UC Berkeley, ME100, Spring 2021
Lab 3: Linear circuits

Plan to complete these tasks during the week of February 15

Checkoff due on Gradescope by Monday March 1, 11:59pm

Introduction and objectives

In this lab you will build some simple RC and RLC circuits and practice using your mini oscilloscope to take measurements from them. The objectives are:

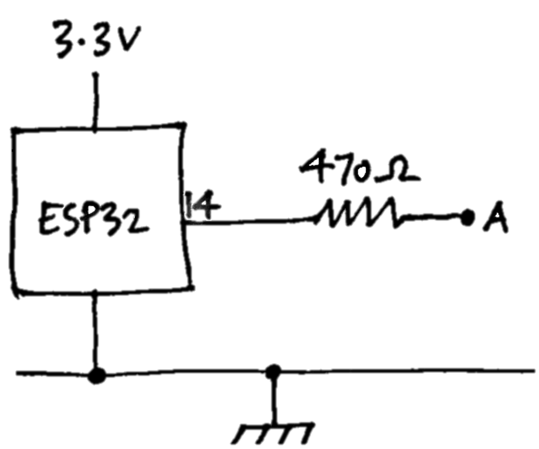
- To gain experience of using an oscilloscope to measure voltage waveforms (i.e. voltages as functions of time);
- To gain practice at assembling discrete components on a breadboard/prototyping board;
- To reinforce understanding of RC and L/R time constants in linear circuits;
- To gain exposure to an RLC circuit, including the oscillations that can occur in them.

Step 1: Set up waveform generator using ESP32

You will use the ESP32 to generate a periodic signal — a square wave — to stimulate the circuits you'll build. Modify `blink.py` from Lab 1 so that:

- Instead of outputting a signal on the LED pin, it does so on GPIO pin 14.
- The script holds the output pin high (value 1) for 0.2 s, then holds it low for 0.2 s, and repeats this cycle 300 times.

Test this script by running it from `shell49`, with the following setup on your breadboard:



Attach your probe cable to the oscilloscope, then connect the hook at the tip of your oscilloscope probe to node A, and the alligator clip to ground. Set the switch on the probe to the "X1" position.

Turn on the oscilloscope, and enter the following settings:

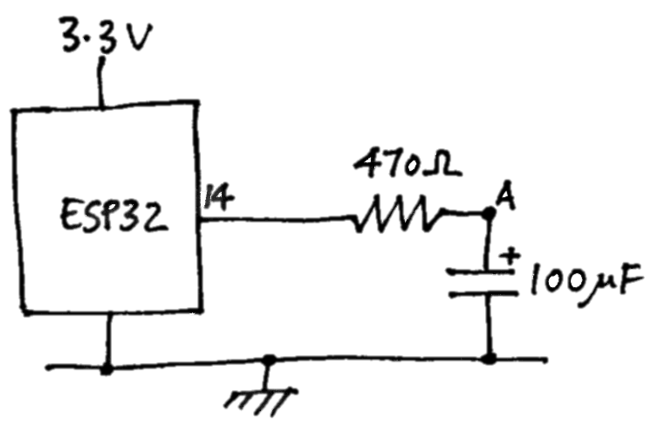
- Ensure 1X is visible in the top left corner of the screen, pressing the “1X10X” button if necessary to toggle between 1X and 10X modes.
- Use the left–right arrow keys to set the time resolution to 50 ms per division: “50 ms” will be visible at the top of the screen.
- Use the up–down arrow keys to set the voltage resolution to 1 V per division: “1 V” will be visible at the top of the screen.
- Ensure DC mode is selected by pressing the AC/DC button until “DC” appears in blue at the top of the screen.
- Press “TRIG” until both “Run” (in green) and “Auto” appear at the top of the screen.

Connect your ESP32 to your computer and run your script in `shell49`.

Verify that you obtain a voltage waveform with the expected amplitude, period and duty cycle (duty cycle is the fraction of time the waveform is high).

Step 2: Capacitor charging/discharging curves

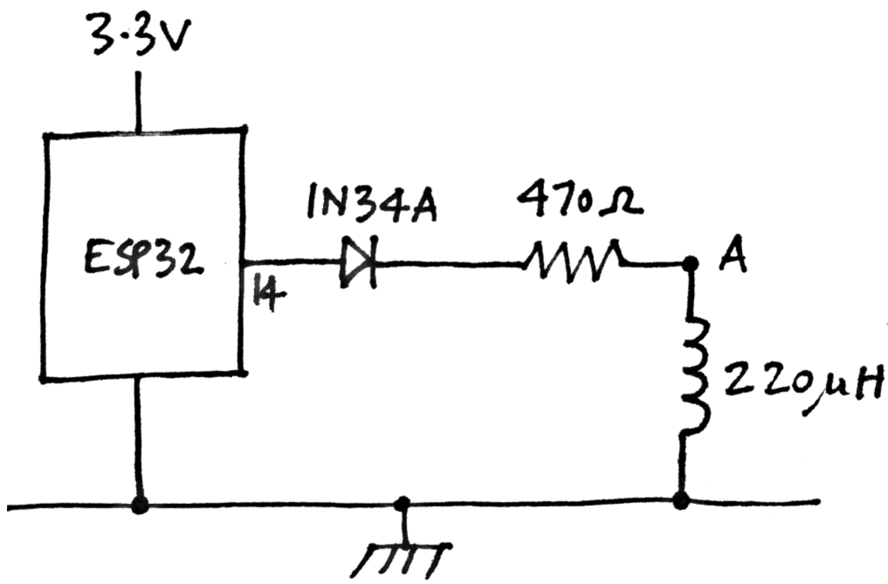
Now add a 100 μF electrolytic capacitor (be careful about polarity) between node A and ground:

