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Innovative Applications of O.R.

The block relocation problem with appointment scheduling

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ABSTRACT

In many container terminals, containers are piled vertically and horizontally in the terminal yard, limited mainly by the dimensions of the yard crane. Import and export containers are typically stacked separately. An external truck can access the terminal to pick up an import container only after making an appointment reserving a pickup time. To reduce truck waiting time inside the terminal, container pickup appointments are normally scheduled on a time window basis. However, when a truck arrives at the terminal yard at the appointed time, it is common for the target container not to be at the top of its stack, resulting in unproductive relocations to remove all the containers stacked above the target container and thus increasing the truck's waiting time. To minimize the number of relocations, the Block Relocation Problem (*BRP*) is usually solved independently, without consideration of appointment scheduling. In this paper, we introduce a new optimization problem—the Block Relocation Problem with Appointment Scheduling (*BRPAS*)—to jointly address the two issues. To solve the problem, two binary IP models are proposed, and examples from the literature are solved to confirm the performance of the two models. The proposed formulations are further extended to cover several operational aspects related to the flexibility of container pickup operations. Results show that the proposed approach can improve container relocation operations at terminal yards by coordinating with appointment scheduling.

1. Introduction

Container terminals serve as global exchange hubs for containerized cargos that flow dynamically through the global supply chain. Such terminals are always seeking ways to improve operations, management techniques, and technologies at all levels, starting at the strategic level, down through the tactical level, and ending at the operational level. Continuous improvement in container terminal operations is no longer a choice but a necessity, especially for terminals looking for a competitive advantage. Typically, terminal operators deal with containers transported from the hinterland through the landside by external trucks or railways and export containers transported from the seaside by ship. Transshipment containers are received from the seaside and loaded again onto vessels to be shipped to other container terminals (de Melo da Silva, Toulouse & Wolfler Calvo, 2018a), as shown in Fig. 1. All containers received from both the seaside and landside are temporarily stored at the terminal yard until pickup time.

Due to the spatial constraints on container storage at these terminals, containers are stacked in bays in the terminal yard (Caserta, Schwarze & Voß, 2011). To conceptualize the situation, we can think of a bay as representing a two-coordinate stacking configuration, where a set of vertical stacks and horizontal tiers of containers are constructed, as shown in Fig. 2. The intersection of a stack and a tier represents a "slot" where a container can be stacked (Caserta, Schwarze & Voß, 2012). To stack/unstack a container to/from a bay, a yard crane is used, with accessibility to the container stacks only from the top. The efficiency of container handling is one of the essential Key Performance Indicators (KPIs) for terminal productivity. This drives terminal operators to strive for reducing the container handling time in order to reduce power consumption, emissions, and operational costs, and therefore maximizing productivity. However, unproductive container moves at the yard are unavoidable, especially when there is a need to pick up containers that do not occupy the topmost slots of a bay (Ku & Arthanari, 2016a). In such a case, the yard crane must remove the upper containers blocking the target container and relocate them to empty slots, then retrieve the target container (Fig. 2). Container-handling delays negatively impact the departure time of both vessels and external trucks. Thus, minimizing the unproductive moves is vital for improving the terminal throughput.

The Block Relocation Problem (*BRP*), or the so-called Container Relocation Problem (*CRP*), describes the problem of minimizing the number of container relocations needed to retrieve a set of target containers stacked in a particular bay (Ting & Wu, 2017). In the typical *BRP*, the retrieval order for each container in the bay

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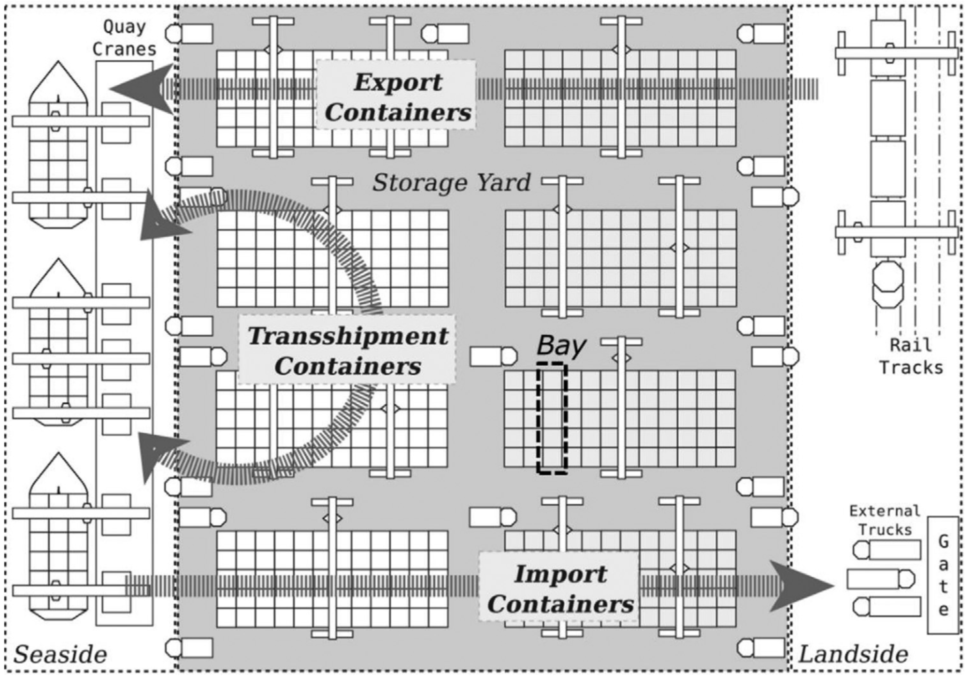


Fig. 1. Container terminal organization and container flow directions (de Melo da Silva et al., 2018a).

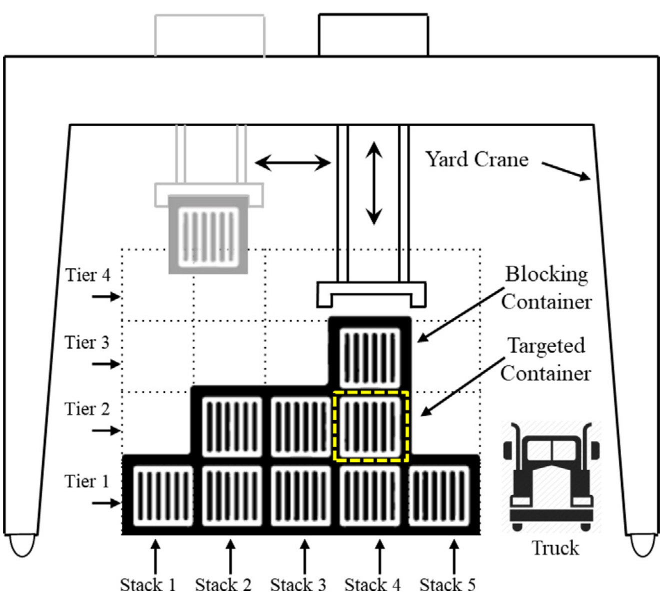


Fig. 2. Bay layout in a container terminal yard and the container relocation operation.

is assumed to be known in advance, and all containers are assumed to be retrieved within the planning horizon. Thus, the *BRP* can be solved for both export containers to be loaded onto a vessel or import containers to be loaded onto external trucks. For export containers, the container retrieval order is determined based on the stowage plan that has been developed for stacking the target containers on the vessel. The stowage plan typically describes the stacking plan for the export containers in determined slots on the vessel, considering the vessel loading constraints (Jovanovic, Tanaka, Nishi & Voß, 2019). On the other hand, the retrieval times for import containers are produced from external truck arrival information or container pickup appointments. The arrival process is usually managed by adopting an appointment scheduling system for the trucks arriving at the terminal. In this paper, we examine appointment scheduling for import container pickup under the *BRP*.

In many terminals, a Truck Appointment System (*TAS*) is implemented to coordinate the arrival of external trucks and mitigate congestion at the terminal (Ramírez-Nafarrate, González-Ramírez, Smith, Guerra-Olivares & Voß, 2017). This is mainly achieved by deciding on the acceptable quota of trucks to be served during each time window (Chen, Govindan, Yang, Choi & Jiang, 2013). *TAS* supports the management of trucking operations for multiple trucking companies and individual truckers by assigning trucks to different time slots with a goal of keeping a balanced terminal workload over time (Torkjazi, Huynh & Shiri, 2018). When a truck arrives at the terminal gate to pick up a container based on the pre-determined appointment time, it is directed to the designated container location in the yard after completing gate operations. If the yard is congested, some trucks may have to wait outside or inside the terminal until the congestion is relieved. If truck appointments and container handling operations are not managed efficiently, the terminal gates and the yard area may become highly congested. In such a situation, congestion alleviation on the landside and in the yard area is critical to terminal productivity enhancement. Coordination between yard operations and the arrival process can lead to significant performance improvements for both the trucking companies and the terminal operators (Zhang, Zeng & Yang, 2019).

Despite the importance of coordinating landside and yard operations, such coordination remains largely under-covered in the literature, as most of the existing work is focused on the use of *BRP* independently from container pickup appointment scheduling. In this paper, we take a more integrated approach, using a relocation-based scheduling method that considers appointment scheduling when seeking to minimize container relocations. To the best of our knowledge, this is the first container relocation optimization problem to consider both operational aspects. The remainder of the paper is organized as follows: Section 1, Section 2 provides a summary of the previous literature. Section 3 discusses in more detail the motivation behind this work. Section 4 introduces the proposed mathematical models for the Block Relocation Problem with Appointment Scheduling (*BRPAS*). Section 5 describes computational experiments and results. Finally, Section 6 offers conclusions and suggests areas for future work.

2. Previous Work

The *BRP* has been studied extensively in the literature from many perspectives; versions can be classified based on the following two metrics: container retrieval priority and container relocation rule. When each container has a unique retrieval priority, the *BRP* is known as *distinct BRP*. The case in which more than one container shares the same retrieval priority is referred to as *duplicate* (group) *BRP*. Regarding container relocation rules, *restricted BRP* assumes that only the containers above the target container can be relocated, whereas, in the *unrestricted* version, containers can be relocated from any slot regardless of the slot of the target container. Under these classifications, various mathematical formulations have been proposed (Caserta et al., 2012; Eskandari & Azari, 2015; Expósito-Izquierdo, Melián-Batista & Marcos Moreno-Vega, 2014; Galle, Barnhart & Jaillet, 2018a;Yat-wah Wan & Liu, 2009), with a greater focus on computational performance in solving the *NP*-hard problem (Caserta et al., 2012). (Interested readers may refer to Caserta et al. (2011) for a comprehensive survey of the *BRP*. For truck arrival scheduling in container terminals, please see Ma, Fan, Jiang and Guo (2019).)

