Experimental lab, test case 2
Analysis of the velocity field in the wake of a circular cylinder

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General Information

Report for the second test case of the Experimental Laboratories within the course of Fluids Labs, by G. V. Messa, Politecnico di Milano.

1 Introduction

Goal of this laboratory is to analyze the velocity field around a cylinder in a water channel flow, focusing on the wake region. The experiments on which the report is based have been performed in the water channel facility of the Hydraulics Laboratory of Politecnico di Milano.

2 Case description

The case refers to experimental data obtained through the Particle Streak Velocimetry (PSV) technique, which essentially resides on seeding the flow with small particles[1] and shoot a video of the light-sheet illumination plane. The camera has a high exposure time[2], intended as the time duration of a frame, so that each particle leaves a streak on the image (see Figure 1). Being the Reynolds number of the cylinder $Re_D = DU_\infty/\nu$ within the range of the "sub-critical regime", the flow is expected to show separation at a certain distance from the stagnation point, causing a recirculation zone behind of it, and a turbulent oscillating wake is created by the shedding of two counter-rotating vortexes.

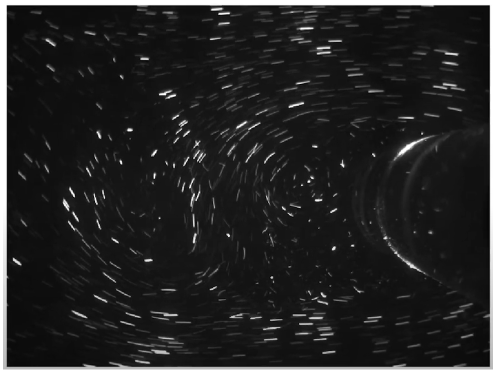


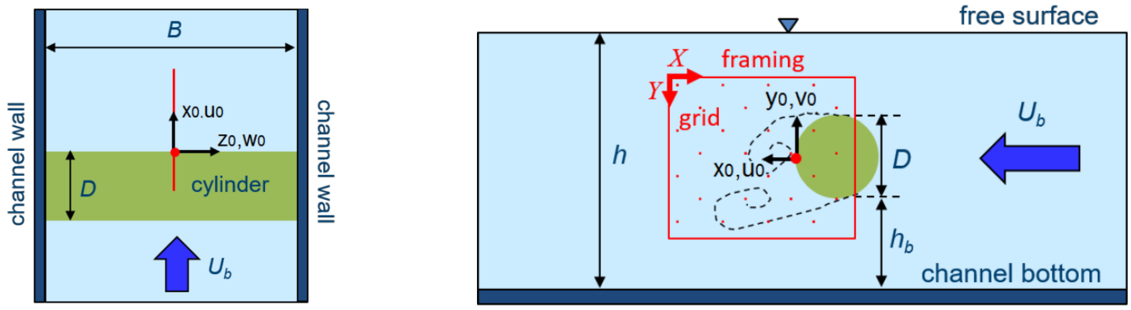
Figure 1: Exemplary image of PSV experiment

2.1 Configuration of the problem

The scheme of the experiment is reported in Figure 2. The flow is from right to left in order to be consistent with the location of the PSV acquisition system in the water channel setup. The framing of the camera, which is the measurement field, is a vertical plane in the mid-section of the channel. The coordinate system of the PSV acquisition is (X,Y), as reported in the figure. Moreover, one can see visual representations of the problem on YouTube, following these links: Video 1 - Overview of the experimental setup, Video 2 - Measurement of water flow around a cylinder using PSV.

[1]Choosing particles with low Stokes number, their motion will be well representative of the fluid flow.

[2]As opposed to PIV technique, where the camera has a short exposure and each frame displays a set of points.



(a) Top view (b) Side view

Figure 2: Sketch of the experiment configuration with coordinates' systems

2.2 Experimental setup

The input data of the problem are summarized in Table 1.

