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Homework 1 #

Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. $(10011011)_2$

$(2^0 \times 1) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 0) + (2^6 \times 0) + (2^7 \times 1) =$
$= 1 + 2 + 0 + 8 + 16 + 0 + 0 + 128 =$
$= 155$

$$(10011011)_2 = (\mathbf{155})_{\mathbf{10}}$$

2. $(456)_7$

$(7^0 \times 6) + (7^1 \times 5) + (7^2 \times 4) =$
$= 6 + 35 + 196 =$
$= 237$

$$(456)_7 = (\mathbf{237})_{\mathbf{10}}$$

3. $(38A)_{16}$

$A_{16} = 10_{10}$
$(16^0 \times 10) + (16^1 \times 8) + (16^2 \times 3) =$
$= 10 + 128 + 768 =$
$= 906$

$$(38A)_{16} = (\mathbf{906})_{\mathbf{10}}$$

4. $(2214)_5$

$(5^0 \times 4) + (5^1 \times 1) + (5^2 \times 2) + (5^3 \times 2) =$
$= 4 + 5 + 50 + 250 =$
$= 309$

$$(2214)_5 = (\mathbf{309})_{\mathbf{10}}$$

B. Convert the following numbers to their binary representation:

1. $(69)_{10}$

   $69 \div 2 = 34$ with a remainder of 1

   $34 \div 2 = 17$ with a remainder of 0

   $17 \div 2 = 8$ with a remainder of 1

   $8 \div 2 = 4$ with a remainder of 0

   $4 \div 2 = 2$ with a remainder of 0

   $2 \div 2 = 1$ with a remainder of 0

   $1 \div 2 = 0$ with a remainder of 1

   By reading the reminders from bottom to top, we obtain the binary representation of the number:

   $$(69)_{10} = (\mathbf{1000101})_{\mathbf{2}}$$

2. $(485)_{10}$

   $485 \div 2 = 242$ with a remainder of 1

   $242 \div 2 = 121$ with a remainder of 0

   $121 \div 2 = 60$ with a remainder of 1

   $60 \div 2 = 30$ with a remainder of 0

   $30 \div 2 = 15$ with a remainder of 0

   $15 \div 2 = 7$ with a remainder of 1

   $7 \div 2 = 3$ with a remainder of 1

   $3 \div 2 = 1$ with a remainder of 1

   $1 \div 2 = 0$ with a remainder of 1

   By reading the reminders from bottom to top, we obtain the binary representation of the number:

   $$(485)_{10} = (\mathbf{111100101})_{\mathbf{2}}$$

3. $(6D1A)_{16}$

   We can solve that by using the table below.

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

The first digit of our number in the hexadecimal number system is 6, which corresponds to 6 in the decimal number system and '0110' in the binary number system.

The second digit of our number in the hexadecimal number system is D, which corresponds to 13 in the decimal number system and '1101' in the binary number system.

The third digit of our number in the hexadecimal number system is 1, which corresponds to 1 in the decimal number system and '0001' in the binary number system.

The fourth digit of our number in the hexadecimal number system is A, which corresponds to 10 in the decimal number system and '1010' in the binary number system.

$(6)_{16} = (0110)_2 = (110)_2$

$(D)_{16} = (1101)_2$

$(1)_{16} = (0001)_2$

$(A)_{16} = (1010)_2$

Finally, we combine the binary numbers above and obtain '110110100011010'.

$$(6D1A)_{16} = (\mathbf{110110100011010})_\mathbf{2}$$

C. Convert the following numbers to their hexadecimal representation:

1. $(1101011)_2$

$(2^0 \times 1) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 1) =$
$= 1 + 2 + 0 + 8 + 0 + 32 + 64 =$
$= 107$

$(1101011)_2 = (107)_{10}$

$107 \div 16 = 6$ with a remainder of 11
$6 \div 16 = 0$ with a remainder of 6

$11_{10} = B_{16}$
$6_{10} = 6_{16}$

By reading the reminders from bottom to top, we obtain the hexadecimal representation of the number:

$$(1101011)_2 = (\mathbf{6B})_{\mathbf{16}}$$

2. 895_{10}

$895 \div 16 = 55$ with a remainder of 15
$55 \div 16 = 3$ with a remainder of 7
$3 \div 16 = 0$ with a remainder of 3

$15_{10} = F_{16}$
$7_{10} = 7_{16}$
$3_{10} = 3_{16}$

By reading the reminders from bottom to top, we obtain the hexadecimal representation of the number:

$$(895)_{10} = (\mathbf{37F})_{\mathbf{16}}$$

Question 2

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8$

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 7566_8 \\ +4515_8 \\ \hline 1\,4303_8 \end{array}$$

Assumption: modulo operator is denoted as % and integer division operator is denoted as //

(a) $6 + 5 = 11$
$11//8 = 1$
$11\%8 = 3$
Therefore, we write down the 3, and the 1 is our carryover.

