

# Global measures of spatial autocorrelation

Bayesian modelling for spatial and spatio-temporal data

MSc in Epidemiology

Week 6

## Measuring spatial autocorrelation

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- Global measures of spatial autocorrelation share a common structure: calculate the similarity of values at locations  $i$  and  $j$ , then weight the similarity by the proximity of locations  $i$  and  $j$ .
- Null hypothesis: spatial randomness (independent observations).
- Form of global measure of spatial autocorrelation:

$$T = c \frac{\sum_{i=1}^N \sum_{j=1}^N w_{ij} \times \text{similarity}_{ij}}{\sum_{i=1}^N \sum_{j=1}^N w_{ij}}$$

- $c$  constant
- $N$  number of areas
- $w_{ij}$  weights reflecting the proximity between areas  $i$  and  $j$
- $\text{similarity}_{ij}$  measure of similarity between data values in areas  $i$  and  $j$

## Global Moran's I test for spatial autocorrelation

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- Moran's I test can be applied to the data directly, or to the residuals from a regression model (as shown in tutorial 6.1).
- Let  $\{Z_i : i, \dots, N\}$  represent spatially referenced data (or residuals) for  $N$  spatial locations.
- The global Moran's I statistic is:

$$I = \frac{N \sum_{i=1}^N \sum_{j=1}^N w_{ij} (Z_i - \bar{Z})(Z_j - \bar{Z})}{\left( \sum_i \sum_j w_{ij} \right) \sum_{i=1}^N (Z_i - \bar{Z})^2}$$

where  $\bar{Z} = (1/N) \sum_i Z_i$  is the spatial mean and  $w_{ij}$  are the spatial weights

## Global Moran's $I$ interpretation

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$I$  tell us if most pairs of neighbouring areas have the same sign regarding their deviation from the mean.

- If there is no spatial dependence,  $I$  will be close to 0 (i.e. spatial pattern is random)
- If  $I > 0$  and significant, then areas close together (as defined by  $w_{ij}$ ) will tend to have similar values (i.e. clustering of like value)
- If  $I < 0$  and significant, clustering of dissimilar (i.e. alternating) values
- “Significance” will be done using Monte Carlo (MC) approach (i.e. the data are repeatedly randomly assigned to different areas, and the statistic calculated under each permutation, yielding a comparison distribution)

- Haining R., Guangquan L. (2020), Modelling Spatial and Spatial-Temporal Data. A Bayesian Approach, CRC Press, Section 6.2.4.2
- Waller L. A., Gotway C.A. (2004), Applied Spatial Statistics for Public Health Data. John Wiley & Sons, Section 7.4