Symbol	Parameter	Value	Units
B	Width of the channel	0.5	m
D	Diameter of the cylinder	0.06	m
f_s	Sampling frequency	50	Hz
h	Water level in the channel upstream of the cylinder	0.42	m
h_b	Distance of the cylinder wall from the channel bottom	0.18	m
Q	Volumetric flow rate of water	35	l/s
res	Resolution of the images	3040	px/m
ρ	Density of water	998	kg/m^3
μ	Dynamic viscosity of water	0.001	Pa·s

Table 1: List of parameters of the problem

2.3 Data postprocessing

For the post-processing of the PSV data, the frame was divided into a regular grid of 30 x 23 uniform cells along the directions X and Y, indicated in Figure 2b. The number of time steps is 1499, and the duration of each of them is equal to 1/fs, being fs the sampling frequency. The complete set of acquisition data is provided in the MATLAB workspace `PSVdata.mat`, which includes the following variables:

Variable	Type	Description
`Grid_Xpx`	23 × 30 matrix	X-coordinates of grid centers in the framing [in px]
`Grid_Ypx`	23 × 30 matrix	Y-coordinates of grid centers in the framing [in px]
`SXpx`	23 × 30 × 1499 matrix	X displacement [in px] for every time step
`SYpx`	23 × 30 × 1499 matrix	Y displacement [in px] for every time step

Table 2: Grid and displacement data used in the analysis

A static snapshot of the PSV frame with a ruler close to the cylinder has been taken before running the experiment, and it is provided as an image file ("ref_image.png"). A small preview of the image file is reported below in Figure 3. The scope of this preliminary step is two-fold. On the one hand, it has been used to estimate the resolution of the image as res = 3040 px/m, which is here already given as input for the sake of simplicity. On the other hand, it is used to turn the "pixel-based" matrices `Grid_Xpx`, `Grid_Ypx`, `SXpx`, and `Sypx` into the corresponding dimensional matrices in the new coordinate system (x0, y0), centered in the rear stagnation point and directed as shown in Figure 2.

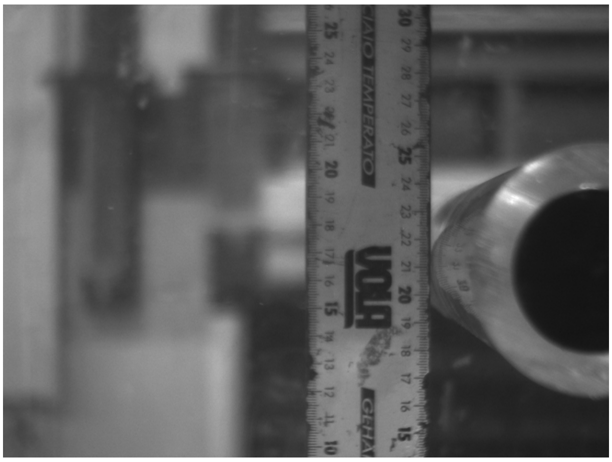


Figure 3: Reference image

Preliminary analyses

Question 1

Question

Calculate the channel bulk velocity U_b based on the measured quantities upstream of the cylinder Q, B, h. Assume that this velocity is the free stream one approaching the cylinder, U_∞, and then calculate the Reynolds number $Re_D = DU_\infty/\nu$. Note that $Ub \neq U\infty$ since the experiments involve a water flow in a finite size channel. Nonetheless, this could be a reasonable first approximation.

Considering the free stream velocity as a good approximation, knowing Q volumetric flow rate, B channel width and h channel thickness, we can easily calculate U_b bulk velocity as follows: $U_b = \frac{Q}{Bh} = 0.3333\ m/s$. Considering the same approximation, we obtain also the corresponding Reynolds number $Re_D = \frac{U_b D}{\nu} = 19960$.

Question 2

Question

Turn the "pixel based" matrices `Grid_Xpx`, `Grid_Ypx`, `SXpx`, and `SYpx` into proper dimensionless matrices. Reference is now made to the coordinate system (x0, y0), which is more physical than that of the PSV frame (X, Y). To this aim, make the following steps:

Point (1)

Find the coordinates of the origin of the new coordinate system (x0, y0), both in pixels and in meters applying the suggested resolution $res = 3040 px/m$. To accomplish this task, use must be made of the image file "ref_image.png" provided, and not to the preview shown previously (Figure 3).

