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Homework 3 #

Question 7

a) Exercise 3.1.1

a) True (A is a set of integers that are a multiple of 3, and 27 is a multiple of 3 (27%3=0))

b) False (B is a set of integers that are perfect squares, and there is no integer y, such that $27 = y^2$ (there $y = \sqrt{27}$, which is not an integer))

c) True (B is a set of integers that are perfect squares, and 100 is a perfect square, since there is an integer y, such that $100 = y^2$ (there $y = 10$))

d) False ($E \nsubseteq C$, because $3 \notin C$, and $6 \notin C$. $C \nsubseteq E$, because $4 \notin E$, $5 \notin E$, and $10 \notin E$)

e) True (A is a set of integers that are a multiple of 3, and 3, 6, and 9 are all multiples of 3 (3%3=0, 6%3=0, 9%3=0))

f) False (For $A \subseteq E$, every element in set A would also have to be an element of set E, and it's not true, since E consists of 3,6,9 only, and A is a set of integers that are a multiple of 3)

g) False ("$E \in A$" suggests that E is an element, and it's not, it's a set; additionally, there is no element E in set A, since set A is a set of integers that are a multiple of 3)

b) Exercise 3.1.2

a) False ("$15 \subset A$" suggests that 15 is a set, and it's not)

b) True (A is a set of integers that are a multiple of 3, and 15 is a multiple of 3 (15%3=0))

c) True (since $\emptyset \subseteq C$, and there are elements that of C that aren't elements of $\emptyset$, the $\emptyset$ is a subset of C)

d) True (Every element in D is also an element in D)

e) False ("$\emptyset \in B$" suggests that $\emptyset$ is an element of B, and it's not, $\emptyset$ it's a set, and B consists of elements that are integers and perfect squares)

c) Exercise 3.1.5

b) $A = \{x \in Z^+ : \text{x is an integer multiple of 3}\}$
Answer: Set A is infinite.

d) $A = \{x \in Z : \text{x is an integer multiple of 10 and } 0 \leq x \leq 1000\}$
$|A| = 101$
Answer: Set A is finite.

d) Exercise 3.2.1

a) True (2 is an element of set X)

b) True (every element of set $\{2\}$, is also an element of set X)

c) False (set $\{2\}$ isn't an element of set X)

d) False (there is no element 3 in set X, there is only set $\{3\}$)

e) True (set $\{1,2\}$ is an element of set X)

f) True (every element of set $\{1,2\}$ is an element of set X)

g) True (every element of set $\{2,4\}$ is an element of set X)

h) False (there is no set $\{2,4\}$ in set X)

i) False (not every element of set $\{2,3\}$ is an element of set X - $3 \notin X$)

j) False (set $\{2,3\}$ is not an element of set X)

k) False ($|X| = 6$)

Question 8

1. Exercise 3.2.4

    b) $A = \{1, 2, 3\}$

    $P(A) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

    $\{X \in P(A) : 2 \in X\}$ means that X is an element of set P(A) that includes an element 2, therefore:

    $X = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

Question 9

a) Exercise 3.3.1

c) $A \cap C = \{-3, 1, 17\}$

d) $A \cup (B \cap C)$
$(B \cap C) = \{-5, 1\}$
$A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

e) $A \cap B \cap C$
$A \cap B = \{1, 4\}$
$A \cap B \cap C = \{1\}$

b) Exercise 3.3.3

a) $\bigcap_{i=2}^{5} A_i$

$\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5$

$A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$
$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$
$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$
$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

$\bigcap_{i=2}^{5} A_i = \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} = \{1\}$

b) $\bigcup_{i=2}^{5} A_i$

$\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5$

$A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$
$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$
$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$
$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

$\bigcup_{i=2}^{5} A_i = \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\} = \{1, 2, 3, 4, 5, 9, 16, 25\}$

e) $\bigcap_{i=1}^{100} C_i$

Since we know that $C_i = \{x \in \mathbb{R} : -\frac{1}{i} \leq x \leq \frac{1}{i}\}$, we can calculate C_1 and C_{100}

$C_1 = \{x \in \mathbb{R} : -\frac{1}{1} \leq x \leq \frac{1}{1}\} = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$

$C_{100} = \{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

We can see that function C is a decreasing function, therefore, if $j \geq i$, then every element of C_j is also an element of C_i.

