

# Compulsory Exercise 2: Age Prediction for Heart Failure

Karianne Strand Bergem

Marte Ragnhild Hotvedt

Erlend Henriksen Winje

04 April, 2025

## Abstract

This is the place for your abstract (max 350 words)

## Introduction: Scope and purpose of your project

Problemstillingen vår kan være om vi kan predikere alder basert på de variablene i Heart Failure-datasettet? Er det vi skal finne ut av liksom

## Descriptive data analysis/statistics

```
data <- read.csv("heart.csv")
```

To better understand the variables and their relationships, we perform descriptive data analysis on the heart failure dataset. Since our goal is to predict a person's age, we focus on identifying variables that might be relevant predictors.

## Summary statistics

```
# Mean, SD, Min, Max etc. across numeric variables

# Summary statistics for numeric variables
summary_stats <- data %>%
  summarise(across(where(is.numeric), list(
    Mean = ~mean(.),
    Median = ~median(.),
    SD = ~sd(.),
    Variance = ~var(.),
    Min = ~min(.),
    Max = ~max(.)
  )), .names = "{.col}_{.fn}")

# Table format
summary_stats_long <- summary_stats %>%
  pivot_longer(cols = everything(),
    names_to = c("Variable", "Statistic"),
    names_sep = "_" ) %>%
  pivot_wider(names_from = Statistic, values_from = value)

kable(summary_stats_long, caption = "Summary statistics for the numeric variables")
```

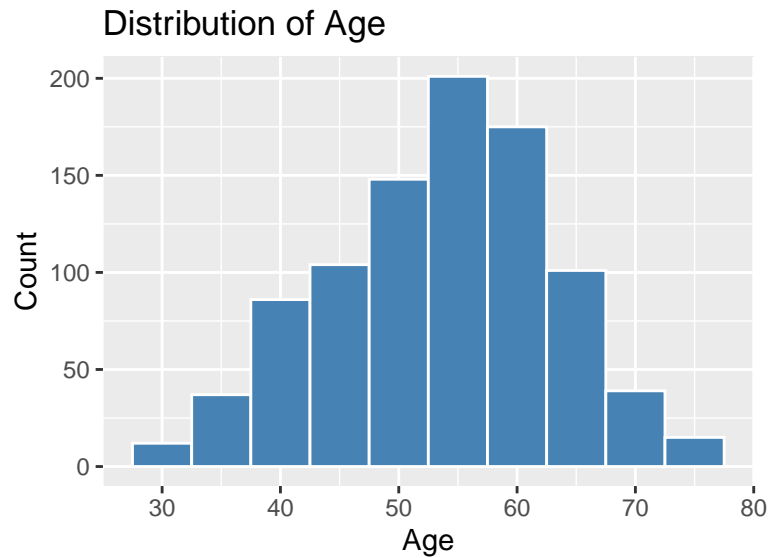
Table 1: Summary statistics for the numeric variables

Variable	Mean	Median	SD	Variance	Min	Max
Age	53.5108932	54.0	9.4326165	8.897425e+01	28.0	77.0
RestingBP	132.3965142	130.0	18.5141541	3.427739e+02	0.0	200.0
Cholesterol	198.7995643	223.0	109.3841446	1.196489e+04	0.0	603.0
FastingBS	0.2331155	0.0	0.4230456	1.789676e-01	0.0	1.0
MaxHR	136.8093682	138.0	25.4603341	6.482286e+02	60.0	202.0
Oldpeak	0.8873638	0.6	1.0665702	1.137572e+00	-2.6	6.2
HeartDisease	0.5533769	1.0	0.4974137	2.474204e-01	0.0	1.0

The table above presents key statistics for all numeric variables. We observe, for example, that Age ranges from 28 to 77, and MaxHR (maximum heart rate) has a relatively high standard deviation, indicating strong variability between individuals. These values give an initial sense of scale, spread, and potential outliers in the data, which are important to consider before modeling.