Fristly, we find the (x_0, y_0) coordinates in meters, using the image provided. Indeed, we take a ruler and align it perfectly with the one in the image, so that the centimeters in the image match real centimeters exactly. Then, starting from the origin, we calculate the position of the coordinates (x_0, y_0) in meters. After that, we perform a simple conversion to find the position in pixels. Alternatively, we realized a MATLAB code which allowed us to indicate the point on the cylinder in the image and store its position in pixel. We obtain that:

- $x_0 = 15.32 * 10^{-2}m = 288.80px$.
- $y_0 = 9.5 * 10^{-2}m = 465.73px$.

Point (2)

Start from `Grix_Xpx` and `Grid_Ypx` and build two new matrixes `Grid_x0m`, `Grid_y0m`, which contain the centres of the cells expressed in meters according to the new coordinate system (x0, y0). This requires converting the old "pixel-based" matrices into dimensional matrices (in meters), and then applying the transformation of coordinate system to switch from (X, Y) to (x0, y0) – not necessarily in this order. Finally, build the dimensionless matrixes `Grid_x0_D` and `Grid_y0_D` as obtained by normalizing the grid point coordinates by the diameter of the cylinder, D.

We make an equivalence, a change of coordinates and then make the matrices adimensional:

```
Grid_Xm=Grid_Xpx./res;
Grid_Ym=Grid_Ypx./res;
Grid_X0m=-(Grid_Xm-x0*ones(n, m));
Grid_Y0m=-(Grid_Ym-y0*ones(n, m));
Grid_X0_D=Grid_X0m./D;
Grid_Y0_D=Grid_Y0m./D;
```

Point (3)

Start from `SXpx`, and `SYpx` and build new matrices `Sx0m`, and `Sy0m`, containing the displacements in meters according to the new coordinate system (x_0, y_0). Obtain the matrices of the "instantaneous" velocity field matrices u_0, v_0 by considering that the displacements were measured during the exposition time $1/f_s$. Finally, make the two matrices dimensionless by dividing by the bulk velocity U_b; let us call the final matrices `u0_Ub, v0_Ub`.

As we already did, we make an equivalence to obtain the meters displacement matrices, then we calculate the istantaneous velocity field, for each time steps, by dividing displacements by exposition time, making then them dimensionless.

```
for tt=1:steps
Sx0m(:, :, tt)= (SXpx(:, :, tt)./res);
Sy0m(:, :, tt)= (SYpx(:, :, tt)./res);
end

u0=zeros(n, m, steps);
v0=zeros(n, m, steps);
for tt=1:steps
    u0(:, :, tt)=(Sx0m(:, :, tt))./(1/fs);
    v0(:, :, tt)=(Sy0m(:, :, tt))./(1/fs);
end
u0_Ub=u0./Ub;
v0_Ub=v0./Ub;
```

Analysis of the Reynolds-averaged flow

Question 3

Question

Calculate the Reynolds-averaged velocity field through matrices U0_Ub and V0_Ub. This requires performing the time average of the "instantaneous"[3]velocity matrices previously calculated in question 2.3. It is recommended to use the MATLAB command "nanmean", so that the many "NaN" cells are ignored. Finally, visualize the Reynolds-averaged velocity field by showing the following quantities.

We start by computing the RA-velocity components as:

```
U0_Ub=nanmean(u0_Ub, 3);
V0_Ub=nanmean(v0_Ub, 3);
```

In this way, the many 'NaN's of the data are excluded from the computation of the mean in time (which is the 3^{rd} dimension of the matrices). We expect that the sampling time of the experiment is enough for the time cumulative mean to achieve a stable value, but we do not verify this hypotesis.

Point (1)

The color plot of the Reynolds-averaged velocity magnitude. To this aim, you can use, for instance, the command using the commands "pcolor" with "shading interp".

Being $M = \sqrt{(U_0/U_b)^2 + (V_0/V_b)^2}$, the suggested commands for visualization return Figure 4. Note that we make use also of following advice, plotting the cylinder shape, placing it such that the rear stagnation point corresponds to the point (0, 0) in the grid.
The obstacle generates a substantial decrease in the velocity magnitude, which approaches very small values right after it. Then, high velocity is instead observed up and down of the cylinder. Finally, in the last part of the image, the field seems to get more uniform.
Another thing we could notice, is the asymmetry of the color plot: this can be due to several factors, as the proximity of the object to the tank floor and the fact that there is no constraint on the upper part of the fluid.