The *BRP* is solved using both exact approaches (de Melo da Silva et al., 2018a; Expósito-Izquierdo, Melián-Batista & Moreno-Vega, 2015; Ku & Arthanari, 2016b; Lu, Zeng & Liu, 2020; Tanaka & Mizuno, 2018; Tanaka & Takii, 2016; Tanaka & Voß, 2021;Bacci, Mattia & Ventura, 2020) and heuristic approaches (Bacci, Mattia & Ventura, 2019; Caserta, Schwarze & Voß, 2009, 2011; Expósito-Izquierdo et al., 2014; Feillet, Parragh & Tricoire, 2019; Forster & Bortfeldt, 2012; Ji, Guo, Zhu & Yang, 2015; Jovanovic & Voß, 2014; Jovanovic, Tuba & Voß, 2017, 2019; Petering & Hussein, 2013; Ting & Wu, 2017; Tricoire, Scagnetti & Beham, 2018; Zhang, Guan, Yuan, Chen & Wu, 2020) under the different classifications noted above. However, most of the existing work has studied the *BRP* with *distinct* priority. This version of the *BRP* is best suited to the export container handling process, where the retrieval priority is mostly matched with the stacking sequence of containers on the vessel obtained in advance from the stowage plan.

Receiving far less attention has been the *BRP* with *duplicate* container retrieval priority, which fits the case in which multiple trucks arrive at the terminal within the same time window to pick up import containers (Kim & Hong, 2006; Ku & Arthanari, 2016a; Tanaka & Takii, 2016). In this case, the target containers are given the same retrieval priority, corresponding to the arrival time window of the corresponding trucks. Recently, the *BRP* with container *group* retrieval priority has been studied under the assumption of a stochastic pickup time window for containers, such as in Galle, Borjian Boroujeni, Manshadi, Barnhart & Jaillet, 2018, or a deterministic pickup time, such as in the work of Zeng, Feng and Yang (2019). The majority of studies solve the problem under the restricted relocation assumption in order to reduce the search space for obtaining the optimal solution (de Melo da Silva et al., 2018a), whereas the unrestricted version yields more optimization opportunities (Tricoire et al., 2018).

Considering more realistic aspects in solving the *BRP* is rarely addressed in the literature. Due to the complexity of the problem, only a small minority of existing papers explicitly consider the terminal's interrelated operations that directly impact the relocation plan obtained from the *BRP*. This would include operational planning such as yard crane scheduling, ship stowage planning, and external truck appointment scheduling, all of which are apt to have important implications for the *BRP* solution. Among the limited studies seeking to address such elements, Ji et al. (2015); Jovanovic, Tuba and Voß (2019), and Tanaka and Voß (2019) examined the *BRP* considering the stowage plan for export containers for ships and yards. The idea is to coordinate the loading of export containers onto the vessel with container retrieval operations in the yard in order to minimize the number of container relocations, vessel time, or crane time. From another perspective, Galle, Barnhart and Jaillet (2018b) combined the *BRP*, or the so-called *CRP*, with yard crane scheduling and introduced a novel optimization problem that considers scheduling yard crane operations along with relocations and retrievals using the distinct priority and restricted *CRP*. Determining the container pickup sequence that reduces container relocations is addressed in the literature as well. Borjian, Galle, Manshadi, Barnhart and Jaillet (2015), Zeng et al. (2019), and, more recently, Feng, Song, Li and Zeng (2020) showed that adopting a flexible policy to reorder the container retrieval sequence for a group of trucks after their arrival at the terminal can reduce the number of relocations, with implications for truck delays. Table 1 summarizes some of the *BRP* literature. The bottom row of the table shows the nature of our work.

As noted above, the few existing studies that address the coordination of the *BRP* with other terminal operations tend to focus only on inter-yard operations and the ship stowage operation at the terminal seaside using distinct priority and the restricted version of the *BRP*. In contrast, we address the coordination between appointment scheduling for import container pickup and container handling operations using the unrestricted *BRP* with duplicate container retrieval priority, which better suits appointment scheduling purposes. (The proposed approach can fit the unrestricted *BRP* with distinct container retrieval priority).

One of the main objectives of the appointment scheduling of external trucks is to alleviate congestion (Chen, Govindan & Yang, 2013) inside and outside the terminal gates by controlling the arrival process and reducing truck turnaround times. A number of research efforts have shown the benefits of adopting appointment systems to manage external truck arrivals (Douma, Schutten & Schuur, 2009; Namboothiri & Erera, 2008; Zehendner & Feillet, 2014; Zhao & Goodchild, 2010). Truck appointment systems have evolved in recent years to meet the many challenges faced by terminal operators and trucking companies. Approaches such as collaborative appointment scheduling (Azab, Karam & Eltawil, 2020; Phan & Kim, 2015, 2016), where appointment scheduling systems are designed to support coordinated decision making between the container terminal on one side, and the trucking companies on the other, have been studied. However, the majority of the literature with this theme focuses simply on controlling truck entries to the terminals; only a few studies deal directly with coordinating truck arrivals with vessel and yard operations (Shiri & Huynh, 2016). Appointment scheduling for external trucks is usually applied to both export and import containers. For export containers, Chen and Yang (2010) proposed coordination strategies involving vessel arrival times and external truck arrival times. For import container operations, most of the scheduling systems consider terminal capacity—for example, gate service rates and container handling rates in the yard area (Li, Chen, Ng, Talley & Jin, 2020; Zhang et al., 2019)—with less focus on coordination with yard operations. In this paper, we address this issue by introducing a relocation-based appointment scheduling approach in which container pickup appointments are mainly determined based on the expected container relocations in the yard. We describe the motivation for this work in detail in the next section.

3. Motivation

In many container terminals that adopt truck appointment systems, individual drivers and trucking companies are encouraged to

Table 1
Summary of the recent BRP literature under different considerations.

Author/s (year)	Retrieval priority		Relocation rule		Coordination aspects			
	Dist.	*Dup.*	*Rest.*	*Unrest.*	*SP*	*YCS*	*CPS*	*AS*
Kim and Hong (2006)	√	√	√					
Wan et al. (2009)	√		√	√				
Zhu, Qin, Lim and Zhang (2012)	√		√					
Caserta et al. (2012)	√		√	√				
Expósito-Izquierdo et al. (2014)	√		√	√				
Ji et al. (2015)	√		√		√	√		
Expósito-Izquierdo et al. (2015)	√		√					
Zehendner, Caserta, Feillet, Schwarze and Voß (2015)	√		√					
Ku and Arthanari (2016a)		√	√					
Tanaka and Takii (2016)	√	√	√					
Tanaka and Mizuno (2018)	√		√	√				
Tricoire et al. (2018)	√			√				
Galle et al. (2018a)	√		√					
Galle et al. (2018b)	√		√			√		
Jovanovic et al. (2019)	√		√		√			
Tanaka and Voß (2019)	√		√	√	√			
Borjian et al. (2015)		√	√	√			√	
Zeng et al. (2019)		√	√				√	
Feng et al. (2020)		√	√				√	
This work	√	√		√			√	√

Note: *Dist.*: Distinct, *Dup.*: Duplicate, *Rest.*: Restricted, *Unrest.*: Unrestricted, *SP*: Stowage Plan, *YCS*: Yard Crane Scheduling, *CPS*: Container Pickup Sequence, *AS*: Appointment Scheduling.

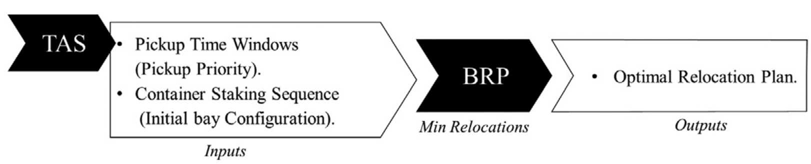


Fig. 3. Classical BRP under the TAS.

submit their preferred arrival times in advance via a web-based information system (Phan & Kim, 2016; Zhang et al., 2019). The preferred times are considered along with terminal capacity and the expected terminal workload per each time window (e.g., 30 minutes) to decide whether the terminal can accommodate the truck during the indicated time. In order to avoid congestion, service rates in the yard area and yard crane capacity are considered. Typically, the queue lengths at the gates and in the terminal yard are the main factors in the appointment scheduling process (Chen et al., 2013). After deciding the appointment schedules, the terminal operators notify the trucking companies/truckers of the final appointment window. In turn, the companies/truckers are expected to be punctual, despite the fact that not all containers will be picked up according to the originally submitted preferred times but rather according to the appointment windows determined by the terminal operators.

At the terminal, the yard operators use the final appointments schedule produced by *TAS* to define a time-window-based pickup priority for each container stacked in a certain bay (Ku & Arthanari, 2016a). When a truck arrives at the designated bay, there is a considerable likelihood that the target container is not in the top slot of a stack. The yard crane is then used to relocate the blocking containers with minimum relocations by solving the *BRP* (Fig. 3.). It is common that several trucks are waiting at the same bay during a particular time window while some of the target containers are buried under one or more blocking containers. Thus, a truck may experience more delays resulting from relocations performed to access its container or from the relocations associated with other target containers with earlier pickup times. The lack of coordination between the pickup appointment times and the container handling operation at the bay can lead to unproductive container moves and, consequently, more truck delays.

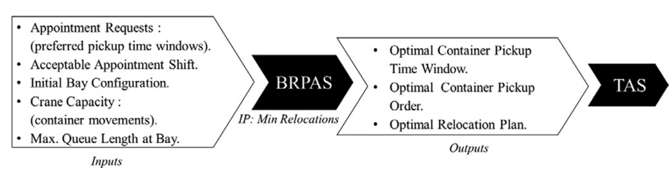


Fig. 4. The proposed block relocation problem with appointment scheduling.

The motivation for this study is to introduce a new optimization problem aimed at coordinating the appointment scheduling for import container pickup with the container relocation process. The proposed *BRPAS* determines the container pickup times from the expected relocations that will be performed if trucks arrive within certain time windows. Two binary *IP* mathematical models are proposed with the objective of minimizing the overall number of container relocations. The adopted version of the *BRP* is unrestricted with duplicate container pickup priority, which is rarely featured in the existing literature but best fits the case of import container pickup scheduling.