(b) $1 + 6 + 1 = 8$
$8//8 = 1$
$8\%8 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(c) $1 + 5 + 5 = 11$
$11//8 = 1$
$11\%8 = 3$
Therefore, we write down the 3, and the 1 is our carryover.

(d) $1 + 7 + 4 = 12$
$12//8 = 1$
$12\%8 = 4$
Therefore, we write down the 4, and the 1 is our carryover.

$$7566_8 + 4515_8 = \mathbf{14303_8}$$

2. $10110011_2 + 1101_2$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\,1\,1} \\ 10110011_2 \\ + \qquad 1101_2 \\ \hline 11000000_2 \end{array}$$

(a) $1 + 1 = 2$
$2//2 = 1$
$2\%2 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(b) $1 + 1 + 0 = 2$
$2//2 = 1$
$2\%2 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(c) $1 + 0 + 1 = 2$
$2//2 = 1$
$2\%2 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(d) $1 + 0 + 1 = 2$
$2//2 = 1$
$2\%2 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(e) $1 + 1 = 2$
$2//2 = 1$
$2\%2 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(f) $1 + 1 = 2$
$2//2 = 1$
$2\%2 = 0$
Therefore, we write down the 0, and the 1 is our carryover.

(g) $1 + 0 = 1$
$1//2 = 0$
$1\%2 = 1$
Therefore, we write down the 1, and there is no carryover.

(h) $1 + 0 = 1$
$1//2 = 0$
$1\%2 = 1$
Therefore, we write down the 1, and there is no carryover.

$$10110011_2 + 1101_2 = \mathbf{11000000_2}$$

3. $7A66_{16} + 45C5_{16}$

$$\begin{array}{r} {\scriptstyle 1\,1}\phantom{\,6\,6_{16}} \\ 7\,\mathrm{A}\,6\,6_{16} \\ +4\,5\,\mathrm{C}\,5_{16} \\ \hline \mathrm{C}\,0\,2\,\mathrm{B}_{16} \end{array}$$

(a) $6 + 5 = 11$
$11//16 = 0$

$11\%16 = 11$

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

$11_{10} = B_{16}$

Therefore, we write down the B, and there is no carryover.

(b) $(6 + C)_{16} = (6 + 12)_{10} = 18_{10}$

$18//16 = 1$

$18\%16 = 2$

Therefore, we write down the 2, and the 1 is our carryover.

(c) $(1 + A + 5)_{16} = (1 + 10 + 5)_{10} = 16_{10}$

$16//16 = 1$

$16\%16 = 0$

Therefore, we write down the 0, and the 1 is our carryover.

(d) $1 + 7 + 4 = 12$

$12//16 = 0$

$12\%16 = 12$

$12_{10} = C_{16}$

Therefore, we write down the C, and there is no carryover.

$$7A66_{16} + 45C5_{16} = \mathbf{C02B_{16}}$$

4. $3022_5 - 2433_5$

$$\begin{array}{r} \scriptstyle 2\,4\,1 \\ \cancel{3}\cancel{0}\cancel{2}2_5 \\ -2433_5 \\ \hline 34_5 \end{array}$$

(a) We can't do 2-3, so we need to borrow. We replace 2 in the second row with the 1. Therefore, we have 2+10, so 12, in the first row.

$(12 - 3)_5 =?$

Base 5: $0, 1, 2, 3, \mathbf{4}, \cancel{10}, \cancel{11}, \cancel{12}, 13, 14, 20...$

$(12 - 3)_5 = 4_5$

Therefore, we write down the 4.

(b) We can't do 1-3, so we need to borrow. We replace the "30" (the 3 in fourth row, and 0 in third row) with "24". That's because in base 5 number system before the 30 comes 24.

Therefore, we have 1+10, so 11, in the second row.