## Age distribution

```
ggplot(data, aes(x = Age)) +
  geom_histogram(binwidth = 5, fill = "steelblue", color = "white") +
  labs(title = "Distribution of Age", x = "Age", y = "Count")
```



The distribution of Age is concentrated in the range 45–65, with a peak around 55 years. The distribution is roughly symmetric, with a few younger and older outliers. Since age is our target variable, this plot helps understand the range and whether any skew might influence model performance.

## Correlation matrix

```
numeric_vars <- data[sapply(data, is.numeric)]

cor_matrix <- round(cor(numeric_vars), 4)

kable(cor_matrix, caption = "Correlation matrix of numeric variables")
```

Table 2: Correlation matrix of numeric variables

	Age	RestingBP	Cholesterol	FastingBS	MaxHR	Oldpeak	HeartDisease
Age	1.0000	0.2544	-0.0953	0.1980	-0.3820	0.2586	0.2820
RestingBP	0.2544	1.0000	0.1009	0.0702	-0.1121	0.1648	0.1076
Cholesterol	-0.0953	0.1009	1.0000	-0.2610	0.2358	0.0501	-0.2327
FastingBS	0.1980	0.0702	-0.2610	1.0000	-0.1314	0.0527	0.2673
MaxHR	-0.3820	-0.1121	0.2358	-0.1314	1.0000	-0.1607	-0.4004
Oldpeak	0.2586	0.1648	0.0501	0.0527	-0.1607	1.0000	0.4040
HeartDisease	0.2820	0.1076	-0.2327	0.2673	-0.4004	0.4040	1.0000

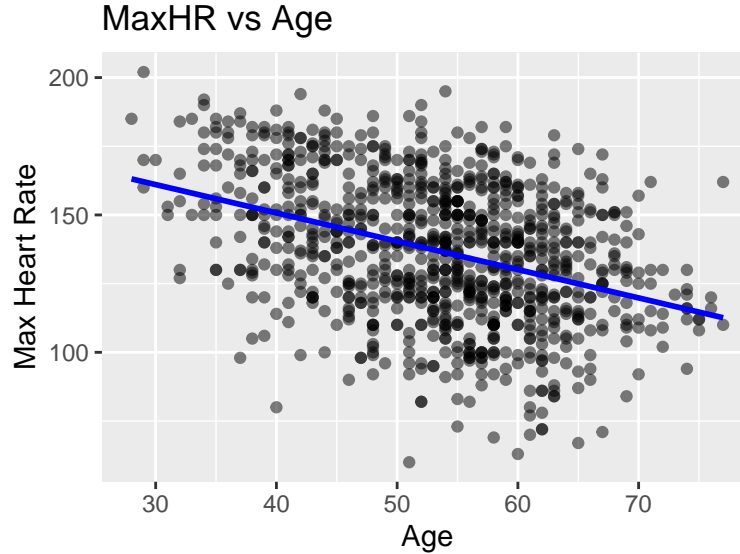
The correlation matrix above highlights the relationships between numeric variables. We see a moderate negative correlation between Age and MaxHR (-0.38), and a mild positive correlation between Age and Oldpeak (0.26). Other variables, such as Cholesterol and RestingBP, appear weakly correlated with age. This suggests that MaxHR and Oldpeak could be useful predictors. The correlation between Age and HeartDisease is also not negligible, and it may be a useful predictor.

