POLITECNICO
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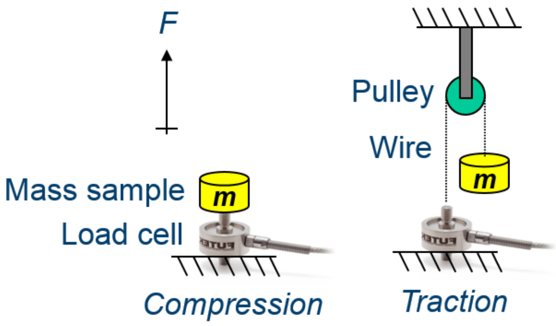
Course of "Fluid Labs" A.A. 2024-2025

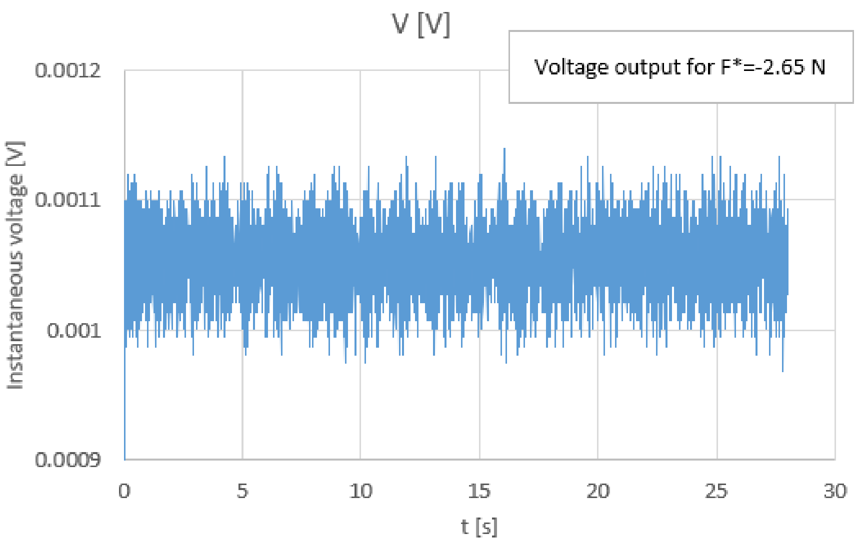
EXP Test Case 1
Calibration of a load cell

Case description and objectives

The load cell is an instrument for measuring a force, which produces an electrical output (voltage) which is approximately proportional to the applied force. In order to estimate the value of a force with a load cell, it is necessary to determine the transfer function from voltage to Newton *V* [volt]→*F* [Newton]. The transfer function is obtained by using mass samples whose weight force F^* is accurately known (see sketch here below). This operation can sometimes be called calibration of the load cell. As a matter of facts, the values of *V* have a random variability, which makes the procedure for determining the transfer function analogous to the one for determining the calibration curve; this justify why the name calibration is used also in this context.



Each mass sample is applied to the load cell for a few seconds and the correspondent electrical output is recorded (the sampling frequency is f_s=100 Hz). This produces a noisy signal like the one shown here below by way of example.



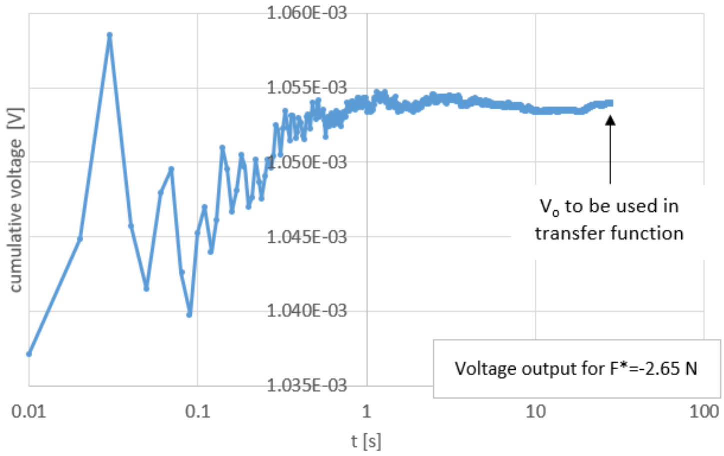
The voltage output histories and the corresponding values of the reference force, F^*, are listed in the table and provided in the form of MATLAB workspaces. Each workspace contains the vector of the readings in Volt (remember that the sampling frequency is f_s=100 Hz). In the table, the sign of F^* is as presented in the sketch, namely, F^*>0 when the force is directed upwards, and F^*<0 when it is directed downwards.

