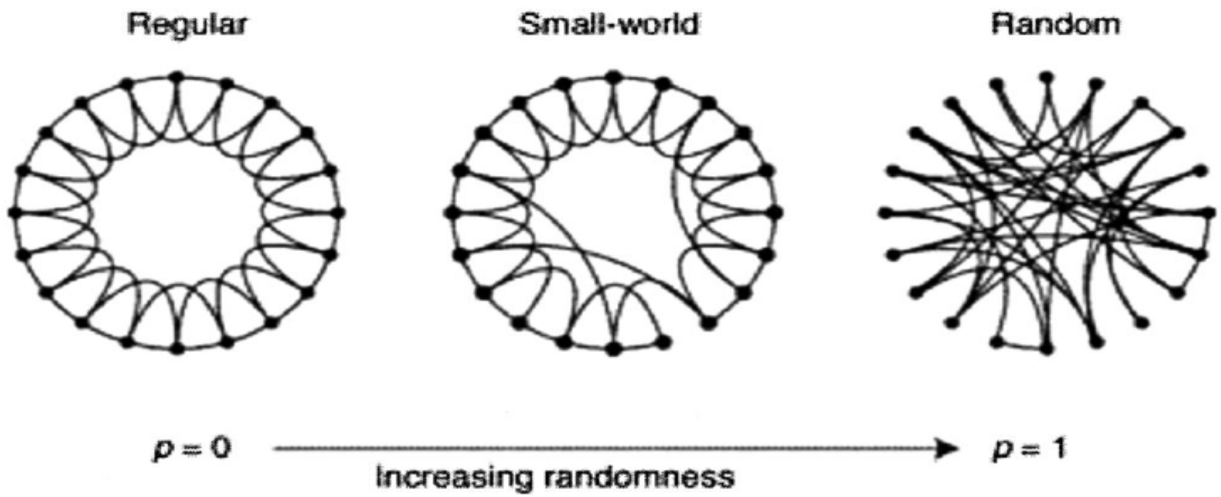
Small World Property

Collective dynamics of 'small-world' networks

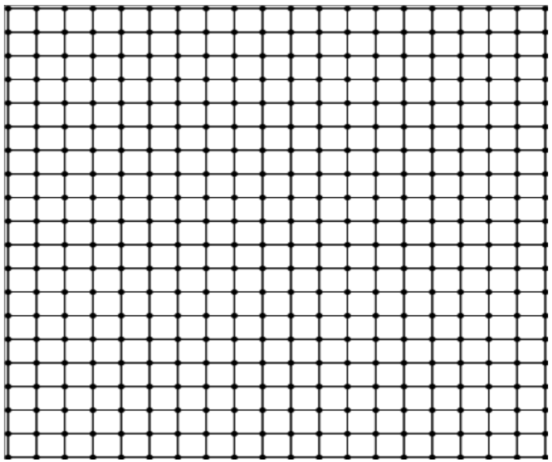
Duncan J. Watts* & Steven H. Strogatz



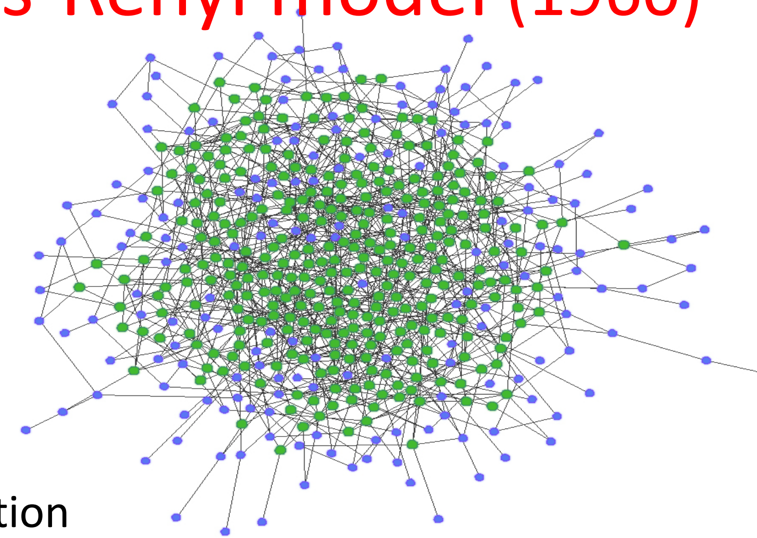
Duncan Watts

WATTS & STROGATZ

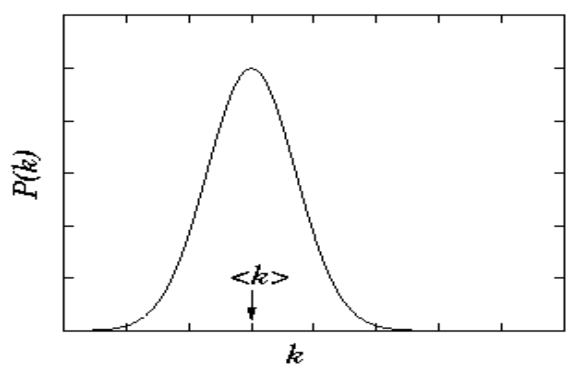
Lattice



Erdös-Rényi model (1960)



Poisson distribution



Small world phenomenon:
applicable to other kinds of networks

Same pattern:

high clustering

low average shortest path

$$C_{\text{network}} >> C_{\text{random graph}}$$

$$l_{\text{network}} \approx \ln(N)$$

Collective dynamics of 'small-world' networks

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Table 1 Empirical examples of small-world networks

	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C. elegans	2.65	2.25	0.28	0.05