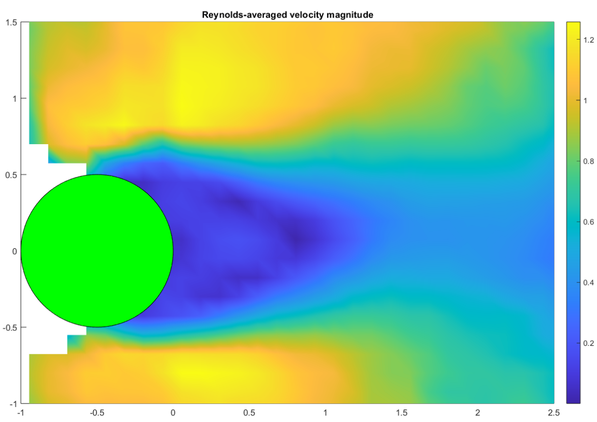


Figure 4: Magnitude of the RA-velocity

[3]Actually, the values are averaged over the sampling time.

Point (2)

The vector map through the MATLAB command "quiver". It is recommended to draw also the cylinder through the MATLAB command "rectangle". Note that spurious vectors will be found also inside of the space occupied by the cylinder. Clearly, these vectors are unphysical and they are the consequence of perspective errors or issues in the PSV algorithm. They should be ignored.

For a nicer rendering, we utilized the function `quiverC2D`, taken from this GitHub repository, which is practically a wrapper of `quiver` function that adds colors to the arrows of the vector map. See Figure 5. In general, what we can observe, also from the plots of `U0_Ub, V0_Ub`, is that the velocity mainly consists of its x-component, while the vertical part is present especially near the cylinder, in the more turbulent region.

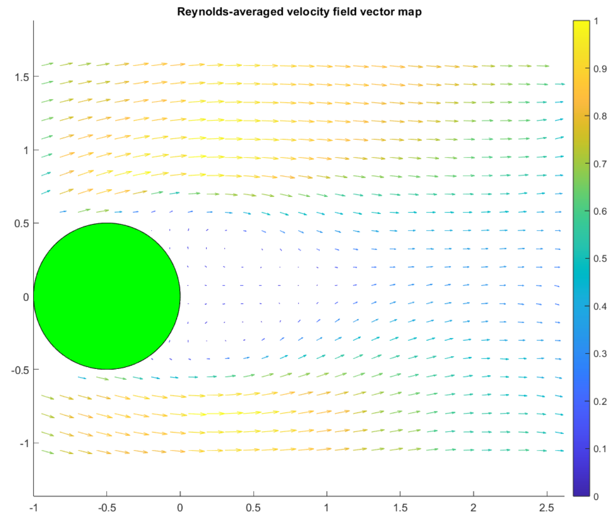


Figure 5: RA velocity vector field

Point (3)

The streamlines with arrows through the MATLAB command "streamslice". Once again, it is recommended to draw also the cylinder through the MATLAB command "rectangle".

The resulting image (Figure 6) perfectly shows two vortexes that spread from the right-most part of the cylinder in the direction of the flow.

Point (4)

The U0/Ub profiles along the horizontal line y0/D = 0 and along the two vertical lines at x0/D = 0.5 and x0/D = 1.5. This might require interpolating the grid values at selected space positions. You might use the MATLAB command "interp2".

Referring to Figure 7, one could say that along the x direction, x-velocity decreases due to the presence of the object, but it grows back later, since mass conservation must hold, while, along the y direction, the velocity, in correspondence of the object, shows a drop, whose deepness reduces getting further downstream (hence as x increases).

