A mapping $f : V \to W$ is a linear mapping between two vector spaces V and W, over the field K, if for any two vectors $a, b \in V$ and any scalar $k \in K$

$$\begin{aligned} f(a+b) &= f(a) + f(b) \ , \\ f(k \cdot a) &= k \cdot f(a) \ . \end{aligned}$$

In the case of $V = W$ the mapping f is called a linear operator.

A linear mapping f between two finite-dimensional vector spaces V and W with basis e and e' can be represented as a transformation matrix $A_f^{e,e'}$. Applying the linear mapping on a vector $v \in V$ can be reduced to multiplying the matrix and the vector's coordinate column in the basis e

$$\overline{f(v)_{e'}} = A_f^{e,e'} \cdot \overline{v}_e \ ,$$

where the bar represents the coordinate column.

The transformation matrix of the linear operator $\varphi_x : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates a three-dimensional vector about the x-axis by a roll angle of α is as follows

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} \ .$$

Transformation matrices for the y- and z-axes for by a pitch angle β and a yaw angle γ are as follows

$$R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \qquad R_z(\gamma) = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \ .$$

Having the coordinates of points that define an object in the standard basis, we can rotate the object by applying the rotation linear operators on each of the points

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = R_x(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma) \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \ ,$$

where (x_0, y_0, z_0) are the initial coordinates of a point and (x_1, y_1, z_1) are the coordinates after the transformation.