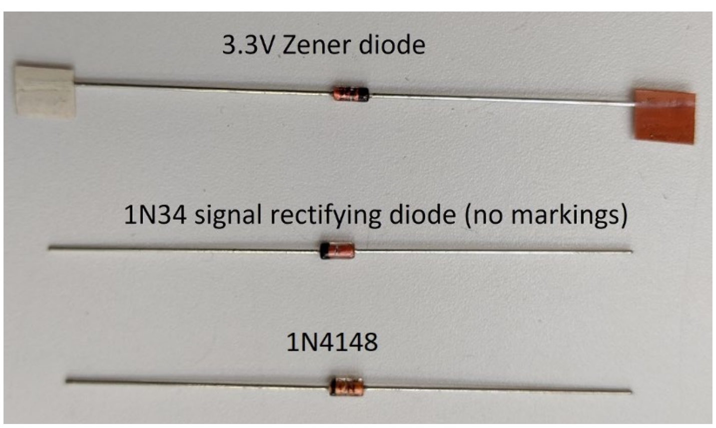


1. Compute the expected time constant for charging and discharging this capacitor, neglecting internal resistance of the ESP32.
2. Compute the expected peak current that must be sourced or sunk by pin 14 of the ESP32. Is this peak within the spec of the ESP32?
3. Record a voltage waveform (you may need to press “TRIG” again first, to ensure both “Run” (in green) and “Auto” appear at the top of the screen). Take a photo of the waveform as it appears on the screen of the oscilloscope (there does not appear to be a way to export saved waveforms from the oscilloscope).
4. Use a graphical method to estimate the charging and discharging time constants from your captured waveform. Are the charging and discharging time constants equal (as far as you can tell) or not? Upload to Gradescope an image or PDF slide showing how you analyzed the captured waveform.
5. Compare your measured time constants to the one you predicted in item 1 above. Do they agree reasonably closely or not, bearing in mind the uncertainty of the graphical method you used?

Step 3: Inductive load

Now modify your circuit as follows, by introducing one of your 1N34 diodes (or an 1N4148 if that is what was supplied in your kit) and your 220 μH inductor in series with the resistor. The diode and inductor can be identified from the photos below (note that the Zener diode, which should not be used for this circuit, is distinguished by paper tabs left on the ends of its leads). The negative terminal of the diode (which should connect directly with the resistor) is marked by black paint on the body of the diode. The diode is to protect the ESP32 from possible voltage spikes that may occur across the inductor when the current changes.



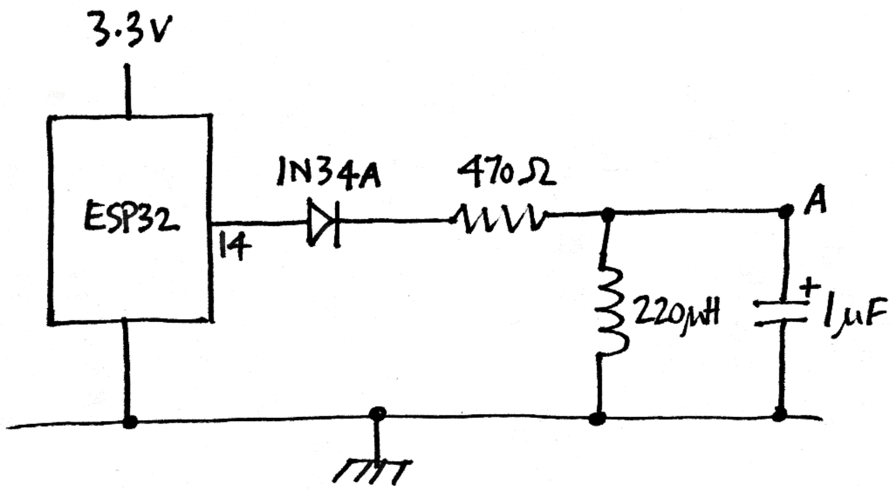


1. Modify your waveform-generation script to hold pin 14 high for 2 ms, then low for 2 ms, and repeat 3000 times.
2. Connect your oscilloscope between node A and ground, and set its voltage resolution to 50 mV/div and the time resolution to 1 ms/div. Set the scope running in auto-trigger mode as before, then run the script from `shell49`.

