



# Container yard template planning under uncertain maritime market



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## ABSTRACT

A yard template determines the assignment of spaces in a yard for arriving vessels. Fluctuation of demand for freight transportation brings new challenges for making a robust yard template when facing uncertain maritime market. A model is proposed for yard template planning considering random numbers of containers that will be loaded onto vessels that visit the port periodically. Traffic congestions and multiple schedule cycle times for vessel arrival patterns are also considered. Moreover, a meta-heuristic method is developed for solving the model in large-scale cases. Numerical experiments are conducted to validate effectiveness and efficiency of the model.

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## 1. Introduction

With advancements of quay side equipments and technologies, the bottleneck of port operations has moved from quay side to yard side (Chang et al., 2010; He et al., 2010). The yard management of a port has significant influences on the competitiveness of the port in a global shipping network. The yard template is a concept applied in container ports, which utilize a consignment strategy (Moorthy and Teo, 2006; Lee et al., 2006; Han et al., 2008; Jiang et al., 2012). This strategy stores export and transshipment containers, which will be loaded onto the same departing vessel, at the same assigned subblocks. The yard template planning is concerned with the assignment of subblocks to vessels. Some dedicated subblocks in the yard are reserved for each vessel. The incoming containers that will be loaded onto the vessel  $V_i$  in the future are discharged from incoming vessels and placed in the subblocks reserved for  $V_i$ . When  $V_i$  arrives at the terminal, all the containers stored in these dedicated subblocks are loaded onto it. This strategy can evidently reduce the number of reshuffles and vessels' turn-around time. The yard template planning aims to minimize the utilization cost of subblocks and the transportation cost for moving containers from their incoming berths to the storage subblocks in the yard and then to their outgoing berths.

In Fig. 1, under the consignment strategy in the terminal, subblocks in the yard are reserved for some vessels. Vessel  $B$  arrives at the port, and the dashed lines denote the unloading process: containers that will be loaded onto other vessels (i.e., Vessel  $D$  and Vessel  $E$ ) in future are unloaded to the subblocks which are reserved for these vessels, i.e.,  $K21$ ,  $23$ ,  $42$ ,  $45$  for Vessel  $D$ , and  $K16$ ,  $36$ ,  $38$  for Vessel  $E$ . Meanwhile, the loading process is denoted by the solid lines in Fig. 1. All the containers in subblocks reserved for Vessel  $B$  (i.e.,  $K9$ ,  $29$ ,  $48$ ,  $50$ ) are loaded onto Vessel  $B$ .

The yard template planning problem is to determine which subblocks should be reserved to which vessels so as to optimize some certain objective functions, which could be the minimization of the number of used subblocks, the route length of all the transshipped container flows in the yard.

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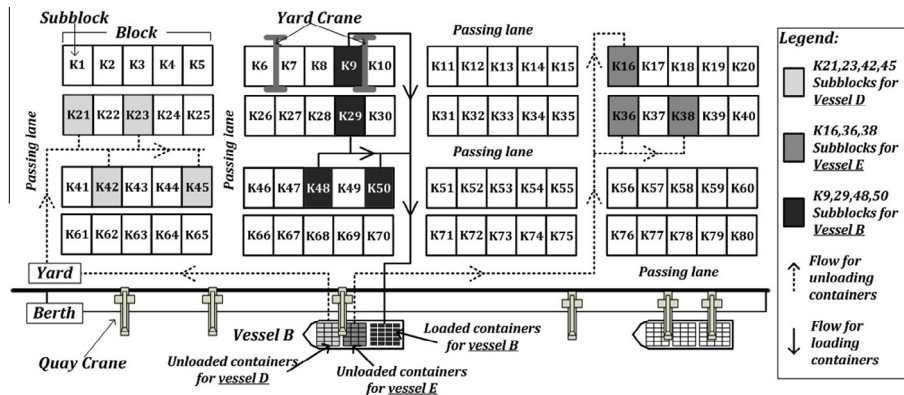


Fig. 1. A typical configuration of transshipment terminal.

Although yard cranes are important resources in yards, the moving time of the yard cranes is not a bottleneck for yard operations when using the yard template (Yan et al., 2011). In the traditional yard management without the yard template, each loading (or unloading) task is usually related with a specific location (block #, row #, column #, tier #) in a yard. When performing a sequence of loading or unloading tasks, yard cranes need to move along the rails back and forth so as to reach the specific locations of the tasks. In addition, reshuffling activities are common in the traditional yard management. The reshuffling can also increase the handling time of yard cranes evidently.

However, by using the yard template, there is no reshuffling; and each loading (or unloading) task is related to a specific location with respect to subblock #. The loading activity is to load all the containers in a subblock onto a vessel, and the unloading activity is to unload a group of containers from a vessel to a subblock. Therefore, when performing loading or unloading tasks, the yard cranes need not to move along the rails for a long distance between two consecutive tasks (He et al., 2013). Based on the above reasons, the handling time of crane movements is not a very important issue for evaluating a yard template, thus the objective for optimizing a yard template mainly considers the route length of transshipped containers, but does not consider the handling time of yard cranes in details.

Most studies on the yard template planning are based on a deterministic environment with respect to numbers of containers that will be loaded onto arriving vessels. However, the global maritime logistic market contains a lot of uncertainties that inherit from the fluctuation of the demand for freight transportation. Shipping liners' vessels visit a port periodically (weekly, 10-days, or biweekly). For a vessel, the numbers of containers loaded onto the vessel are different in each period. The numbers of containers unloaded from the vessels in each period also fluctuate along the time. The randomness contained in the uncertain maritime market has brought new challenges for making a robust yard template so as to improve the efficiency of port operations.

The yard template planning belongs to a tactical level decision in a port. Once determined, the yard template is usually not modified in the planning horizon, and is used as a basis for making some operational level schedules in the port. This paper investigates how to obtain a robust yard template under uncertain numbers of containers that will be loaded onto vessels that visit the port periodically. A comprehensive analysis on all types of costs in a yard template is given in this paper. The traffic congestions in the yard and the multiple schedule cycle times for vessel arrival patterns are also considered in the proposed model. Moreover, a meta-heuristic method is developed for solving the above problem in large-scale realistic environments. Numerical experiments are conducted to validate the effectiveness and efficiency of the proposed model.

The remainder of the paper is organized as follows. Section 2 reviews the related works. Section 3 elaborates the problem backgrounds and analyzes objectives and constraints for the problem. A mathematical model is formulated in Section 4. Then a meta-heuristic solution method is proposed in Section 5 for solving the proposed model. Section 6 shows the results of some numerical experiments. Closing remarks and conclusions are then outlined in the last section.

## 2. Literature review

For an introduction to the general port operations, we refer readers to the review works given by Vis and de Koster (2003), Steenken et al. (2004), Stahlbock and Voß (2008). The yard template planning, i.e., the theme of this study, is a tactical level decision problem, but is closely related to the widely studied storage allocation problems, which is operational level decision problems. In addition, the yard template is based on one kind of yard management policies. Another highlight of this study is that the uncertainty is considered. Thus, this section mainly reviews the related studies through these four topics, i.e., storage allocation, yard management policies, yard template planning, and port operations under uncertainty.

### 2.1. Storage allocation problems

This paper is related to the strategies of allocating storage space in a yard to arriving containers. Most of studies in this research area are yard storage allocation problems, e.g., Kim et al. (2000), Preston and Kozan (2001), Zhang et al. (2003). These papers are oriented to generic terminals, i.e., gateway ports, which are mainly for import and export activities. In addition, these storage allocation decisions mainly belong to the operational level decision problems, but the yard template planning, which is the theme of this study, belongs to a tactical level decision problem. Some recent studies were performed with focusing on the particular needs of transshipment hubs in the storage allocation activities. For example, Nishimura et al. (2009) studied a storage allocation problem for transshipment hubs in order to minimize the movement time of container transshipment flow and the waiting time of feeders. Moccia et al. (2009) also formulated a storage allocation model based on generalized assignment models. A column generation heuristic is proposed to minimize the movement of transshipped container flows in a yard. These two studies belong to some generalized studies on yard management in transshipment hubs, but do not consider some realistic constraints (e.g., yard traffic congestions, etc.) and different yard management policies (e.g., consignment strategy, house-keeping strategy, etc.). However, they act as basis for further studies by other scholars, and some common issues (e.g., minimization on container transshipment routes) are considered in these further studies.

### 2.2. Yard management policies

The yard template is based on a consignment strategy, which is one kind of yard management policies. The ports around the world use different yard management policies, which have specific influences on yard storage allocation decisions. For example, Kim and Kim (1999) studied a storage allocation problem under a yard space segregating policy. Cordeau et al. (2007) investigated a tactical level yard management problem under a house-keeping strategy, in which the container relocations in yards are the main concerns for terminal operators. Different from the above studies, the yard management in this paper is based on a consignment strategy, which was studied by Chen et al. (1995) and Dekker et al. (2006). The most important advantage for this strategy is that there are no reshuffling and relocation operations for containers in the yards, which can improve the efficiency of port operations for transshipment hubs evidently.

### 2.3. Yard template planning

The yard template, i.e., the theme of this study, is based on the above mentioned consignment strategy. In academia, the concept of yard template was first mentioned in a paper by Moorthy and Teo (2006), but their study mainly focused on the berth allocation planning. Lee et al. (2006) studied how to optimize yard storage allocation plans with a yard template given. They extended their work in Han et al. (2008) to optimize the yard template and the yard storage allocation plans simultaneously. Then they also proposed a two-space sharing method to improve the space utilization of a yard template (Jiang et al., 2012). Zhen et al. (2011b) studied how to optimize a yard template with considering berth allocation plans simultaneously. It should be noted that all of these papers are the studies on the yard template planning under deterministic environments.