### Relationship between numeric variables and age

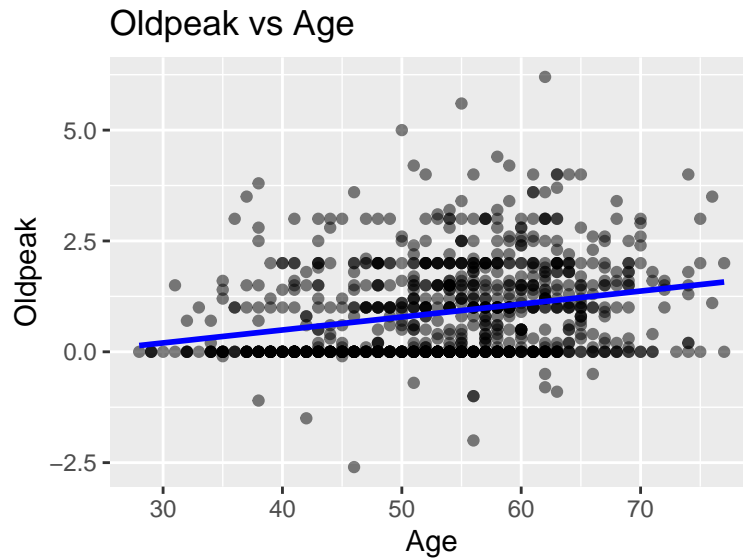
Based on the correlation matrix, MaxHR and Oldpeak were the numeric variables most strongly related to age. While HeartDisease also appears as a numeric variable, it is a binary indicator and better treated as a categorical variable. Therefore, it was not included among the continuous variables explored through scatter plots, but rather in the analysis of the categorical variables.

*# Numeric variables, scatter plots*

```
ggplot(data, aes(x = Age, y = MaxHR)) + geom_point(alpha = 0.5) + geom_smooth(method = "lm", se = FALSE)
labs(title = "MaxHR vs Age", x = "Age", y = "Max Heart Rate")
```



```
ggplot(data, aes(x = Age, y = Oldpeak)) + geom_point(alpha = 0.5) + geom_smooth(method = "lm", se = FALSE)
labs(title = "Oldpeak vs Age", x = "Age", y = "Oldpeak")
```



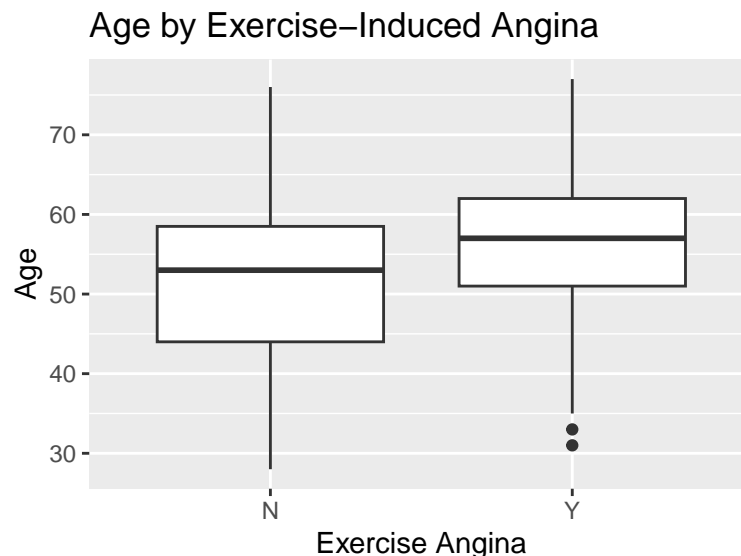
The scatter plot of MaxHR against Age reveals a clear negative linear trend: as age increases, maximum heart rate tends to decrease. This is physiologically expected and supports using MaxHR as a predictor. Oldpeak shows a slight upward trend with age, indicating a mild positive association. Both variables are therefore potentially useful in predicting age.