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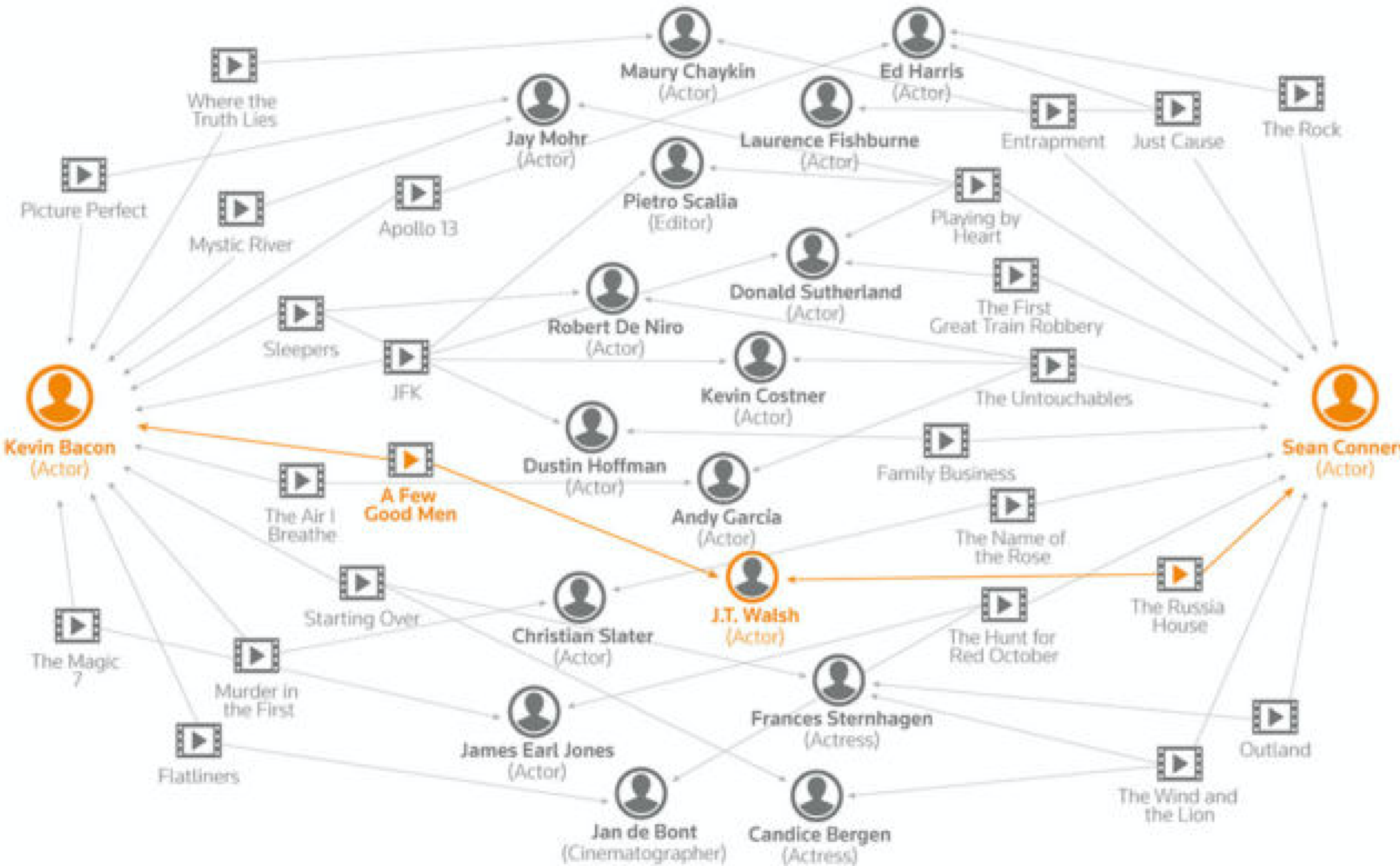
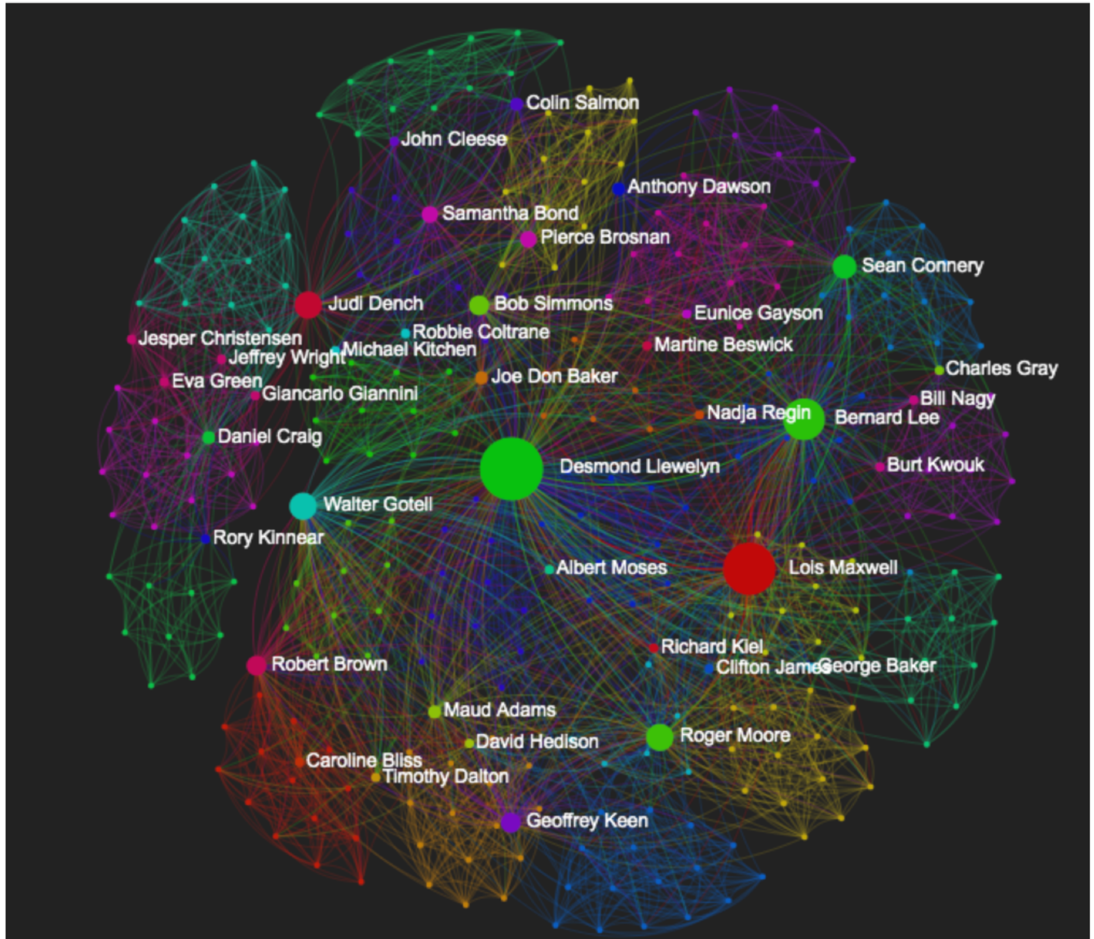