4. *BRP* with appointment scheduling (*BRPAS*)

Unlike the classical *BRP*, which assumes that the actual container pickup time/priority is already decided in advance, the *BRPAS* (Fig. 4) assumes that the trucking companies will submit appointment requests to pick up their containers at least one day before heading to the terminal. An appointment request gives information about the preferred pickup time window for each container and the container/truck *ID*. Following this submission, the *BRPAS* determines the optimal pickup time window and pickup or-

Table 2
IP model parameters and indices for the BRPAS.

N	Number of containers in the bay initial configuration.
i	Index for container, $i \in \{1, \ldots, N\}$.
C	Number of stacks.
s	Index for stack, $s \in \{1, \ldots, C\}$.
H	Maximum Height of bay.
r	Index for tier, $r \in \{1, \ldots, H\}$.
T	Number of time windows.
t	Index for time window, $t \in \{1, \ldots, T\}$.
L	Maximum Queue length at bay (appointments per time window).
G	Maximum number of containers moves (*retrievals and relocations*) per time window.
k	Index of the stage, $k \in \{1, \ldots, G\}$, where each stage k represents one container move.
I_{isr}	Whether container i occupies slot (s, r) in the initial bay layout, $I_{isr} \in \{0, 1\}$.
p_i	Preferable pickup time for each container, $i \in \{1, \ldots, N\}$.
δ	Acceptable shift of container pickup appointment time window.

der within the time window for each container. The main goal of the *BRPAS* is to provide a relocation-based appointment schedule that achieves the minimum number of relocations where both the relocation plan and appointment schedule are to be determined simultaneously. The container retrieval times determine the appointment schedules to be shared with the trucking companies; the expected relocation plan is given to the yard crane operators. It is worth mentioning that both plans are decided in advance, before the arrival of the first truck at the terminal.

Note that the terminal operator uses the *BRPAS* to reschedule the container pickup times submitted in the appointment requests and determines the final appointments that achieve the problem objective. However, to keep the gap between the preferred pickup times (i.e., the preferred times submitted to the online appointment system) and the final appointment times within reason, it is assumed that an acceptable appointment shift has been agreed to by the company side and the terminal side. Therefore, the proposed *BRPAS* assumes that the trucking companies will accept the appointment plan and follow the pickup order, considering their arrival preferences. This shift allowance is to be applied equally for all containers in the bay to guarantee fairness in the appointment scheduling process. The proposed *BRPAS* (Fig. 4) recognizes, as well, scheduling factors related to yard crane capacity, defined here by the number of container moves that a crane can perform during a given time window at the designated bay. In addition, the maximum queue length at the bay, which is the maximum number of trucks that can wait at the designated bay to receive their containers, is specified.

To formulate the *BRPAS*, the following assumptions are made: (1) the initial bay configuration is known in advance; (2) all containers in the bay will be picked up within the planning horizon, and no containers are received during the retrieval process; (3) each container has a predefined preferable pickup time window which may be changed due to scheduling; (4) and the *BRPAS* mainly adopts the unrestricted version of *BRP*, where containers can be relocated from any slot in the bay.

4.1. Binary IP model for BRPAS: model 1

The basic binary IP mathematical model of the *BRPAS* is inspired by the mathematical formulation proposed by Caserta et al. (2012) for the *BRP* with distinct priority, which defines the decision variables as the binary status of containers in certain bay slots at a certain time. In our *BRPAS*, two mathematical formulations are proposed. The indices and parameters for both models are defined in Table 2. Each container is given a unique index $i \in \{1, \ldots, N\}$ which is used primarily for tracking the container status, since the actual container pickup time, pickup order, and stacking sequence change dynamically during the solution process. In the *BRPAS*, the preferred pickup time window p_i and the initial position I_{isr} for each container in the bay are assumed to be known in advance. The planning horizon T is divided into smaller time windows $t \in \{1, \ldots, T\}$, where T is the latest time a container can be picked up from the bay. The time window length is not specified in this work and is expressed as a time unit for generalization. In a bay of C stacks and H tiers (max height), container i can occupy slot (s, r), where stack $s \in \{1, \ldots, C\}$ is indexed from left to right and tier $r \in \{1, \ldots, H\}$ is indexed from bottom to top (see Fig. 5a). To identify the initial bay layout, the input parameter I_{isr} is a binary encoding of the initial stacking for N containers in different bay slots (Fig. 5b). Both the p_i and I_{isr} parameters are used to describe the initial bay configuration and layout as shown in Fig. 5. In this paper, the bay layout refers to the stacking sequence of containers in the bay slots, while the bay configuration refers to the layout where each container has a pickup time attached to it.

In the *BRPAS*, assuming that only trucks with the same arrival time window at the designated bay will be kept waiting at that bay during the retrieval process, the maximum queue length at that bay is defined in terms of the maximum number of retrievals per time window using the parameter L. The crane capacity, i.e., the maximum number of container moves that a crane can perform per time window, is considered by defining parameter G where $G \geq L$. Note that the bay configuration changes with each container move (either relocation or retrieval); to update the configuration, each container move is referred to as a *stage,* with $k \in \{1, \ldots, G\}$ indicating the stage number. Defining the stages based on each move makes it easier to formulate the problem and update the configuration dynamically. Consequently, we have the following characterization for container status in the bay: First, container i is in its slot (s, r); second, container i is relocated from the slot (s, r); third, container i is relocated to slot (s', r'): $s' \in \{1, \ldots, C\} \mid s' \neq s, r' \in \{1, \ldots, H\}$; fourth, container i is picked up (by the waiting truck) and removed permanently from the slot (s, r). In the first mathematical formulation of the $BRPAS_{(\mathbf{1})}$, the four container statuses are expressed as the binary decision variables of the model, as shown in Table 3.

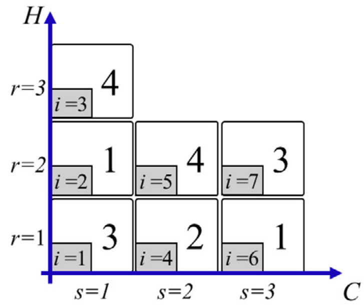
The first of the two binary *IP* mathematical formulations of the *BRPAS* under the conditions noted above is described below.

$BRPAS_{(\mathbf{1})}$ Model:
Objective:

$$Min \sum_{i=1}^{N} \sum_{s=1}^{C} \sum_{r=1}^{H} \sum_{k=1}^{G} \sum_{t=1}^{T} y_{isrk}^{t} \tag{1}$$

Subjected to :

$$\left| p_i - \sum_{s=1}^{C} \sum_{r=1}^{H} \sum_{k=1}^{G} \sum_{t=1}^{T} t \ v_{isrk}^{t} \right| \leq \delta, \ \forall i \in \{1, \ldots, N\} \tag{2}$$



a. initial bay configuration.

(s,r) \ i	1	2	3	4	5	6	7
(1,1)	1	0	0	0	0	0	0
(1,2)	0	1	0	0	0	0	0
(1,3)	0	0	1	0	0	0	0
(2,1)	0	0	0	1	0	0	0
(2,2)	0	0	0	0	1	0	0
(2,3)	0	0	0	0	0	0	0
(3,1)	0	0	0	0	0	1	0
(3,2)	0	0	0	0	0	0	1
(3,3)	0	0	0	0	0	0	0

b. binary encoding for initial bay layout (I_{isr}).

Fig. 5. Container bay configuration for BRPAS.

Table 3
Decision variables of the BRPAS$_{(1)}$.

$$u^t_{isrk} = \begin{cases} 1 & if\ container\ i\ occupies\ the\ slot\ (s,r)\ at\ stage\ k\ of\ time\ window\ t \\ 0 & otherwise \end{cases}$$

$$x^t_{isrk} = \begin{cases} 1 & if\ container\ i\ is\ relocated\ from\ slot\ (s,r)\ at\ stage\ k\ of\ time\ window\ t \\ 0 & otherwise \end{cases}$$

$$y^t_{isrk} = \begin{cases} 1 & if\ container\ i\ is\ relocated\ to\ slot\ (s,r)\ at\ stage\ k\ of\ time\ window\ t \\ 0 & otherwise \end{cases}$$

$$v^t_{isrk} = \begin{cases} 1 & if\ container\ i\ is\ picked\ up\ from\ slot\ (s,r)\ at\ stage\ k\ during\ time\ widow\ t \\ 0 & otherwise \end{cases}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k=1}^{G} v^t_{isrk} \le L,\ \forall t \in \{1,\dots,T\} \tag{3}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H} v^t_{isrk} + \sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H} x^t_{isrk} \le 1,\ \forall k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{4}$$

$$\sum_{i=1}^{N} x^t_{isrk} \le \sum_{i=1}^{N}\left(u^t_{isrk} - u^t_{is(r+1)k}\right),\ \forall s \in \{1,\dots,C\},\ r \in \{1,\dots,H-1\},\ k \in \{1,\dots,G\}, t \in \{1,\dots,T\} \tag{5}$$

$$\sum_{s'=1, s'\neq s}^{C}\ \sum_{r=1}^{H} y^t_{is'rk} \ge \sum_{r=1}^{H} x^t_{isrk},\ \forall i \in \{1,\dots,N\},\ s \in \{1,\dots,C\},\ k \in \{1,\dots,G\},\ t \in \{1,\dots.,T\} \tag{6}$$

$$\sum_{i=1}^{N} v^t_{isrk} + \sum_{i=1}^{N} y^t_{isrk} + \sum_{i=1}^{N} x^t_{isrk} \le 1,\ \forall s \in \{1,\dots,C\},\ r \in \{1,\dots,H\},\ k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{7}$$

$$u^1_{isr1} = I_{isr},\ \forall\ i\ \{1,\dots,N\}, s \in \{1,\dots,C\},\ r \in \{1,..,H\} \tag{8}$$

$$u^t_{isrk+1} = u^t_{isrk} + y^t_{isrk} - x^t_{isrk} - v^t_{isrk},\ \forall\ i \in \{1,\dots,N\}, s \in \{1,\dots,C\},\ r \in \{1,\dots,H\},\ k \in \{1,\dots,G-1\}, t \in \{1,\dots,T\} \tag{9}$$