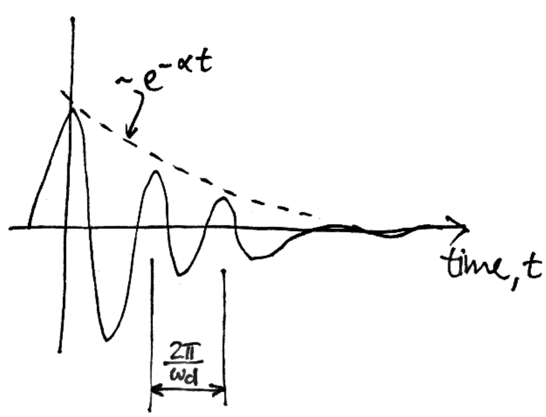
3. You will probably see some spikes of voltage appear periodically. Take a photo of the waveform that appears. You may need to press “STOP” on the scope to get a stable image.
4. By analyzing the captured waveform, estimate the peak dI/dt through the inductor.
5. What, approximately (order-of-magnitude) is the time constant for decay of the voltage spikes you observe? What, physically, determines the value of this time constant? Upload a labeled copy of your captured waveform, showing how you extracted the time constant.
6. What is the time period of the spikes that appear? If you get the same results as I did, you will see only significant positive spikes of voltage. Does it surprise you that there are no negative spikes? Explain your observations.

Step 4: RLC load

Finally, add 1 μF electrolytic capacitor in parallel with the inductor:



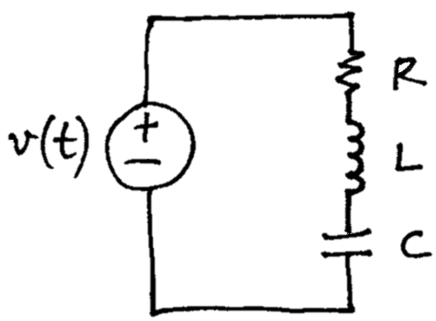
1. Set your scope to 50 mV/div and 50 μs/div. Set the scope running in auto-trigger mode as before.
2. Run the same waveform-generation script as in Step 3 (i.e. 4 ms period, 3000 repetitions).
3. You will likely see periodic decaying, oscillating voltage pulses such as in the sketch below. This would be a sign of an *underdamped* response of the RLC circuit. Whatever behavior you see, press ‘STOP’ on the scope and take a photograph of the captured waveform.



4. Using your captured waveform and the appendix for reference, estimate the parasitic resistance in the LC branch of the circuit. Explain your method — we suggest by uploading a sketch. (Is the discrete 470 Ω resistance involved in the oscillations you observe? Explain.)
5. Based on your answer for parasitic resistance from item 4 above, on the nominal L and C values used, and on the information in the appendix, estimate the angular frequency ω_d of the oscillations. Is this estimated frequency approximately consistent with what you observe from the captured waveform?

Appendix: RLC oscillator

Consider the RLC circuit below, whose behavior is governed by a second-order differential equation:



KVL:

$$v_R + v_L + v_c = v(t)$$

$$Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int_0^t i(t)\mathrm{d}t = v(t)$$

Assuming:

$$v(t) = \begin{cases} 0 & t \leq 0 \\ V_0 & t > 0 \end{cases}$$

$$\frac{\mathrm{d}^2 i(t)}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{LC}i(t) = 0$$

for $t > 0$.

Let $2\alpha = \frac{R}{L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$.

α is the Neper frequency or attenuation, and describes how quickly transients die away when there is a change in input. Substituting:

$$i'' + 2\alpha i' + \omega_0^2 i = 0$$

Damping factor:

$$\zeta = \frac{\alpha}{\omega_0} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

For $\zeta < 1$, the system is under-damped and will oscillate. For $\zeta = 1$, the system is critically damped and transients will gracefully decay away to zero in the minimum possible time without oscillating. For $\zeta > 1$ the system is overdamped and transients will decay without oscillation, but more slowly than in the $\zeta = 1$ case.

Characteristic equation [trial solution $i = e^{st}$]:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\omega_0\left(\zeta \mp \sqrt{\zeta^2 - 1}\right)$$

General form of solution:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

In the overdamped case, *s* are real:

$$i(t) = B_1 e^{-\omega_0\left(\zeta + \sqrt{\zeta^2 - 1}\right)t} + B_2 e^{-\omega_0\left(\zeta - \sqrt{\zeta^2 - 1}\right)t}.$$

In the underdamped case, oscillations occur:

$$i(t) = C_1 e^{-\alpha t}\cos(\omega_d t) + C_2 e^{-\alpha t}\sin(\omega_d t)$$

where $\omega_d = \omega_0\sqrt{1 - \zeta^2}$.

In the critically damped case:

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}.$$