### 2.4. Port operation problems under uncertainty

However, the global maritime logistic market contains a lot of uncertainties. Recently the studies on port operations were extended to the problem backgrounds under uncertainties. For example, Zhen et al. (2011a) and Han et al. (2010) studied the berth allocation problems with considering uncertain arrival time and operation time. For the yard template planning under uncertain vessel arrival time and berthing positions, Zhen (2013) proposed a mixed integer programming model and a solution method. However, the model (Zhen, 2013) assumes the numbers of unloading and loading containers are deterministic and does not consider the periodicity of vessel arrivals, which disobeys the realistic maritime market environments. Therefore, this paper makes a further study in this field. The considerations of uncertain unloading/loading container numbers and vessel arrivals' periodicities complicate the model in this study, which cannot be easily extended by adding some parameters on the basis of the model (Zhen, 2013). For example, the numbers of used subblocks for a vessel during multiple periods are different. However, the model (Zhen, 2013) is a single period model; and all the vessels have the same cycle time for their periods. All of these issues incur that this study cannot be a simple extension on the work of Zhen (2013).

### 2.5. Summary

The above summarizes the historical developments for the research streams on the yard management in container ports during the past decades. The research trend shows that the yard template, as a new and effective yard management policy for increasing ports' operational efficiency, has attracted more and more attentions from academia. However, this area lacks the studies considering some uncertain factors in the global maritime logistic market. Therefore, this study is performed by following this trend. From the perspective of mathematical modeling, the challenges of this study lie in the parameter definition for periodicity of vessel arrivals, the heterogeneous arrival patterns of vessels, the objective formulation for the route lengths of container flows, constraint formulation for yard traffic congestions, etc. These factors are seldom considered in the

literature. The objectives and constraints in this paper are also different from the traditional yard management studies, most of which actually belong to operational level decisions. For the yard template planning under uncertainties, this paper performs an explorative study on this tactical level decision problem.

### 3. Problem backgrounds and analysis

#### 3.1. Yard template planning under uncertainties

The unforeseen global maritime market contains some uncertainties for terminal operators to plan their yard templates. Among these uncertainties, the most significant one is concerned with the random number of unloading (and loading) containers from (and to) arriving vessels, which brings a lot of challenges for the yard template planning. This paper formulates the yard template planning model with considering the uncertain number of unloading and loading containers.

For the deterministic model on yard template planning, the numbers of unloading (and loading) containers from (and to) vessels are known data. Hence the number of subblocks that are reserved to each vessel is also deterministic. However, when considering the uncertainty on the number of unloading and loading containers, the number of subblocks that are actually used by each vessel becomes uncertain. For each vessel, some subblocks are exclusively reserved to it for a relatively long time, which is usually stated in contracts between terminal operators and shipping liners. In reality, the actually number of containers that are loaded on to the vessel may be greater than maximum capacity of the reserved subblocks. In this case, more subblocks are needed for some periods.

Based on the above analysis, the subblocks in a yard template under uncertain environments can be categorized under two types:

(Type 1) **Exclusive mode**: some subblocks are exclusively reserved to each vessel, as the example in Fig. 1. This arrangement of subblocks usually takes effect for a relatively long time.

(Type 2) **Sharing mode**: some other subblocks are dynamically shared by all the arriving vessels. Some vessels require additional subblocks when they face some unpredictable increase of the loading containers.

Fig. 2 illustrates the above two modes in a yard template for considering the uncertainties.

For making a yard template under uncertainty, the terminal operator determines which subblocks belong to the exclusive mode and the sharing mode, respectively. Among the subblocks of the exclusive mode, the terminal operator also needs to decide which subblocks are reserved to each vessel.

The periodicity of vessel arrival is a key feature for the yard template planning. For a terminal, the arriving vessels may have different cycle time. For example, some vessels have weekly arrival pattern, some may have 10-days or biweekly patterns (Moorthy and Teo, 2006). The weekly pattern is the most common in usual practice. It should be mentioned that this study can support the multiple cycle time based yard template planning problem.

In addition, the uncertainty source in the yard template planning problem is modeled from the perspective of the number of containers that will be loaded onto each vessel (e.g., Vessel  $i$ ) in a cycle time. This uncertain number is denoted by  $n_{i,t}$ , which denotes the number of loaded containers for Vessel  $i$  during the  $t$ th period.  $n_{i,t}$  will be used in the remainder part of this paper. Fig. 3 illustrates the periodicity of the process for collecting arriving containers, vessel arrival, and loading the containers. For the interest of simplicity, we assume the containers that will be loaded onto a vessel (e.g., Vessel  $i$ ) are accumulated under a constant rate during each period (e.g., 1 week, 10 days, 2 weeks, etc.). When Vessel  $i$  arrives at the end of each period, all of these containers are loaded onto it in a relatively short time (e.g., a number of hours).

Fig. 3 does not mean the process of unloading containers from Vessel  $i$  happens firstly, and then the process of loading containers onto Vessel  $i$  happens. Actually, when a vessel stays in the port, unloading containers from the vessel and loading containers onto the vessel can occur in a parallel way rather than a sequential way.

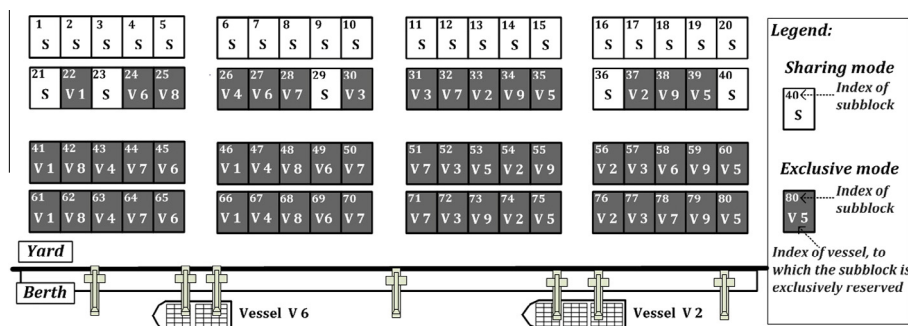


Fig. 2. Two modes of subblocks in a yard template for considering the uncertainties.





subblocks that are exclusively reserved to Vessel  $i$ . Then  $\text{avg}_{k \in R_i} D_{k,i}^L$  can be used to reflect the length of loading routes for Vessel  $i$ .

The calculation on the length of the unloading routes for Vessel  $i$  is based on another known data  $D_k^U$ , which is the average length of the unloading route from all the berths to Subblock  $k$ . The reason for considering all the berths is: the containers that are loaded onto Vessel  $i$  are unloaded from other vessels, which may moor at any possible berths in the port. For the interest of simplicity, we define  $D_k^U$ , but do not distinguish the transshipped container flows according to their unloading vessels. Then  $\text{avg}_{k \in R_i} D_k^U$  can be used to reflect the length of unloading routes for Vessel  $i$ .

Fig. 4 illustrates the above two parameters on route lengths, i.e.,  $D_{k,i}^L$  and  $D_k^U$ . In addition, as shown in Fig. 4, the unloading route (dashed line with arrow) and loading route (solid line with arrow) between the same pair of berth-subblock are different.

We define  $c^T$  is the cost of transporting a TEU container for a kilometer in the yard. The transportation cost in the exclusive mode is formulated as:

$$\text{Cost}_2 = c^T \sum_{\forall i \in V} \sum_{\forall t \in T_i} [\min(y_i P, n_{i,t}) \cdot \text{avg}_{k \in R_i} (D_k^U + D_{k,i}^L)] \quad (2)$$

Formula (2) only reflects laden trips of prime movers without considering empty trips, which are related to dispatching schedules of prime movers and belong to operational-level planning. Thus this study utilizes the route length of laden trips to reflect the cost of transportation in yard. Assuming that the route length of empty trips is approximately proportional to the length of laden trips, the cost of empty trips can be taken into account by setting a proper coefficient  $c^T$ .

The above analysis is based on an assumption that the berthing positions of vessels are deterministic. In reality, some shipping liners may have contracts with terminals. In these service contracts, the berthing positions of vessels owned by the shipping liners are determined. In addition, the proposed methodology in this study can be extended to consider the problem cases, in which the vessels' berthing positions and berthing time are uncertain. Section 7 addresses some details for this extension. For the interest of simplicity, the main part of this paper is based on the assumption of the deterministic berthing position. However, it should be noted that this is not a hard assumption but can be relaxed by extending the model.

### 3.2.3. (Cost\_3) The utilization cost of subblocks in the sharing mode

In the usual practice, the contracts between shipping liners and the terminal operator may claim the assignment of subblocks to vessels. The subblocks in the yard are assigned to each vessel for a relatively long time, and the locations of the subblocks are usually near the berthing positions of the vessels. Besides these exclusively assigned subblocks, the terminal operators also use some subblocks for dynamically sharing among all the vessels, because the number of loading containers for a vessel may vary along the planning horizon. We define  $c^S$  is the cost of occupying a subblock in the sharing mode for a time step, which is different from the previously defined cost rate  $c^F$ . In addition,  $c^S$  should be greater than  $c^F$ , as the cost rate for operating and maintaining a subblock (including labors, resources, equipments related with the subblock) for a relatively long time (i.e., the exclusive mode) is usually cheaper than the cost rate for temporarily operating and maintaining a subblock (i.e., the sharing mode). The reason is that the dynamically scheduling resources may incur higher cost than the regularly scheduling resources. If all the subblocks are dynamically assigned to vessels, there is no yard template, which is not only contradict with usual practice in reality but also incurs a higher total cost than the method by using a yard template that manages and schedules the yard resources in a stable and regular way. This statement will be later validated by numerical experiments in Section 6.3.