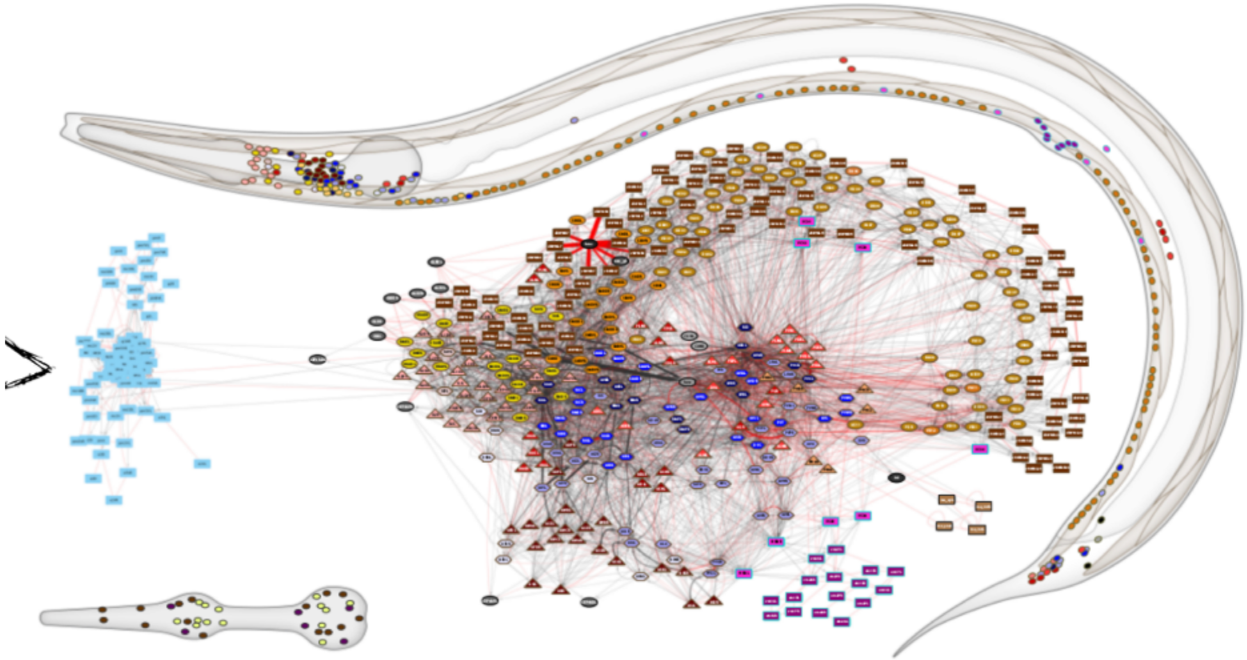