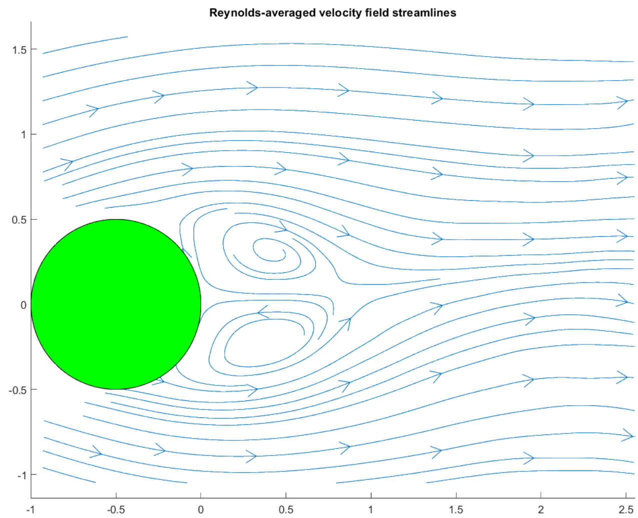


Figure 6: Streamlines of the flow

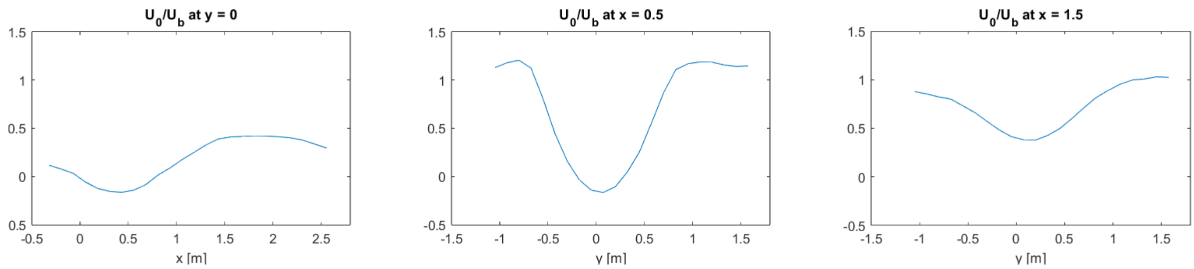


Figure 7: U_0/U_b along chosen lines

Question 4

Question

Calculate the Reynolds-averaged vorticity field applying the “curl” command to both normalized Reynolds-averaged velocities. Show the color plot of this variable (once again, you might use “pcolor” with “shading interp”). For better analysis, overlap the Reynolds-averaged streamlines (“streamslice” or “streamline” commands) and the cylinder.

Since our velocity field consists of only the x and y component, which means it is just a $2D$ vector field, the only non-zero component of the curl would be the z component. Thus, we compute:

```
[vorticity, ~] = curl(Grid_X0_D, Grid_Y0_D, U0_Ub, V0_Ub);
```

And obtain a scalar field, that we can plot with the same commands as before.

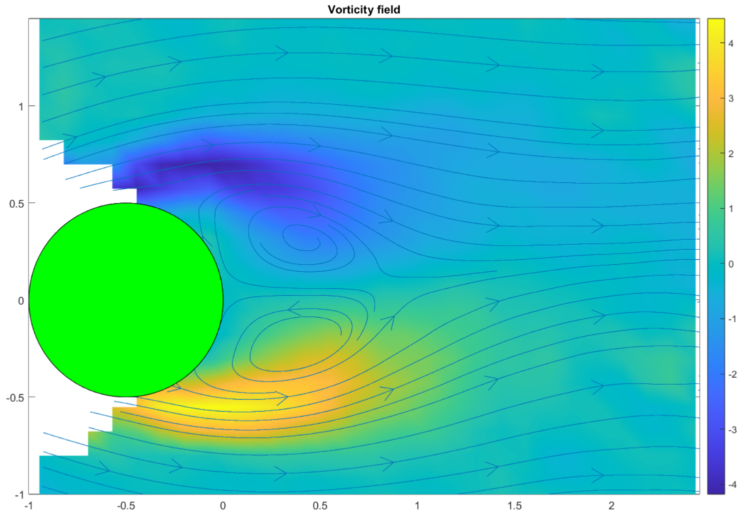


Figure 8: RA vorticity field

Analysis of the dynamic evolution of the flow

Question 5

Question

Investigate the dynamic evolution of the flow, as follows.

Point (1)

Plot the time history of the normalized vertical velocity v0/Ub in a point around the rear of the cylinder.

To choose a point near the rear stagnation point of the cylinder, we find the cells where $0 < x < \varepsilon$ and $|y| < \varepsilon$. Imposing $\varepsilon = 0.15$, a single cell satisfies the criterion, and that is in the position [22, 15]. In that point, the graph of the normalized vertical velocity over time is shown in Figure 9. We obviously obtain a prominent oscillatory movement, because of the fact that in this region vortexes periodically swirl in opposite directions, detaching from the cylinder.