$(11 - 3)_5 =?$

Base 5: $0, 1, 2, \mathbf{3}, \cancel{4}, \cancel{10}, \cancel{11}, 12, 13, 14, 20...$

$(11 - 3)_5 = 3_5$

Therefore, we write down the 3.

(c) $24_5 - 24_5 = 0_5$

Therefore, we don't have to write anything.

$$3022_5 - 2433_5 = 34_5$$

Question 3

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. 124_{10}

   $124 \div 2 = 62$ with a remainder of 0
   $62 \div 2 = 31$ with a remainder of 0
   $31 \div 2 = 15$ with a remainder of 1
   $15 \div 2 = 7$ with a remainder of 1
   $7 \div 2 = 3$ with a remainder of 1
   $3 \div 2 = 1$ with a remainder of 1
   $1 \div 2 = 0$ with a remainder of 1

   In 8-bit 2's complement, the first digit from the left represents whether the number is positive or negative. Since we have a positive number, the first digit is 0. The rest of the digits are obtained by reading the remainders from bottom to top. Therefore, we have:

   $$124_{10} = (\mathbf{01111100})_{\text{8-bit 2's complement}}$$

2. -124_{10}

   We already know that
   $124_{10} = (01111100)_{\text{8-bit 2's complement}}$

   Therefore, we can obtain 8-bit two's complement representation of the number -124_{10} by flipping the digits of $(01111100)_{\text{8-bit 2's complement}}$ (replacing 1s with 0s and 0s with 1s) and adding 1_2 to it.

   Flipped number: 10000011_2

   $10000011_2 + 1_2 = 10000100_2$

   $$-124_{10} = (\mathbf{10000100})_{\text{8-bit 2's complement}}$$

3. 109_{10}

   $109 \div 2 = 54$ with a remainder of 1
   $54 \div 2 = 27$ with a remainder of 0
   $27 \div 2 = 13$ with a remainder of 1

$13 \div 2 = 6$ with a remainder of 1
$6 \div 2 = 3$ with a remainder of 0
$3 \div 2 = 1$ with a remainder of 1
$1 \div 2 = 0$ with a remainder of 1

In 8-bit 2's complement, the first digit from the left represents whether the number is positive or negative. Since we have a positive number, the first digit is 0. The rest of the digits are obtained by reading the remainders from bottom to top. Therefore, we have:

$$109_{10} = (\mathbf{01101101})_{\text{8-bit 2's complement}}$$

4. -79_{10}

$79 \div 2 = 39$ with a remainder of 1
$39 \div 2 = 19$ with a remainder of 1
$19 \div 2 = 9$ with a remainder of 1
$9 \div 2 = 4$ with a remainder of 1
$4 \div 2 = 2$ with a remainder of 0
$2 \div 2 = 1$ with a remainder of 0
$1 \div 2 = 0$ with a remainder of 1

In 8-bit 2's complement, the first digit from the left represents whether the number is positive or negative. Since we have a positive number, the first digit is 0. The rest of the digits are obtained by reading the remainders from bottom to top. Therefore, we have:

$$(79)_{10} = (01001111)_{\text{8-bit 2's complement}}$$

Therefore, we can obtain 8-bit two's complement representation of the number -79_{10} by flipping the digits of $(01001111)_{\text{8-bit 2's complement}}$ and adding 1_2 to it.

Flipped number: 10110000_2

$10110000_2 + 1_2 = 10110001_2$

$$-79_{10} = (\mathbf{10110001})_{\text{8-bit 2's complement}}$$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{\text{8-bit 2's complement}}$

$(2^0 \times 0) + (2^1 \times\ 1) + (2^2 \times 1) + (2^3 \times 1) + (2^4 \times 1) =$
$= 0 + 2 + 4 + 8 + 16 =$
$= 30$

$$(00011110)_{\text{8-bit 2's complement}} = (\mathbf{30})_{\mathbf{10}}$$

2. $11100110_{\text{8-bit 2's complement}}$

$(2^0 \times 0) + (2^1 \times\ 1) + (2^2 \times 1) + (2^3 \times 0) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 1) =$
$= 0 + 2 + 4 + 0 + 0 + 32 - 64 =$
$= -26$

$$(11100110)_{\text{8-bit 2's complement}} = (\mathbf{-26})_{\mathbf{10}}$$

3. $00101101_{\text{8-bit 2's complement}}$

$(2^0 \times 1) + (2^1 \times\ 0) + (2^2 \times 1) + (2^3 \times 1) + (2^4 \times 0) + (2^5 \times 1) =$
$= 1 + 4 + 8 + 32 =$
$= 45$

$$(00101101)_{\text{8-bit 2's complement}} = (\mathbf{45})_{\mathbf{10}}$$

4. $10011110_{\text{8-bit 2's complement}}$

The number is negative, as the leftmost digit is 1. Therefore, we can obtain the 8-bit 2's complement representation of that positive number, by subtracting 1_2 from it, and then flipping the digits of the result.