TestID	*F** [N]	TestID	*F** [N]
calibr01	0.000	calibr09	-5.598
calibr02	-0.314	calibr10	-10.501
calibr03	-0.411	calibr11	0.126
calibr04	-0.695	calibr12	1.097
calibr05	-0.891	calibr13	2.078
calibr06	-1.186	calibr14	4.039
calibr07	-1.676	calibr15	8.942
calibr08	-2.656	calibr16	13.845

Questions

Determine the transfer function for the load cell, and estimate the uncertainty of the instrument.

1. The noisy behaviour of the voltage signal is due to the variability of the electric output. The electric noise must be filtered out before finding the values of V_o which are to be correlated with F^* through the transfer function. This is achieved by calculating the cumulative average plots of the instantaneous voltage output. Such plots show, for each t_i, the average voltage output from 0 to t_i, with the time on the horizontal axis in log-scale, and they look as in the example below.



By visual inspection of the curves, it can be assessed whether the time of acquisition of the signal is long enough to provide a "reasonably stable" cumulative average voltage. If this is the case, the value at the end of the curve, V, will be the voltage V_o associated with F^* in the next calculations (alternatively, one could take an average of the values in the "stable" region). The operation must be repeated for all tests. A gross estimate of the time of acquisition required to filter out the electric noise might be also obtained.

2. Note that the values of V_o are, in themselves, random. In fact, repetition of the same test produces different values of V_o. A formula can be derived, which estimates the limiting mean of the V_o values, say μ_{Vo}, as a function of the "reference" force value, F^*. This can be obtained by linear regression of the (F^*-V_o) pairs. The formula will have the following form:

$$\mu_{Vo} = mF^* + b$$

Find out the values of $\boldsymbol{m}$ and $\boldsymbol{b}$.

3. However, the practical interest is in a formula which, given a voltage reading V_o, provides an estimate of the corresponding "true" value $\boldsymbol{F}$. This formula is called (linear) transfer function, and it can be obtained by rearranging the previous equation, as follows:

$$\widetilde{\boldsymbol{F}} = \frac{\boldsymbol{V_o} - \boldsymbol{b}}{\boldsymbol{m}}$$

The tilde over the $\widetilde{\boldsymbol{F}}$ indicates that, for a given $\boldsymbol{V_o}$, the formula here above does not produce the real value of $\boldsymbol{F}$, but just an estimate. This is due to the random nature of the measurement process (note that, in principle, the same $\boldsymbol{V_o}$ could correspond to every possible $\boldsymbol{F}$). We now would like to provide an estimate of the uncertainty associated with the estimate $\widetilde{\boldsymbol{F}}$, that is, we would like to find $\boldsymbol{U}$ such that, for every $\boldsymbol{V_o}$, the corresponding force lies within the range $\widetilde{\boldsymbol{F}} \pm \boldsymbol{U}$.

3.1. A first approach is to set $\boldsymbol{U}$ as the maximum absolute difference between $\widetilde{\boldsymbol{F}}$ and $\boldsymbol{F}^*$ for all the 16 calibration cases. What would be the value of $\boldsymbol{U}$ in this case?

3.2. A second approach is to refer to the confidence intervals. For a perfect Gaussian PDF, 95% of the readings falls in the range $\boldsymbol{\mu} \pm \boldsymbol{2\sigma}$. According to the theory of linear regression, we can estimate $\boldsymbol{U}$ at a 95% confidence as

$$2\bar{s}_{\widetilde{F}} \quad \textbf{where} \quad \bar{s}_{\widetilde{F}}^2 = \frac{1}{N-2} \sum_{\substack{\text{calibration} \\ \text{data}}} \left(\frac{\boldsymbol{V_o} - \boldsymbol{b}}{\boldsymbol{m}} - \boldsymbol{F}^* \right)^2$$

What would be the value of $\boldsymbol{U}$ in this case? What could you infer by comparing the values of $\boldsymbol{U}$ obtained with the two methods? [Note that, in principle, the formula here above is reasonable only for very large populations, which cannot be the case here. For moderate populations, a better method is available, which relies on the *t*-distribution. Such method will not be subject of discussion in the Fluid Labs course.]

Optional question for further individual study

Could you estimate the resolution of the load cell? Remember that the resolution of an instrument is the smallest change that can be measured. The resolution makes the possible voltage outputs of the load cell discrete, with a "step-wise" fashion. What is the resolution of the load cell in terms of voltage? And in terms of force?