When facing unexpected increasing number of arriving containers for some vessels, the terminal operator may need to temporarily assign and relocate resources so as to open more subblocks for the vessels. These temporary assignments are beyond the previously determined baseline schedules for all types of resources; so the unit cost for the sharing mode is greater than the exclusive model. On the contrary, if  $c^S < c^F$ , it will result in all the subblocks are dynamically allocated to arriving vessels, and there is no need to make a yard template in advance, which is not suitable for yard management and is also different from the realistic practices. In reality, some subblocks are reserved and dedicated to shipping liners.

In the sharing mode, subblocks are allocated to a vessel just when the number of containers, which will be loaded onto the vessel, exceeds the full capacity of the subblocks that are exclusively reserved to the vessel. When the vessel arrives at the port and loads up their containers, these allocated subblocks in the sharing mode will be released for other vessels' usage in future. For Vessel  $i$ , the length of the time interval for occupying subblocks in the sharing mode is  $(n_{i,t} - y_i P)^+ L_i / n_{i,t}$ , as shown in Fig. 5. Recalling that  $c^S$  is a cost rate for a time step, the cost of occupying one subblock in sharing mode during Period  $t$  should be  $c^S (n_{i,t} - y_i P)^+ L_i / n_{i,t}$ . The number of subblocks occupied in the sharing mode during Period  $t$  is  $\lceil (n_{i,t} - y_i P)^+ / P \rceil$ . Then the utilization cost of subblocks in the sharing mode is formulated as:

$$\text{Cost}_3 = c^S \sum_{\forall i \in V} \sum_{\forall t \in T_i} \left\lceil \frac{(n_{i,t} - y_i P)^+}{P} \right\rceil \cdot \frac{(n_{i,t} - y_i P)^+}{n_{i,t}} \quad (3)$$

### 3.2.4. (Cost\_4) The transportation cost in the sharing mode

The calculation of the transportation cost in the sharing mode is similar as the cost in the exclusive mode. For Vessel  $i$ , the number of the containers that are stored in the subblocks of the sharing mode during Period  $t$  is  $(n_{i,t} - y_i P)^+$ . If the length of

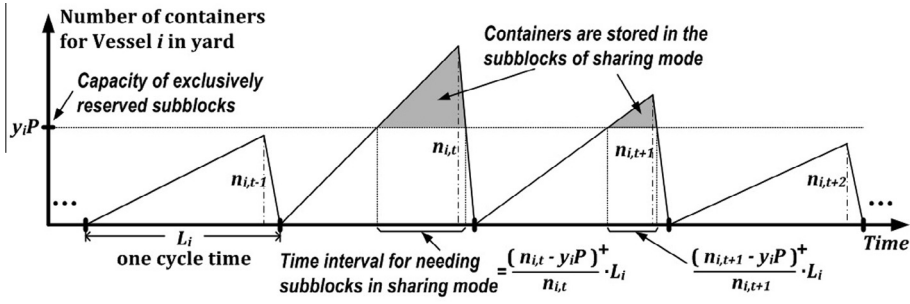


Fig. 5. The time interval for occupying subblocks in sharing mode.

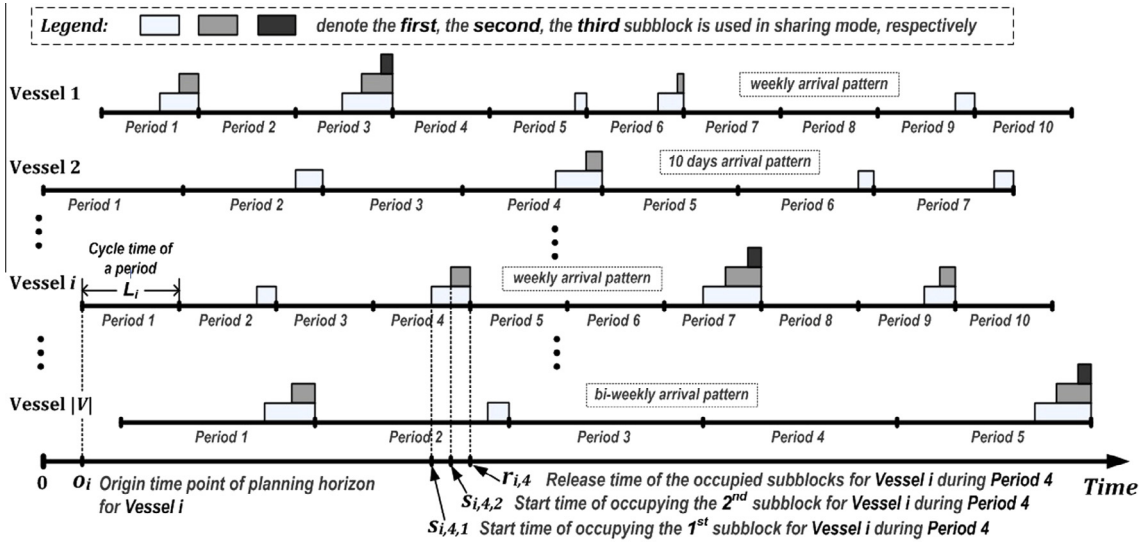


Fig. 6. The dynamic allocation of subblocks for vessels in the sharing mode.

unloading and loading routes in the sharing mode for these containers that are loaded onto Vessel  $i$  during Period  $t$  is denoted by  $\tilde{D}_{i,t}^S$ , the transportation cost in the sharing mode is formulated as:

$$\text{Cost}_4 = c^T \sum_{\forall i \in V} \sum_{\forall t \in T_i} \left[ (n_{i,t} - y_i P)^+ \cdot \tilde{D}_{i,t}^S \right] \quad (4)$$

The calculation on the route length in sharing mode is much more difficult than the exclusive mode. In the sharing mode, subblocks are dynamically allocated to vessels; thus the lengths of unloading and loading routes for a vessel are different for each period. The calculation on  $\tilde{D}_{i,t}^S$  cannot be formulated by some closed-form formulae.  $\tilde{D}_{i,t}^S$  can only be calculated by some procedure.

Before explaining the procedure, some parameters need to be defined as follows:  $o_i$  is the origin time point of planning horizon for Vessel  $i$ ;  $s_{i,t,h}$  is the start time of occupying the  $h$ th subblock for Vessel  $i$  during Period  $t$ ;  $r_{i,t}$  is the release time of the occupied subblocks for Vessel  $i$  during Period  $t$ . Fig. 6 illustrates the dynamic allocation of subblocks for vessels in the sharing mode.

These parameters  $s_{i,t,h}$  and  $r_{i,t}$  can be calculated in advance according to the following formulae:

$$s_{i,t,h} = o_i + [t - 1 + (y_i + h - 1)(P/n_{i,t})]L_i \quad h = 1, 2, \dots, \lceil (n_{i,t} - y_i P)^+ / P \rceil \quad (5)$$

$$r_{i,t} = o_i + tL_i \quad (6)$$

The procedure for calculating  $\tilde{D}_{i,t}^S$  is as follows:

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The procedure for calculating  $\tilde{D}_{i,t}^S$   $\tilde{D}_{i,t}^S = f(x_{i,k}, y_i, n_{i,t})$

---

**For** each empty subblock  $k$  in the sharing mode

Set  $\pi(k) = \text{Empty}$ ;  $\pi(k)$  denotes the status of Subblock  $k$  in the sharing mode

**End For**

**For** each vessel  $i$ , each period  $t$

Calculate  $s_{i,t,h}$  and  $r_{i,t}$ ;  $h = 1, 2, \dots, (n_{i,t} - y_i P)^+ / P$

**End For**

Sort all the time point  $s_{i,t,h}$  and  $r_{i,t}$  by the increasing order;

**For** each time point in the above sequence

**If** the time point is  $s_{i,t,h}$

Among empty subblocks, select the subblock  $k$  that has the minimum value of ' $D_k^U + D_{k,i}^L$ ', with satisfying some constraints;  $\pi(k)$  The constraints are elaborated in the next subsection

Set  $\pi(k) = \text{Occupied}$ ;

$\beta_{i,t,h} = D_k^U + D_{k,i}^L$ ;  $\beta_{i,t,h}$  is a temporary variable

**End If**

**If** the time point is  $r_{i,t}$

As to the subblocks  $k$  that are allocated to Vessel  $i$  during Period  $t$ , set  $\pi(k) = \text{Empty}$ ;

**End If**

**End For**

**For** each vessel  $i$ , each period  $t$

$\tilde{D}_{i,t}^S = \text{Avg}_{\forall h} \{\beta_{i,t,h}\}$ ;

**End For**

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### 3.2.5. Summary of costs considered in the objective

The above four subsections elaborate the subblock utilization cost and transportation cost for the two modes (i.e., exclusive and sharing modes), respectively. The sum of the above four parts of costs will be the objective of the model for optimizing the yard template. In the objective, the transportation cost actually reflects the route lengths (or traveling time) of transshipped container flows in the yard, which is considered in traditional studies on yard management (Cordeau et al., 2007; Moccia et al., 2009; Nishimura et al., 2009; Zhen et al., 2011b).

## 3.3. Constraints for the yard template planning

### 3.3.1. Basic constraints

There are some basic constraints for the yard template planning. For example, there exists a minimum number for subblocks that should be reserved for each vessel in the exclusive mode. In addition, shipping liners have their favorable scopes (ranges) of subblocks. For each vessel, their reserved subblocks in the exclusive mode should be selected from a given set of subblocks, which are requested by their shipping liners in advance.

### 3.3.2. Constraints on yard crane contention in one block

With the consignment strategy used in transshipment activities, the loading process is crucial for the efficiency of port operations, because all the containers in some subblocks need to be loaded onto vessels within a limited length of time.

One block, which consists of five subblocks, is usually deployed with two yard cranes. In loading process, one yard crane is dedicated for a subblock. In each block, the number of subblocks that are reserved for one vessel (or a group of simultaneously loading vessels) would better not exceed one. In this way, another spared yard crane can be utilized by possible unloading activities in the block.

