SOLUTIONS

Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Exercise 12

1. **Decomposition of stochastic economic dispatch**

We consider a power system with 1 wind farm (G_2), 2 thermal generators (G_1, G_3), and 1 inflexible load (D_1) equal to 200MWh. The generation of the thermal generators and the wind farm are controllable. In order to ensure that production meets demand, their generation must be dispatched in the day-ahead. All generators are remunerated for their day-ahead dispatch at a uniform (marginal) price.

However, the *available* wind power is uncertain and can take 4 distinct values in real-time. The normalized production for each scenario and their associated probabilities is provided in Table 1. Therefore, the generation of the thermal and wind generators may need to be

Table 1: Wind production scenarios

	1	**2**	**3**	**4**
Production	0.60	0.70	0.75	0.85
Probability	0.25	0.25	0.4	0.1

adjusted in real-time based on the realized *available* wind power, to ensure that the demand is met. Their day-ahead generation can be adjusted upward or downward by up to their adjustment capacity, as long as their final generation respects their generation capacity bounds. Based on the realized *available* wind power, all generators are remunerated for their real-time adjusted generation at a uniform (marginal) price, which may differ from the day-ahead price.

The day-ahead dispatch costs, upward and downward adjustment costs, generation capacity, upward and downward adjustment capacity of the generators, and the inflexible demand are summarized in Table 1. We observe that the upward adjustment costs of the generators

Table 2: Generators parameters

	G_1	G_2	G_3
Day-ahead cost (DKK/MWh)	75	5	80
Upward adjustment cost (DKK/MWh)	77	7	82
Downward adjustment cost (DKK/MWh)	74	4	79
Generation capacity (MW)	100	150	50
Upward adjustment capacity (MW)	10	150	50
Downward adjustment capacity (MW)	10	150	50

is greater than their day-ahead dispatch cost, and their downward adjustment cost is lower than their day-ahead dispatch cost. This entails that any upward adjustment to generator i's day-ahead dispatch will incur a positive penalty cost equal to $(c_i^{UP} - c_i^{G})$DKK/MWh, and any downward adjustment will incur a positive penalty cost equal to $(c_i^{G} - c_i^{DW})$DKK/MWh.

(a) The system operator aims at dispatching the generators to cover the inflexible load for all realizations of the *available* wind power, at the lowest *expected* cost. Formulate this ED as a stochastic optimization problem.

Solution:
Formulation with implicit non-anticipativity:

Nomenclature:

c_i^G Day-ahead dispatch cost of generator $i \in \mathcal{G}$ (in DKK/MWh)

c_i^{UP} Real-time upward adjustment cost of generator $i \in \mathcal{G}$ (in DKK/MWh)

c_i^{DW} Real-time downward adjustment cost of generator $i \in \mathcal{G}$ (in DKK/MWh)

$\overline{P}_i^G$ Generation capacity of generator $i \in \mathcal{G}$ (in MW)

$\overline{P}_i^{UP}$ Real-time upward adjustment capacity of generator $i \in \mathcal{G}$ (in MW)

$\overline{P}_i^{DW}$ Real-time downward adjustment capacity of generator $i \in \mathcal{G}$ (in MW)

ρ_{is}^W Normalized available wind power of wind producer $i \in \mathcal{W}$ in scenario $s \in \mathcal{S}$

$\boldsymbol{p_i^G}$ Day-ahead dispatch of generator $i \in \mathcal{G}$ (in MWh)

$\boldsymbol{p_{is}^{UP}}$ Real-time upward adjustment of generator $i \in \mathcal{G}$ in scenario $s \in \mathcal{S}$ (in MWh)

$\boldsymbol{p_{is}^{DW}}$ Real-time downward adjustment of generator $i \in \mathcal{G}$ in scenario $s \in \mathcal{S}$ (in MWh)