Point (2)

Use the MATLAB function `fft_of_v0_Ub_velocity` provided to compute the FFT (Fast Fourier Transfom) of the v0/Ub velocity signal and find out the frequency spectrum. Identify the vortex shedding frequency, f_{vortex}, as the main peak of the frequency spectrum.

We identify the vortex frequency, f_{vortex}, as the point of maximum of the Fast Fourier Transform of the signal: the graph in Figure 10 clearly exhibits the peak of the signal and we obtain $f_{vortex} = 0.5674$.

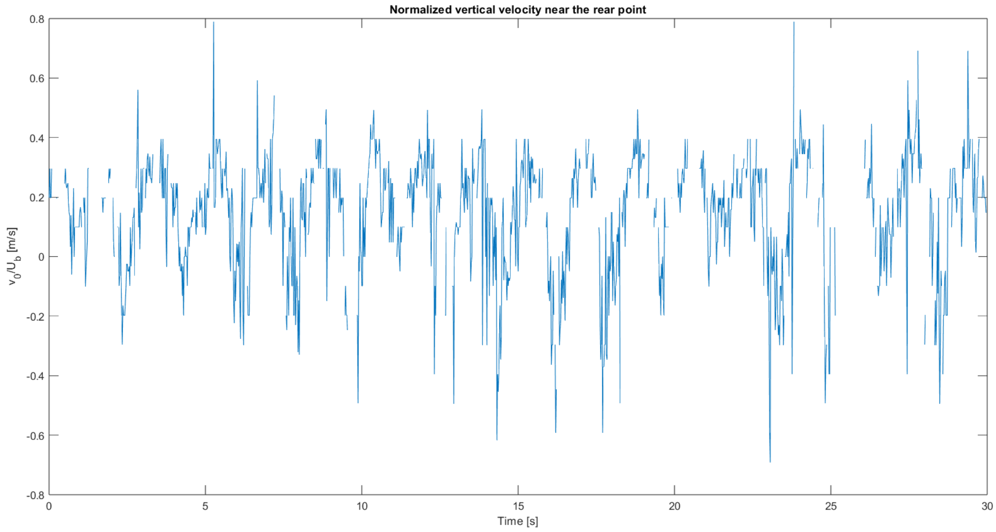


Figure 9: v_0/U_b near the rear point

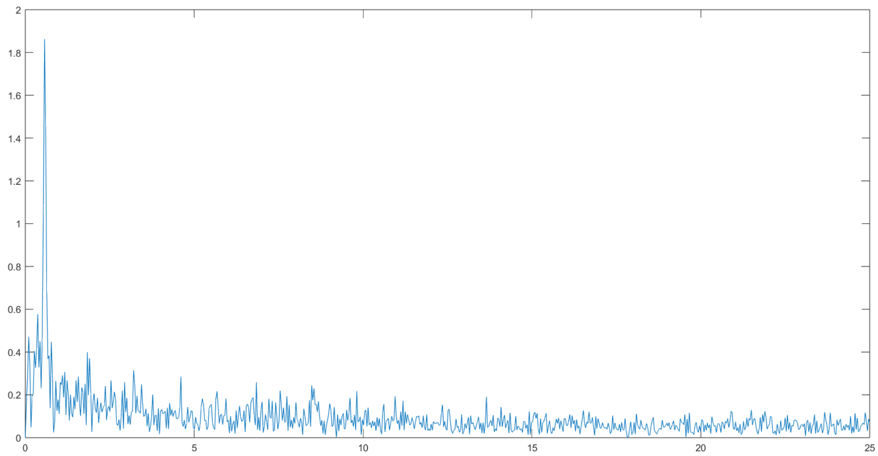


Figure 10: FFT of the vertical velocity near the rear point w.r.t. frequencies

Point (3)

Estimate the characteristic Strouhal number of the oscillating wake, as $Sr = f_{vortex} \cdot D/U_b$, and compare with the reference value from the literature at the same Reynolds number for the unbounded case. As a reference solution, the plot provided in the paper by Fey et al. (1998) might be used, as reported in Figure 11.

Evaluating the formula $Sr = f_{vortex} \cdot D/U_b$, we find that $Sr \simeq 0.2043$.
Being $\frac{1}{\sqrt{Re}} \simeq 0.0100$, the reference plot in Figure 11 would give us approximately $Sr \simeq 0.196$, which is comparable to our result.