$10011110_{\text{8-bit 2's complement}} - 1_2 = 10011101_2$

Flipped number: 01100010_2

$x = (01100010)_{\text{8-bit 2's complement}}$
$x = (2^1 \times 1) + (2^5 \times 1) + (2^6 \times 1) =$
$x = 2 + 32 + 64 =$
$x = 98$
$-x = -98$

$$(10011110)_{\text{8-bit 2's complement}} = (\mathbf{-98})_{\mathbf{10}}$$

Question 4

1. Exercise 1.2.4

   b) $\neg(p \lor q)$

p	q	$(p \lor q)$	$\neg(p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

   c) $r \lor (p \land \neg q)$

p	q	r	$\neg q$	$p \land \neg q$	r $\lor (p \land \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4

   b) $(p \to q) \to (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

   d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5

1. Exercise 1.2.7

   b) (B and D) or (B and M) or (D and M)
   Solution: $(\mathbf{B} \wedge \mathbf{D}) \vee (\mathbf{B} \wedge \mathbf{M}) \vee (\mathbf{D} \wedge \mathbf{M})$

   c) (B or (D and M)
   Solution: $(\mathbf{B} \vee (\mathbf{D} \wedge \mathbf{M})$

2. Exercise 1.3.7

   b) p if (s or y)
   Solution: $(s \vee y) \rightarrow p$

   c) y is necessary for p
   Solution: $p \rightarrow y$

   d) p if and only if (s and y)
   Solution: $p \leftrightarrow (s \wedge y)$

   e) p implies that (s or y)
   Solution: $p \rightarrow (s \vee y)$

3. Exercise 1.3.9

   c) c only if p
   $c \rightarrow p$

   d) p is necessary for c
   $c \rightarrow p$

Question 6

1. Exercise 1.3.6

   b) q: Joe has maintained a B average.
   p: Joe is eligible for the honors program.
   Maintaining a B average is necessary for Joe to be eligible for the honors program.
   q is necessary for p ≡ if p, then q
   Solution: If Joe is eligible for the honors program, then he has maintained a B average.

   c) p: Rajiv can go on the roller coaster.
   q: Rajiv is at least four feet tall.
   Rajiv can go on the roller coaster only if he is at least four feet tall.
   p only if q ≡ if p, then q
   Solution: If Rajiv can go on the roller coaster, then he is at least four feet tall.

   d) q: Rajiv can go on the roller coaster.
   p: Rajiv is at least four feet tall.
   Rajiv can go on the roller coaster if he is at least four feet tall.
   q if p ≡ if p, then q
   Solution: If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10

   c) $(p \vee r) \leftrightarrow (q \wedge r)$
   $(T \vee r) \leftrightarrow (F \wedge r)$
   $(r \vee T) \leftrightarrow (r \wedge F)$
   $(T) \leftrightarrow (F)$
   F
   Solution: The logical expression is false.

   d) $(p \wedge r) \leftrightarrow (q \wedge r)$
   $(T \wedge r) \leftrightarrow (F \wedge r)$
   $(r \wedge T) \leftrightarrow (r \wedge F)$
   $(r) \leftrightarrow (F)$
   Solution: Unknown. If r is true, then the logical expression is false, and if r is false - true.

   e) $p \rightarrow (r \vee q)$
   $T \rightarrow (r \vee F)$
   $T \rightarrow (r)$
   Solution: Unknown. If r is true, then the logical expression is true, and if r is false - false.

f) $(p \wedge q) \rightarrow r$
$(T \wedge F) \rightarrow r$
$(F) \rightarrow r$
Solution: The expression is true. If r is true, then the logical expression is true, and if r is false the logical expression is also true.

