Experimental Lab, Test Case 3
Analysis of the hydrodynamic forces acting on a cylinder in steady free surface flow

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General Information

Report for the third test case of the experimental laboratories within the course of Fluids Labs, by G. V. Messa, Politecnico di Milano.

1 Introduction

Goal of this test case is to investigate the drag and lift forces acting on a cylinder submerged in a steady-state free-surface water flow. The experiments have been performed in the water channel facility of the Hydraulics Laboratory of Politecnico di Milano, and the two force components (horizontal and vertical) have been measured through a balance.

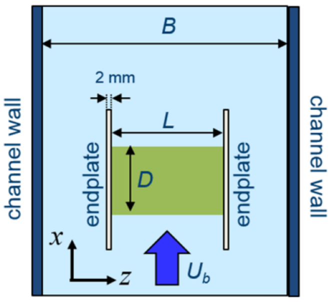
2 Case description

The scheme of the experiment is reported in Section 2.1.
Being the Reynolds number of the case study within the subcritical regime, the flow is expected to separate at a certain distance from the front stagnation point, causing a recirculation zone behind it, and that an oscillating wake is created by the shedding of two counter-rotating vortexes. As a result, also drag and lift will show an oscillating behavior.
The balance is fixed to the ground and connected to the cylinder through the endplates. The hydrodynamically shaped endplates allow to hold the cylinder and ensure some sort of "two-dimensional flow" around it by suppressing the three-dimensional effects at the cylinder ends. Two load cells are installed inside the balance, and they provide an output voltage linearly proportional to the applied force.
The calibration of the balance has been performed after the installation of the endplates and the cylinder on the balance, by applying weight standards to the cylinder without water in the channel. When developing the calibration function, it was taken into account that the real forces experienced by the cells are not only those produced by the applied load, but they also include the weight of the structure, and other small contributions related with the deformability of the structure. The calibration function has been determined in such a way that the condition F_x=0, F_y=0 corresponds to the absence of applied (external) forces, net of the weight and the small deformability-related contributions.
When water flows in the channel, assuming that the dynamic forces on the endplates is negligible since they are hydrodynamically shaped, the horizontal external force F_x is equal to the drag force acting on the cylinder, F_D. Conversely, the vertical external force F_y is equal to the sum of the lift force acting on the cylinder F_L and the buoyancy force acting on the cylinder and the endplates, F_b. Thus, whereas F_D is simply taken as the calibration output F_x, F_L will be given as F_y - F_b. The buoyancy force, F_b, could be theoretically calculated by multiplying

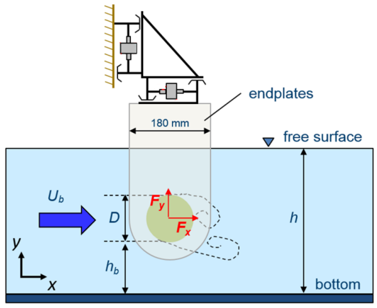
the volume of the immersed parts (cylinder and part of the endplates) by the specific weight of water. However, since knowing all the geometrical details with high accuracy is not trivial, directly measuring F_B with the balance appears a preferred option. This is achieved by making a test in still water with the same level of the flowing-water test. Since no drag and lift forces play a role in the static test, in this case the horizontal external force F_x will be zero and the vertical external force F_y will be equal to F_b.

2.1 Configuration of the problem

The scheme of the experiment is reported in Figure 1.
Moreover, one can see visual representations of the problem on YouTube, following these links: Video 1 - Overview of the experimental setup, Video 2 - Forces on a cylinder in a water flow.



(a) Top view



(b) Side view

Figure 1: Sketch of the experiment configuration

2.2 Experimental setup

The input data of the problem are summarized in Table 1.