### Relationship between categorical variables and age

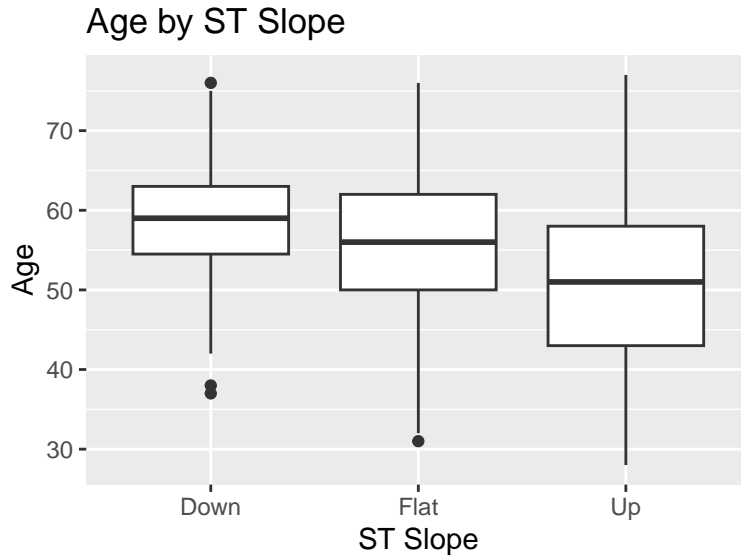
To assess the relationship between categorical variables and age, we plotted boxplots for each one. ExerciseAngina and ST\_Slope showed the most distinct group differences in age and were therefore chosen as the most relevant to include. HeartDisease is also included because of the results found in the correlation matrix.

*# Categorical variables, box plots*

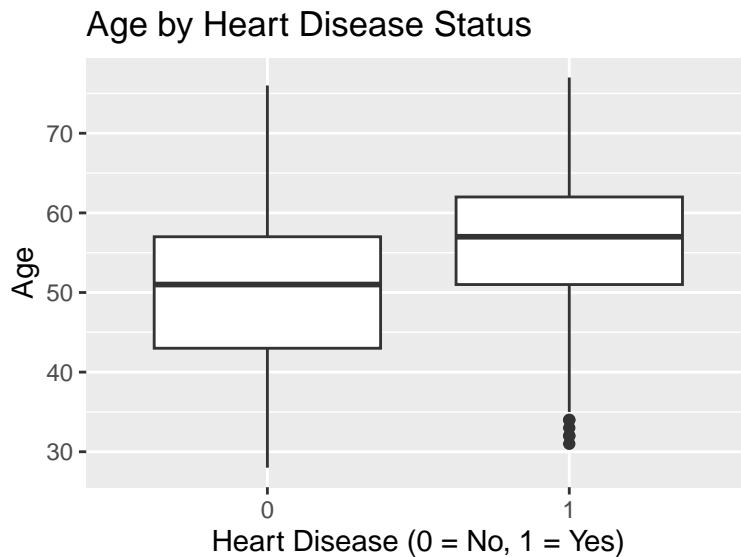
```
ggplot(data, aes(x = ExerciseAngina, y = Age)) + geom_boxplot() + labs(title = "Age by Exercise-Induced Angina")
```



```
ggplot(data, aes(x = ST_Slope, y = Age)) + geom_boxplot() + labs(title = "Age by ST Slope", x = "ST Slope")
```



```
ggplot(data, aes(x = factor(HeartDisease), y = Age)) +
  geom_boxplot() +
  labs(title = "Age by Heart Disease Status", x = "Heart Disease (0 = No, 1 = Yes)", y = "Age")
```



In the boxplot for ExerciseAngina, we observe that patients with angina induced by exercise (Y) tend to be older than those without (N). Similarly, the boxplot for ST\_Slope shows that patients with a “Down” slope tend to be older than those with a “Flat” or “Up” slope. These differences suggest that both variables are potentially informative for modeling age and may capture patterns related to heart diseases. The boxplot for HeartDisease shows that patients with heart disease tend to be older on average than those without. This supports the idea that age is associated with heart disease presence.