$$u^t_{isr1} = u^{t-1}_{isrG} + y^{t-1}_{isrG} - x^{t-1}_{isrG} - v^{t-1}_{isrG},\ \forall\ i \in \{1,\dots,N\}, s \in \{1,\dots,C\},\ r \in \{1,\dots,H\}, t \in \{2,\dots,T\} \tag{10}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k'=k+1}^{G} u^t_{isrk'} + \sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k'=1}^{G}\sum_{t'=t+1}^{T} u^{t'}_{isrk'} \le G\cdot T\left(1 - \sum_{s=1}^{C}\sum_{r=1}^{H} v^t_{isrk}\right),\ \forall\ i \in \{1,\dots,N\},\ k \in \{1,\dots,G\}, t \in \{1,\dots,T\} \tag{11}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k=1}^{G}\sum_{t=1}^{T} v^t_{isrk} = 1,\ \forall i \in \{1,\dots,N\} \tag{12}$$

$$\sum_{i=1}^{N} u^t_{isrk} \le 1,\ \forall s \in \{1,\dots,C\},\ r \in \{1,\dots,H\},\ k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{13}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H} u^t_{isrk} \le 1,\ \forall i \in \{1,\dots,N\}, k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{14}$$

$$u^t_{isrk},\ x^t_{isrk},\ y^t_{isrk}\ and\ v^t_{isrk}\ \in \{\ 0,\ 1\ \},\ \forall\ i \in \{1,\ \dots,N\},\ s \in \{1,\dots,C\}, r \in \{1,\dots,H\}, k \in \{1,\dots,G\}, t \in \{1,\dots,T\} \tag{15}$$

The objective function (1) here is to minimize the total number of relocations needed for all of the containers in the bay to be picked up. Constraint (2) is used to restrict the shifting of the container pickup appointment time from the preferred pickup time submitted by the trucking company. To control this shift, parameter δ limits the pickup time shift to $\pm\delta$ time windows from the original preferred time for each container. In constraint (3), the queue length at the bay is limited to L trucks/container retrievals, assuming that a truck can pick up only one container. Constraint (4) controls crane capacity by limiting the number of container moves per time window to the value of the parameter G. This constraint is also used to define the stage k at which each container move can take place. Constraints (5), (6), and (7) describe the relocation process. Under constraint (5), when relocating containers, the topmost blocking container must be relocated first, before the containers below it, following the Last-In-First-Out (*LIFO*) rule. Constraint (6) requires that the relocated container must go to a different stack. In constraint (7), when a container is moved from or to a slot, it is either relocated or retrieved. This constraint prevents transitive and cyclic container moves within the bay. Constraints (8), (9), and (10) are used to update the bay layout when containers are moved: Constraint (8) initiates the bay layout before the first container move, constraint (9) updates the bay layout from one stage to the next within a time window, and constraint (10) updates the layout transition from the last stage of time window $t-1$ to the first stage in the next time window t. Constraints (11) through (14) are logical constraints. Constraint (11) establishes that if a container is retrieved, it can no longer occupy any slot in the configuration. Constraint (12) guarantees that each container must be retrieved. Constraint (13) states that each slot must be occupied by at most one container; similarly, constraint (14) specifies that a container must not belong to more than one slot. Finally, the constraints in (15) define the binary domain of the decision variables.

4.2. Numerical example

Fig. 6 shows a numerical example for a set of $N=7$ containers stacked in a bay of dimensions $C \times H = 3 \times 3$, with a planning horizon of $T=4$ time windows. The preferred container pickup times p_i can be advanced or delayed by a $\delta = 1$ time window; maximum bay queue length is set as $L=2$, with crane capacity $G=3$ container moves per time window. The preferred pickup time windows for containers are intentionally shown in faded font to indicate that these times are not necessarily the final appointment times. Once the model determines the target containers that minimize container relocations, the final pickup appointment times are uncovered, and the related retrieval and relocation operations are performed. As shown in Fig. 6, at time window $t1$, container $i2$ is the target container. Since the pickup time of container $i3$ cannot be advanced to $t1$ due to the strict appointment shifting allowance of ($\delta = 1$), container $i3$ needs to be relocated. This differs from the case of containers $i6$ and $i7$ at time window $t2$, where the pickup time can be shifted by one time window, allowing both containers to be picked up without a relocation. However, at time window $t3$, container $i4$ cannot remain in the bay, constrained by the $\delta = 1$ limit in constraint (2). Thus, $i4$ is a target container at $t3$. The reason that containers $i3$, $i5$, and $i4$ cannot be retrieved together at $t3$ despite satisfying the allowable appointment shift is that the number of containers to be retrieved is limited by the bay queue length constraint (3), with $L=2$. In this case, only $i3$ and $i4$ can be picked up, while $i5$ must be relocated and then retrieved with $i1$ at $t4$. Note that the model can give multi-optimal solutions for the same instance. This is because the *BRP* is a combinatorial optimization problem, motivating the *BRPAS* proposed in this paper. For example, at $t1$ in Fig. 6, we started with container $i2$ as a target container. However, in an alternative optimal solution, we could pick up container $i6$ first as a target container at $t1$ and then delay container $i2$ one time window, to be picked up later at $t2$.

In a general sense, shifting the container appointment pickup times from the preferred times may result from one or more of the following scenarios: (1) when a container is blocking the target container and changing the pickup time by $\pm\delta$ will prevent relocation; (2) when a container is blocked by other containers and changing the pickup appointment time will prevent the relocation of containers above it; (3) when the number of containers that can be retrieved exceeds the corresponding queue length at the bay (In this situation, the model tends to shift the pickup times of some containers to avoid the excessive queue length); (4) when the number of container moves exceeds the limit and the model changes the pickup times to keep the crane capacity under control. Note that coordinating truck appointments with container relocations can significantly reduce the number of relocations compared to having trucks arriving at the terminal at their original preferred times. For instance, in the numerical example in Fig. 6, the initial bay configuration shows that containers $i3$,$i5$, and $i7$ are blocking containers $i2$, $i4$, and $i6$, respectively. This will lead to "four relocations" to retrieve all containers based on the preferred pickup times (*i.e.*, $\delta = 0$).

4.3. BRPAS: model 2

Although the first formulation (shown above) is informative and provides important details regarding both the relocation and appointment plan in an organized manner, the model is quite large. To deal with this issue, we introduce a second formulation, $BRPAS_{(2)}$, in which the number of variables and constraints is reduced by replacing the retrieval variable v^t_{isrk} with binary variables z^t_{ik} to indicate whether container i is retrieved at stage k of time window t. In the new formulation, the variable x^t_{isrk} does not distinguish between a container retrieval and a container relocation; rather, it defines whether the container is moved from its slot at a certain stage and a certain time window.

***BRPAS$_{(2)}$* Model:**

The $BRPAS_{(2)}$ model uses the same objective function (1) and constraints (5), (8), (13), and (14) from the $BRPAS_{(1)}$ model. For the remainder of the $BRPAS_{(2)}$ model, constraints (16)–(26) are formulated using the new decision variables z^t_{ik} and the new definition of x^t_{isrk}, as shown below:

$$\left| p_i - \sum_{k=1}^{G}\sum_{t=1}^{T} t\, z^t_{ik} \right| \le \delta,\ \forall i \in \{1,..,N\} \tag{16}$$

$$\sum_{i=1}^{N}\sum_{k=1}^{G} z^t_{ik} \le L,\ \forall t \in \{1,\dots,T\} \tag{17}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H} x^t_{isrk} \le 1,\ \forall k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{18}$$

$$\sum_{r=1}^{H} y^t_{isrk} + \sum_{r=1}^{H} x^t_{isrk} \le 1,\ \forall i \in \{1,\dots,N\}, s \in \{1,\dots,C\},\ k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{19}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H} y^t_{isrk} + z^t_{ik} = \sum_{s=1}^{C}\sum_{r=1}^{H} x^t_{isrk},\ \forall i \in \{1,\dots,N\}, k \in \{1,\dots,G\},\ t \in \{1,\dots,T\} \tag{20}$$

$$u^t_{isrk+1} = u^t_{isrk} + y^t_{isrk} - x^t_{isrk},\ \forall\, i \in \{1,\dots,N\}, s \in \{1,\dots,C\},\ r \in \{1,\dots,H\},\ k \in \{1,\dots,G-1\}, t \in \{1,\dots,T\} \tag{21}$$

$$u^t_{isr1} = u^{t-1}_{isrG} + y^{t-1}_{isrG} - x^{t-1}_{isrG},\ \forall\, i \in \{1,\dots,N\}, s \in \{1,\dots,C\},\ r \in \{1,\dots,H\}, t \in \{2,\dots,T\} \tag{22}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k'=k+1}^{G} u^t_{isrk'} + \sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k'=1}^{G}\sum_{t'=t+1}^{T} u^{t'}_{isrk'} \le G \cdot T\left(1 - z^t_{ik}\right),\ \forall\, i \in \{1,\dots,N\},\ k \in \{1,\dots,G\}, t \in \{1,\dots,T\} \tag{23}$$

