Tropical Geometry

Exercises

Exercise 1 - Tropical curve

Draw the tropical hypersurface defined by the tropical polynomial

$$(4 \odot X^3) \oplus (1 \odot XY) \oplus (4 \odot Y^3) \oplus (1 \odot Y^2) \oplus (1 \odot Y) \oplus 6.$$

How does the dual subdivision of the Newton polytope look like?

Exercise 2 - Non-regular subdivision

Give an example of a point configuration and a subdivision which is not regular.

Exercise 3 - Smooth cubic curves

Prove that every smooth cubic curve has a unique bounded region, and that this region can have either three, four, five, six, seven, eight, or nine vertices.

Exercise 4 - Kapranov's Theorem

Prove Kapranov's Theorem for $f = x + y + t \in \mathbb{C}\{\{t\}\}[x^{\pm 1}, y^{\pm 1}]$.

Exercise 5 - Quadratic curve

Compute and draw the quadratic curve through the points $(0, 5)$, $(1, 0)$, $(4, 2)$, $(7, 3)$, $(9, 4)$.