Symbol	Parameter	Value	Units
B	Width of the channel	0.5	m
D	Diameter of the cylinder	0.06	m
L	Width of the cylinder	0.185	m
t	Thickness of the endplates	0.002	m
b	Length of the endplates	0.180	m
h	Water level upstream of the cylinder	0.45	m
h_b	Distance of the cylinder wall from channel bottom	0.18	m
f_s	Sampling frequency	200	Hz
Q	Volumetric flow rate of water	75	l/s
ρ	Density of water	998	kg/m^3
μ	Dynamic viscosity of water	0.001	Pa·s

Table 1: List of parameters of the problem

2.3 Data postprocessing

The complete set of acquisition data is provided in the following MATLAB workspaces (Table 2). In each workspace, the values of F_x and F_y are provided in the form of vectors. These values have already been converted from the voltage output of the cells through the calibration functions, as explained previously.

Filename	**Condition**
`FORCEdata_stillwater.mat`	Cylinder and endplates, still water in the channel
`FORCEdata_flow.mat`	Cylinder and endplates, flowing water in the channel
`FORCEdata_NatOsc.mat`	Cylinder and endplates, still water in the channel, cylinder hit once manually

Table 2: Acquisition data of the experiments

Question 1

Question

Calculate the channel bulk velocity U_b based on the measured quantities upstream of the cylinder Q, B, h. Assume that this velocity is the free stream one approaching the cylinder, U_∞, and then calculate the Reynolds number $Re = \rho D U_\infty / \mu$.

The bulk velocity is easily computed as $U_b = \frac{Q}{Bh} = 0.333\ m/s$. We set it, as indicated, equal to the free stream velocity, U_∞, that approaches the cylinder horizontally. Then, we can calculate the Reynolds number as: $Re = \frac{D \cdot U_\infty \cdot \rho}{\mu} = 1.996 \cdot 10^4$. Thus, as anticipated, this flow totally falls in the subcritical regime [1], that is identified by $300 < Re < 3 \cdot 10^5$.

Question 2

Question

Calculate the buoyancy force F_b acting on the immersed parts (cylinder and part of the endplates).

Following the suggestion in Section 2, the buoyancy force F_b is just given by the mean in time of F_y, vector taken from `FORCEdata_stillwater.mat` workspace, which refers to the vertical external force exerted on the balance in static conditions. We obtain $F_b \simeq 7.3236\ N$.

Question 3

Question

Calculate the Reynolds-averaged ($\langle \cdot \rangle$) drag and lift forces acting on the cylinder, $<F_D>$ and $<F_L>$, and the Reynolds averaged drag and lift coefficients, $<C_D>$ and $<C_L>$. Note that the drag and lift forces acting on the endplates are considered negligible.

To answer this question, we rely on the `FORCEdata_flow.mat` workspace, containing all measurements of horizontal and vertical external forces exerted on the balance with the fluid in motion. The only thing we need to do is to average the vectors[2] (since they contain different values over time) and apply the formula for the two coefficients.

[1] Note that our classification refers to unconfined flows.

[2] Supposing that the total duration of the experiment is enough for the averaged variables to be stable in time.

$$
\begin{aligned}
F_D &= mean(F_x) \simeq 0.9223\ N \\
F_L &= mean(F_y) - F_b \simeq 0.9223\ N \\
C_{D,L} &= \frac{F_{D,L}}{\frac{1}{2}\rho U_\infty^2 A}, \quad with\ A = DL^3 \\
&\Rightarrow C_D \simeq 1.4987 \\
&\Rightarrow C_L \simeq -0.02796
\end{aligned}
$$

Question 4

Question

Calculate the uncertainties of C_D and C_L starting from the definition of these quantities and applying the error propagation law:

$$
y = y(x_1, x_2, \ldots, x_n) \;\Rightarrow\; u(y) = \sqrt{\left(\frac{\partial y}{\partial x_1}u(x_1)\right)^2 + \left(\frac{\partial y}{\partial x_2}u(x_2)\right)^2 + \cdots + \left(\frac{\partial y}{\partial x_n}u(x_n)\right)^2}
$$

Make reference to the values of uncertainty in the following table (3), and try to estimate the missing values.

