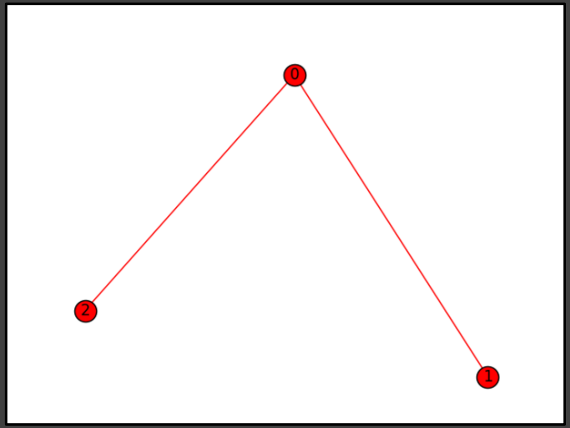
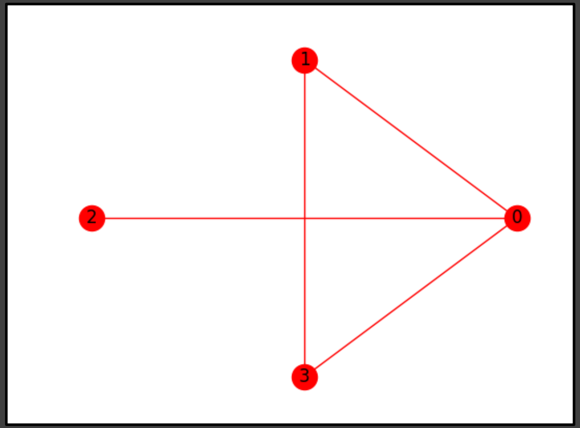
Introduction to Graph Theory

What is a graph?

- A graph is a set of vertices and edges, denoted G(V,E).
- A vertex, or node, represents an object, and an edge denotes a relation between two vertices.



V = {0,1,2}
E = {(0,1), (0,2)}

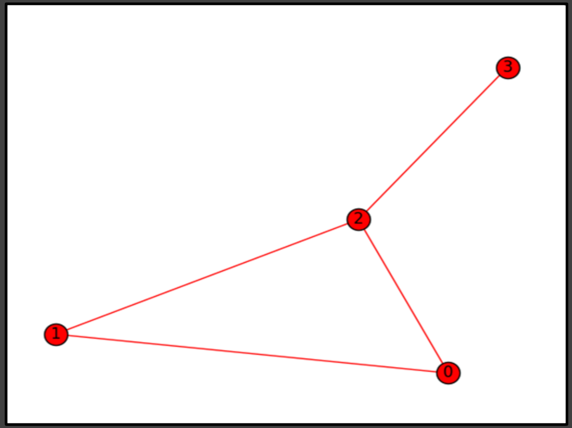


V = {0,1,2,3}
E = {(0,1),(0,2),(0,3),(3,1)}

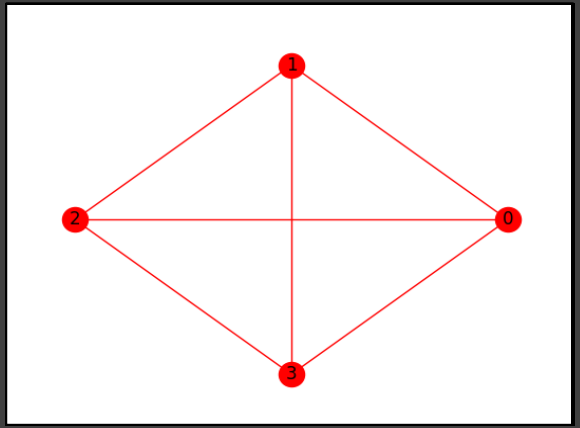
Introduction to Graph Theory

What is a clique?

- A clique, in an undirected graph G= {V,E}, is a subset of the vertices such that every two distinct vertices are adjacent.
- We say that we have a k-clique if the subset of the vertices has size k.



(0,1,2) forms a 3-clique



(0,1,2,3) forms a 4-clique

Ramsey Numbers

Ramsey numbers answer the question of the minimum number of guests that must be invited R(m,n) such that at least m will know each other or at least n will not know each other.

Answers how many elements of some structure must there be to guarantee that a particular property will hold.

Mathematically, we want to find the minimum size for a graph such that we must find either a k-clique in the graph or it’s complement.

Goal of the Project

- What we want to do is find a method which will improve the lower bound for the 5th Ramsey number.
  - So far, our techniques have included:

Stimulated annealing

Genetic algorithms

Swarm algorithms

Algorithms

- Why don’t we just brute force search?
  - Note that the total number of graphs of size n is $2^{\binom{n}{2}}$.
  - If we wanted to brute force search through the graphs of size 43, we would have to search through

**6762169998536515153309949246931412563441245773262355483237897
0755414259527260782012725408753620120050518322559136912470896
9404876163437487680689892432562658442734955518726507735976342
6258258445478710181225103211573094762147219990257131480304218
066899066093835491046378700 8**

different graphs.

Algorithms

- Genetic Algorithms
  - A metaheuristic inspired by nature which attempts to weed out “bad” objects through a sort of natural selection (filter).
  - Often used in optimization and search problems that have a large sample space to cycle through.
- Typically, a genetic algorithm requires some form of a fitness measure and some form of mutation. You have a population which you iterate over however many times, each time mutating and breeding the things in your population in order to improve the fitness.

Algorithms

- Simulated Annealing
  - An metaheuristic inspired by the controlled cooling of temperatures on metal.
- Pick a starting temperature and a starting point. Find the “neighbor” and calculate the probability of selecting that neighbor using the current temperature and the distance from the fitness of the neighbor to the fitness of the current value. This allows for some flexibility at the start but slowly converges to a greedy algorithm as the temperature goes to 0.
- The probability function is $e^{\frac{\text{best fitness} - \text{neighbor fitness}}{\text{temperature}}}$.

Algorithms

- What is a swarm algorithm?
  - A metaheuristic inspired by insects who work in swarms in order to find things
- Specifically, we’re focused on using the bee metaheuristic.
- The bee metaheuristic has three core classes to consider: the worker bee, the scout bee, and the lazy bee. The worker bee takes a greedy path, the scout bee finds a new path and greedily goes as far as it can, and the lazy bee uses the work of the worker bee and the scout bee and makes slight changes. The bees then do a “waggle dance” and communicate to each other on what they find, and then readjust accordingly.

Future Work

- Neural networks to create counterexamples from scratch.
- Developing a more general algorithm.
- Have the code run in parallel.
- Combine algorithms.