Where the Truth Lies
Maury Chaykin (Actor)
Ed Harris (Actor)
Jay Mohr (Actor)
Laurence Fishburne (Actor)
Entrapment
Just Cause
The Rock
Picture Perfect
Mystic River
Apollo 13
Pietro Scalia (Editor)
Playing by Heart
Donald Sutherland (Actor)
The First Great Train Robbery
Sleepers
Robert De Niro (Actor)
JFK
Kevin Costner (Actor)
The Untouchables
Kevin Bacon (Actor)
Sean Connery (Actor)
A Few Good Men
Dustin Hoffman (Actor)
Family Business
The Air I Breathe
Andy Garcia (Actor)
The Name of the Rose
J.T. Walsh (Actor)
The Russia House
Starting Over
Christian Slater (Actor)
The Hunt for Red October
The Magic 7
Murder in the First
Frances Sternhagen (Actress)
James Earl Jones (Actor)
Outland
Flatliners
The Wind and the Lion
Jan de Bont (Cinematographer)
Candice Bergen (Actress)

Movie Actors Network

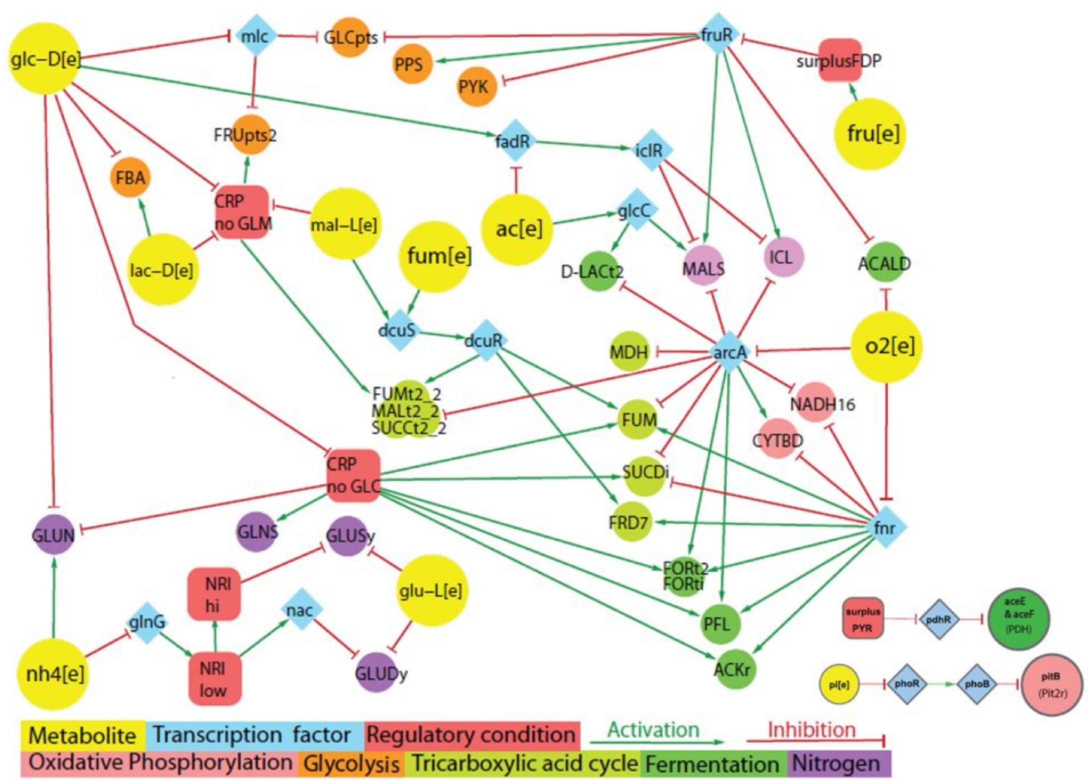