Question 7

1. Exercise 1.4.5

   b) First expression: If Sally did not get the job, then she was late for her interview or did not update her resume.
   "if not j then (l or not r)"
   $\neg j \rightarrow (l \vee \neg r)$
   Second expression: If Sally updated her resume and was not late for her interview, then she got the job.
   "if (r and not l) then j"
   $(r \wedge \neg l) \rightarrow j$
   Equivalence-check:

j	l	r	$\neg j$	$\neg r$	$(l \vee \neg r)$	$\neg j \rightarrow (l \vee \neg r)$
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

j	l	r	$\neg l$	$(r \wedge \neg l)$	$(r \wedge \neg l) \rightarrow j$
T	T	T	F	F	T
T	T	F	F	T	T
T	F	T	T	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	T	F	F
F	F	F	T	T	T

   Answer: The two expressions are equivalent as they have the same truth values for all possible combinations of truth values of their variables.

   c) First expression: If Sally got the job then she was not late for her interview.
   "if j then not l"
   $j \rightarrow \neg l$
   Second expression: If Sally did not get the job, then she was late for her interview.
   "if not j, then l"
   $\neg j \rightarrow l$
   Equivalence-check:

j	l	$\neg l$	$j \to \neg l$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

j	l	$\neg j$	$\neg j \to l$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Answer: The two expressions aren't equivalent as they don't have the same truth values for all possible combinations of truth values of their variables (for example, when the truth value of j is true, and the truth value of l is true, the first expression is false, and the second - true)

d) First expression: If Sally updated her resume or she was not late for her interview, then she got the job.

"if (r or not l), then j"

$(r \vee \neg l) \to j$

Second expression: If Sally got the job, then she updated her resume and was not late for her interview.

"if j, then (r and not l)"

$j \to (r \wedge \neg l)$

Equivalence-check:

j	l	r	$\neg l$	$(r \vee \neg l)$	$(r \vee \neg l) \to j$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	T	T	F
F	F	F	T	T	F

j	l	r	$\neg l$	$(r \wedge \neg l)$	$(l \to (r \wedge \neg l)$
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	F	T

Answer: The two expressions aren't equivalent as they don't have the same truth values for all possible combinations of truth values of their variables (for example, when the truth values of j, l, and r are true, the first expression is true, and the second - false).

Question 8

1. Exercise 1.5.2

   c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

      $(p \to q) \land (p \to r)$
      $(\neg p \lor q) \land (\neg p \lor r)$ (Conditional identity)
      $\neg p \lor (q \land r)$ (Distributive law)
      $p \to (q \land r)$ (Conditional identity)

      $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

   f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

      $\neg(p \lor (\neg p \land q))$
      $\neg((p \lor \neg p) \land (p \lor q))$ (Distributive law)
      $\neg(T \land (p \lor q))$ (Complement law)
      $\neg((p \lor q) \land T)$ (Commutative law)
      $\neg(p \lor q)$ (Identity law)
      $\neg p \land \neg q$ (De Morgan's law)

      $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

   i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

      $(p \land q) \to r$
      $\neg(p \land q) \lor r$ (Conditional identity)
      $(\neg p \lor \neg q) \lor r$ (De Morgan's law)
      $\neg p \lor (\neg q \lor r)$ (Associativee law)
      $\neg p \lor (r \lor \neg q)$ (Commutative law)
      $(\neg p \lor r) \lor \neg q)$ (Associative law)
      $(\neg p \lor \neg\neg r) \lor \neg q$ (Double negation law)
      $\neg(p \land \neg r) \lor \neg q$ (De Morgan's law)
      $(p \land \neg r) \to \neg q$ (Conditional identity)

      $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

2. Exercise 1.5.3

   c) $\neg r \lor (\neg r \to p)$
      $\neg r \lor (r \lor p)$ (Conditional identity)
      $(\neg r \lor r) \lor p$ (Associative law)

$(r \vee \neg r) \vee p$ (Commutative law)
$T \vee p$ (Complement law)
$p \vee T$ (Commutative law)
T (Domination law)

$\neg r \vee (\neg r \rightarrow p) \equiv T$

d) $\neg(p \rightarrow q) \rightarrow \neg q$
$\neg(\neg p \vee q) \rightarrow \neg q$ (Conditional identity)
$(\neg\neg p \wedge \neg q) \rightarrow \neg q$ (De Morgan's law)
$(p \wedge \neg q) \rightarrow \neg q$ (Double negation law)
$\neg(p \wedge \neg q) \vee \neg q$ (Conditional identity)
$(\neg p \vee \neg\neg q) \vee \neg q)$ (De Morgan's law)
$(\neg p \vee q) \vee \neg q$ (Double negation law)
$\neg p \vee (q \vee \neg q)$ (Associative law)
$\neg p \vee T$ (Complement law)
T (Domination law)