As the berthing time of vessels are deterministic, the information about whether Vessel  $i$  has loading activities in time step  $m$  or not is also known in advance, which is denoted by a binary parameter  $l_{i,m}$ . The  $l$  parameters will be used later in the model formulation.

### 3.3.3. Constraints on traffic congestion in neighbor subblocks

As aforementioned, the loading process is crucial for the efficiency of port operations because all the containers in some subblocks need to be loaded onto vessels within a limited length of time. When a subblock performs loading activities, the yard cranes and prime movers are under high workload, and the traffic is very heavy in the area near the subblock. The traffic congestion may happen when too much workload needs to be handled in a small area at the same time.

As shown in Fig. 7, if there are a lot of container movements in the subblocks K27 and K42, many prime movers will wait or move nearby, which may incur traffic congestion. In addition, the high workload in K27 may impact prime movers that are going to K28. To ensure a smooth traffic flow, port managers have to impose several restrictions during the yard template



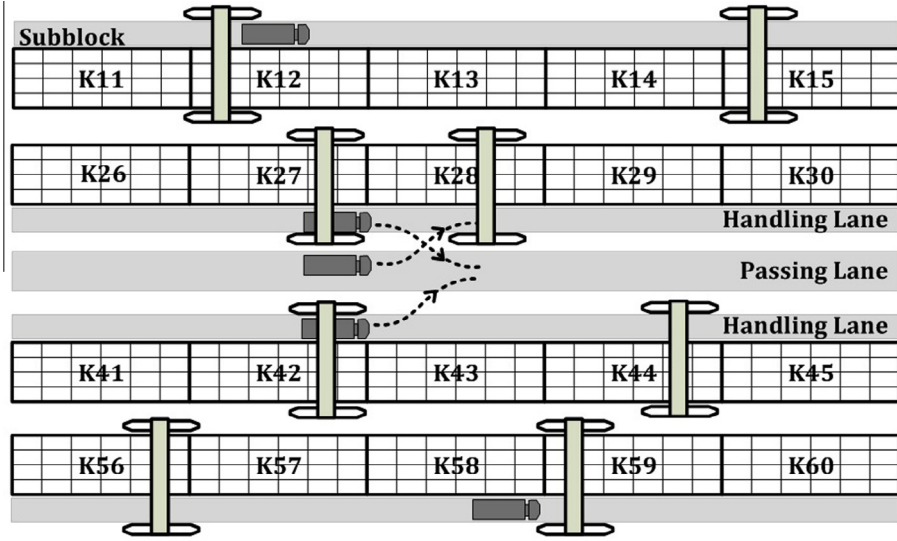


Fig. 7. Traffic congestion in neighbor subblocks.

planning stage. Among these practical restrictions, the most important one is: there should not be two or more neighbor subblocks which have loading activities simultaneously. Here a subblock is a neighbor of another one if they are adjacent and share the same truck path. As shown in Fig. 7, K27 is a neighbor of K26, K28 and K42, but not a neighbor of K12 although they are back to back. When planning the yard template, the subblocks reserved for one vessel (or vessels that are loading simultaneously during a length of time) should be scattered, so that two neighbor subblocks will not have loading process simultaneously. Here we only consider the loading process as the workload of loading activity in a subblock is much higher than unloading. In loading process, all the containers in the subblocks need to be loaded onto vessels within a limited length of time. The unloading process has greater flexibility than loading. When a container is unloaded, it may be stored at any one of the subblocks reserved for a vessel, onto which the container will be loaded in future. So the unloaded container can be flexibly distributed to a subblock with low workload nearby.

### 3.3.4. Constraints on traffic flow in vertical passing lanes

Besides the traffic congestions incurred by loading activities in neighbor subblocks areas, the traffic flows of prime movers in passing lanes can result in another type of congestions in the yard. For the mathematical model formulation in the next section, this study only considers some constraints on the traffic flows of prime movers for loading activities in the vertical passing lanes. The reason for only considering the loading activities is similar as the previous analysis; and the reason for only considering the vertical passing lane lies that the traffic flows in horizontal passing lanes have been limited by the constraints defined in the previous two sub-sections.

As shown in Fig. 8, for each vertical passing lane  $a$ , we define a binary parameter  $w_{i,k,a}$  to denote whether the loading flow from Subblock  $k$  to the berth where Vessel  $i$  moors passes Lane  $a$ . As the berthing positions of vessels are known, the parameters  $w_{i,k,a}$  can be determined in advance. Based on these parameters, the total number of traffic flows that pass each lane (e.g., Lane  $a$ ) can be limited within an upper bound (e.g.,  $W_a$ ) when making a yard template.

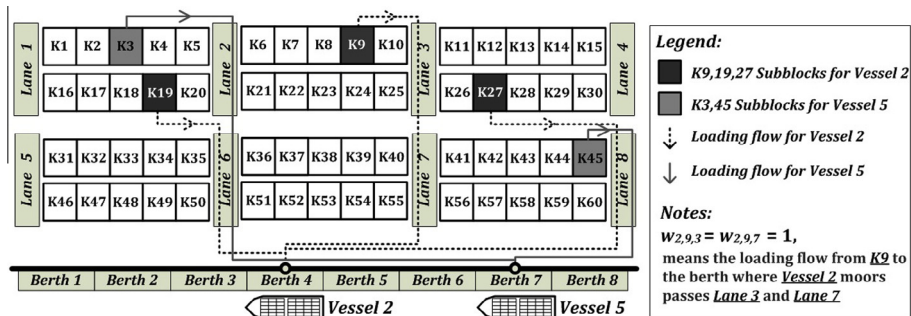


Fig. 8. Traffic flows for loading activities in vertical passing lanes.

#### 4. Model formulation

A stochastic programming model is formulated for the yard template planning problem under uncertainty. The uncertainty is represented by a finite set of scenarios. Each scenario (denoted by  $\omega$ ) is composed of collective random number of containers loaded onto each vessel in each period, i.e.,  $n_{i,t}(\omega)$ . The objective is to minimize the total cost that is elaborated in the previous section.

##### 4.1. Notations

Index:

$i$	vessels;
$k$	subblocks;
$m$	time steps;
$\omega$	scenarios
$t$	periods;
$a$	passing lanes;

Sets:

$V$	set of all vessels;
$K$	set of all subblocks;
$M$	set of all time steps;
$\Omega$	set of all scenarios;
$T_i$	set of all periods in the planning horizon for Vessel $i$ ;
$A$	set of all lanes.

In the above definition, a time step ( $m$ ) is a further discretization of a period ( $t$ ). For example, a period of a vessel is 1 week, a time step is 8 h, and then this period contains 21 time steps. The relationship between them is illustrated in the time axis of Fig. 3.

Input data:

$n_{i,t}(\omega)$	number of containers loaded onto Vessel $i$ during Period $t$ in Scenario $\omega$
$L_i$	cycle time of a period for Vessel $i$ , the unit is time step
$c^F$	fixed cost of occupying a subblock in the exclusive mode for a time step
$c^S$	cost of occupying a subblock in the sharing mode for a time step; $c^F < c^S$
$c^T$	cost of transporting a TEU container for a kilometer in the yard
$P$	capacity of a subblock in terms of TEUs, which is 240 (5 tiers $\times$ 6 lanes $\times$ 8 slots)
$D_{k,i}^L$	length of the loading route from Subblock $k$ to the berth where Vessel $i$ moors
$D_k^U$	average length of the unloading routes from all the berths to Subblock $k$
$l_{i,m}$	equal 1 if Vessel $i$ has loading activities in Time Step $m$ , and 0 otherwise
$w_{i,k,a}$	equal 1 if loading route from Subblock $k$ to the berth where Vessel $i$ moors passes Lane $a$
$W_a$	maximum number of routes passing Lane $a$ simultaneously
$e$	pair of subblocks that are neighbors; e.g., $e = \{21, 41\}$ means K21 and K41 are neighbors
$E$	set of all the pairs $e$ , $e \in E$
$g$	group of five subblocks that belong to the same block; e.g., $g = \{1, 2, 3, 4, 5\}$ means K1, K2, K3, K4, and K5 belong to a block
$G$	set of all the blocks, i.e., the groups of subblocks $g$ , $g \in G$
$Q_i$	set of candidate subblocks, from which some subblocks are selected and assigned to Vessel $i$
$Y_i^{LB}$	minimum number of subblocks that should be assigned to Vessel $i$ in the exclusive mode
$\rho(\omega)$	probability of Scenario $\omega$

Variables:

$x_{i,k}$	set to 1 if Subblock $k$ is reserved for Vessel $i$ in the exclusive mode, and 0 otherwise
$y_i$	number of subblocks that are reserved for Vessel $i$ in the exclusive mode, and 0 otherwise
$\bar{D}_{i,t}^S(\omega)$	average length of unloading and loading routes in the sharing mode for the containers loaded onto Vessel $i$ during Period $t$ in Scenario $\omega$ ; its value depends on $x_{i,k}$ , $y_i$ , $n_{i,t}(\omega)$