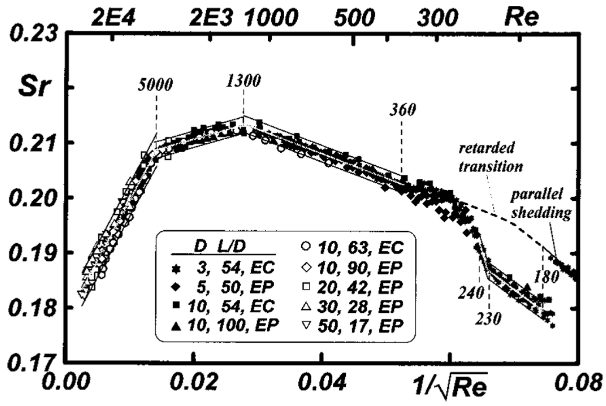


Figure 11: Strouhal number versus Reynolds number for unbounded flow over a circular cylinder, from Fey et al. [1].

Point (4)

Investigate the sensitivity of the estimated Strouhal number with respect to the initial choice of the monitoring point. Overlap the frequency spectra for all the points in the domain and discuss the resulting picture (note that for some points the `fft_of_v0_Ub_velocity` function will return an error due to the excessive number of 'NaN' values – just exclude those points). Analyze the colour plot distribution of the vortex shedding frequency.

Over the $23 \times 30 = 690$ points of the grid, only 600 of them (86.96%) allowed us to apply the `fft_of_v0_Ub_velocity` function. The resulting graph (Figure 12), showing the frequency spectra, is a quite messy one, due to the high number of lines overlapping.

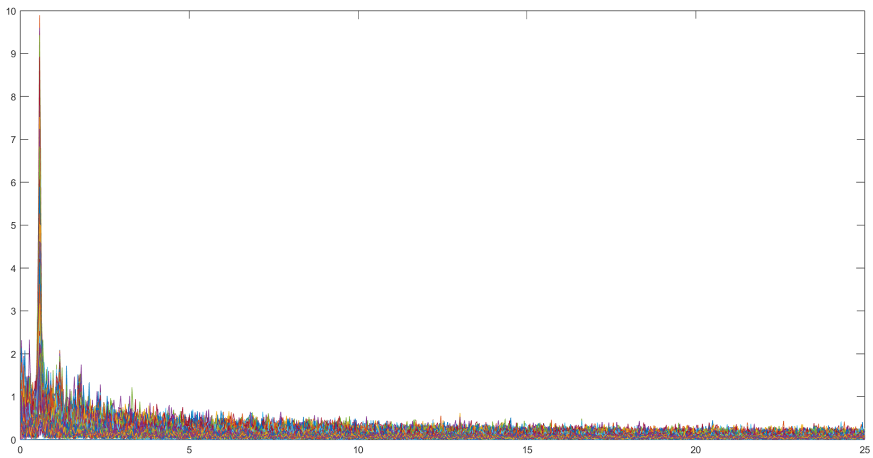


Figure 12: FFT of the vertical velocity for all of the grid points

Anyway, one thing seems to be evident: the only appearing peak is at the same point of f_{vortex}, no other one is particularly noticeable. Investigating the values, we see that 387 have 0 as peak frequency, thus are not interested by oscillatory behaviour. Between the remaining 213, there are only 3 points (meaning 0.94%) whose peak frequence has a relative distance from f_{vortex} that is greater than 0.1.

As for the color plot of the vortex shedding frequencies (Figure 13), the plot doesn't look so meaningful, but still we can confirm that values are either zero or another number, which, from our analysis, is very close to our first computed f_{vortex}.