$\neg(p \rightarrow q) \rightarrow \neg q \equiv T$

Question 9

1. Exercise 1.6.3

   c) There is a number that is equal to its square.
   $\exists x(x = x^2)$

   d) Every number is less than or equal to its square plus 1.
   $\forall x(x \leq (x^2 + 1))$

2. Exercise 1.7.4

   b) Everyone was well and went to work yesterday.
   $\forall x(\neg S(x) \wedge W(x))$

   c) Everyone who was sick yesterday did not go to work.
   $\forall x(S(x) \rightarrow \neg W(x))$

   d) Yesterday someone was sick and went to work.
   $\exists x(S(x) \wedge W(x))$

Question 10

1. Exercise 1.7.9

   c) $\exists x((x = c) \to P(x))$
   Answer: True, for example a.

   d) $\exists x(Q(x) \wedge R(x))$
   Answer: True, for example e.

   e) $Q(a) \wedge P(d)$
   $T \wedge T$
   Answer: True.

   f) $\forall x((x \neq b) \to Q(x))$
   Answer: True.

   g) $\forall x(P(x) \vee R(x))$
   Answer: False. The counterexample is c.

   h) $\forall x(R(x) \to P(x))$
   Answer: True.

   h) $\exists x(Q(x) \vee R(x))$
   Answer: True, for example a.

2. Exercise 1.9.2

   b) $\exists x \forall y Q(x, y)$
   Answer: True. If x=2, then Q(x,1), Q(x,2), and Q(x,3) are all true.

   c) $\exists y \forall x P(x, y)$
   Answer: True. If y=1, then P(1,y), P(2,y), and P(3,y) are all true.

   d) $\exists x \exists y S(x, y)$
   Answer: False. S(x,y) is never true.

   e) $\forall x \exists y Q(x, y)$
   Answer: False. The counterexample is x=1.

   f) $\forall x \exists y P(x, y)$
   Answer: True. If y=1, then P(1,y), P(2,y), and P(3,y) are all true.

   g) $\forall x \forall y P(x, y)$
   Answer: False. The counterexample is P(1,2).

   h) $\exists x \exists y Q(x, y)$
   Answer: True. The example is Q(2,1).

   i) $\forall x \forall y \neg S(x, y)$
   Answer: True. S(x,y) is never true.

Question 11

1. Exercise 1.10.4

   c) There are two numbers whose sum is equal to their product
   $\exists x \exists y(x + y = xy)$

   d) The ratio of every two positive numbers is also positive
   $\forall x \forall y(((x > 0) \land (y > 0)) \rightarrow (\frac{x}{y} > 0))$

   e) The reciprocal of every positive number less than one is greater than one
   $\forall x(((x > 0) \land (x < 1)) \rightarrow (\frac{1}{x} > 1))$

   f) There is no smallest number
   $\neg \exists x \forall y(x \leq y)$

   g) Every number other than 0 has a multiplicative inverse
   $\forall x \exists y((x \neq 0) \rightarrow (y = \frac{1}{x}))$

2. Exercise 1.10.7

   c) There is at least one new employee who missed the deadline.
   $\exists x(N(x) \land D(x))$

   d) Sam knows the phone number of everyone who missed the deadline
   $\forall x(D(x) \rightarrow P(Sam, x))$

   e) There is a new employee who knows everyone's phone number
   $\exists x \forall y(N(x) \land P(x, y))$

   f) Exactly one new employee missed the deadline.
   $\exists x \forall y((N(x) \land D(x)) \land (((x \neq y) \land N(y)) \rightarrow \neg D(y)))$

3. Exercise 1.10.10

   c) Every student has taken at least one class other than Math 101
   $\forall x \exists y((y \neq Math101) \land T(x, y))$

   d) There is a student who has taken every math class other than Math 101
   $\exists x \forall y((y \neq Math101) \rightarrow T(x, y))$

   e) Everyone other than Sam has taken at least two different math classes
   $\forall x \exists y \exists z((x \neq Sam) \rightarrow ((y \neq z) \land T(x, y) \land T(x, z)))$

   f) Sam has taken exactly two math classes.
   $\exists x \exists y \forall z((y \neq x) \land (T(Sam, x) \land T(Sam, y) \land ((z \neq x \land z \neq y) \rightarrow \neg T(Sam, z)))$