#### 4.2. Mathematical model

$$(M_0) \text{ Minimize } Z = c^F \sum_{\forall i \in V} |T_i| L_i y_i + \sum_{\forall \omega \in \Omega} \rho(\omega) \sum_{\forall i \in V} \sum_{\forall t \in T_i} \left\{ c^T \left[ \min \left( P, \frac{n_{i,t}(\omega)}{y_i} \right) \sum_{\forall k \in K} x_{i,k} (D_k^U + D_{k,i}^L) \right] + c^S L_i \right. \\ \left. \times \frac{(n_{i,t}(\omega) - y_i P)^+}{P} \frac{(n_{i,t}(\omega) - y_i P)^+}{n_{i,t}(\omega)} + c^T [(n_{i,t}(\omega) - y_i P)^+ \tilde{D}_{i,t}^S(\omega)] \right\} \quad (7)$$

$$s.t. \sum_{i \in V} x_{i,k} \leq 1 \quad \forall k \in K \quad (8)$$

$$\sum_{k \in Q_i} x_{i,k} = y_i \quad \forall i \in V \quad (9)$$

$$y_i \geq Y_i^{LB} \quad \forall i \in V \quad (10)$$

$$\sum_{k \in \{K - Q_i\}} x_{i,k} = 0 \quad \forall i \in V \quad (11)$$

$$\sum_{k \in e} \sum_{i \in V} l_{i,m} x_{i,k} \leq 1 \quad \forall e \in E, \forall m \in M \quad (12)$$

$$\sum_{k \in g} \sum_{i \in V} l_{i,m} x_{i,k} \leq 1 \quad \forall g \in G, \forall m \in M \quad (13)$$

$$\sum_{i \in V} \sum_{\forall k \in K} x_{i,k} l_{i,m} w_{i,k,a} \leq W_a \quad \forall a \in A, \forall m \in M \quad (14)$$

$$\tilde{D}_{i,t}^S(\omega) = f(x_{i,k}, y_i, n_{i,t}(\omega)) \quad \forall i \in V, \forall t \in T_i, \forall \omega \in \Omega \quad (15)$$

$$x_{i,k} \in \{0, 1\}; y_i \text{ integer} \quad \forall i \in V, \forall k \in K \quad (16)$$

Objective (7) is to minimize the four types of costs that have been elaborated in Section 3.2. Constraint (8) ensures each subblock is reserved to at most one vessel in the exclusive mode. Constraint (9) connects the two decision variables  $x_{i,k}$  and  $y_i$ . Constraint (10) guarantees the requirement on the minimum number of subblocks reserved for each vessel in the exclusive model. Constraint (11) limits the reserved subblocks should be selected from a given set that is required by shipping liners in advance. Constraint (12) ensures two neighbor subblocks cannot have loading activities simultaneously. Constraint (13) guarantees that in each block, the number of subblocks that are in loading process cannot exceed one. Constraint (14) is concerned with the limitation on the number of loading flows that pass each vertical lane. Constraint (15) states that the variable  $\tilde{D}_{i,t}^S(\omega)$  depends on  $x_{i,k}$ ,  $y_i$ , and  $n_{i,t}(\omega)$ . It cannot be calculated by some closed-form formulae, but by a procedure that has been elaborated in Section 3.2.4. Constraint (16) defines decision variables.

Given arbitrary probability distributions of the number of containers loaded onto Vessel  $i$  during Period  $t$ , we can obtain a set of samples on  $n_{i,t}$ . The probability distributions of  $n_{i,t}$  can be calibrated from historical data, which includes how many containers are loaded onto the vessel in the past periods. A random sample of  $|\Omega|$  scenarios is generated to denote  $n_{i,t}(\omega)$ ,  $\omega = 1, \dots, |\Omega|$ . In this case, the probability of Scenario  $\omega$ , i.e.,  $\rho(\omega)$  in the above model, equals  $1/|\Omega|$ .

By using the proposed model M\_0, a plan built for an upcoming horizon is in enforcement till the end of that horizon. The M\_0 mainly applies to the situation that we know the probability distribution for the numbers of loading containers such that we can generate a set of scenarios ( $\Omega$ ) for the M\_0. In other words, if the probability distribution is not very precise, the performance of the M\_0 is worse than a rolling-horizon based method, which rebuilds the plan at a regular interval  $T$ . When the interval  $T$  becomes shorter and shorter, this rolling-horizon based method will approach to a real-time scheduling method, which is surely better than the M\_0 under the situations that decision makers cannot obtain a very precise probability distribution for the numbers of loading containers during periods.

#### 5. Meta-heuristic for solving the model

The above model (M\_0) cannot be solved directly by any commercial solvers (e.g., CPLEX, LINDO) due to some challenges inherited in the model. For example, the objective function contains a lot of extremely nonlinear forms that are difficult to be linearized. Furthermore, a procedure (i.e.,  $\tilde{D}_{i,t}^S(\omega)$ ) is involved in the model. The procedure cannot be transformed to some closed-form formulae so as to be embedded in the model directly, not to mention the linearization of the model. All of the above challenges result in that the proposed model is hard to be handled by some commonly used OR methodologies so as to develop an exact solution method. Therefore, this study designs a meta-heuristic approach for obtaining a good solution for the model M\_0 efficiently.

### 5.1. General framework

The solution to the model (M\_0) mainly contains two parts: (1) the number of subblocks reserved for each vessel in the exclusive mode, i.e.,  $y_i$ ; and (2) which subblocks should be reserved for each vessel, i.e.,  $x_{i,k}$ . Our proposed approach has a two-loop iteration framework. The outer loop determines values for all the  $y_i$  variables; while the inner loop determines values for all the  $x_{i,k}$  variables with given the  $y_i$  values. The well known simulated annealing method is used for the outer loop to change the  $y_i$  values in iterations. For the inner loop, a sub-model is solved by CPLEX so as to obtain the  $x_{i,k}$  values. In this way, a good feasible solution to the model (M\_0) can be obtained by the meta-heuristic approach within a reasonable time period.

The simulated annealing (SA) method is a generic probabilistic meta-heuristic for the global optimization problem. The typical simulated annealing approach involves a pair of nested loops and some additional parameters, e.g., cooling rate,  $0 < r < 1$ , and temperature length,  $R$ , which represents the size of neighborhood of a solution. The following describes the procedure:

- 
- Step 1:* Obtain an initial solution, let  $T = T_0$  (the initial temperature in SA).  
*Step 1.1:* Generate an initial setting for  $y_i$  variables (denoted by  $y$ ).  
*Step 1.2:* Based on  $y$ , solve a model by CPLEX, obtain solution for  $x_{i,k}$  variables (denoted by  $x$ ).  
*Step 1.3:* Calculate the objective value of the solution  $(x, y)$ , denoted by  $F(y)$ .  
*Step 2:* Repeat the following steps until one of stopping conditions becomes true.  
*Step 2.1:* Generate  $R$  neighbors of  $y$ , i.e.,  $y^{(n)}$ ,  $n \in \{1, 2, \dots, R\}$ .  
*Step 2.2:* For  $n = 1$  to  $R$ , repeat the following steps:  
*Step 2.2.1:* Based on  $y^{(n)}$ , solve a model by CPLEX, obtain solution for  $x_{i,k}$  variables.  
*Step 2.2.2:* Calculate the objective value, denoted by  $F(y^{(n)})$ .  
*Step 2.2.3:* Let  $\delta = F(y^{(n)}) - F(y)$ .  
*Step 2.2.4:* If  $\delta < 0$ , set  $y = y^{(n)}$ .  
*Step 2.2.5:* If  $\delta \geq 0$ , generate a random number  $x \in (0, 1)$ ; If  $x < e^{-\delta/T}$ ,  $y = y^{(n)}$ .  
*Step 2.3:* Set  $T = r \times T$ .
- 

The stopping criterions: (a) the fitness value of a solution is zero; (b) temperature becomes less than a given threshold value; (c) the best value of the fitness has not been improved during a given number of external loops.

There are some key issues for implementing the above SA procedure: (1) how to obtain an initial setting for  $y_i$  variables; (2) how to obtain solution for  $x_{i,k}$  variables by solving a model; (3) how to evaluate the objective of a solution; (4) How to define the neighborhood set of a solution.

The following Sections 5.2–5.5 will address the above four issues respectively in detail.

### 5.2. Generating an initial solution

For the outer loop, the initial setting for the  $y_i$  values is the starting point of the proposed meta-heuristic approach, and is critical for the efficiency of the solution process. Here we use an enumeration method to find the best number of reserved subblocks for each vessel according to a criterion which is a part of the original objective. The criterion is as follows. For  $\forall i \in V$ :

$$y_i^{st} = \arg \min_{y_i \geq Y_i^{LB}} \left\{ c^F |T_i| L_i y_i + \sum_{\omega \in \Omega} \rho(\omega) \left\{ c^S \sum_{\forall t \in T_i} L_i \left[ \frac{(n_{i,t}(\omega) - y_i P)^+}{P} \right] \cdot \frac{(n_{i,t}(\omega) - y_i P)^+}{n_{i,t}(\omega)} \right\} \right\} \quad (17)$$

The above formula enumerates all the possible  $y_i$  values that satisfy ' $y_i \geq Y_i^{LB}$ ' for each vessel so as to find a local optimal value (i.e.,  $y_i^{st}$ ), which will be used as the initial solution for the following procedure. There exists possibility that the sum of the  $y_i^{st}$  may exceed the number of all the available subblocks, i.e.,  $\sum_{i \in V} y_i^{st} > |K|$ . It should be mentioned that that situation seldom emerges if the port yard has a sufficient capacity for these transshipped containers (i.e.,  $n_{i,t}(\omega)$ ). However, if that situation emerges, we have to random choose some vessels'  $y_i$  values to decrease so that the sum of them becomes less than  $|K|$ .