Reconstructed biological neural network

Course Content

Transcriptional Regulatory Networks



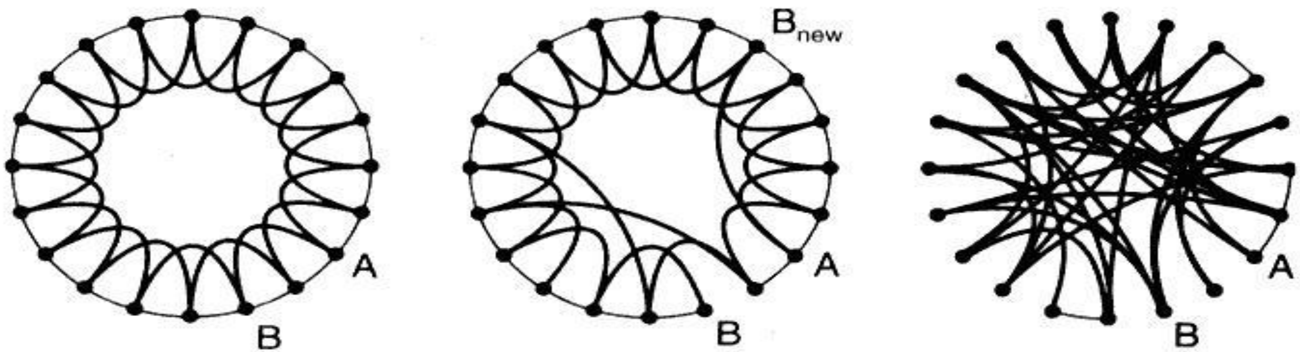
Comparison with "random graph" used to determine whether real-world network is "small world"

Network	size	av. shortest path	Shortest path in fitted random graph	Clustering (averaged over vertices)	Clustering in random graph
Film actors	225,226	3.65	2.99	0.79	0.00027
MEDLINE co-authorship	1,520,251	4.6	4.91	0.56	1.8×10^{-4}
E.Coli substrate graph	282	2.9	3.04	0.32	0.026
C.Elegans	282	2.65	2.25	0.28	0.05

Small world phenomenon:
Watts/Strogatz model

Reconciling two observations:

- **High clustering:** my friends' friends tend to be my friends
- **Short average paths**

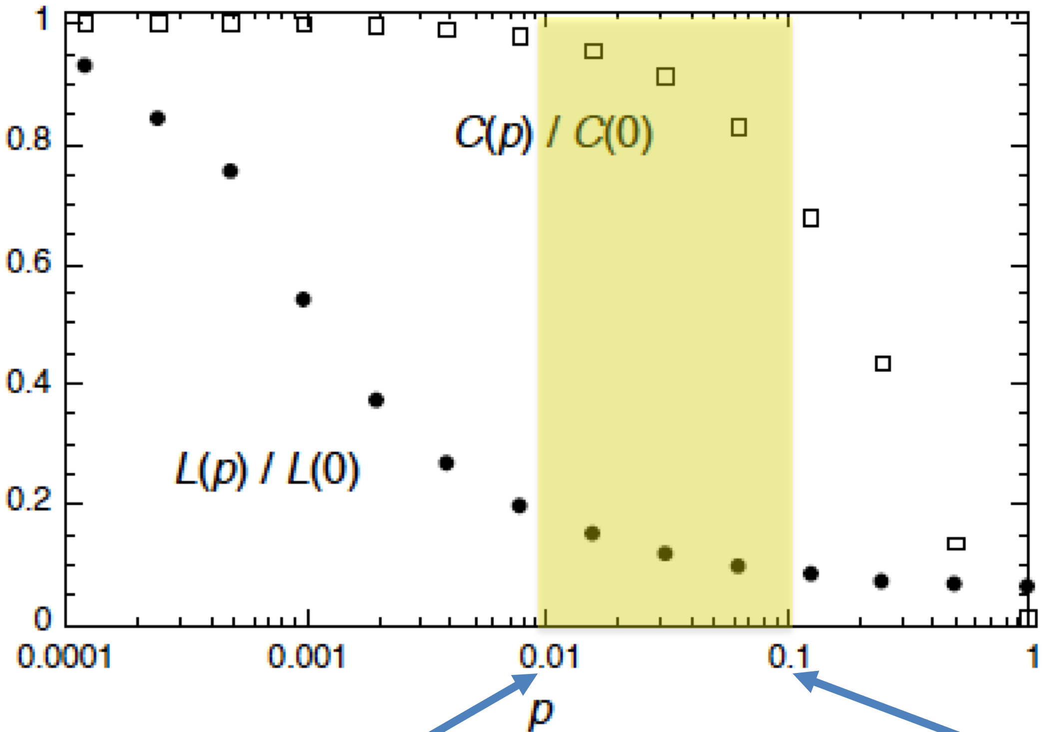


Source: Watts, D.J., Strogatz, S.H.(1998) Collective dynamics of 'small-world' networks. Nature 393:440-442.