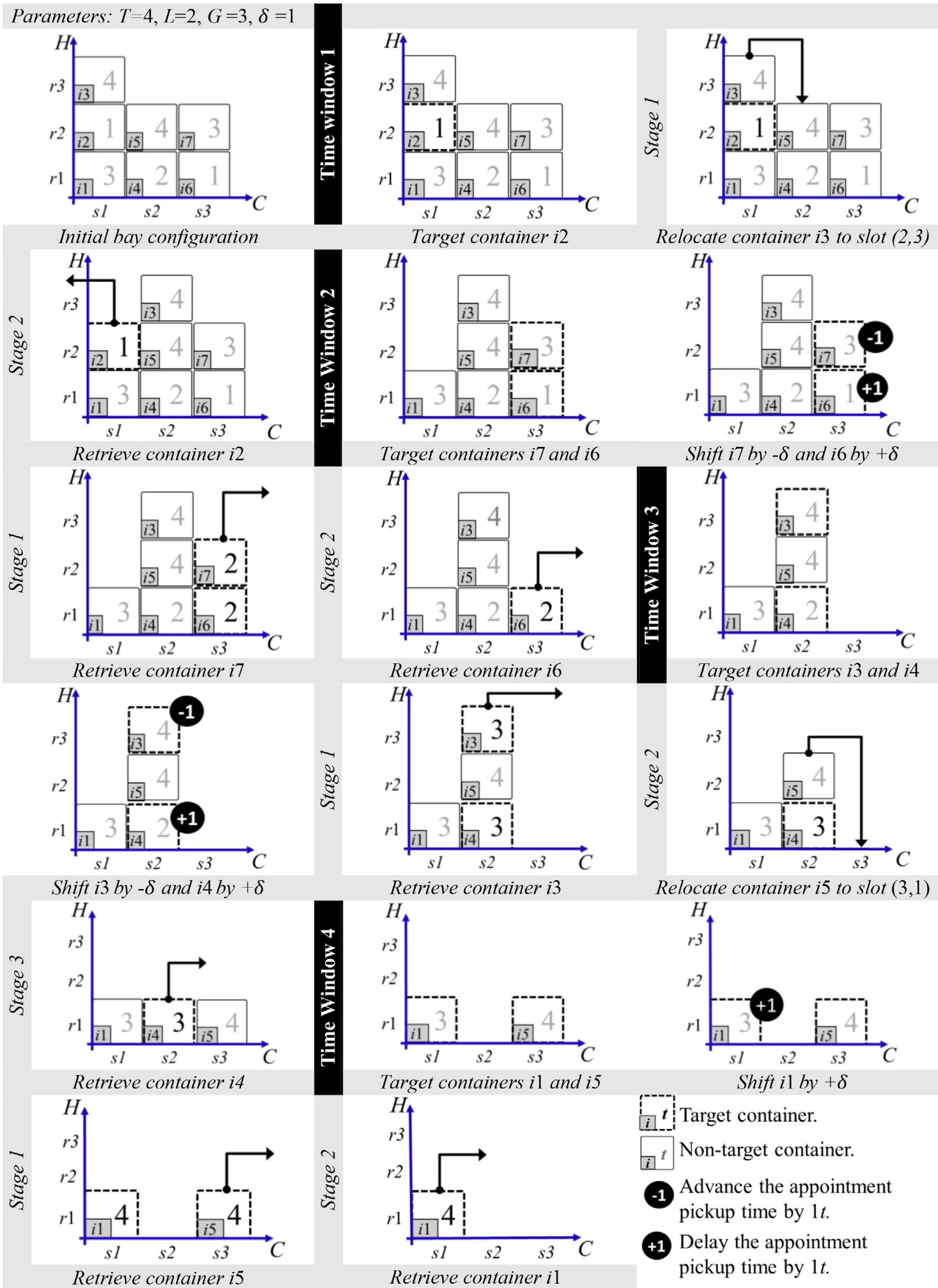


Fig. 6. A numerical example for BRPAS.

$$\sum_{k=1}^{G}\sum_{t=1}^{T} z_{ik}^{t} = 1,\ \forall i \in \{1,\ldots,N\} \tag{24}$$

$$u_{isrk}^{t},\ x_{isrk}^{t},\ \textit{and}\ y_{isrk}^{t} \in \{\,0,\ 1\,\},\ \forall\, i \in \{1,\ \ldots,N\}, s \in \{1,\ldots,C\},$$
$$r \in \{1,\ldots,H\}, k \in \{1,\ldots,G\}, t \in \{1,\ldots,T\} \tag{25}$$

$$z_{ik}^{t}\ \in\ \{\,0,\ 1\,\},\ \forall\, i \in \{1,\ \ldots,N\}, k \in \{1,\ldots,G\}, t \in \{1,\ldots,T\} \tag{26}$$

In the BRPAS$_{(2)}$ model, constraints (16), (17), and (18) correspond to constraints (2), (3), and (4) in the BRPAS$_{(1)}$ model, respectively. Note that constraint (18) defines the stage using only the variable x_{isrk}^{t}. Constraints (19) and (20) are equivalent to constraints (6) and (7), respectively. In constraint (19), the container cannot occupy the same stack s when moved from that stack. Constraint (20) defines the variable x_{isrk}^{t} using variables y_{isrk}^{t} and z_{ik}^{t} while implying the same condition as constraints (7) of the BRPAS$_{(1)}$ model. Constraints (21), (22), (23), and (24) correspond to constraints (9), (10), (11), and (12) of the BRPAS$_{(1)}$ model, respectively. Finally, constraints (25) and (26) are equivalent to constraint (15) in the BRPAS$_{(1)}$ model; constraint (26) defines the binary domain of the decision variable z_{ik}^{t}.

4.4. The flexible BRPAS

In the *BRPAS* formulations, the adopted modeling approach tracks container status after each move and updates the bay layout. In this sense, the approach forces specific container relocation and retrieval orders based on the minimization of the total number of relocations. This provides a substantial amount of information about the solution behavior of the model through the obtained optimal solution and makes it simpler for the terminal operator to adopt the solution. However, flexibility in retrieving containers might be required in order to load containers onto the waiting trucks based on a First-Come-First-Served (*FCFS*) basis rather than being forced to retrieve containers in the order prescribed by the *basic* mathematical models (*BRPAS*$_{\mathbf{(1)}}$ and *BRPAS*$_{\mathbf{(2)}}$). This flexibility can be relatively achieved for the *BRPAS* if the bay configuration is updated based on container *relocation* rather than on every container *move* (*both relocation and retrieval*). The idea is to give the target containers occupying the topmost slots in the bay an equal retrieval priority (to be retrieved at the same stage without assigning a unique pickup order/stage for each). Updating the bay layout based on container *relocations* helps achieve retrieval flexibility while leaving the optimal solution of the *BRPAS* unchanged.

For such retrieval flexibility under the *BRPAS*, the *stage* definition no longer defines a *container move,* but rather it defines a container relocation; more than one container can be moved in one stage. As a result, a container subgroup schedule is obtained, unlike the individual container schedule per each stage in the original *BRPAS*. For example, in Fig. 7*a*, suppose containers *i*2, *i*3, *i*5, and *i*6 are the target containers at time window 1, and we have two stages (maximum number of relocations) to flexibly retrieve the four containers. Containers *i*3, *i*5, and *i*7 could form the first container subgroup to be moved at time window $t = 1$ at stage $k = 1$ (Fig. 7b), while containers *i*2 and *i*6 would comprise the second container subgroup (Fig. 7*c*). The flexibility in this example means that the crane operator could retrieve *i*3 or *i*5 at the first stage of time window *t*1 when either of the corresponding trucks arrives first at the bay. Similarly, at the second stage, after the relocation of *i*7 at the first stage, if the truck picking up container *i*6 arrives first, it will pick up its container and leave the terminal without the need to wait for the truck picking up container *i*2 to be served first.

4.4.1. The flexible BRPAS formulation

To formulate the flexible *BRPAS*, we introduce a new parameter μ that defines the stage based on the maximum number of container relocations. We use μ in the second *BRPAS* formulation (the *BRPAS*$_{\mathbf{(2)}}$ model) to introduce the *BRPAS(flex)* model.

***The BRPAS(flex)* Model:**

$$Min\ \sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k=1}^{\mu}\sum_{t=1}^{T} y_{isrk}^{t} \tag{27}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H} y_{isrk}^{t} \le 1,\ \forall k \in \{1,\ldots,\mu\},\ t \in \{1,\ldots,T\} \tag{28}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k=1}^{\mu} x_{isrk}^{t} \le G,\ \forall t \in \{1,\ldots,T\} \tag{29}$$

$$\sum_{i=1}^{N} x_{isrk}^{t} \le \sum_{i=1}^{N}\left(u_{isrk}^{t} - u_{is(r+1)k}^{t}\right),\ \forall s \in \{1,\ldots,C\},$$
$$r \in \{1,\ldots,H-1\},\ k \in \{1,\ldots,\mu\}, t \in \{1,\ldots,T\} \tag{30}$$

$$u_{isr1}^{1} = I_{isr},\ \forall\, i\, \{1,\ldots,N\}, s \in \{1,\ldots,C\},\ r \in \{1,..,H\} \tag{31}$$

$$\sum_{i=1}^{N} u_{isrk}^{t} \le 1,\ \forall s \in \{1,\ldots,C\},\ r \in \{1,\ldots,H\},\ k \in \{1,\ldots,\mu\},\ t \in \{1,\ldots,T\} \tag{32}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H} u_{isrk}^{t} \le 1,\ \forall i \in \{1,\ldots,N\}, k \in \{1,\ldots,\mu\},\ t \in \{1,\ldots,T\} \tag{33}$$

$$\left| p_i - \sum_{k=1}^{\mu}\sum_{t=1}^{T} t\ z_{ik}^{t} \right| \le \delta,\ \forall i \in \{1,..,N\} \tag{34}$$

$$\sum_{i=1}^{N}\sum_{k=1}^{\mu} z_{ik}^{t} \le L,\ \forall t \in \{1,\ldots,T\} \tag{35}$$

$$\sum_{r=1}^{H} y_{isrk}^{t} + \sum_{r=1}^{H} x_{isrk}^{t} \le 1,\ \forall i \in \{1,\ldots,N\}, s \in \{1,\ldots,C\},$$
$$k \in \{1,\ldots,\mu\},\ t \in \{1,\ldots,T\} \tag{36}$$

$$\sum_{s=1}^{C}\sum_{r=1}^{H} y_{isrk}^{t} + z_{ik}^{t} = \sum_{s=1}^{C}\sum_{r=1}^{H} x_{isrk}^{t},\ \forall i \in \{1,\ldots,N\}, k \in \{1,\ldots,\mu\},\ t \in \{1,\ldots,T\} \tag{37}$$

$$u_{isrk+1}^{t} = u_{isrk}^{t} + y_{isrk}^{t} - x_{isrk}^{t},\ \forall\, i \in \{1,\ldots,N\}, s \in \{1,\ldots,C\},$$
$$r \in \{1,\ldots,H\},\ k \in \{1,\ldots,\mu-1\}, t \in \{1,\ldots,T\} \tag{38}$$

$$u_{isr1}^{t} = u_{isr\mu}^{t-1} + y_{isr\mu}^{t-1} - x_{isr\mu}^{t-1},\ \forall\, i \in \{1,\ldots,N\}, s \in \{1,\ldots,C\},$$
$$r \in \{1,\ldots,H\}, t \in \{2,\ldots,T\} \tag{39}$$