### 5.3. Solving the assignment of subblocks to vessels

Given the numbers of reserved subblocks (i.e.,  $y_i^{st}$ ) for all the vessels, a model (M\_1) is solved so as to obtain the detailed assignment of subblocks to the vessels.

$$(M.1) \quad \text{Minimize} \quad \sum_{\omega \in \Omega} \rho(\omega) \left\{ c^T \sum_{\forall i \in V} \sum_{\forall t \in T_i} \left[ \min \left( P, \frac{n_{i,t}(\omega)}{y_i^{st}} \right) \sum_{\forall k \in K} x_{i,k} (D_k^U + D_{k,i}^L) \right] \right\} \quad (18)$$

$$s.t. \sum_{i \in V} x_{i,k} \leq 1 \quad \forall k \in K \quad (19)$$

$$\sum_{k \in Q_i} x_{i,k} = y_i^{*'} \quad \forall i \in V \quad (20)$$

$$\sum_{k \in \{K-Q_i\}} x_{i,k} = 0 \quad \forall i \in V \quad (21)$$

$$\sum_{k \in e} \sum_{i \in V} l_{i,m} x_{i,k} \leq 1 \quad \forall e \in E, \forall m \in M \quad (22)$$

$$\sum_{k \in g} \sum_{i \in V} l_{i,m} x_{i,k} \leq 1 \quad \forall g \in G, \forall m \in M \quad (23)$$

$$\sum_{i \in V} \sum_{k \in K} x_{i,k} l_{i,m} w_{i,k,a} \leq W_a \quad \forall a \in A, \forall m \in M \quad (24)$$

$$x_{i,k} \in \{0, 1\}; \quad \forall i \in V, \forall k \in K \quad (25)$$

In the above model, Constraints (22)–(24) are the bottlenecks for the solving process as the numbers of constraints are a bit large. For increasing the solving speed, we can replace the set  $M$  in the constraints with a reduced set. The set  $M$  contains all the time steps in the planning horizon; while the reduced set just contains the time steps that cover all the vessels' first periods. This reduction does not change the optimality of the model (M\_1) because the parameter  $l_{i,m}$  has a characteristics of periodicity, which means the loading time steps for each vessel are periodically repeated during all the periods. In this way, the above model can be solved by CPLEX directly within a very short time even for large scale problem cases.

#### 5.4. Evaluating solutions

After the previous two steps, a solution of yard template planning is obtained. The solution (i.e.,  $x_{i,k}$  and  $y_i$  variables) is substituted in the objective formula (7) so as to calculate the objective value of the solution. In this process, the main challenge is to calculate  $\bar{D}_{i,t}^S(\omega)$  according to the procedure  $f(x_{i,k}, y_i, n_{i,t}(\omega))$ , which is elaborated in Section 3.2.4. If the problem scale is large, the procedure may not be finished instantaneously. When the model contains a number of scenarios, the CPU time for calculating the objective value for a solution may be a bit long because the procedure  $f(x_{i,k}, y_i, n_{i,t}(\omega))$  needs to run for  $|\Omega|$  (i.e., the number of scenarios) times.

#### 5.5. Neighborhood of a solution in SA

The above sections address how to obtain a yard template plan with given a set of values for  $y_i$  variables, which represent the number of subblocks reserved for each vessel. The SA heuristic is used to change the set of values for  $y_i$  variables so as to improve the quality of the obtained yard template plan.  $Y$  denotes the set of solutions that satisfy  $y_i \geq Y_i^{LB}$ . The SA heuristic explores the solution space by moving at each iteration from the current solution  $y$  to another (usually the best) solution in its neighborhood  $N(y)$ . The neighborhood  $N(y)$  of a solution  $y$  is defined by applying a simple operation that increase or decrease one in a randomly chosen element  $y_i$  of the solution  $y$ , i.e.,  $y = \langle y_1, \dots, y_i, \dots, y_{|V|} \rangle$ . For a solution, the size of its complete neighborhood is  $2|V|$ . As the solving time for the model (M\_1) and the procedure  $f(x_{i,k}, y_i, n_{i,t}(\omega))$  is not trivial, the whole process of the meta-heuristic is time consuming especially when the number of vessels is large. Thus we only select  $R$  neighbors randomly among the complete neighborhood of a solution in each iteration.  $R$  is also the temperature length in the SA method.

### 6. Computational experiments

We conduct experiments to validate the effectiveness of the proposed method. The model M\_1, embedded in the solution approach, is implemented and solved by CPLEX 12.1 in a PC with 1.6 GHz quad-core processor and 4G memory. For the SA meta-heuristic, we set the initial temperature  $T = 10,000$ ; the temperature length  $R = 6$ ; the cooling rate  $r = 0.6$ ; the stopping criterions are:  $T < 0.01$ , or the best value of the fitness has not been improved during five external loops.

#### 6.1. Generation of test instances

The planning horizon is 10 weeks (i.e., 70 days). Each day is divided into three time steps of 8 h. Hence the whole planning horizon has 210 time steps. The periodicity of vessel arrival is also considered. The arriving vessels have different cycle time, e.g., weekly arrival pattern, 10-days pattern, and biweekly pattern, which mean the planning horizon contains ten periods, seven periods, and five periods, respectively. Most of the vessels in the experimental cases belong to the weekly arrival pattern.

For the number of vessels and the size of the yard template, five types of instances are generated in this experiment: 20 vessels, 120 subblocks; 30 vessels, 160 subblocks; 40 vessels, 240 subblocks; 50 vessels, 300 subblocks; and 60 vessels, 360 subblocks. The configuration of the yard layout is shown in Fig. 9. For the vessels, we distinguish them between three classes,



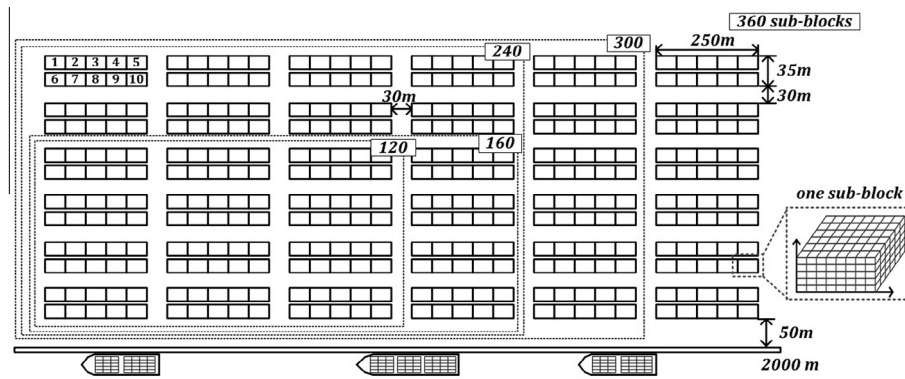


Fig. 9. Yard configurations for the cases.

namely feeder, medium, and jumbo (Meisel and Bierwirth, 2009; Zhen et al., 2011b). These classes of vessels differ in their capacity specifications. For generating  $n_{i,t}(\omega)$ , i.e., the number of containers loaded onto Vessel  $i$  during Period  $t$  in Scenario  $\omega$ , their average numbers (denoted by  $\bar{n}_{i,t}$ ) for different vessel classes and arrival patterns are listed in Table 1.

The stochastic feature of the problem is modeled on the basis of scenarios. In this study, a scenario is a set of actual loading container numbers in each period for all the vessels. For generating a series of scenarios, first we set a lower bound and an upper bound for each vessel (e.g., Vessel  $i$ ), denoted by  $n_i^{LB}$  and  $n_i^{UB}$ , which are set according to the average numbers in Table 1. Then, the actual loading container numbers for Vessel  $i$  in different scenarios are randomly generated by following the uniform distributions  $U(n_i^{LB}, n_i^{UB})$ . The range of the uncertainty set (i.e.,  $n_i^{UB} - n_i^{LB}$ ) reflects the 'magnitude of variation' of loading container number for each vessel. They are parameters for the experiment setting. Later, the influence of these parameters on results will be investigated. As to the probability of each scenario, it is assumed to equal the reciprocal of the number of scenarios for the interest of simplicity.

For the yard configuration, an illustrative example is shown in Fig. 9. The yard has a length of 2000 m quay, and 72 blocks (360 subblocks). The depth of each block is six containers (TEU), and the length is 40 containers. Every block is further divided into five subblocks, where the length of each subblock is eight containers. The length of a subblock is around 50 m. The stacking height is five containers. Thus the capacity of a subblock is about 240 ( $=6 \times 8 \times 5$ ) TEUs. The basic unit for yard storage allocation process and yard template planning is at the subblock level. The width of the passing lanes in the yard is set at 30 m. According to the configuration in Fig. 9, the loading and unloading route length (i.e.,  $D_{k,i}^L$  and  $D_k^U$ ) can be calculated in advance. The neighborhood matrix between any pair of subblocks (i.e.,  $e$  and  $E$ ) and the subordinate relationships between subblocks and blocks (i.e.,  $g$  and  $G$ ) can also be defined on the basis of the yard configuration.

For the cost parameters, we set  $c^F$  is 40 USD/subblock · timestep;  $c^S$  is 60 USD/subblock · timestep;  $c^T$  is 5 USD/TEU · kilometer. The setting on these parameters is based on consulting with practitioners in terminal operators; it is also in accordance with the parameter setting in our previous related research (Zhen et al., 2011b). In addition, this setting on parameters ensures that the subblock utilization cost and the container transportation cost are of the same order of magnitude in the objective function during our numerical experiments.

We assume the arrival time of all the vessels in each period is deterministic input data. According to the known arrival time, the time steps of loading activities for each vessel (i.e.,  $l_{i,m}$ ) can be estimated in advance. It should be mentioned that the proposed model can be extended to consider the uncertain arrival time of the vessels. The extensions of the model are discussed Section 7.

## 6.2. Convergence analysis under different numbers of scenarios

The number of scenarios is an important parameter in the proposed method for planning a yard template. We consider two types of test cases, i.e., the numbers of vessels and subblocks are set as 20 and 120; 30 and 180, respectively. As to each type of cases and a given number of scenarios (from 10 to 10,000), ten different cases are generated randomly, and are solved

Table 1  
Average of  $n_{i,t}(\omega)$  for different vessel classes and arrival patterns.

