Tractable inference for latent ability in competitive games: Approximating Bayesian rank-ordered models using a simpler proportion model

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1 Introduction

In research but also in more applied contexts it may be the case that the only available data are given by rankings, where items or people are ordered upon a specific criterion. Such data are present for example in sports, in consumer studies or in election studies. The basic idea is that the position in the ranking of each item is derived from properties of that item that are not directly observable, and the goal of the research is to make inference about the relative position of the items on this latent scale. The frame of reference for my project is competitive games, but the methods and analysis can also be extended to other contexts and fields. Example of data could be given by the ranks of skiers in different competitions, with the aim of inferring the relative ability of skiers. There are several known probability models for ranking data in the literature. The oldest to have been developed belongs to the class called "order statistics models" whose pioneer was Thurstone (1923), who assumed a normal distribution for the unobserved utilities of the items. Later models assumed other types of distributions. The most famous is that of Luce (1959) and Plackett (1975), whereby utilities follow a Gumbel (type 1 extreme value) distribution. This model has become particularly used in numerous extensions and applications since it leads to a closed form for the probability of each possible ranking of the objects. Rank-Ordered Logit Models (ROL) extend the Plackett-Luce model by adding covariates to it, either item-specific ones, judge-specific or specific on their combination and interactions, making it suitable also for prediction. These models can also be used to handle cases of partial or subset of rankings where the order of only a subset of items is known and those cases where ties exist. All the models mentioned above respect the Indipendence of Irrelevant Alternatives (IIA) axiom, postulated by Trversky (1972), and therefore models have also been developed to accommodate those situations where this assumption is not plausible. Although the validity of the rank-ordered logit models model has been widely acknowledged, their execution can be complex in the practice for two reasons: the first one concerns the implentation, which can be computationally intensive in those cases where the number of items to be ranked is very large (Alvo et Philip, L. H. (2014). report 15 items as the threshold in this sense); the second point concerns the interpretability of parameters in this model which is not straightforward, especially in those cases where a lot of variables are inserted in the model as covariates. ((should add something about how variables are interpreted) For this reason, a new model will be tested in this project in order to produce reliable inference for the latent ability parameters of players in competitive games. This new approach is based on the transformation of the outcome variable of interest: from the

rank of players into a proportion of competitors beaten for each player . In this way a proportion model can be used , which represents a simpler method to produce the wanted inference. In this approach, the canonic assumption of Independent and identically distributed random variables would imply that each proportion is independent from the others within the same game. It's difficult to imagine that this condition can be realistic in those cases where there are few players in the game but the assumption could become less relevant for increasing number of players. My project will try to evaluate this and therefore answer the following research question: when is a proportion model a reasonable approximation for a complex rank-ordered model?

2 Analytic strategy

In order to answer the research question, a simulation study will be conducted to compare the regression model with the rank-ordered logit one.The analysis will be conducted in a Bayesian framework, using cmvstan program which allows for a great flexibility in the choice of the prior distribution for the parameters. Data will be simulated generating values from a rank-ordered logit model with known abiity parameters and the beta regression model will then be fitted to the data. Performance of this model will be evaluated by looking at the precision of the estimates and their credible intervals. Conditions that will be changed and tested concern the number of players to be ranked, the entities and varia, the possibility to have ties and partial rankings. If circumstances permit an existing dataset will be used to try to make predictions R softwere version 4.2.1 will be used to conduct the analysis (130)