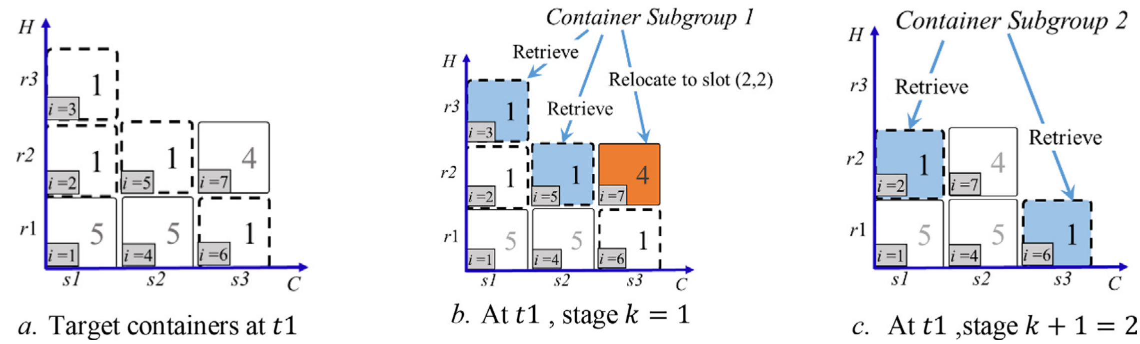


Fig. 7. Updating bay layout based on relocations in the flexible BRPAS.

$$\sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k'=k+1}^{\mu} u^{t}_{isrk'} + \sum_{s=1}^{C}\sum_{r=1}^{H}\sum_{k'=1}^{\mu}\sum_{t'=t+1}^{T} u^{t'}_{isrk'} \le \mu \cdot T\left(1 - z^{t}_{ik}\right),$$
$$\forall\, i \in \{1,\ldots,N\},\; k \in \{1,\ldots,\mu\}, t \in \{1,\ldots,T\} \quad (40)$$

$$\sum_{k=1}^{\mu}\sum_{t=1}^{T} z^{t}_{ik} = 1,\; \forall i \in \{1,\ldots,N\} \quad (41)$$

$$u^{t}_{isrk},\; x^{t}_{isrk},\; and\; y^{t}_{isrk} \in \{\,0,\;1\,\},\; \forall\, i \in \{1,\;\ldots,N\}, s \in \{1,\ldots,C\},$$
$$r \in \{1,\ldots,H\},\; k \in \{1,\ldots,\mu\}, t \in \{1,\ldots,T\} \quad (42)$$

$$z^{t}_{ik} \;\in\; \{\,0,\;1\,\},\; \forall\, i \in \{1,\;\ldots,N\},\; k \in \{1,\ldots,\mu\}, t \in \{1,\ldots,T\} \quad (43)$$

To produce the *BRPAS(flex)* model, three main modifications are made to the $BRPAS_{(2)}$ formulation. First, parameter μ is defined and used instead of parameter G (number of container moves per time window) to define the *stage* in the *BRPAS(flex)* model so that the stage index $k \in \{1,\ldots,\mu\}$. This appears clearly in the objective function (27), subject to constraints (28)–(43). Second, to define the stage based on the relocation decision variable y^{t}_{isrk}, constraint (18) in the $BRPAS_{(2)}$ model is replaced by constraint (28) in the *BRPAS(flex)* model. Third, in the $BRPAS_{(2)}$ model, crane capacity (which is a key characteristic of the proposed *BRPAS*) is tacitly considered in constraint (18). To ensure that the total number of container moves remains within the crane capacity in the *BRPAS(flex)* model, constraint (29) is introduced. In the remainder of the *BRPAS(flex)* model formulation, constraints (30), (31), (32), and (33) correspond to constraints (5), (8), (13), and (14), respectively. Constraints (34) and (35) correspond to constraints (16) and (17), respectively. Finally, constraints (36)–(43) correspond to constraints (19)–(26) in sequence.

As can be noted from Fig. 7b, containers $i3$ and $i5$ can be retrieved in any order, but container $i7$ cannot be relocated to slot $(2,2)$ until the retrieval of container $i5$. In the flexible *BRPAS*, the relocation order is not distinguished from the retrieval order when the blocking container is in the same subgroup (*stage*) with the target container(s). Therefore, we propose an online post-processing algorithm that can be implemented to guide the crane operator to the optimal sequence of moves using the actual arrival information of trucks. In the proposed Algorithm 1, we refer to the container and the truck by the same index i, where the containers subgroup is defined with its stage k. Note that the *BRPAS(flex)* model, with its post-processing algorithm, applies the *FCFS* policy only for containers in the same subgroup; however, containers within the same time window may not be served following the *FCFS* if this will increase the pre-determined optimal relocations. This can be seen clearly in Fig. 7, where containers $i2$ and $i6$ are not picked up before containers $i3$ and $i5$ even if their trucks arrive earlier than the trucks for $i3$ and $i5$.

4.4.2. A numerical example of BRPAS(flex)

A detailed example of the flexible *BRPAS* is shown in Fig. 8. Sixteen containers are planned to be picked up within 5 time windows from a 4×5 bay. The following parameters are assumed for the solved example: $L = 4,\ \mu = 3,\ G = 7,\ \delta = 1$.

The initial bay configuration in Fig. 8. shows that several containers will be blocked if they are to be picked up according to their preferred times. Moreover, the number of containers with preferred pickup times of $t1$ and $t2$ exceeds the maximum queue length of 4 container retrievals per time window. The *BRPAS(flex)* model tends to shift the pickup times for some containers and form a container subgroup schedule per stage. For instance, at $t1$, containers $i4$, $i8$, $i12$ and $i16$ are targeted for retrieval; however, $i4$ is blocked by $i5$. Consequently, at stage $k = 1$ of $t1$, containers $i8$, $i12$ and $i16$ will be picked up and container $i4$ will be relocated to the slot that container $i5$ is occupying (slot $(2,3)$). In this situation, the online post-processing algorithm helps the crane operator retrieve the three target containers based on the arrival of their trucks but not relocate $i5$ until the retrieval of $i8$. After relocating $i5$, container $i4$ will be retrieved at the second stage. At $t2$, all remaining containers from time window $t1$ must be picked up in order to satisfy the $\delta = 1$ limit. Thus, container $i1$, $i2$, $i3$ and $i11$ will be retrieved in three stages, given that the flexible *BRPAS* approach can group only the topmost containers at once. Similarly, the remaining containers ($i9$ and $i13$) from $t2$ will be shifted to $t3$, while $i15$ is shifted to $t3$ from $t4$ to avoid relocating it when $i3$ is being picked up. At $t3$, we relocate containers $i10$ and $i14$ to enable pickin up $i9$ and $i13$. For $t4$ and $t5$, the remaining containers will be retrieved, one container per stage, without a relocation.

5. Computational experiments and results

To evaluate the proposed models, different subsets of instances from Tanaka and Takii (2016) and de Melo da Silva et al. (2018a); de Melo da Silva et al. (2018b) are solved as container group priority cases, and a subset of instances from Caserta et al. (2012) is solved as a distinct container priority case. The container retrieval group/distinct priority in the selected instances is assumed to be the preferred pickup time window for the containers and not the final pickup priority. All selected subsets of instances, which are modified to fit the *BRPAS* approach, can be found in the dataset repository of Azab, Ahmed (2021). In this dataset, for all instances, the modeling parameters are defined so as to provide an opportunity for future benchmark studies.

To solve the proposed models, we used the IBM ILOG CPLEX Optimization Studio 12.9. on a PC with Intel® Core™ i7-8700 CPU 3.20 GHz and 32.0 GB of RAM running under OS 64-bit Windows

Algorithm 1
Online post-processing for the *Flexible BRPAS* solution.

```
Input: BRPAS(flex) optimal solution
  for time window t ∈ {1..T} do
    for container subgroup at the stage k ∈ {1..μ} do
      for container i belongs to subgroup k do
        if container i is a target container and truck i arrived at the bay do
          Retrieve container i to truck i according to FCFS
        end if
        if container i is a blocking container do
          Check if the decided relocation slot is occupied by another container j that belongs to the same subgroup k
          if the relocation slot is occupied by the container j do
            Do not relocate container i until the retrieval of container j
          else
            Relocate container i to the relocation slot
          end if
        end if
      end for
    end for
  end for
Output: Retrieval and relocation order for all containers in the bay
```

Table 4
Solving $\mathrm{BRPAS}_{(1)}$ and $\mathrm{BRPAS}_{(2)}$ models using a subset of instances from Tanaka and Takii (2016).

Inst.	***BRPAS*** $_{(1)}$ 6×3 [(a)] *Opt.*	*Sec.*	7×3 [(b)] *Opt.*	*Sec.*	8×3 [(c)] *Opt.*	*Sec.*	6×4 [(d)] *Opt.*	*Sec.*	***BRPAS*** $_{(2)}$ 6×3 [(a)] *Opt.*	*Sec.*	7×3 [(b)] *Opt.*	*Sec.*	8×3 [(c)] *Opt.*	*Sec.*	6×4 [(d)] *Opt.*	*Sec.*
1	3	9.37	1	4.39	7	526.25	3	449.64	3	2.10	1	1.77	7	110.10	3	20.40
2	3	7.04	2	3.22	2	25.70	4	574.49	3	1.46	2	1.68	2	3.16	4	4.27
3	0	0.71	5	64.52	6	666.06	7	8603.64	0	0.55	5	9.98	6	400.71	7	854.61
4	3	6.69	5	76.54	2	164.79	6	1079.37	3	1.24	5	15.13	2	4.44	6	159.96
5	1	3.05	3	45.85	3	374.40	6	251.62	1	1.08	3	11.59	3	29.76	6	73.92
6	2	3.11	2	2.48	4	287.44	3	400.48	2	4.15	2	1.50	4	31.81	3	13.73
7	0	0.99	5	262.15	4	383.19	9	2516.17	0	0.67	5	36.52	4	26.41	9	2133.36
8	4	18.56	3	198.45	4	816.41	3	79.07	4	2.79	3	3.72	4	14.94	3	14.46
9	1	2.63	3	11.32	4	311.51	6	1915.72	1	1.26	3	4.03	4	33.84	6	652.92
10	1	3.59	3	26.78	5	798.82	3	243.64	1	0.98	3	8.10	5	134.60	3	5.49
Avg.	1.8	5.57	3.2	69.57	4.1	435.46	5	1611.38	1.8	1.63	3.2	9.40	4.1	78.98	5	393.31

Note: $\delta = 1$,[(a) $T = 6,\ N = 15$, (b) $T = 7,\ N = 18$, (c) $T = 8,\ N = 21$, (d) $T = 8,\ N = 20$].