Vessel classes	Arrival patterns		
	Weekly (percentage about 70%)	10 days (percentage about 15%)	Biweekly (percentage about 15%)
Feeder (percentage about 1/3)	800 TEUs/Period	1100 TEUs/Period	1600 TEUs/Period
Medium (percentage about 1/3)	1200 TEUs/Period	1700 TEUs/Period	2400 TEUs/Period
Jumbo (percentage about 1/3)	1600 TEUs/Period	2300 TEUs/Period	3200 TEUs/Period

in the proposed stochastic programming model. The maximum, minimum, and average of the ten cases are recorded, as shown in each row of Table 2.

From the results in Table 2, we can see that the gap between the maximum and the minimum, and the standard deviation decrease evidently when the number of scenarios increases, which shows that the solutions of the proposed model converge as the number of scenarios is large enough. In addition, the results in Table 2 also show that the CPU time increases evidently with the number of scenarios growing.

### 6.3. Comparisons with other methods of yard management

Some comparative experiments are performed to validate the effectiveness of the proposed model. Two other methods are compared.

(1) *Dynamic assignment (DA)*: it means there is no exclusive mode, and all the subblocks are dynamically assigned to arriving vessels under the sharing mode. For a fair comparison, the objective of the proposed model M\_0 is applied for evaluating the dynamic assignment method, which means all the variables  $x_{i,k}$  and  $y_i$  are zeros.

$$(DA) \quad Z_{DA} = \sum_{\omega \in \Omega} \rho(\omega) \sum_{\forall i \in V \forall t \in T_i} \left\{ c^S L_i \left\lceil \frac{n_{i,t}(\omega)}{P} \right\rceil + c^T [n_{i,t}(\omega) \tilde{D}_{i,t}^S(\omega)] \right\} \quad (26)$$

$$s.t. \quad \tilde{D}_{i,t}^S(\omega) = f(x_{i,k}, y_i, n_{i,t}(\omega)) \quad \forall i \in V, \forall t \in T_i, \forall \omega \in \Omega \quad (27)$$

$$x_{i,k} = 0, y_i = 0 \quad \forall i \in V, \forall k \in K \quad (28)$$

The dynamic assignment method actually means there is no need to make a yard template in advance. Thus, the gap between the objective values of the proposed model M\_0 and the dynamic assignment method under the same test case can be used to evaluate the value of yard template.

$$\text{Val\_YarTp} = Z_{DA} - Z_{M_0} \quad (29)$$

(2) *Deterministic model (DM)*: it means to solve a deterministic model of yard template planning based on estimated numbers of loaded containers for vessels (denoted by  $n_{i,t}^E$ ).  $n_{i,t}^E$  can be estimated as the expected value of  $n_{i,t}(\omega)$ , i.e.,  $n_{i,t}^E = \sum_{\omega \in \Omega} \rho(\omega) n_{i,t}(\omega)$ . The deterministic model is the same as the model M\_0, but just has one scenario. The expected values ( $n_{i,t}^E$ ) are used in the scenario. We solve M\_0 with just one scenario (i.e., the deterministic model), and obtain a yard template (denoted by  $Y_{P_{DM}}$ ). Then same objective of the proposed model M\_0 with a series of scenarios is applied for evaluating the performance of  $Y_{P_{DM}}$  when facing the uncertain realistic environments.

The gap between the objective values of the proposed model M\_0 and the deterministic model method under the same test case can be used to evaluate the value of stochastic programming model (Avriel and Williams, 1970).

**Table 2**

Results of test cases under different numbers of scenarios.

	Num. of scenarios	Mean value	Standard deviation	Min value	Max value	Gap (max–min)	Avg. CPU time (s)
20 vessels & 120 subblocks	10	1,484,746	5301	1,475,792	1,493,487	17,695	206
	20	1,485,350	3978	1,477,305	1,493,076	15,771	242
	50	1,486,893	3204	1,480,332	1,492,870	12,538	259
	70	1,488,352	2645	1,484,721	1,492,342	7621	317
	100	1,488,847	1986	1,486,008	1,491,437	5429	354
	200	1,490,322	1670	1,488,975	1,492,035	3060	435
	500	1,489,597	1236	1,488,907	1,491,153	2246	612
	700	1,489,030	962	1,489,504	1,491,096	1592	769
	1000	1,489,835	867	1,489,922	1,491,061	1139	1221
	2000	1,490,023	504	1,489,677	1,490,523	846	1531
	5000	1,489,458	364	1,489,228	1,489,758	530	2185
	7000	1,489,609	289	1,489,549	1,489,803	254	3284
	10,000	1,489,691	136	1,489,655	1,489,815	160	5399
30 vessels & 160 subblocks	10	2,293,348	9043	2,282,312	2,308,385	26,073	223
	20	2,294,429	7932	2,284,814	2,306,044	21,230	237
	50	2,295,774	7008	2,286,688	2,304,861	18,173	314
	70	2,294,368	6201	2,290,028	2,300,308	10,280	426
	100	2,294,570	4325	2,289,117	2,298,423	9306	439
	200	2,295,349	3827	2,293,108	2,297,990	4882	729
	500	2,295,021	2173	2,293,253	2,296,789	3536	1235
	700	2,293,342	1551	2,292,814	2,295,070	2256	1693
	1000	2,293,126	1429	2,292,780	2,294,473	1693	2082
	2000	2,294,041	820	2,293,054	2,294,228	1174	2972
	5000	2,293,594	505	2,293,139	2,293,853	714	4461
	7000	Out of memory		N.A.			N.A.

**Table 3**

Results of comparative experiments under different uncertainty degrees.

Case scale	Uncertain degree	M_0				Dynamic assignment			Deterministic model		
		$Z_{M,0}$	$Z_{DA}$	$GAP_{DA}$ ( $\frac{Z_{DA}-Z_{M,0}}{Z_{M,0}}$ ) (%)	Val_YarTp ( $Z_{DA} - Z_{M,0}$ )	$Z_{DM}$	$GAP_{DM}$ ( $\frac{Z_{DM}-Z_{M,0}}{Z_{M,0}}$ ) (%)	Val_StoMd ( $Z_{DM} - Z_{M,0}$ )			
20 vessel & 120 subblock	200	1,472,527	2,082,988	41	610,461	1,783,345	21	310,818			
	400	1,455,752	2,046,842	41	591,090	1,772,393	22	316,641			
	600	1,456,883	2,044,598	40	587,715	1,776,283	22	319,400			
	800	1,462,919	2,030,348	39	567,429	1,823,287	25	360,368			
	1000	1,508,488	2,051,304	36	542,816	1,875,304	24	366,816			
	1200	1,588,760	2,136,135	34	547,375	2,005,806	26	417,046			
			Avg.	39		Avg.	23				
30 vessel & 160 subblock	200	2,112,117	2,937,440	39	825,323	2,557,648	21	445,531			
	400	2,157,390	2,956,476	37	799,086	2,623,340	22	465,950			
	600	2,183,979	2,978,737	36	794,758	2,693,448	23	509,469			
	800	2,280,436	3,070,210	35	789,774	2,822,739	24	542,303			
	1000	2,186,828	2,902,240	33	715,412	2,771,235	27	584,407			
	1200	2,457,932	3,215,259	31	757,327	3,163,521	29	705,589			
			Avg.	35		Avg.	24				
40 vessel & 240 subblock	200	3,344,501	4,460,934	33	1,116,433	4,009,110	20	664,609			
	400	3,389,401	4,537,472	34	1,148,071	4,152,633	23	763,232			
	600	3,387,544	4,519,892	33	1,132,348	4,207,047	24	819,503			
	800	3,419,527	4,485,602	31	1,066,075	4,235,907	24	816,380			
	1000	3,450,685	4,489,374	30	1,038,689	4,234,783	23	784,098			
	1200	3,472,999	4,431,823	28	958,824	4,328,793	25	855,794			
			Avg.	32		Avg.	23				
50 vessel & 300 subblock	200	4,572,189	6,008,532	31	1,436,343	5,497,783	20	925,594			
	400	4,560,255	5,980,688	31	1,420,433	5,455,538	20	895,283			
	600	4,556,057	6,016,323	32	1,460,266	5,666,852	24	1,110,795			
	800	4,786,448	6,096,935	27	1,310,487	5,982,266	25	1,195,818			
	1000	4,609,025	5,882,247	28	1,273,222	5,763,864	25	1,154,839			
	1200	4,653,420	5,792,824	24	1,139,404	5,799,852	25	1,146,432			
			Avg.	29		Avg.	23				
60 vessel & 360 subblock	200	5,736,923	7,429,319	30	1,692,396	7,082,038	23	1,345,115			
	400	5,757,983	7,461,985	30	1,704,002	7,154,459	24	1,396,476			
	600	5,734,366	7,389,983	29	1,655,617	7,032,468	23	1,298,102			
	800	5,701,689	7,309,409	28	1,607,720	7,156,364	26	1,454,675			
	1000	5,980,972	7,500,259	25	1,519,287	7,527,078	26	1,546,106			
	1200	5,926,843	7,334,170	24	1,407,327	7,537,560	27	1,610,717			
			Avg.	28		Avg.	25				

$$ValstoMd = Z_{M,0}(YP_{DM}) - Z_{M,0}(YP_{M,0}) \quad (30)$$

Here  $YP_{M,0}$  is a yard template solution obtained by solving the proposed model M\_0.

In addition, the reason why other models in the literature are not used in experiments lies in that their objective and constraints are different from the model M\_0, the comparisons between them and M\_0 are meaningless.

### 6.3.1. Comparisons under different uncertain degrees

The comparative experiments between the proposed model M\_0 and the above two other methods are performed under a series of cases with different uncertain degrees, which are reflected by the average range of  $n_{i,t}(\omega)$ 's uncertain set (i.e.,  $n_i^{UB} - n_i^{LB}$ ) for all the vessels. The stochastic parameters  $n_{i,t}(\omega)$  in the experiments are randomly generated by following uniform distributions  $U(n_i^{LB}, n_i^{UB})$ . The results are listed in Table 3.

