MARTA BRZEZIŃSKA

TOPOLOGICAL PHASES AND TOPOLOGICAL PHASE TRANSITIONS IN LOW-DIMENSIONAL SYSTEMS

TOPOLOGICAL PHASES AND TOPOLOGICAL PHASE TRANSITIONS IN LOW-DIMENSIONAL SYSTEMS

A dissertation submitted to attain the degree of

DOCTOR OF SCIENCES of WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

2020

DOI:

. . .

ABSTRACT

Abstract

ABSTRAKT

Abstrakt

ACKNOWLEDGEMENTS

I would like to thank . . .

CONTENTS

ABBREVIATIONS

QH	quantum Hall TI
topological insulator	

1 INTRODUCTION

In recent years, we have witnessed impressive developments in the field of condensed matter physics, both from theoretical and experimental perspectives. In particular, the discoveries of new phases of matter beyond the Landau paradigm - starting from the Integer Quantum Hall effect and, subsequently, its fractional version, through unconventional superconductors and more, have stimulated many people to find broader and more general classifications schemes.

To understand the physics of QH systems, back in the 80s people tried to employ the notion of topology as a way to understand some peculiar physical properties that cannot be captured by the local order parameters. Later on, the seminal paper by Haldane showed the concept of QH systems without the Landau levels - what is called now the Chern insulator. A reignited interest in topological aspects of quantum states appeared in 2005 with the papers by Kane and Mele (QSH in graphene) as well as Bernevig-Hughes-Zhang (QW in HgTe) .

In 2008, the idea of classification based on the spatial dimensionality and the presence (or absence) of internal symmetries - time-reversal, particle-hole and chiral, was provided using different methods (K-theory, ...), the so-called ten-fold way. Later attempts were trying to capture the relevance of crystal symmetries in the protection of topological states. Also non-Hermiticity extends this classification [bla bla].

In this thesis, we would like to present and discuss novel topological states which fall beyond the ten-fold way. To familarize the reader with basic concepts, we start with a short introduction to topological band theory in Chapter 2. In Chapter 3 we discuss the IQHE on a class of self-similar lattices. Chapter 4 is devoted to the phases of intermediate stability, which are not defined by strong topological indices. Non-Hermitian physics will be covered in Chapter 5. Finally, the summary and further remarks are given in Chapter 6.

2

TOPOLOGICAL BAND THEORY

If the system has a translational invariance, then the states can be represented in the form of Bloch states labeled by the band index n and momenta k.

For two-dimensions, it is given by the (first) Chern number:

$$C = \frac{1}{2\pi} \tag{2.1}$$

There's another striking consequence of non-trivial (non-zero) bulk topological index - the presence of protected gapless edge modes in the open geometry. Even though there's no rigorous proof for the bulk-boundary correspondence, it works remarkably well.

3 TOPOLOGICAL STATES IN FRACTAL LATTICES

Underlying geometry in quantum lattice models plays an important role in defining their electronic properties (

Interestingly, the authors in pointed out that topological states can be realized even in amorphous solids. Fractal lattices comprise of many interesting features: they are aperiodic, but scale invariant. Also, there is no sharp notion between bulk and edge.

Here, we are interested in the lattice regularization of two fractals, Sierpiński carpet (SC) and triangle (or gasket) (SG). This approach is relevant for potential experimental realization as it introduces the distance between nearest-neighbouring sites (lattice constant) to be a natural cutoff.

The reason why we investigate these lattices is motivated by their distinct Hausdorff dimensions ($d_H = \ln A / \ln L$, where A is the area and L the linear size) and connectivity properties. Firstly, $d_H = 1.892\ldots$ for SC and $d_H = 1.585\ldots$ for SG.

We consider tight-binding model of spinless electrons exposed to a magnetic field. The Hamiltonian reads

$$H = -t \sum_{\langle i,j \rangle} e^{\mathrm{i}A_{ij}} c_i^\dagger c_j + \text{h.c.}, \tag{3.1}$$

where we set $t = 1$.

One of the difficulties is to compute topological invariants in that systems as they do not exhibit translational invariance. One may therefore employ real-space methods. Other methods (for example, the Bott index) may be numerically insufficient. Here, we used the real-space expression for the Chern number:

$$\mathcal{C} = 12\pi i \sum_{j \in A} \sum_{k \in B} \sum_{l \in C} \left(P_{jk} P_{kl} P_{lj} - P_{jl} P_{lk} P_{kj} \right), \tag{3.2}$$

where P is the projector operator onto occupied states and i, j, l label the lattice sites.

As an ultimate probe, we study potential topological phase transition with the level spacing statistics. Depending whether states are extended or localized, they follow Wigner-Dyson or Poisson distribution, respectively.