This problem can be modelled as a two-stage stochastic optimization problem, such that:

$$\min_{\boldsymbol{p^G},\boldsymbol{p^{UP}},\boldsymbol{p^{DW}}} \quad \sum_{i\in\mathcal{G}} \left[c_i^G \boldsymbol{p_i^G} + \sum_{s\in\mathcal{S}} \pi_s \left(c_i^{UP}\boldsymbol{p_{is}^{UP}} - c_i^{DW}\boldsymbol{p_{is}^{DW}}\right)\right] \tag{1a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} \boldsymbol{p_i^G} = P_1^D \tag{1b}$$

$$0 \le \boldsymbol{p_i^G} \le \overline{P}_i^G \quad , \forall i \in \mathcal{G} \tag{1c}$$

$$\sum_{i\in\mathcal{G}} (\boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}}) = 0 \quad , \forall s \in \mathcal{S} \tag{1d}$$

$$0 \le \boldsymbol{p_{is}^{UP}} \le \overline{P}_i^{UP} \quad , \forall i \in \mathcal{G}, s \in \mathcal{S} \tag{1e}$$

$$0 \le \boldsymbol{p_{is}^{DW}} \le \overline{P}_i^{DW} \quad , \forall i \in \mathcal{G}, s \in \mathcal{S} \tag{1f}$$

$$0 \le \boldsymbol{p_i^G} + \boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}} \le \overline{P}_i^G \quad , \forall i \in \mathcal{G} \setminus \mathcal{W}, s \in \mathcal{S} \tag{1g}$$

$$0 \le \boldsymbol{p_i^G} + \boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}} \le \rho_{is}^W \overline{P}_i^G \quad , \forall i \in \mathcal{W}, s \in \mathcal{S}. \tag{1h}$$

The first-stage variables $\boldsymbol{p^G} = \{\boldsymbol{p_i^G}\}_{i\in\mathcal{G}}$ represent the day-ahead dispatch of all generators (3 first-stage variables). The second-stage variables $\boldsymbol{p^{UP}} = \{\boldsymbol{p_{is}^{UP}}\}_{i\in\mathcal{G},s\in\mathcal{S}}$ and $\boldsymbol{p^{DW}} = \{\boldsymbol{p_{is}^{DW}}\}_{i\in\mathcal{G},s\in\mathcal{S}}$ represent the real-time upward and downward adjustment of all generators over all scenarios ($3 * 4 = 12$ second-stage variables). The objective (1a) of this optimization problem is to minimize the day-ahead dispatch cost and expected real-time adjustment cost. The first-stage constraints include the day-ahead balance equation (1b) and lower- and upper-bounds on the day-ahead dispatch of all generators (1c). The second-stage constraints are formulated for all scenarios of wind power $s \in \mathcal{S}$, and include the real-time balance equations (1d), the upper- and lower-bounds on the upward and downward adjustment of all generators (1e)-(1f), and the upper- and lower-bounds on the real-time production of all generators (1g)-(1h). In this formulation, non-anticipativity for the first-stage decision variables is enforced implicitly by defining a single day-ahead dispatch decision $\boldsymbol{p_i^G}$ per generator i, over all scenarios $s \in \mathcal{S}$. This is clear by the fact that these decision variables are not indexed byt the scenarios $s \in \mathcal{S}$.

Formulation with explicit non-anticipativity constraints:

Alternatively, we can enforce non-anticipativity for the first-stage decision variables explicitly as constraints. We introduce the (second-stage) day-ahead dispatch decisions $\boldsymbol{p_{is}^G}$ for each generator i and scenario $s \in \mathcal{S}$, and the first-stage auxiliary variables $\overline{p}_i^G$ for each generator i . The ED problem can then be modelled as a two-stage stochastic optimization problem, such that:

$$\min_{\boldsymbol{p^G}, \boldsymbol{\overline{p}^G}, \boldsymbol{p^{UP}}, \boldsymbol{p^{DW}}} \quad \sum_{i \in \mathcal{G}} \sum_{s \in \mathcal{S}} \pi_s \left[c_i^G \boldsymbol{p_{is}^G} + c_i^{UP} \boldsymbol{p_{is}^{UP}} - c_i^{DW} \boldsymbol{p_{is}^{DW}} \right] \tag{2a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{G}} \boldsymbol{p_{is}^G} = P_1^D \quad , \forall s \in \mathcal{S} \tag{2b}$$

$$0 \leq \boldsymbol{p_{is}^G} \leq \overline{P}_i^G \quad , \forall i \in \mathcal{G}, s \in \mathcal{S} \tag{2c}$$

$$\sum_{i \in \mathcal{G}} (\boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}}) = 0 \quad , \forall s \in \mathcal{S} \tag{2d}$$

$$0 \leq \boldsymbol{p_{is}^{UP}} \leq \overline{P}_i^{UP} \quad , \forall i \in \mathcal{G}, s \in \mathcal{S} \tag{2e}$$