Variable	Instrument	Uncertainty[4]	Units
B	Ruler	$\boxed{1}$	mm
D	Vernier caliper ($0.05mm$ scale)	$\boxed{0.05}$	mm
L	Ruler	$\boxed{1}$	mm
h	Piezometer	1	mm
Q	Proline Promag 50L (DN200 pipe)	$\boxed{0.05 * Q + 0.001 m/s * A_{pipe}}$	of reading $[m^3/s]$
ρ	–	≈ 0	–
F_x	From calibration of frame	0.045	N
F_y	From calibration of frame	0.025	N

Table 3: Measurement instruments and associated uncertainties

Firstly, we analyze the inserted uncertainties in the table:

- B and L $\rightarrow$ being these quantities measured by means of a ruler, we expect the error to be around $1mm$, since that is the sensibility of a standard ruler;
- D $\rightarrow$ the Vernier caliper utilized for this measure has a $0.05mm$ scale, meaning it can measure up to two decimal places in mm; the typical uncertainty for this instrument is ± 0.05 mm (from [1], [2] and the provided `caliber-accuracy` sheet);
- Q $\rightarrow$ we can say that, for the *Proline Promag 50L (DN200 pipe)* -looking at sites like [3], where one can buy the product[5], but also at the provided instructions manual- the standard accuracy involves an error of $\pm 0.5\%$ of reading $+ \pm 1mm/s$, where the 2^{nd} term is derived from the sensitivity on the velocity measurement, hence, in terms of volumetric flow rate, it must be multiplied by the cross section area of the pipe where the flow properties are assessed. In this case the pipe has a diameter of 200mm, specified by "DN200 pipe".

[3] This is the projected area of only the cylinder, since the endplates are considered negligible.

[4] Items in a $\boxed{\text{box}}$ are the ones inserted a posteriori, the ones requested by the laboratory text.

[5] Not sponsored, it is just an example.

Secondly, we apply the *error propagation formula* to our desired quantities:

$$C_{D,L} = \frac{F_{D,L}}{\frac{1}{2}\rho U_\infty^2 A} = \frac{F_{D,L}}{\frac{1}{2}\rho(\frac{Q}{Bh})^2 DL} = f(F_{D,L}, \rho, Q, B, h, D, L)$$

In MatLab, we compute all the necessary derivatives $\frac{\partial C_{D,L}}{\partial F_{D,L}}, \frac{\partial C_{D,L}}{\partial Q}, \frac{\partial C_{D,L}}{\partial B}, \frac{\partial C_{D,L}}{\partial h}, \frac{\partial C_{D,L}}{\partial D}, \frac{\partial C_{D,L}}{\partial L}$, since we neglect the uncertainty on ρ, and we obtain:

$$u(C_D) \simeq 0.1683$$

$$u(C_L) \simeq 0.07317$$

Question 5

Question

Use the MATLAB function `fft_of_force` provided to compute the FFT (Fast Fourier Transform) of the F_L signal and find out the frequency spectrum. Identify the vortex shedding frequency, f, as the main peak of the frequency spectrum, and calculate the Strouhal number $S_r = \frac{f \cdot D}{U_b}$. Compare it with the reference value from the literature at the same Reynolds number for the unbounded case. As a reference solution, the plot provided in the paper by Fey et al. (1998) might be used, as reported here below in Figure 2.

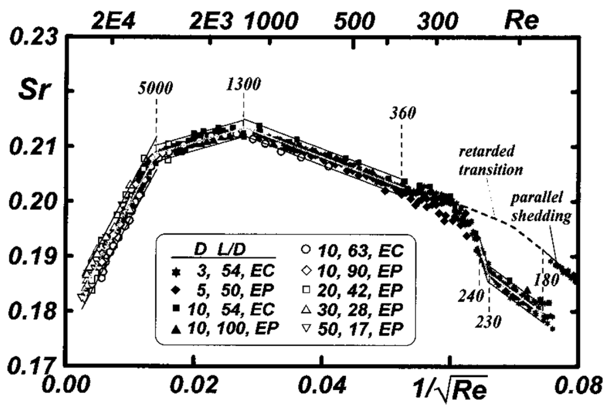


Figure 2: Strouhal number versus Reynolds number for unbounded flow over a circular cylinder, from Fey et al. [4].