## Methods

We used the “Heart Failure Clinical Records” dataset and selected **Age** as the response variable. All other variables were used as predictors. Before we can apply the methods to the dataset, we need to split the data into training and test sets. Splitting the data ensures that we can train the model on one subset and evaluate its performance on unseen data, helping us assess how well the model generalizes. We randomly assign 70% of the data to the training set and the remaining 30% to the test set.

```

n <- nrow(data) # Number of observations

# Indexes for the training set (70% of the data)
train_idx <- sample(1:n, size = round(0.7 * n), replace = FALSE)

# Split the data
train_data <- data[train_idx, ]
test_data <- data[-train_idx, ]

```

## Multiple Linear Regression

In this section, we will consider Multiple Linear Regression (MLR) as a method to predict age for the heart failure data set. This choice of algorithm was due to the fact that MLR is easy to interpret. Another strength of using this method is its ability to extend to bivariate and categorical covariates. This suits the covariates we are evaluating, such as **Sex** and **ChestPainType**. With MLR we can also consider interaction and non-linear terms. However, a weakness of MLR is occurrence of collinearity when two or more predictors are correlated. This results in a reduced accuracy of the estimated regression.

### Model Assumptions

First, we will consider the training data and assume that

$$\mathbf{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon,$$

where  $\mathbf{Y}$  is a  $(n \times 1)$ -vector of responses (**Age**),  $X_j$  is the  $j^{\text{th}}$  predictor, and  $\beta_j$  is the respective coefficient. From the training data set, we have  $n = 643$  patients, that is, 643 sampling units  $(x_{1i}, \dots, x_{pi}, y_i), 1 \leq i \leq n$ , with  $p = 15$  attributes for each patient.  $\epsilon$  is a  $(n \times 1)$ -vector of random errors. We assume that  $E(\epsilon_i) = 0$ , that all  $\epsilon_i$  have the same variance, and that  $\epsilon_i$  are independent of each other and normally distributed.

To solve the prediction of age by using MLR, we used the `lm()`-function to estimate the regression coefficients  $\beta$ .

```
mlr_mod <- lm(Age ~ ., data = train_data)
```

We checked that the model assumptions of a MLR actually hold. To check this we used the model checking tools Tukey-Anscombe diagram and the QQ-diagram.

### Evaluation of Multiple Linear Regression Performance

To get an idea of which the predictors  $X_1, \dots, X_n$  are useful in predicting the response, we checked if we could as well omit all predictor variables at the same time. We formulated the test as

$$\begin{aligned}
 H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0 \\
 \text{vs.} \\
 H_1 : \text{at least one } \beta_j \text{ is non-zero.}
 \end{aligned}$$

We used  $F$ -statistics to do this test,

$$F = \frac{(\text{TSS} - \text{RSS})/p}{\text{RSS}/(n - p - 1)} \sim F_{p, (n-p-1)},$$

where TSS is the total sum of squares and RSS is the residual sum of squares. Under the assumptions we have made,  $F$  follows an  $F_{p,(n-p-1)}$ -distribution. The `lm()`-function calculates the  $F$ -statistics. If  $H_0$  is true, the  $F$ -statistics is expected to be  $\approx 1$ .

To do the model selection between MLR and Ridge regression, we evaluate the model performance by Mean Squared Error (MSE) and  $R^2$  (coefficient of determination). We use the test set to do this in a valid way. MSE measures the average of the squared differences between the actual and predicted values. It is calculated as:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

where  $y_i$  is the actual value and  $\hat{y}_i$  is the predicted value. A low MSE indicates that the model's predictions are close to the actual values, making it a useful measure of model accuracy.  $R^2$  measures the proportion of the variance in the dependent variable that is explained by the model.

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

The  $R^2$  value ranges from 0 to 1, where higher values indicate better explanatory power.

```
# Model assessment
mlr_pred <- predict(mlr_mod, test_data)
mse_mlr <- mean((test_data$Age - mlr_pred)^2)

r2_mlr <- 1 - sum((test_data$Age - mlr_pred)^2) / sum((test_data$Age - mean(test_data$Age))^2)
```

## Ridge regression

We now aim to predict **Age** using a different method, in order to compare the results with those obtained from multiple linear regression. We have chosen to use Ridge regression, both to gain a better understanding of this technique and to optimize the model through the use of a hyperparameter.