$$0 \leq \boldsymbol{p_{is}^{DW}} \leq \overline{P}_i^{DW} \quad , \forall i \in \mathcal{G}, s \in \mathcal{S} \tag{2f}$$

$$0 \leq \boldsymbol{p_{is}^G} + \boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}} \leq \overline{P}_i^G \quad , \forall i \in \mathcal{G} \setminus \mathcal{W}, s \in \mathcal{S} \tag{2g}$$

$$0 \leq \boldsymbol{p_{is}^G} + \boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}} \leq \rho_{is}^W \overline{P}_i^G \quad , \forall i \in \mathcal{W}, s \in \mathcal{S}. \tag{2h}$$

$$\boldsymbol{p_{is}^G} = \boldsymbol{\overline{p}_i^G} \quad , \forall i \in \mathcal{W}, s \in \mathcal{S}. \tag{2i}$$

In the objective (2a), the first-stage constraints (2b)-(2c), and the second-stage constraints (2d)-(2h) the first-stage decision variables, i.e. the day-ahead dispatch $\boldsymbol{\overline{p}_i^G}$, are replaced by their second-stage counterparts $\boldsymbol{p_{is}^G}$. Besides, non-anticipativity constraints (2i) are added to enforce that the second-stage variables $\boldsymbol{p_{is}^G}$ are equal to a single value $\boldsymbol{\overline{p}_i^G}$ over all scenarios $s \in \mathcal{S}$. Alternatively, the variable $\boldsymbol{\overline{p}_i^G}$ can be replaced by the average of the dispatch decisions $\boldsymbol{p_{is}^G}$ over all scenario $s \in \mathcal{S}$, $\sum_{s \in \mathcal{S}} \pi_s \boldsymbol{p_{is}^G}$, for each generator i.

(b) Is this optimization problem decomposable? If so, specify the complicating variables/constraints and the number of subproblems into which it can be decomposed.

Solution: This stochastic ED problem is decomposable. Using the formulation in (1), the complicating variables are the first-stage variables, i.e. the day-ahead dispatch $\boldsymbol{p_i^G}$. By fixing these complicating variables, Problem (1) can be decomposed into $|\mathcal{S}|$ subproblems, i.e. one per scenario.

Alternatively, using the formulation in (2), the complicating constraints are the non-anticipativity constraints, (??). By relaxing these complicating constraints, Problem (2) can be decomposed into $|\mathcal{S}|$ subproblems, i.e. one per scenario.

(c) Formulate the Benders decomposition algorithm to decompose this stochastic ED problem. Detail the steps of the algorithm, the formulation of the sub-problems and (uni- and multi-cut) master problem at each iteration, and the convergence criteria.

Solution:
The Benders algorithm is summarized in Algorithm ??.
Subproblems:

A each iteration $\nu \geq 1$, the subproblem $SUB_s^{(\nu)}$ for scenario $s \in \mathcal{S}$ is formulated as:

$$\min_{\boldsymbol{p^G},\boldsymbol{p^{UP}},\boldsymbol{p^{DW}}} \quad \sum_{i\in\mathcal{G}} \left\{ c_i^{UP}\boldsymbol{p_{is}^{UP}} - c_i^{DW}\boldsymbol{p_{is}^{DW}} \right\} \tag{3a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} \left(\boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}}\right) = 0 \tag{3b}$$

$$0 \leq \boldsymbol{p_{is}^{UP}} \leq \overline{P}_i^{UP} \quad , \forall i \in \mathcal{G} \tag{3c}$$

$$0 \leq \boldsymbol{p_{is}^{DW}} \leq \overline{P}_i^{DW} \quad , \forall i \in \mathcal{G} \tag{3d}$$

$$0 \leq \boldsymbol{p_i^G} + \boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}} \leq \overline{P}_i^G \quad , \forall i \in \mathcal{G}\setminus\mathcal{W} \tag{3e}$$

$$0 \leq \boldsymbol{p_i^G} + \boldsymbol{p_{is}^{UP}} - \boldsymbol{p_{is}^{DW}} \leq \rho_{is}^W\overline{P}_i^G \quad , \forall i \in \mathcal{W} \tag{3f}$$

$$\boldsymbol{p_i^G} = p_i^{G(\nu)} \quad , \forall i \in \mathcal{G} \ : \ \boldsymbol{\lambda_i^G} \tag{3g}$$