Examining the dataset from `FORCEdata_flow.mat`, we find out that the signal peaks at a frequency $f_{vortex} \simeq 1.0080\ Hz$, hence we get $S_r = \frac{f \cdot D}{U_b} \simeq 0.1814$.
To compare our result to the literature for flow in unbounded domain, we need to determine $x = \frac{1}{\sqrt{Re}} \simeq 0.007078$: after that, we can approximately say that the corresponding Strouhal number is about 0.181, which definitely matches our outcome. We could say that, taking this value, the relative percentage distance[6] between the two instances is of circa 0.2406%, but this is not too indicative, because the value extracted from the plot cannot not be very precise, as it's just deduced by eye.

[6]It's computed as $dist(a, b) = abs(a - b)/mean(a, b) * 100$.

Question 6

Question

It is necessary to ensure that the vortex shedding frequency is significantly different from the natural frequencies of the system. Otherwise, the structure could vibrate under the fluid loading, making the assumption of fixed body fail. In order to measure the natural frequencies of the structure in water, the cylinder is hit along the x-direction once using a stick, and left free to vibrate in still water. Based on the results of this test (`FORCEdata_NatOsc`), and using the MATLAB function `fft_of_force`, calculate the natural frequencies of the balance structure in still water and verify that no risk of resonance-induced vibration occurs.

Relying on `FORCEdata_NatOsc.mat`, we can observe that the signal F_x has peaks[7] in the natural frequencies $f_{nat} = (0.0207,\ 0.0271,\ 0.0621,\ 0.0940) = (f_1, f_2, f_3, f_4)$. All of these frequencies seem very distant from f_{vortex}: as a rule of thumb, resonance doesn't occur if

$$|f_{vortex} - f_{nat_i}| > 0.2 \ \forall i = 1, 2, 3, 4$$

We get that $\forall i = 1, 2, 3, 4$, such relative distance is greater than 9, hence we can conclude that the assumption of fixed body is justified.

Question 7 (optional)

Question

Setting the same water level in static and dynamic tests is important for the accurate measurement of the dynamic force. Provide an estimate of the errors in the force coefficients $<C_D>$ and $<C_L>$ due to a different water level in the static and dynamic tests (assume $\Delta h = 2mm$, for example). Consider that the water-section area of each of the two endplates, as observed from the top, is a rectangle with size $180mm \times 2mm$.

We start by considering that a different water level in the experimental configuration would produce a different volume of the immersed objects. Since the cylinder is supposed to be fully under water throughout the whole observation, the volume change would only be due to a different portion of the endplates immersed, that is $\Delta W = \Delta h \times 2 \times 0.002\ m \times 0.180\ m$. Then, this would generate a different buoyancy force F_b, so only the vertical total force F_y would be affected and, consequentially, the lift force F_L and the lift coefficient C_L[8].
For $\Delta h = 0.002\ m$, we can calculate:

$$\Delta F_L = \Delta F_b = \rho g \Delta W \simeq 0.01410\ N$$
$$\Delta C_L = \frac{\Delta F_L}{\frac{1}{2}\rho U_\infty^2 A} \simeq 0.022908$$
$$relative\ error = \frac{\Delta C_L}{C_L} \cdot 100 = 81.93\%$$

By the last computation, we can witness that the water level is a pretty relevant parameter to correctly set the case and achieve accurate results.

[7] Obtained as the points where the Fourier transform of the force reaches 90% of its maximum amplitude.
[8] If the water level change is symmetric, it doesn't introduce horizontal forces or flow changes, which would have an impact on the drag force F_D and the drag coefficient C_D.

References

[1] Physics Stack Exchange. Least count error is random or systematic?

[2] Wikipedia. Vernier scale.

[3] Endress + Hauser. Promag 50L.

[4] Uwe Fey, Michael König, and Helmut Eckelmann. A new strouhal–reynolds-number relationship for the circular cylinder in the range $47 < Re < 2 \times 10^5$. *Physics of Fluids*, 10(7):1547–1549, 07 1998.