Watts-Strogatz model:
Generating small world graphs

- Each node has K>=4 nearest neighbors (local)
- tunable: vary the probability *p* of rewiring any given edge
- small *p*: regular lattice
- large *p*: classical random graph

Watts/Strogatz model:
Change in clustering coefficient and average path length as a function of the proportion of rewired edges



Between 1% and 10% rewired the shortest path decreases and the C stays high!!

Source: Watts, D.J., Strogatz, S.H.(1998) Collective dynamics of 'small-world' networks. Nature 393:440-442.

Definition of Clustering Coefficient

clustering(*G, nodes=None, weight=None*) [source]

Compute the clustering coefficient for nodes.

For unweighted graphs, the clustering of a node u is the fraction of possible triangles through that node that exist,

$$c_u = \frac{2T(u)}{deg(u)(deg(u) - 1)},$$

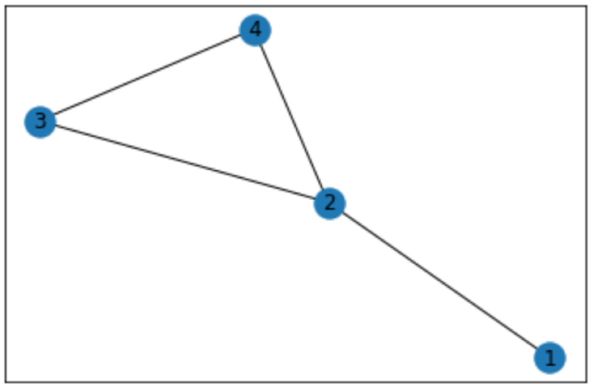
where $T(u)$ is the number of triangles through node u and $deg(u)$ is the degree of u.

Note there are extensions of the definitions for Weighted and Directed Graphs

Next class we study the calculations of Paths and clustering coefficient in ipynb and cover Routing in Networks

In the social sciences, the word **"clique"** is used to describe a group of 2 to 12 (averaging 5 or 6) "persons who interact with each other more regularly and intensely than others in the same setting."

Example 1 Average Clustering Coefficient




```
nx.clustering(g)
```

```
{'1': 0, '2': 0.3333333333333333, '3': 1.0, '4': 1.0}
```

```
nx.average_clustering(g)
```

```
0.5833333333333333
```

```
(0+0.333333+1+1)/4
```

```
0.58333325
```

T(1) = 0 2*0/(1)(0) =0

T(2)=1 2*1/(3)(2)=0.333

$$\frac{2T(u)}{deg(u)(deg(u)-1)}$$

<C>=0.5833

T(3)=1 2*1/(2)(1)=1

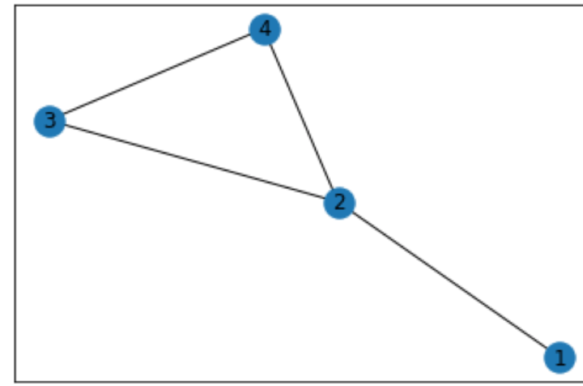
T(4)=1 2*1/(2)(1)=1

Example 1, Average Shortest Path

```
1 has  3 paths
['1', '2'] length  1
['1', '2', '3'] length  2
['1', '2', '4'] length  2
2 has  3 paths
['2', '1'] length  1
['2', '3'] length  1
['2', '4'] length  1

3 has  3 paths
['3', '2'] length  1
['3', '4'] length  1
['3', '2', '1'] length  2

4 has  3 paths
['4', '2'] length  1
['4', '3'] length  1
['4', '2', '1'] length  2
```