Ridge Regression is an extension of linear regression that includes a penalty for large coefficient values. The goal is to reduce model variance and improve generalization. Instead of minimizing only the Residual Sum of Squares (RSS), Ridge adds an additional term to the loss function:

$$\text{Loss} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p \beta_j^2$$

Where:

- $y_i$  are the observed values
- $\hat{y}_i$  are the model's predictions
- $\lambda$  is a hyperparameter that controls the strength of the penalty on large coefficient values  $\beta_j$

When  $\lambda = 0$ , the model is equivalent to ordinary linear regression.

In practice, Ridge regression shrinks the coefficients toward zero, but never exactly to zero— this means that all variables are retained in the model.

To implement the method to the Heart dataset, we used `model.matrix()` to create design matrices where all predictors are included. The categorical variables are automatically encoded as dummy variables. Ridge was implemented using the `glmnet` package in R, using  $\alpha = 0$ . Then, we used `cv.glmnet()` with 10-fold cross-validation to find the optimal value of lambda (the hyperparameter). The value `lambda.min` was used for the final models.

```

library(glmnet)

# Create design matrices
x_train <- model.matrix(Age ~ ., data = train_data)[, -1]
y_train <- train_data$Age
x_test <- model.matrix(Age ~ ., data = test_data)[, -1]
y_test <- test_data$Age

ridge_mod <- glmnet(x_train, y_train, alpha = 0) # `alpha=0` is the ridge penalty

# Cross-validation to find the best lambda
set.seed(123)
cv_ridge <- cv.glmnet(x_train, y_train, alpha = 0)
best_lambda <- cv_ridge$lambda.min

```

## Ridge Regression - Advantages and disadvantages

Ridge regression is especially useful when two or more of the predictors are correlated with each other or when the number of predictors is large compared to the number of observations. The method helps prevent overfitting by adding just enough bias to make the model's estimates more stable and robust. It also helps reduce model complexity without completely removing variables.

In our case, the most important advantages are that the method works well when the predictors are correlated. In our dataset, we can expect that variables such as **RestingBP**, **Cholesterol**, and **MaxHR** are related to each other. Additionally, Ridge regression is useful because we do not want to exclude any explanatory variables, but rather improve the precision of our predictions.

The general disadvantages of Ridge regression are that it includes all predictors in the final model, which can make interpretation challenging when the number of predictors is large. Although the penalty term shrinks all coefficients toward zero, it never sets any of them exactly to zero. This limitation can make Lasso a more suitable alternative in cases where variable selection and model simplification are important.

In our case, the number of predictors is not very large, and it is not necessarily beneficial to exclude any of them. Therefore, we argue that Ridge regression is well-suited for this particular dataset.

## Evaluation of Ridge Regression Performance

To evaluate the performance of the model, we use Mean Squared Error (MSE) and  $R^2$  (coefficient of determination) as performance metrics.

Mean Squared Error (MSE) measures the average of the squared differences between the actual and predicted values. It is calculated as:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

where  $y_i$  is the actual value and  $\hat{y}_i$  is the predicted value. A low MSE indicates that the model's predictions are close to the actual values, making it a useful measure of model accuracy.  $R^2$  measures the proportion of the variance in the dependent variable that is explained by the model.

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

The  $R^2$  value ranges from 0 to 1, where higher values indicate better explanatory power. In this project, we evaluate model performance on the test data using both MSE and  $R^2$ .



```
# Predict Age on the test set
ridge_pred <- predict(cv_ridge, s = best_lambda, newx = x_test)

mse <- mean((y_test - ridge_pred)^2)
r2 <- 1 - sum((y_test - ridge_pred)^2) / sum((y_test - mean(y_test))^2)
```

## Results and interpretation

Evaluere modellene på testsettet. Sammenligne metodene – hvilken ga best resultater? Diskutere hvilke variabler som har størst betydning for prediksjon av alder. Diskutere bias-variance tradeoff.