Thus, our intersection, in which every next set is a subset of the previous set, is equal to the set with the highest i, in our case $i = 100$. Therefore:

$\bigcap_{i=1}^{100} = C_{100} = \{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

f) ${}_{i=1}^{100} C_i$

We already know that function C is a decreasing function (therefore, if $j \geq i$, then every element of C_j is also an element of C_i), and:

$C_1 = \{x \in \mathbb{R} : -\frac{1}{1} \leq x \leq \frac{1}{1}\} = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$
$C_{100} = \{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

Thus, our union, in which every next set is a subset of the previous set, is equal to the set with the lowest i, in our case $i = 1$. Therefore:

$\bigcup_{i=1}^{100} = C_1 = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$

Exercise 3.3.4

b) $A \cup B = \{a, b, c\}$

   $P(A \cup B) = \{\ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

d) $P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

   $P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

   $P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Question 10

a) Exercise 3.5.1

  b) For example: (foam, tall, non-fat)

  c) $B \times C = \{$(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)$\}$

b) Exercise 3.5.3

  b) True (Since the set of all integers $\mathbb{Z}$ is a subset of set of all real numbers $\mathbb{R}$ (because every integer is also a real number), so $\mathbb{Z} \subseteq \mathbb{R}$, then $\mathbb{Z}^2 \subseteq \mathbb{R}^2$)

  c) True (Set $\mathbb{Z}^2$ consists of ordered pairs, while the set $\mathbb{Z}^3$ consists of ordered triples, therefore, they don't share any elements)

  e) True (Since $A \times C$ means that $a \in A$, and $c \in C$, and $B \times C$ means that $b \in B$, and $c \in C$, and we know that $A \subseteq B$ (so a is also an element of B), then $(a, c) \in A \times C$ and $(a, c) \in B \times C$. Therefore, $A \times B \subseteq A \times C$)

c) Exercise 3.5.6

  d) $X = \{0, 00\}$
  $Y = \{1, 11\}$
  $xy = \{01, 011, 001, 0011\}$

  e) $X = \{aa, ab\}$
  $Y = \{a, aa\}$
  $xy = \{aaa, aaaa, aba, abaa\}$

d) Exercise 3.5.7

  c) $A \times B = \{ab, ac\}$
  $A \times C = \{aa, ab, ad\}$
  $(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

  f) $A \times B = \{ab, ac\}$
  $P(A \times B) = \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$

  g) $P(A) = \{\emptyset, \{a\}\}$
  $P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$
  $P(A) \times P(B) =$
  $= \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

Question 11

a) Exercise 3.6.2

b) $(B \cup A) \cap (\overline{B} \cup A)$
$(A \cup B) \cap (A \cup \overline{B})$ (Commutative law)
$A \cup (B \cap \overline{B})$ (Distributive law)
$A \cup \emptyset$ (Complement law)
A (Identity law)

c) $\overline{A \cap \overline{B}}$
$\overline{A} \cup \overline{\overline{B}}$ (De Morgan's law)
$\overline{A} \cup B$ (Doule complement law)

b) Exercise 3.6.3

b) If $A = \{1, 2\}$, and $B = \{1\}$, then $(B \cap A) = \{1\}$, and then $A - (B \cap A) = \{2\}$. Since $A - (B \cap A) = \{2\}$ and not {1,2}, it's false that $A - (B \cap A) = A$

d) If $A = \{1, 2\}$, and $B = \{1, 3\}$, then $(B - A) = \{3\}$, and then $(B - A) \cup A = \{1, 2, 3\}$. Since $(B - A) \cup A = \{1, 2, 3\}$ and not {1,2}, it's false that $(B - A) \cup A = A$

c) Exercise 3.6.4

b) $A \cap (B - A)$
$A \cap (B \cap \overline{A})$ (Set subtraction law)
$A \cap (\overline{A} \cap B)$ (Commutative law)
$(A \cap \overline{A}) \cap B$ (Associative law)
$\emptyset \cap B$ (Complement law)
$B \cap \emptyset$ (Commutative law)
$\emptyset$ (Domination law)

c) $A \cup (B - A) = A \cup B$
$A \cup (B \cap \overline{A})$ (Set subtraction law)
$(A \cup B) \cap (A \cup \overline{A})$ (Distributive law)
$(A \cup B) \cap U$ (Complement law)
$(A \cup B)$ (Identity law)