10. Comparisons of the formulations considering the various modeling aspects proposed in Section 4 are presented below, along with some analytical insights. The abbreviations used here are as follows: ***Inst.***, the instance number; ***Obj.***, the objective function value for the solved instances; ***Opt.***, the optimal value of the objective function for instances that solved to optimality within the time limit; **Sec.**, the solution time in seconds; ***#Feas.***, the number of instances that had a feasible solution within the time limit; and ***#Opt.***, the number of instances that solved to optimality within the time limit. ***#LB.*** is the number of instances that had a lower bound for the *LP* relaxation of IP models within the time limit, and ***lb.*** is the lower bound obtained within the time limit for the *LP* relaxation of the IP models.

The $BRPAS_{(1)}$ and $BRPAS_{(2)}$ formulations were solved using a subset of instances from Tanaka and Takii (2016) without any time limitation (Table 4) and a subset of instances from de Melo da Silva et al. (2018b) for which a time limit of 3600 seconds was set (Table 5). From Tanaka and Takii (2016), we selected four bay sizes to be solved, with ten instances for each, as shown in Table 4. From de Melo da Silva et al. (2018b), 360 instances were selected, with 12 different bay sizes and 30 instances for each bay size, as illustrated in Table 5. This subset of instates was also used to solve the *BRPAS*(*flex*) model, as shown in Table 6. Note that de Melo da Silva et al. (2018b) developed their instances in order to study the Block Retrieval Problem where each container subgroup in the bay is given a retrieval priority, and only the target subgroup (e.g., the subgroup with a priority of 1) is picked up, while the other container subgroups remain in the bay. We used their instances, assuming that all container subgroups will be picked up. It is worth mentioning that we selected the most difficult subset of instances from de Melo da Silva et al. (2018b)—instances where the bay is 80% occupied (that is, the number of empty slots is 20% of the total number of slots), with a planning horizon of $T = 5$ (the number of container groups)—rather than the easier instances, where the occupation rates are 70% and 75% and $T = 3$ and 4, which can be solved in a relatively short time.

Typically, examples of the duplicate *BRP* in the literature give only the bay configuration; accordingly, the remainder of our model parameters are assumed as follows: To allow a balanced retrieval workload at the bay over time windows, and because all containers are assumed to be retrieved within the planning horizon of T time windows, the maximum queue length L is derived from $L = \lceil N/T \rceil$. For queue length L (container retrievals), crane capacity (G) is assumed to equal $L + 2$ container moves. The acceptable shift of a container pickup time (δ) is set to 1 (i.e., one time window) for all the solved instances. (Later in this section, we investigate the impact of changing this parameter on the total number of relocations.)

The results in Table 4 and Table 5 show that both the $BRPAS_{(1)}$ and $BRPAS_{(2)}$ models were able to produce the same optimal solutions for the solved instances. It is noteworthy that the $BRPAS_{(2)}$ model, having fewer decision variables, computationally outperformed the $BRPAS_{(1)}$ model for most instances. For the de Melo da Silva et al. (2018b) instances, more instances were solved to optimality with $BRPAS_{(2)}$, and more have feasible solutions with $BRPAS_{(2)}$, especially for the larger-size instances. For the smaller-

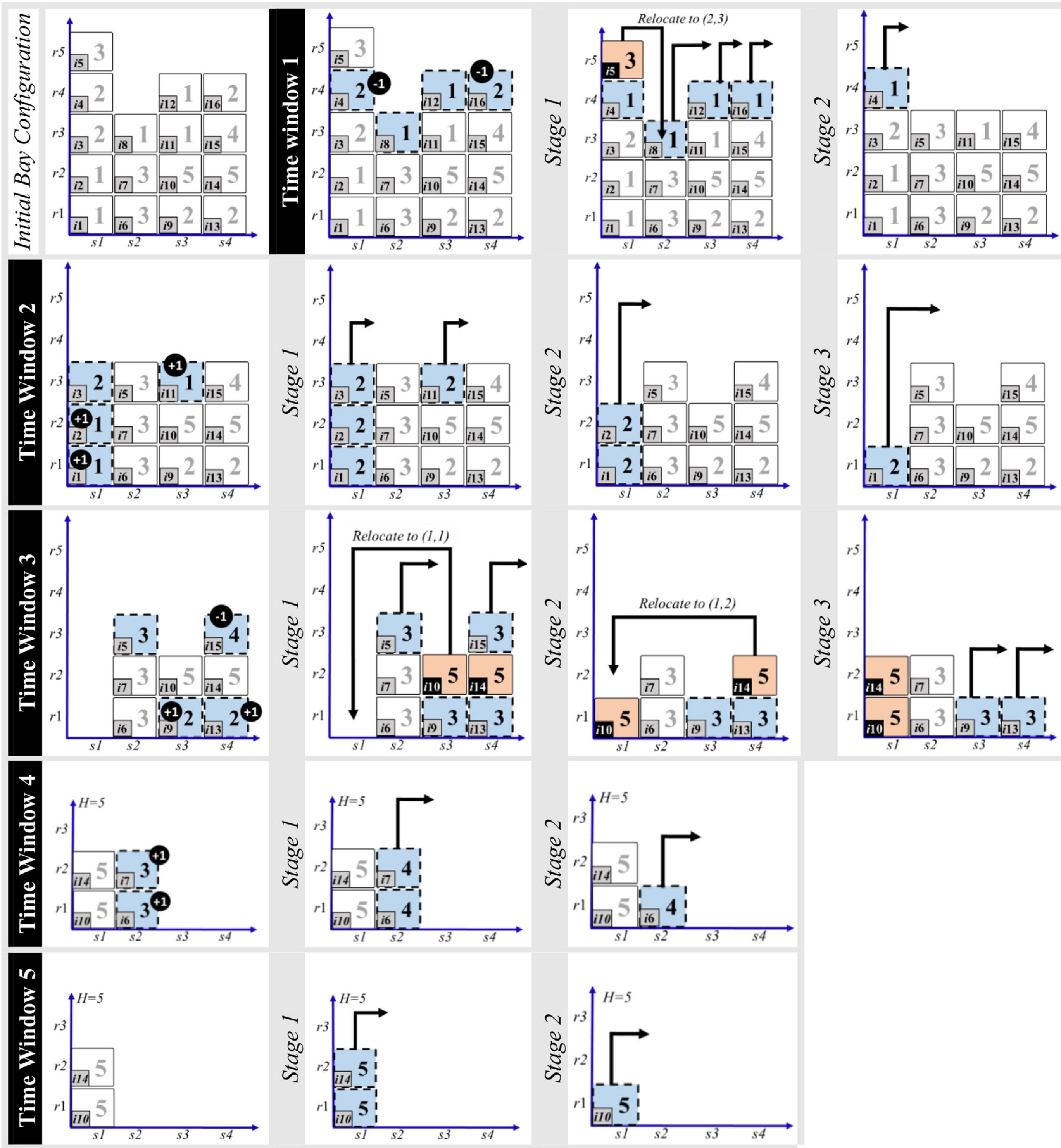


Fig. 8. A numerical example of the flexible *BRPAS*.

size instances, the two models were both able to produce the optimal solution within the time limit for most instances; however, for only a few instances was the *BRPAS*$_{(1)}$ model able to produce the optimal solution in less time than *BRPAS*$_{(2)}$. For the LP relaxation of the IP models, the *BRPAS*$_{(1)}$ and *BRPAS*$_{(2)}$ models both exhibited a relatively low level of performance for obtaining a good *lower bound* (compared to the optimal solution), although the *BRPAS*$_{(1)}$ formulation produced relatively better lower bound values compared to the *BRPAS*$_{(2)}$ formulation, as shown in Table 5. Despite this shortcoming, almost all instances were solved easily under LP relaxation within the time limit.

Table 6 shows results for the *BRPAS(flex)* model. For this experiment, to ensure the model's input equivalence to the inputs of *BRPAS* models, we assumed that parameter $\mu = G = L + 2$. As can be seen here, the flexible *BRPAS* formulation produced the exact same optimal solutions as the original *BRPAS* formulations and had similar lower bounds to those of the linearly relaxed *BRPAS*$_{(2)}$ model. Note that the *BRPAS(flex)* model showed better computa-

Table 5
Solving $BRPAS_{(1)}$ and $BRPAS_{(2)}$ using a subset of instances from de Melo da Silva et al. (2018b).

		$BRPAS_{(1)}$							$BRPAS_{(2)}$						
		IP Model				LP relaxation			IP Model				LP relaxation		
$C \times H$	N	#Feas.	#Opt.	Avg. Opt.	Avg. Sec.	#LB	Avg. lb.	Avg. Sec.	#Feas.	#Opt.	Avg. Opt.	Avg. Sec.	#LB	Avg. lb.	Avg. Sec.
4 × 4	12	30	30	0.83	3.09	30	0.19	1.00	30	30	0.83	1.20	30	0.12	0.96
4 × 5	16	30	30	1.13	165.43	30	0.17	3.50	30	30	1.13	20.42	30	0.10	3.09
4 × 6	19	28	28	2.25	447.92	30	0.30	8.53	29	28	2.25	84.81	30	0.25	5.85
6 × 4	19	30	29	1.86	221.50	30	0.27	10.19	30	30	2.00	71.66	30	0.21	5.54
6 × 5	24	28	26	1.58	297.33	30	0.33	34.50	28	28	1.79	159.39	30	0.23	17.43
6 × 6	28	25	20	1.40	706.58	30	0.32	73.66	27	25	1.68	754.64	30	0.25	45.67
8 × 4	25	29	27	0.93	168.00	30	0.24	38.98	30	28	1.04	70.26	30	0.17	20.17
8 × 5	32	20	16	1.06	593.24	30	0.33	146.16	25	21	1.29	396.31	30	0.24	100.06
8 × 6	38	14	12	0.67	896.18	30	0.33	337.74	17	15	0.80	452.94	30	0.29	231.22
10 × 4	32	22	14	1.14	596.73	30	0.35	134.32	29	23	1.83	783.77	30	0.28	82.98
10 × 5	40	12	9	0.56	1446.60	30	0.39	451.34	17	13	1.08	916.04	30	0.35	916.04
10 × 6	48	7	5	0.00	1772.74	29	0.51	1191.16	8	6	0.17	615.32	30	0.48	760.61
Total		275	246			359			300	277			360		

Note: $\delta = 1, T = 5$, bay occupation rate is 80%.