Question 12

1. Exercise 1.8.2

   b) Every patient was given the medication or the placebo or both.
   $\forall x(D(x) \vee P(x))$
   Negation: $\neg\forall x(D(x) \vee P(x))$
   De Morgan's law: $\exists x(\neg D(x) \wedge \neg P(x))$
   English: There exists a patient that wasn't given the medication and wasn't given the placebo.

   c) There is a patient who took the medication and had migraines
   $\exists x(D(x) \wedge M(x))$
   Negation: $\neg\exists x(D(x) \wedge M(x))$
   De Morgan's law: $\forall x(\neg D(x) \vee \neg M(x))$
   English: Every patient wasn't given the medication or didn't have migraines or both.

   d) Every patient who took the placebo had migraines.
   $\forall x(P(x) \to M(x))$
   Negation: $\neg\forall x(P(x) \to M(x))$
   Conditional identity: $\neg\forall x(\neg P(x) \vee M(x))$
   De Morgan's law: $\exists x(P(x) \wedge \neg M(x))$
   English: There exists a patient that was given the placebo and didn't have migraines.

   e) There is a patient who had migraines and was given the placebo.
   $\exists x(M(x) \wedge P(x))$
   Negation: $\neg\exists x(M(x) \wedge P(x))$
   De Morgan's law: $\forall x(\neg M(x) \vee \neg P(x))$
   English: Every patient didn't have migraines or wasn't given the placebo or both.

2. Exercise 1.9.4

   c) $\exists x\forall y(P(x,y) \to Q(x,y))$

   $\neg\exists x\forall y(P(x,y) \to Q(x,y))$
   $\forall x\exists y\neg(P(x,y) \to Q(x,y))$ (De Morgan's law for nested quantifiers)
   $\forall x\exists y\neg(\neg P(x,y) \vee Q(x,y))$ (Conditional identity)
   $\forall x\exists y(\neg\neg P(x,y) \wedge \neg Q(x,y))$ (De Morgan's law)
   $\forall x\exists y(P(x,y) \wedge \neg Q(x,y))$ (Double negation law)

   $\neg\exists x\forall y(P(x,y) \to Q(x,y)) \equiv \forall x\exists y(P(x,y) \wedge \neg Q(x,y))$

d) $\exists x \forall y (P(x, y) \leftrightarrow P(y, x))$

$\neg \exists x \forall y (P(x, y) \leftrightarrow P(y, x))$
$\forall x \exists y \neg (P(x, y) \leftrightarrow P(y, x))$ (De Morgan's law for nested quantifiers)
$\forall x \exists y \neg ((P(x, y) \rightarrow P(y, x)) \wedge (P(y, x) \rightarrow P(x, y))$ (Conditional identity)
$\forall x \exists y \neg ((\neg P(x, y) \vee P(y, x)) \wedge (\neg P(y, x) \vee P(x, y))$ (Conditional identity)
$\forall x \exists y (\neg (\neg P(x, y) \vee P(y, x)) \vee \neg (\neg P(y, x) \vee P(x, y))$ (De Morgan's law)
$\forall x \exists y ((\neg \neg P(x, y) \wedge \neg P(y, x)) \vee (\neg \neg P(y, x) \wedge \neg P(x, y))$ (De Morgan's law)
$\forall x \exists y ((P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge \neg P(x, y))$ (Double negation law)
$\forall x \exists y ((P(x, y) \wedge \neg P(y, x)) \vee (\neg P(x, y) \wedge P(y, x))$ (Commutative law)

$\neg \exists x \forall y (P(x, y) \leftrightarrow P(y, x)) \equiv \forall x \exists y ((P(x, y) \wedge \neg P(y, x)) \vee (\neg P(x, y) \wedge P(y, x))$

e) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

$\neg (\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y))$
$(\neg \exists x \exists y P(x, y) \vee \neg \forall x \forall y Q(x, y))$ (De Morgan's law)
$(\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y))$ (De Morgan's law for nested quantifiers)

$\neg (\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)) \equiv (\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y))$