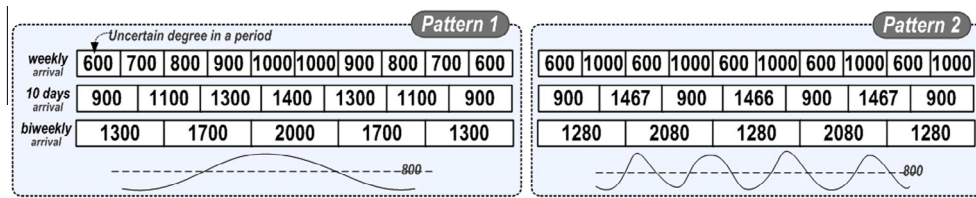
The results in Table 3 validate the effectiveness of the proposed model, which outperforms the other two methods evidently. The value of the yard template (i.e., Val\_YarTp) and the value of the stochastic model (i.e., Val\_StoMd) are evaluated in a quantitative way. Some phenomena are also observed from the results in Table 3.

- The value of yard template decreases with the uncertain degree increasing for each scale of problem cases. It means a yard template is more necessary for the situations with lower uncertain degrees. The demerit of the dynamic assignment method weakens gradually when the situations contain more and more uncertainties.
- The outperformance to the dynamic assignment method (i.e., average value of  $GAP_{DA}$  for each scale of problem cases) decreases with the problem scale growing. It implies that a robust yard template is especially necessary for some yards with small sizes when facing uncertainties.
- The value of stochastic model increases with the uncertain degree increasing for each problem scale. The proposed stochastic programming model is more necessary for the situations with greater uncertain degrees by comparing with

**Table 4**

Comparisons with considering the heterogeneity of uncertain degrees (60-vessel cases).

Std. dev. of uncertain degrees	M_0	Dynamic assignment			Deterministic model		
	$Z_{M,0}$	$Z_{DA}$	$GAP_{DA}$ (%)	Val_YarTp	$Z_{DM}$	$GAP_{DM}$ (%)	Val_StoMd
50	5,700,012	7,210,342	26	1,510,330	7,112,315	25	1,412,303
100	5,713,248	7,263,210	27	1,549,962	7,307,561	28	1,594,313
150	5,743,205	7,304,852	27	1,561,647	7,286,340	27	1,543,135
200	5,797,631	7,427,345	28	1,629,714	7,302,843	26	1,505,212
250	5,842,473	7,485,294	28	1,642,821	7,329,475	25	1,487,002
300	5,891,021	7,592,303	29	1,701,282	7,357,346	25	1,466,325
350	5,921,542	7,691,690	30	1,770,148	7,397,430	25	1,475,888
400	5,973,482	7,942,577	33	1,969,095	7,384,734	24	1,411,252
450	6,021,118	8,562,439	42	2,541,321	7,492,375	24	1,471,257
500	6,094,747	8,739,923	43	2,645,176	7,598,291	25	1,503,544

**Fig. 10.** Two example patterns of uncertain demand for transportation.**Table 5**

Comparisons under different uncertainty patterns (60-vessel cases).

Uncertainty patterns	M_0	Dynamic assignment			Deterministic model		
	$Z_{M,0}$	$Z_{DA}$	$GAP_{DA}$ (%)	Val_YarTp	$Z_{DM}$	$GAP_{DM}$ (%)	Val_StoMd
Pattern 0	5,851,432	7,508,434	28	1,657,002	7,298,358	25	1,446,926
Pattern 1	5,940,084	7,552,149	27	1,612,065	7,724,006	30	1,783,922
Pattern 2	6,022,398	7,584,505	26	1,562,107	8,040,854	34	2,018,456

Note: 'Pattern 0' denotes the situation with no variance of uncertainty degrees along the time.

a deterministic model, whose demerit becomes more and more significant when the randomness of situations increases.

- (d) The problem scale has no significant relationship with the proposed model's outperformance to the deterministic model (i.e., average value of  $GAP_{DM}$  for each scale of problem cases). It stays at a certain level about 23–25% for different problem scales.

### 6.3.2. Analysis on influence of the heterogeneity with respect to vessels' uncertain degrees

The above comparison assumes the uncertain degrees for vessels are similar with each other. In reality, there exist obvious differences with respect to the uncertain degrees of vessels. For some vessels, the forecasts on  $n_{i,t}$  are very reliable; while for some other vessels, the uncertain ranges of  $n_{i,t}$  are very wide due to some difficulties in forecasting  $n_{i,t}$ . Thus experiments are performed to investigate how the deviation of vessels' uncertain degrees influences the comparison results between the aforementioned three methods. For a set of cases under a certain problem scale, their averages of vessels' uncertain degrees equal to 800; but the standard deviations (Std. dev.) of the vessels' uncertain degrees range from 50 to 500. The results are shown in Table 4.

From Table 4, it is observed that the value of yard template increases evidently with the standard deviation growing, which is different from the implication (a) obtained in Section 6.3.1. It means a robust yard template is especially necessary for the situations with significant heterogeneity with respect to vessels' uncertain degrees. Similar with the above implication (d), the heterogeneity of vessels' uncertain degrees has no obvious relationship with the proposed model's outperformance to the deterministic model.

### 6.3.3. Analysis on influence of vessel uncertainty patterns along the time

To further investigate the uncertainty's influence on the results, we distinguish the uncertain range of a vessel among different periods. More specifically, we assume  $n_{i,t}(\omega) \sim U(n_{i,t}^{LB}, n_{i,t}^{UB})$ ; for each vessel, the average of uncertain degrees during all

the periods equals to 800, but there exist variations with respect to the uncertain degrees during different periods. Fig. 10 shows two examples of the uncertainty patterns. The demand uncertainty varies in a more smooth and gradual way in Pattern 1 than Pattern 2. The results of the comparisons between the above three methods under the different uncertainty patterns are listed in Table 5.

From Table 5, it is observed that the value of yard template decreases when the variance of uncertain degree fluctuates along the time periods in a more and more significant way. However, the value of stochastic model increases with the fluctuation growing; and the increasing trend is more significant than the trend under the uncertain degree growing, which is reflected by 'GAP<sub>DM</sub>' in Table 3. It implies that the proposed stochastic programming model (M\_0) is especially necessary for the situations with great fluctuation of the uncertain degree along the time periods.

## 7. Extensions of the model

The M\_0 only considers the uncertain number of loaded containers for vessels in each period, but it can be extended easily to consider uncertain berthing positions and berthing time of vessels (Zhen, 2013). The berthing positions of vessels are related with the parameters  $D_{k,i}^L$  and  $w_{i,k,a}$ ; the berthing time of vessels is related with the parameter  $l_{i,m}$ . Thus,  $D_{k,i}^L$  can be extended to  $D_{k,i}^L(\omega)$  to denote the length of the loading route from Subblock  $k$  to the berth where Vessel  $i$  moors in Scenario  $\omega$ ;  $w_{i,k,a}$  can be extended to  $w_{i,k,a}(\omega)$  to denote whether or not the loading route from Subblock  $k$  to the berth where Vessel  $i$  moors in Scenario  $\omega$  passes Lane  $a$ . The remainder of the model M\_0 is not needed to revise. Moreover,  $l_{i,m}$  can be extended to  $l_{i,m}(\omega)$  to denote whether or not Vessel  $i$  has loading activities during Time Step  $m$  in Scenario  $\omega$ . In the new model, each scenario contains a set of realizations for actual berthing time, actual berthing position, and actual numbers of loaded containers for all the vessels in all the periods.

## 8. Conclusions

This paper develops a stochastic programming model for yard template planning under uncertain maritime market, which contains fluctuations on the volume of loading (and unloading) containers to (and from) arriving vessels. A meta-heuristic method is developed for solving the problem in large-scale realistic environments. Some experiments are conducted to validate the effectiveness of the proposed model and the efficiency of the meta-heuristic algorithm. The results of the numerical experiments show that the proposed stochastic programming model outperforms the dynamic assignment method and the deterministic model based method. The outperformance of the proposed model is more significant in the cases with a smaller size yard and lower uncertain degree. This study also demonstrates that a robust yard template is especially necessary for the situations with significant heterogeneity with respect to vessels' uncertain degrees, and the situations with great fluctuation of the uncertain degree along the time periods. Moreover, this study also discusses the possibility for extending the proposed model so as to consider uncertain berthing positions and berthing time of vessels.

By comparing with other scholars' work in this area, the major contribution mainly includes two aspects:

- (1) Most researches on yard template planning concern how to obtain an optimal yard template in a static and deterministic environment. This paper proposes a mathematical model on yard template planning under uncertainty. Traffic congestions in the yard and multiple schedule cycle times for vessel arrival patterns are also considered in the model.
- (2) The proposed model cannot be solved directly. We propose a meta-heuristic approach for solving the above problem in large-scale realistic environments. Numerical experiments are conducted to validate the effectiveness of the proposed method.

However, there are limitations for the current model. In this study, the issues on traffic congestion of prime movers and yard cranes contention during the *unloading* activities have not been considered in the model specially. In the objective of the model, the cost of transportation between subblocks and berths is formulated on the basis of laden trips, which has not considered the empty trips in the realistic environment. The proposed solution approach is still a bit straightforward, and needs improvements in the future study.

In addition, terminal operators may face the problem on insufficient data for calibrating a stochastic programming model such as M\_0. When limited or no data are available for the number of containers loaded onto vessels, i.e.,  $n_{i,t}$ , the terminal operators can only rely on estimates of vessels' least-most number of loaded containers. Another interesting research direction in the future is to study a robust optimization model for yard template planning that minimizes the worst possible cost for all realizations of vessels' loaded container numbers within the uncertainty sets.

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