Factorization of Matrices with Grades

Martin Hrabal

Abstract

We present an approach to decomposition and factor analysis of matrices with ordinal data.[**1**]

Summary

$$\mu_A(x) \Rightarrow \mu_B(y) = \begin{cases} 1 & \text{, když } \mu_A(x) \leq \mu_B(y) \\ \mu_B(y) & \text{, když } \mu_A(x) > \mu_B(y) \end{cases}$$

In this paper Radim Bělohlávek and Vilém Vychodil propose an algorithm to decompose a matrix with graded values which show how strongly objects fulfill certain criteria. Unlike traditional methods designed for binary matrices, this approach operates on matrices with graded values taken from an ordered set of values (e.g., $0, 0.25, 0.5, 0.75, 1$) structured to support specific aggregation operations.

Problem

The central goal is to factor a matrix I, whose entries are elements of a bounded scale L, into two smaller matrices A and B such that:

$$I = A \circ B$$

Here, the symbol $\circ$ denotes a generalized matrix product defined over L using a t-norm, such as the minimum or product. Matrix A has dimensions $m \times k$ and B is $k \times n$, with k representing the number of latent factors. The aim is to minimize k while making I reconstructible (at least approximately) from A and B.

$R =$

	1	0.75	0.5	0.25	0
0	1	1	1	1	1
0.25	1	1	1	1	0
0.5	1	1	1	0.25	0
0.75	1	1	0.5	0.25	0
1	1	0.75	0.5	0.25	0

References

1. Belohlavek, R., & Vychodil, V. (2015). Factorization of matrices with grades. *Fuzzy Sets and Systems*, 292, 85–97. https://doi.org/10.1016/j.fss.2015.03.020