Assignment 3: Real-Time Implementation of 2D Digital Waveguide

ECS7012P Music and Audio Programming

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Abstract

Sample abstract.

1 Introduction

A specific class of digital musical instruments, or DMI, aims to decouple the human playing interaction and the physical response of the acoustic body. The instrument is thereby seen as a mute controller interface to a digital sound-producing device. This design paradigm allows the control of virtual instruments based on physical models. It might be seen as bizarre to choose to cut off the physics of an instrument, only to then try and reproduce them as accurately as the technology allows. However, the designer can then alter or augment the physics of the instrument arbitrarily; for example, they could map a drum or a set of percussions onto a guitar's body.

The work presented in this report aims to cover the first steps in the design of such an instrument. We are going to try and run a virtual vibrating membrane on Bela, and control it with the signal coming from one axis of an accelerometer.

The major challenge of this task is to ensure a correct implementation of the membrane's equations, a requirement that can prove very tricky when programming on an embedded target as a black box. Therefore, most of our analysis will be performed in simulation on standard C++ code; on-target evaluation will be the last step, a rather ambitious moment of truth at the end of the development efforts.

2 Background

2.1 Prior Work

The numerical reproduction of percussion instruments has been extensively studied, all the way to advanced techniques modelling the non-linearities of such systems. Good surveys of all available techniques come from the works by Rossing *et al.* [Rossing, 2004], Bilbao [Bilbao, 2009] and Mehes *et al* [Mehes, 2017].

Implementations of purely linear model are both less computationally intensive and easier to understand and implement. Two methods seem to have been quite successful in reproducing membranes in real time and are featured in working implementations of musical instruments: the Digital Waveguide model by Fontana and Rocchesso [Fontana, 1998] and the Transfer Function methods analysed in Trautmann *et al.* [Trautmann, 2001].

The two-dimensional Digital Waveguide model is a well-established model that extends the principle of the Digital Waveguide, widely used to model tubes such as wind instruments, to a vibrating surface. The article referenced above provides a clear breakdown of all the continuous-time and discrete-time physics involved, and it provides solutions to problems such as mesh stability, mesh excitation, energy attenuation and modelling of air loading for a more accurate real-world drum reproduction. Thererfore, we chose to base the implementation of our project upon the Digital Waveguide method.

A reference implementation of a Digital Waveguide mesh is already available in an early version (0.62)[1] of the Sound Design Toolkit [Baldan, 2017]. The topology of the mesh available in the toolkit is rectilinear; however the authors describe how using a triangular mesh optimises the dispersion error for most geometries, and especially for the circular membranes of drums [Fontana, 1998]. We will implement a triangular mesh from first principles following this suggestion.

2.2 Theory

The following summary of the theory behind digital waveguides in a triangular mesh structure is based on [Fontana, 1998]; some ideas and concepts are taken from the implementation of triangular membranes for room acoustics in [Murphy, 2000]. A more complete review of the whole process applied to virtual drums can also be found in [Laird, 2001].

Waveguide basics. The principle of the one-dimensional waveguide comes from the well-known D'Alembert solution of the wave wave equation for the transverse velocity $v(t,x)$, indicating two velocity waves travelling in opposite directions:

$$v(t,x) = v_+(x - ct) + v_-(x + ct) \tag{1}$$

We can assume that waves propagate following the equation above in ideal strings. When a number N of identical strings is joined together, we can write an equation for the transmission and scattering of the waves among those strings at their junction point i:

$$v_{i-}(t) = \frac{2}{N} \sum_{k=1}^{N} v_{k+}(t) - v_{i+}(t) \tag{2}$$

v_{i-} being the outgoing wave from the junction, as a function of the incoming waves v_{k+} from the strings joined together.

Both equations can be discretised by sampling the space into Δs spatial samples and time as $t = nT$ periods:

$$v(nT,x) = v_+(x - cnT) + v_-(x + cnT) \tag{3}$$

$$v_{i-}(nT) = \frac{2}{N} \sum_{k=1}^{N} v_{k+}(nT) - v_{i+}(nT) \tag{4}$$

We are going to assume that the mesh is perfectly homogeneous (no difference in impedance in the interfaces) and the boundaries are perfectly rigid, that is, the wave is fully reflected back ($r = 1$) and any point beyond the boundary will have ($v = 0$).

[1] `http://www.soundobject.org/SDT/downloads/SDT_src-062.zip`

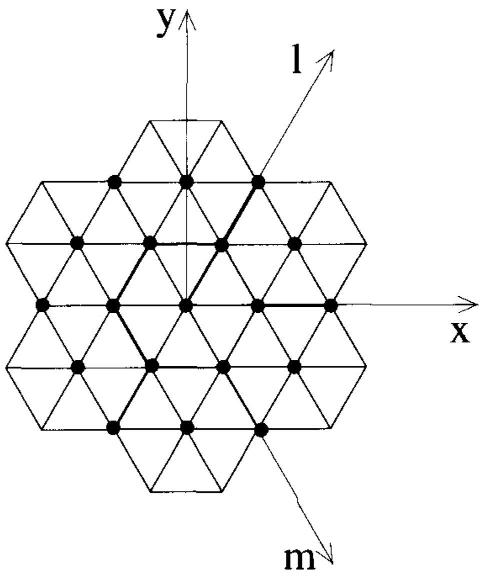

Figure 1: Triangular mesh coordinates [Fontana, 1998].

Triangular mesh. Joining six waveguides we have a structure that can effectively sample a two-dimensional space along directions x, $l = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$ and $m = \frac{1}{2}x - \frac{\sqrt{3}}{2}y$. Figure 1 depicts this structure and the related coordinates.

We can then rewrite equation 2.2 in two parts and restrict the number of waveguides to a maximum of 6, assuming impedance is either homogeneous or infinite (no transmission). We can then divide the computation into a *scattering equation* and a *junction output* (after [Murphy, 2000]):

$$v_i(nT) = \frac{2}{N} \sum_{k=1}^{N} v_{k+}(nT) \tag{5}$$

$$v_{k-}(nT) = v_i(nT) - v_{k+}(nT) \quad \text{for}\, k = 1 \ldots N \tag{6}$$

Junctions having fewer than six waveguides will just ignore the directions that aren't connected to another junction point. A good convention to enumerate the coordinates of the junction points comes from [Murphy, 2000]. We will count clockwise from the first point at top-right: North-East, East, South-East, South-West, West and North-West.

Constraints. The digital waveguide method imposes a relationship between spatial and temporal sampling. When designing the mesh or even running an offline simulation, one can leave the speed of the medium c unspecified. However, in a real-time system, the temporal sampling period T is defined by the system, and so must be the medium speed. The stability of the mesh is enforced by the Courant condition, which in this case takes the following form [Fontana, 1998]:

$$\Delta x = \Delta l = \Delta m = \sqrt{2}cT \tag{7}$$

Absorption, air loading, excitation. The sources referenced provide a good breakdown of all the more advanced problems around lossy junctions, air loading in a cylindrical drum as a spring-mass model, excitation models. We will limit ourselves to the simplest case of a lossless stable membrane, excited directly with an arbitrary audio signal $y(nT)$, using the excitation model outlined in [Murphy, 2000]. The source signal is injected as incoming waves into a designated junction:

$$v_{k+}(nT) = \frac{y(nT)}{2} \tag{8}$$

3 Design

3.1 Mesh Topology

3.2 Processing

3.3 Evaluation Framework

4 Bela Implementation

5 Evaluation

5.1 Unit Tests

5.2 Simulation

5.3 On-Target Evaluation

6 Conclusion

References

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