Table 6
Flexible BRPAS solutions for the instances from Silva, de M. da, Erdoğan, Battarra & Strusevich, 2018b.

		***BRPAS(flex)* model**				***LP relaxation of BRPAS(flex)* model**		
$C \times H$	N	#Feas.	#Opt.	Avg. Opt.	Avg. Sec.	#LB	Avg. lb.	Avg. Sec.
4 × 4	12	30	30	0.83	1.17	30	0.12	0.84
4 × 5	16	30	30	1.13	16.88	30	0.10	2.89
4 × 6	19	29	28	2.25	88.76	30	0.25	6.53
6 × 4	19	30	30	2.00	83.24	30	0.21	5.96
6 × 5	24	28	28	1.79	151.52	30	0.23	18.49
6 × 6	28	28	27	1.81	480.65	30	0.25	45.05
8 × 4	25	30	29	1.17	36.26	30	0.17	21.87
8 × 5	32	26	22	1.32	201.22	30	0.24	98.50
8 × 6	38	20	18	1.22	703.23	30	0.27	238.21
10 × 4	32	27	26	2.04	490.60	30	0.28	85.03
10 × 5	40	21	18	1.39	448.80	30	0.34	284.73
10 × 6	48	9	7	0.29	832.75	30	0.48	796.32
Total		308	293			360		

Note: $\delta = 1,\ T = 5$, bay occupation rate is 80%, $\mu = G = L + 2$.

tional performance and solved more instances to optimality within the time limit.

As mentioned in Section 2, there are two versions of the classical *BRP*: *BRP* with distinct priority and *BRP* with group priority. This paper mainly adopts the duplicate container retrieval priority since it is more compatible with time-window-based appointment scheduling. However, the models can also handle the distinct retrieval priority. In the distinct priority case, the number of containers to be picked up per time window is reduced to one container. This would be applicable when the terminal operator needs to precisely control container pickup with shorter time intervals instead of a time-window-based retrieval process. Here, the parameter L in the mathematical formulations is set to 1 in constraints (3) and (17) in the $BRPAS_{(1)}$ and $BRPAS_{(2)}$ models, respectively. Since the container pickup priority is not duplicated in the distinct *BRP*, parameter T can be set equal to N, the total number of containers. Table 7 gives the results when the distinct retrieval priority is used in solving the subset of instances from Caserta et al. (2012), with different bay sizes and with $\delta = \{0,\ 1, 2\}$. In the Caserta et al. (2012) instances, the two highest tiers are left empty to allow container relocations, and all stacks are filled with a similar number of containers, equal to $H - 2$. Therefore, in the initial bay configuration, assuming that the lowest container in a particular stack is blocked, the number of stages (parameter G) required for relocation and retrieval is set equal to $H - 2$ when using the *BRPAS* models. Table 7 gives the results under each δ value in the following form: "*Objective function value(Solution time in seconds).*" As is evident here, the δ value can, quite reasonably, impact the number of relocations. Thus, the acceptable appointment shift δ is an essential parameter in the *BRPAS*. Note that when $\delta = 0$, this is equivalent to the unrestricted *BRP* with a distinct retrieval priority for containers.

To investigate the impact of the acceptable appointment shifts on the objective function value in the case of container group pickup, we solved 180 instances with different bay configurations from Tanaka and Takii (2016). Specifically, a (7×3) bay layout with $(N, T) = (18, 10), (19, 11)$ and $(20, 12)$, and a (6×3) bay layout with $(N, T) = (15, 9), (16, 9)$ and $(17, 10)$ were solved. Under each combination of bay layout and (N, T), 30 instances were solved using allowable appointment shift values of $\delta = \{0, .., 5\}$. The solved subset of instances with *BRPAS* parameters is also included in the dataset repository mentioned above (Azab, Ahmed (2021)). It should be noted that the average number of relocations can be reduced by accepting more appointment shifts. For example, Table 8 shows that accepting an appointment shift of one time window (i.e., changing from $\delta = 0$ to $\delta = 1$) can reduce container relocations by an average of 42% for (6×3) bays and 36.5% for (7×3) bays. In addition, zero relocations can be obtained for most instances when $\delta = 5$. However, the appointment shift value should be acceptable to the other stakeholders, including the individual truckers and trucking companies.

Table 7
Solutions using the $BRPAS_{(2)}$ model and distinct priority for instances from Caserta et al. (2012).

Inst.	$(C \times H) = (3 \times 5),\ N = 9$			$(C \times H) = (4 \times 5), N = 12$			$(C \times H) = (5 \times 5), N = 15$		
	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 1$	$\delta = 2$
1	6 (1.66)	4 (3.22)	2 (1.28)	5 (11.77)	4 (57.14)	2 (10.95)	6 (255.31)	5 (295.52)	4 (157.14)
2	5 (0.55)	3 (1.19)	2 (1.33)	3 (1.20)	2 (1.75)	0 (1.844)	7 (25.88)	7 (1963.45)	6 (3181.42)
3	2 (0.52)	1 (0.66)	0 (0.86)	7 (7.63)	5 (26.28)	4 (73.92)	8 (64.74)	4 (16.30)	4 (73.39)
4	4 (0.75)	3 (1.02)	1 (1.00)	5 (1.74)	4 (4.91)	2 (5)	6 (22.84)	5 (417.70)	4 (772.34)
5	1 (0.52)	0 (0.59)	0 (0.91)	6 (4.16)	4 (5.44)	3 (24.97)	9 (23.88)	9* (*TL*)	6 (1329.03)
6	6 (0.28)	5 (1.56)	3 (1.59)	7 (3.27)	6 (90.06)	4 (47.84)	7 (11.53)	7 (2828.86)	5 (2644.03)
7	6 (0.55)	3 (1.06)	3 (1.99)	10 (5.33)	7 (127.47)	4 (42.42)	7 (25.56)	5 (149.11)	5 (2969.14)
8	2 (0.56)	1 (0.69)	0 (0.84)	5 (19.92)	4 (12.39)	2 (11.86)	10 (10.98)	8 (41.05)	7 (2269.91)
9	8 (0.33)	5 (2.78)	4 (3.95)	4 (1.25)	4 (3.58)	3 (3.70)	8 (201.17)	7 (1734.56)	7* (*TL*)
10	5 (0.61)	2 (0.89)	1 (1.02)	10 (1.95)	10 (360.2)	7 (795.13)	4 (4.64)	2 (7.11)	2 (7.50)
11	3 (0.56)	2 (1.14)	1 (0.89)	7 (4.52)	5 (23.63)	4 (33.58)	11 (7.84)	11* (*TL*)	8 (3284.39)
12	5 (0.52)	3 (0.95)	1 (0.94)	7 (14.49)	6 (177.88)	5 (174.08)	6 (3.36)	4 (5.91)	3 (17.53)
13	8(0.45)	5 (2.92)	3 (5.48)	3 (1.52)	2 (2.30)	1 (3.02)	11 (322.86)	9 (3319.55)	7* (*TL*)
14	7 (0.33)	4 (1.31)	3 (3.42)	7 (1.69)	5 (13.77)	5 (172.3)	4 (4.06)	4 (10.77)	4 (50.39)
15	6 (0.45)	5 (6.84)	3 (4.33)	5 (1.66)	3 (2.95)	2 (6.05)	7 (13.75)	5 (170.36)	4 (38.34)
Avg.	4.93 (0.57)	3.1 (1.79)	1.8 (1.99)	6.07 (5.47)	4.7 (60.65)	3.2 (93.78)	7.4 (66.56)	5.54 (843.1)	4.77 (1291.9)

Note: Time Limit $(TL) = 3600$ s., * Not optimal, $T = N$, $L = 1$, $G = H - 2$.

Table 8
Impact of acceptable appointment shifts on the number of relocations Tanaka and Takii (2016).

Bay conf.	$(C \times H) = (7 \times 3)$						$(C \times H) = (6 \times 3)$					
	(N=18, T=10)		(N=19, T=11)		(N=20, T=12)		(N=15, T=9)		(N=16, T=9)		(N=17, T=10)	
δ	*Avg. Opt.*	*Avg. Sec.*	*Avg. Opt.*	*Avg. Sec.*	*Avg. Opt.*	*Avg. Sec.*	*Avg. Opt.*	*Avg. Sec.*	*Avg. Opt.*	*Avg. Sec.*	*Avg. Opt.*	*Avg. Sec.*
0	6.17	42.28	6.07	42.35	7.10	80.99	4.50	3.2	5.67	21.35	5.43	9.19
1	3.77	114.13	3.70	362.81	4.47	464.67	2.9	12.4	2.97	12	3.17	52.91
2	2.43	51.24	2.13	125.96	2.87	342.50	1.4	6.1	1.5	3.6	1.4	32.87
3	0.93	10.07	1.17	24.29	1.53	275.72	0.4	2.8	0.4	2.9	0.6	26.94
4	0.03	4.65	0.20	6.57	0.57	13.65	0.0	2.4	0.0	2.8	0.1	3.74
5	0.0	5.91	0.0	6.63	0.13	8.67	0.0	2.7	0.0	3.2	0.0	4.22

6. Conclusions

We have proposed here a new optimization problem: the *BRPAS*. *BRPAS* adds several new aspects to the classical *BRP*, with consideration given to the preferred appointment time window for each container pickup, acceptable appointment shift, yard crane capacity, and maximum queue length at the yard bay. To formulate the problem, two binary integer programming models, $BRPAS_{(1)}$ and $BRPAS_{(2)}$, were proposed. The proposed models are extended to give more flexibility to yard operators servicing arriving trucks within the same appointment time window using the *FCFS* strategy under relocation minimization. To demonstrate the method, several cases involving different bay sizes and configurations were solved. It was found that $BRPAS_{(2)}$ formulation outperformed $BRPAS_{(1)}$ formulations in terms of computational time. The flexible *BRPAS* achieved container retrieval flexibility with high computational performance. Results also showed that coordinating appointments with container handling operations at the bay can reduce the number of relocations, which, in turn, impacts truck delays. Future research should include the development and use of more efficient heuristic algorithms to improve computational efficiency for large-size instances. In addition, the potential impact of coordinating yard crane schedules with appointment scheduling operations on both yard and trucking operations should be investigated.

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