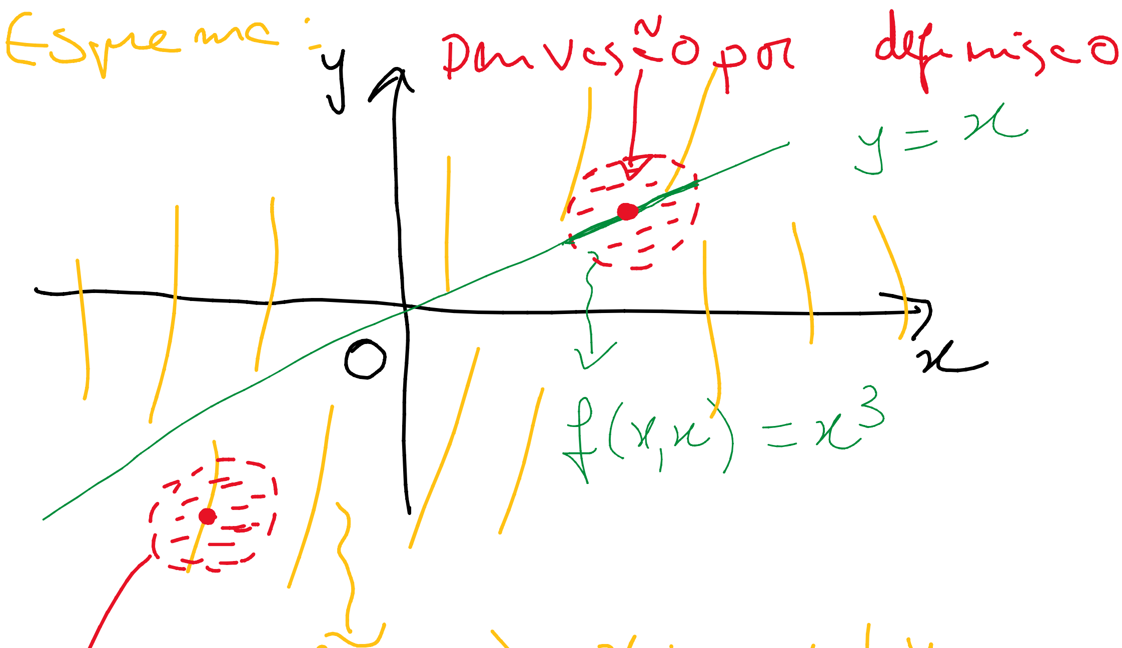
Sessão 6 TP4-9

17 de abril de 2020 07:48

Exemplo (2), slide 28

Esquema:



Podemos usar regras de derivação

$$\frac{\partial f}{\partial x}(1,1) = \lim_{h \to 0} \frac{f(1+h,1) - f(1,1)}{h}$$

$1+h \neq 1$

$$= \lim_{h \to 0} \frac{(1+h)\cdot 1 - 1^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1}+h - \cancel{1}}{h}$$

$$= \lim_{h \to 0} 1$$

$$= 1$$

$$\frac{\partial f}{\partial y}(3,4) = \lim \frac{f(3,4+h) - f(3,4)}{}$$

$$\frac{\partial f}{\partial y}(3,4) = \lim_{h \to 0} \frac{f(3,4+h) - f(3,4)}{h}$$

$$= \dots$$

$$\frac{\partial}{\partial y}(xy) = x, \quad x \neq y$$

$$\frac{\partial f}{\partial y}(3,4) = 3$$

$$\underbrace{\frac{\partial f}{\partial y}(2,2)}_{\text{Não existe}} = \lim_{h \to 0} \frac{f(2,2+h) - f(2,2)}{h}$$

$$= \lim_{h \to 0} \frac{2(2+h) - 2^3}{h}$$

$$= \lim_{h \to 0} \frac{4 + 2h - 8}{h}$$

$$= \lim_{h \to 0} \left(- \frac{4}{\underset{h \to 0}{h}} + 2 \right)$$

Não existe.

Derivadas de ordem superior:

$$f(x,y) = 2x^2 y + x, \quad (x,y) \in \mathbb{R}^2$$

$$\frac{\partial f}{\partial x}(x,y) = 4yx + 1$$

$$\frac{\partial f}{\partial y}(x,y) = 2x^2, \quad (x,y) \in \mathbb{R}^2$$

$$\frac{\partial f}{\partial y}(x,y) = 2x^2 \; , (x,y) \ldots$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)(x,y)$$

$$= \frac{\partial}{\partial x}(4yx+1)$$

$$= 4y \; , \; (x,y) \in \mathbb{R}^2$$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)(x,y)$$

$$= \frac{\partial}{\partial y}(4yx+1)$$

$$= 4x \; , \; (x,y) \in \mathbb{R}^2$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)(x,y)$$

$$= \frac{\partial}{\partial y}(2x^2)$$

iguais

$$= 0 \; , \; \forall (x,y) \in \mathbb{R}^2$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}(2x^2)$$

$$= 4x \; , \; \forall (x,y) \in \mathbb{R}^2$$

Neste exemplo é no

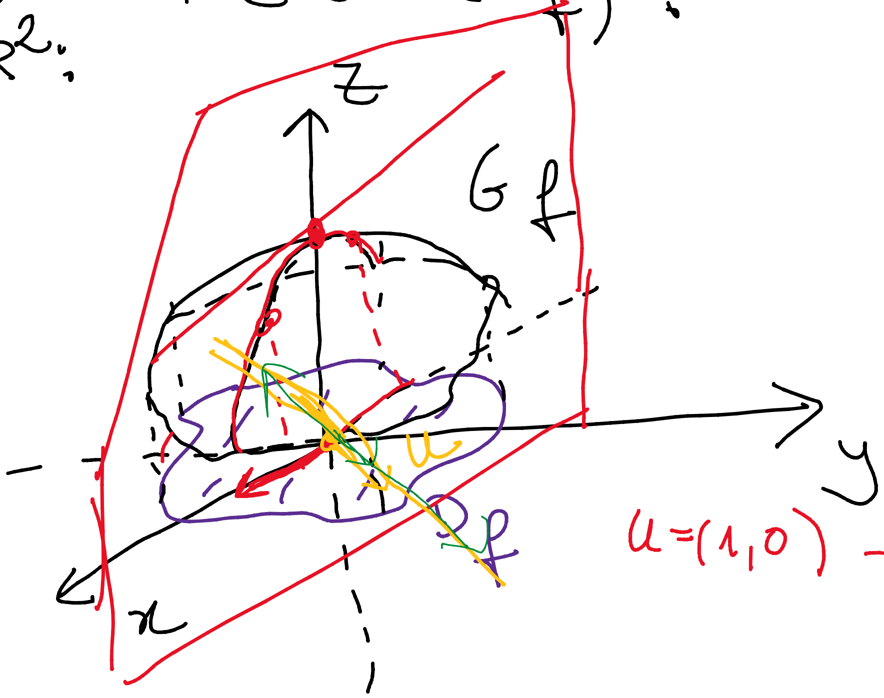
Neste exemplo e no exemplo do slide 31:

$$f \in \mathcal{C}^2(\mathbb{R}^2)$$

f é de classe $\mathcal{C}$ dois em $\mathbb{R}^2$. Ou seja, f tem todas as derivadas até à ordem 2 contínuas em qualquer ponto de $\mathbb{R}^2$.

Derivada Direcional de f segundo um vetor u num ponto $P \in \operatorname{int}(D_f)$:

Em $\mathbb{R}^2$:



$u = (1,0)$ - - - → $\dfrac{\partial f}{\partial x}$