Uni-cut master problem:

A each iteration $\nu \geq 2$, the uni-cut master problem $M^{(\nu)}$ is formulated as:

$$\min_{\boldsymbol{p^G},\boldsymbol{\gamma}} \quad \sum_{i\in\mathcal{G}} c_i^G\boldsymbol{p_i^G} + \boldsymbol{\gamma} \tag{4a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} \boldsymbol{p_i^G} = P_1^D \tag{4b}$$

$$0 \leq \boldsymbol{p_i^G} \leq \overline{P}_i^G \quad , \forall i \in \mathcal{G} \tag{4c}$$

$$\boldsymbol{\gamma} \geq LB^{(1)} \tag{4d}$$

$$\boldsymbol{\gamma} \geq \sum_{i\in\mathcal{G}}\sum_{s\in\mathcal{S}} \pi_s \left\{ c_i^{UP}p_{is}^{UP(k)} - c_i^{DW}p_{is}^{DW(k)} + \lambda_i^{G(k)}\left(\boldsymbol{p_i^G} - p_i^{G(k)}\right) \right\}$$
$$, \forall k \in \{2, ..., \nu-1\} \tag{4e}$$

Multi-cut master problem:

A each iteration $\nu \geq 2$, the multi-cut master problem $\tilde{M}^{(\nu)}$ is formulated as:

$$\min_{\boldsymbol{p^G},\boldsymbol{\gamma}} \quad \sum_{i\in\mathcal{G}} c_i^G \boldsymbol{p_i^G} + \sum_{s\in\mathcal{S}} \pi_s \boldsymbol{\gamma_s} \tag{5a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} \boldsymbol{p_i^G} = P_1^D \tag{5b}$$

$$0 \leq \boldsymbol{p_i^G} \leq \overline{P}_i^G \quad , \ \forall i \in \mathcal{G} \tag{5c}$$

$$\boldsymbol{\gamma_s} \geq LB^{(1)} \quad , \ \forall s \in \mathcal{S} \tag{5d}$$

$$\boldsymbol{\gamma_s} \geq \sum_{i\in\mathcal{G}} \left\{ c_i^{UP} p_{is}^{UP^{(k)}} - c_i^{DW} p_{is}^{DW^{(k)}} + \lambda_i^{G^{(k)}} (\boldsymbol{p_i^G} - p_i^{G^{(k)}}) \right\}$$
$$, \ \forall s \in \mathcal{S}, \ \forall k \in \{2, ..., \nu - 1\} \tag{5e}$$

Convergence Criteria

A each iteration $\nu \geq 2$, the upper- and lower-bounds of the Benders decomposition algorithm are formulated as:

$$UB^{(\nu)} = \sum_{i\in\mathcal{G}} \left[c_i^G p_i^{G^{(\nu)}} + \sum_{s\in\mathcal{S}} \pi_s \left\{ c_i^{UP} p_{is}^{UP^{(\nu)}} - c_i^{DW} p_{is}^{DW^{(\nu)}} \right\} \right] \tag{6a}$$

$$LB^{(\nu)} = \sum_{i\in\mathcal{G}} c_i^G p_i^{G^{(\nu)}} + \gamma^{(\nu)} \tag{6b}$$

$$= \sum_{i\in\mathcal{G}} c_i^G p_i^{G^{(\nu)}} + \sum_{s\in\mathcal{S}} \pi_s \gamma_s^{(\nu)} \tag{6c}$$

and the convergence criteria is $\left| UB^{(\nu)} - LB^{(\nu)} \right| \leq \epsilon$ for a chosen small-enough $\epsilon > 0$.

(d) Solve this stochastic ED problem using the Benders decomposition algorithm. Present and discuss the convergence of the algorithm with the uni- and multi-cut formulations of the master problem. Discuss the pros and cons of each approach.

Solution:

We solve this optimization problem using the uni-cut and multi-cut Benders decomposition algorithms, using the convergence criteria parameter $\epsilon = 0.001$. Both algorithms converge to the optimal solutions of the original optimization problem. However, as illustrated in Figures **??**-**??**, the multi-cut Benders algorithm converges in fewer iterations. However, the master problems at each iteration are larger in the multi-cut Benders algorithm. As a result, for this specific instance, the total convergence time of the multi-cut Benders algorithm is slightly longer than the one of the uni-cut Benders algorithm.