## Multiple Linear Regression

The MLR using the `lm()`-function is summarized in the following.

```
##
## Call:
## lm(formula = Age ~ ., data = train_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -22.9982  -5.5147   0.2984   5.4142  25.7556
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    61.157004   3.597620  16.999 < 2e-16 ***
## SexM           -0.478152   0.832998  -0.574  0.5662
## ChestPainTypeATA -1.511517   0.998769  -1.513  0.1307
## ChestPainTypeNAP  1.150915   0.883941   1.302  0.1934
## ChestPainTypeTA   1.444529   1.589264   0.909  0.3637
## RestingBP        0.078874   0.017384   4.537 6.83e-06 ***
## Cholesterol     -0.004308   0.003231  -1.333  0.1829
## FastingBS       2.276642   0.808269   2.817  0.0050 **
## RestingECGNormal -4.375206   0.829786  -5.273 1.85e-07 ***
## RestingECGST     -2.149916   1.015703  -2.117  0.0347 *
## MaxHR           -0.110458   0.015006  -7.361 5.77e-13 ***
## ExerciseAnginaY  0.186851   0.807360   0.231  0.8171
## Oldpeak         1.142148   0.368024   3.103  0.0020 **
## ST_SlopeFlat    -1.258493   1.431131  -0.879  0.3795
## ST_SlopeUp      -1.063116   1.617031  -0.657  0.5111
## HeartDisease     0.786363   0.968457   0.812  0.4171
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.918 on 627 degrees of freedom
## Multiple R-squared:  0.2866, Adjusted R-squared:  0.2696
## F-statistic: 16.8 on 15 and 627 DF, p-value: < 2.2e-16
```

## Checking Model Assumptions

The model checking tools Tukey-Anscombe diagram and the QQ-diagram gave the following plots shown in the Figures 1 and 2.

We observe that the residuals seem to be spread around a horizontal line and with no particular pattern.

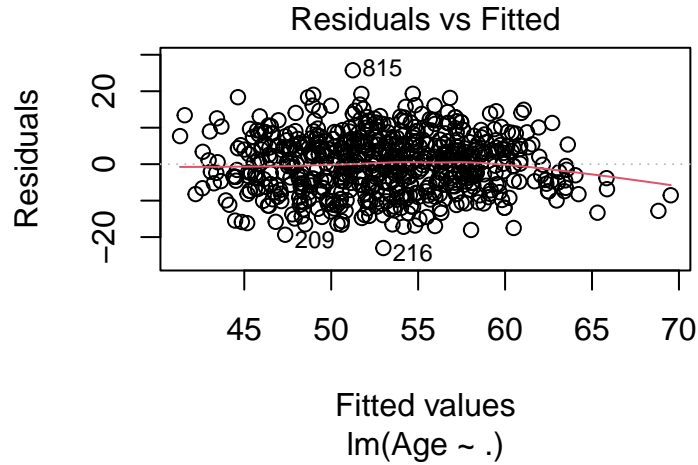


Figure 1: Tukey-Anscombe Diagram.

This indicates that there is a linear relationship between the `Age` and the covariates, which we assume in MLR. We also notice that the residuals are spread around zero, which we assumed to be the mean value. In addition, it seems that the residuals have the same variance approximately being  $\sqrt{20}$ .