To do so, we add the on-site disorder term $\sum_i V_i c_i^\dagger c_i$ in Eq. (3.1), where V_i is drawn from a uniform distribution $[-W/2, W/2]$. Having computed the level spacings, we calculate their variance and average over 500 disorder realizations for each disorder strength W.

4

OBSTRUCTED ATOMIC LIMITS IN TWO-DIMENSIONAL SYSTEMS

Those systems can be understood using the Wannier functions. Given a set of occupied Bloch states, one may construct the Wannier function as their Fourier transform:

$$W(R) \tag{4.1}$$

The rotation matrix U_{nm} is determined in such a way the minimizes the spread of the Wannier function.

Obstructed atomic limits admit the Wannier function representation, (with Wannier functions being exponentially localized and symmetry-preserving) which is in contrast to strong topological phases. However, the Wannier centers do not coincide with atomic positions (as in the case of trivial atomic limit), but they are rather localized on other symmetric points in the unit cell called the Wyckoff positions.

4.1 MATERIAL CANDIDATES

As potential experimental realization, we propose group-V honeycomb monolayers of bismuth, antimony and arsenic. They share the very same crystal structure.

With non-zero buckling, these systems preserve C_3 and $\mathcal{I}$.

5

NON-HERMITIAN SYSTEMS

In quantum mechanics, the condition that the observable must be a Hermitian operator has a deep physical reasoning - the corresponding expectation value has to be a real-valued number. However, this strong assumption can be relaxed - it is possible to have a non-Hermitian operator with real spectrum. This observation gave rise to the concept of PT-symmetric Hamiltonians, where the real spectrum is guaranteed by the product of parity and time-reversal symmetries.

Another motivation comes from the open systems. Instead of a full treatment with Lindblad formalism, for instance, nH Hamiltonians can effectively capture the coupling of the system with its environment, where the non-Hermiticity models the gains and losses.

Such system exhibits interesting phenomena without Hermitian counterpart: exceptional points, the skin effect and, as a consequence, the breakdown of bulk-boundary correspondence.

Novel features of nH systems are seen at the level of 2×2 matrices. Consider a matrix M: M =

$$\begin{pmatrix} 0 & \alpha \\ 1 & 0 \end{pmatrix} \tag{5.1}$$

If $\alpha \neq 1$, M is not diagonalizable, and only admits the Jordan block form. NH matrices have distinct left- and right- eigenvectors. Therefore, a remedy for some problems may be to consider quantities of interests within the biorthogonal quantum mechanics. For instance, the norm is then given by the inner product between left and right eigenvectors. This attempt allowed to restore BB correspondence in some models. Another way is to consider the singular value decomposition (SVD) instead of eigenvalue problem. However, the interpretation of the singular values is not physical (in contrast to the eigendecomposition, where the eigenvalues are the energies).

In non-Hermitian case, the topology is already manifested in single-band systems (in contrast to Hermitian systems where at least two bands are needed). Also, the winding number for 1D systems is defined through the eigenvalues, not the eigenstates.

5.1 EXCEPTIONAL POINTS

5.2 SKIN EFFECT

nH Hamiltonians are sensitive to the boundary conditions. Eigenstates localization properties may change dramatically. All states for the system in an open geometry may be exponentially localized on the one edge, which is dubbed the skin effect (note: this has nothing in common with a typical skin effect, where the electrons in a conductor prefer to flow far from the middle due to electron-electron repulsion).

6

SUMMARY

A

APPENDIX

PUBLICATIONS

Articles in peer-reviewed journals:

1. Schindler, F., Brzezińska, M., Benalcazar, W. A., Iraola, M., Bouhon, A., Tsirkin, S. S., Vergniory, M. G. & Neupert, T. Fractional corner charges in spin-orbit coupled crystals. *Phys. Rev. Research* **1**, 033074 (3 2019).
2. Guguchia, Z., Gawryluk, D. J., Brzezinska, M., Tsirkin, S. S., Khasanov, R., Pomjakushina, E., von Rohr, F. O., Verezhak, J. A. T., Hasan, M. Z., Neupert, T., Luetkens, H. & Amato, A. Nodeless superconductivity and its evolution with pressure in the layered dirac semimetal 2M-WS2. *npj Quantum Materials* **4**, 50 (2019).
3. Brzezińska, M., Cook, A. M. & Neupert, T. Topology in the Sierpiński-Hofstadter problem. *Phys. Rev. B* **98**, 205116 (20 2018).
4. Brzezińska, M., Bieniek, M., Woźniak, T., Potasz, P. & Wójs, A. Entanglement entropy and entanglement spectrum of Bi_{1-x} Sb_x (111) bilayers. *Journal of Physics: Condensed Matter* **30**, 125501 (2018).
5. Radosz, W., Mielnik-Pyszczorski, A., Brzezińska, M. & Sznajd-Weron, K. *Q*-voter model with nonconformity in freely forming groups: Does the size distribution matter? *Phys. Rev. E* **95**, 062302 (6 2017).