<L>=16/12

Number of Paths in Undirected Graph:
nodes(nodes-1)

For Monday 02/08

- Finalize Problem 1 Assignment 1

- Optional: Read article uploaded in Lecture 3, answer Participation Slides 3

Lecture 3 Class Lab

Community structure and ethnic preferences in school friendship networks

Reading_SocialNetwork.ipynb

SchoolEdges.csv

SchoolNodes.csv

Data and Article uploaded in Lecture 3 folder

In ReadingSocialNetwork.ipynb we analyze the Results of a School Survey

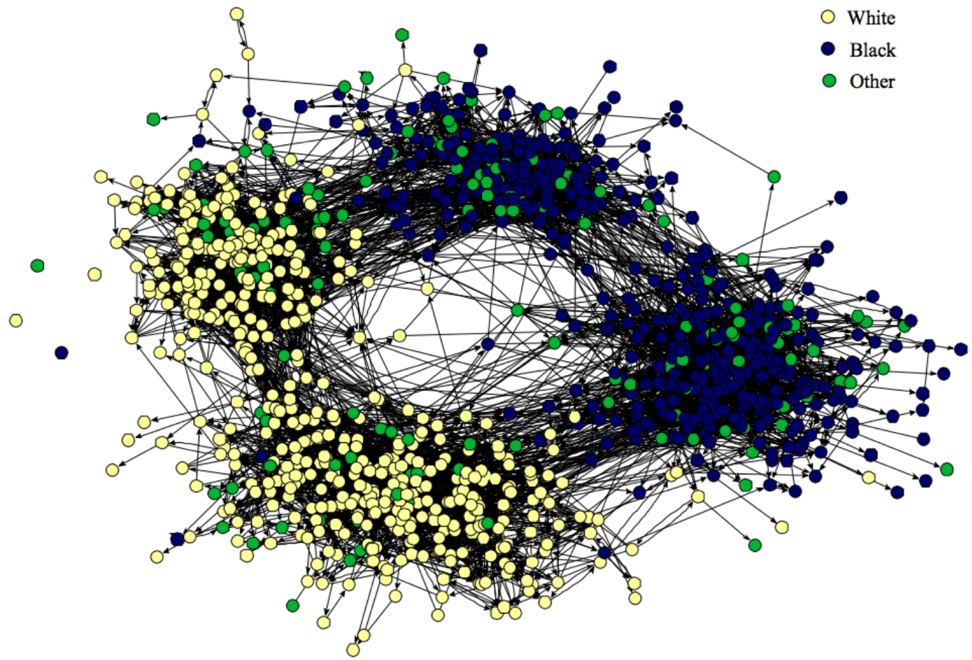


Fig. 3.4 *Friendship network of children in a U.S. school. Friendships are determined by asking the participants, and hence are directed, since A may say that B is their friend but not vice versa. Vertices are color coded according to race, as marked, and the split from left to right in the figure is clearly primarily along lines of race. The split from top to bottom is between middle school and high school, i.e., between younger and older children. Picture courtesy of James Moody.*

Lab (Group Exercise)

For the 4 nodes network MyFirstNwtwork.ipybn, report:

1- Calculate the Clustering Coefficient of each node and of the network

2- Calculate the Clustering Coefficient of of the network

3-Calculate the average shortest path of the network

(you can verify the answers of slides 16 and 17)

Remember: always consult Networkx

https://networkx.github.io/

NetworkX

Stable (notes)

2.4 — October 2019
download | doc | pdf

Latest (notes)

2.5 development
github | doc | pdf

Software for complex networks

NetworkX is a Python package for the creation, manipulation, and study of the structure, dynamics, and functions of complex networks.