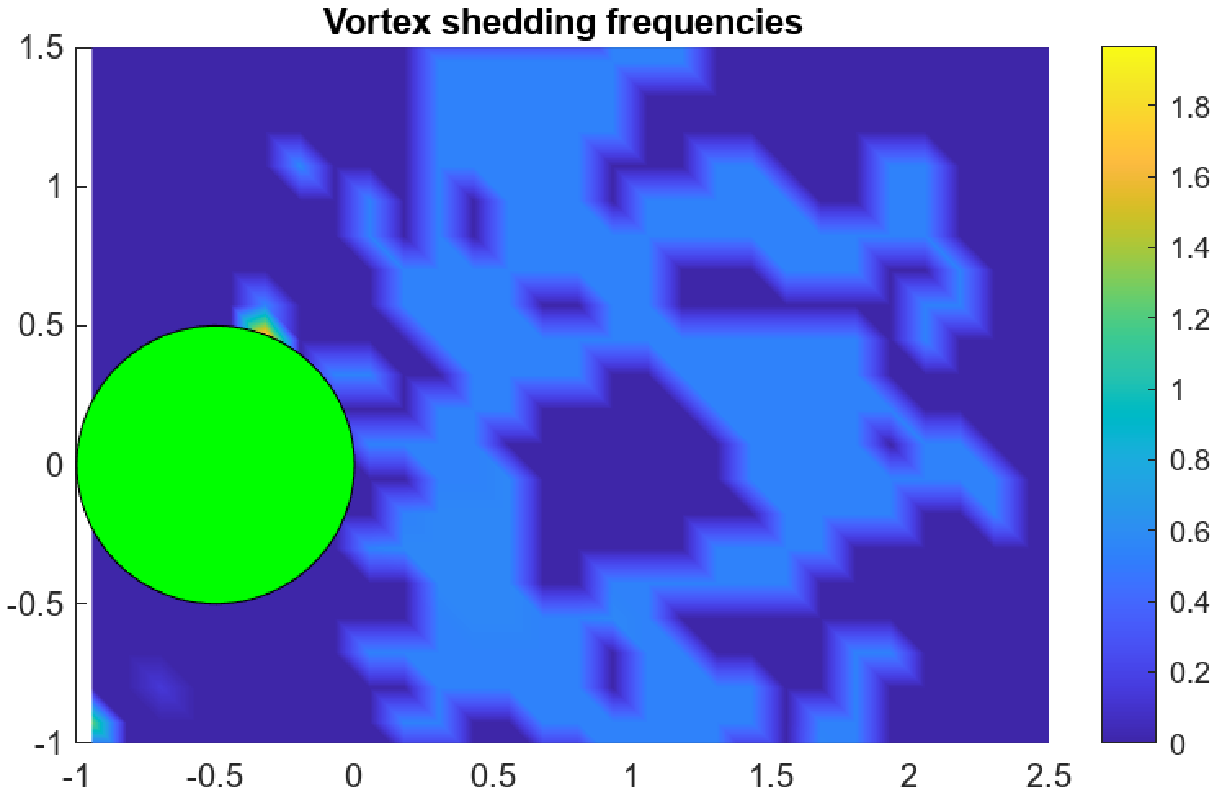


Figure 13: Caption

Point (5)

Apply a temporal moving average to the instantaneous velocity field (u0_Ub, v0_Ub), considering an averaging window of about $0.2 - 0.25T$, being $T = 1/f_{vortex}$ the vortex shedding period. It is suggested to make use of the MATLAB command "movmean" with the option "omitnan" to exclude 'NaN' values from the calculation. Analyze the time evolution of the velocity magnitude field, the velocity vector field, the streamslices, and the vorticity field at different time instants during a period.

We compute the number of time steps to consider as neighbour (taken as input for the average in each point) as $k = \lceil 0.225 \times \frac{f_{sample}}{f_{vortex}} \rceil = 20$ and calculate the moving average across the third dimension of the vectors:
`u0_Ub, v0_Ub, mag = sqrt(u0_Ub.^2 + v0_Ub.^2), vorticity`
where the latter is computed as:

```
for t = 1:length(times)
    [vorticity(:,:,t),~] = curl(Grid_X0_D,Grid_Y0_D,...
                           u0_Ub(:,:,t),v0_Ub(:,:,t));
end
```

After that, to get a broad view of the data, we plot the color plots and the other requested graphs for each of the sample time steps. The computational time is rather high, but we develop significant videos, showing with considerable accuracy (and without being too chaotic[4])

[4] As it happens instead when displaying the raw quantities, with no moving average over time.

the behaviour of the flow and, in particular, the shedding of the two aforementioned vortexes in the wake.

References

[1] Uwe Fey, Michael König, and Helmut Eckelmann. A new strouhal–reynolds-number relationship for the circular cylinder in the range $47 < Re < 2 \times 10^5$. *Physics of Fluids*, 10(7):1547–1549, 07 1998.