

# Homework 1

September 6, 2023

## Instructions for Homework.

- Homework 1 is due by Sep 15.
- You are allowed to collaborate with your classmates as a group (up to three students in each group). Each group only needs to submit one copy of their homework solutions.
- You can find solutions to some problems in the textbook. You are allowed to read these solutions, but not allowed to copy and paste solutions. Concretely, if you read the textbook solutions, you need to wait for 24 hours before writing your solutions.
- You are not allowed to find solutions online or ask for solutions from anyone other than your group members.

## 1 Problem

**Problem 1.1.** Suppose  $L_1, L_2 \in \text{NP}$ . Please prove (or disprove) that  $L_1 \cup L_2 \in \text{NP}$  and  $L_1 \cap L_2 \in \text{NP}$ .

**Problem 1.2.** Assume that  $\text{SAT} := \{\psi : \psi \text{ is a satisfiable CNF}\}$  is NP-complete. Prove that  $3\text{SAT} := \{\psi : \psi \text{ is a satisfiable 3-CNF}\}$  is NP-complete.

**Problem 1.3.** Prove that  $\text{SPACE}(n^3) \not\subseteq \text{SPACE}(n^2)$