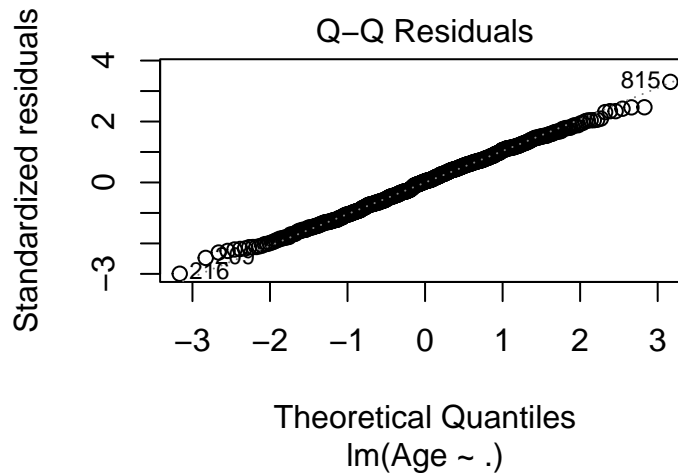


Figure 2: QQ-Diagram.

We observe that the points approximately seem to follow a straight line, which indicates that the data are fairly normally distributed. We can conclude that the model assumptions are fulfilled and that we may use MLR on this dataset.

### Evaluation of the Multiple Linear Regression

The  $F$ -test is shown in the summary of the MLR. We observe from the R output that the  $F$  is higher than 1 and that the respective  $p$ -value is very small, which means the predictors are useful in predicting the response. This also coincides with the  $p$ -values for the estimates of the  $\beta$ -coefficients.

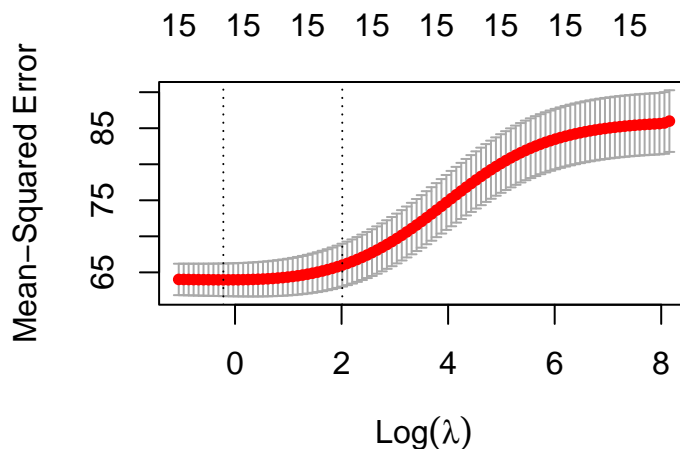
We do notice that not all  $p$ -values suggest significant coefficients. This means that perhaps only a subset of the predictors are useful. We could try to reformulate the test to check whether just some predictors are useful. In addition, we could try to extend the model to also consider interactions and non-linear terms. However, we would like to compare MLR to another model (Ridge regression), thus we will continue with the full model.

The MLR obtained a test MSE and a coefficient of determination approximately equal to 64.12 and 0.25, respectively. The model measures on average a difference of  $\sqrt{64.12} \approx 8$  years between the estimated values of **Age** and the true value. In addition, 25% variability in the data is explained by the model.

## Ridge Regression

Using 10-fold cross-validation with `cv.glmnet()`, the optimal value for the shrinkage parameter  $\lambda = 1.195248$ .

```
plot(cv_ridge)
```



The Ridge regression model achieved a test  $\text{MSE} = 70.92$  and a coefficient of determination  $R^2 = 0.27$ . This means that, on average, the model's predictions are off by about  $\sqrt{70.92} \approx 8$  years, and it explains roughly 27% of the variation in age.

This suggests that while some variables are somewhat related to age, the dataset is not particularly well-suited for accurate age prediction. This is perhaps not surprising, as many of the variables (e.g., blood pressure, cholesterol, etc.) are influenced by both age and numerous other factors — and there is no variable in the dataset that directly indicates age. One way to improve the model would be to add predictors that are strongly correlated with age.

Nevertheless, the results show that Ridge regression is able to capture some underlying patterns and performs as expected by helping to control overfitting. The relatively low  $R^2$  also highlights the importance of variable selection and data quality when building predictive models.

## Model Selection

### Summary