

Assignment 3: Real-Time Implementation of 2D Digital Waveguide

ECS7012P Music and Audio Programming

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1 Introduction

A specific class of digital musical instruments, or DMI, aims to decouple the human playing interaction and the physical response of the acoustic body. The instrument is thereby seen as a mute controller interface to a digital sound-producing device. This design paradigm allows the control of virtual instruments based on physical models. It might be seen as bizarre to choose to cut off the physics of an instrument, only to then try and reproduce them as accurately as the technology allows. However, the designer can then alter or augment the physics of the instrument arbitrarily; for example, they could map a drum or a set of percussions onto a guitar's body.

The work presented in this report aims to cover the first steps in the design of such an instrument. We are going to try and run a virtual vibrating membrane on Bela, and control it with the signal coming from one axis of an accelerometer.

The major challenge of this task is to ensure a correct implementation of the membrane's equations, a requirement that can prove very tricky when programming on an embedded target as a black box. Therefore, most of our analysis will be performed in simulation on standard C++ code; on-target evaluation will be the last step, a rather ambitious moment of truth at the end of the development efforts.

2 Background

2.1 Prior Work

The numerical reproduction of percussion instruments has been extensively studied, all the way to advanced techniques modelling the non-linearities of such systems. Good surveys of all available techniques come from the works by Rossing *et al.* [Rossing, 2004], Bilbao [Bilbao, 2009] and Mehes *et al* [Mehes, 2017].

Implementations of purely linear model are both less computationally intensive and easier to understand and implement. Two methods seem to have been quite successful in reproducing membranes in real time and are featured in working implementations of musical instruments: the Digital Waveguide model by Fontana and Rocchesso [Fontana, 1998] and the Transfer Function methods analysed in Trautmann *et al.* [Trautmann, 2001].

The two-dimensional Digital Waveguide model is a well-established model that extends the principle of the Digital Waveguide, widely used to model tubes such as wind instruments, to a vibrating surface. The article referenced above provides a clear breakdown of all the continuous-time and discrete-time physics involved, and it provides solutions to problems such as mesh stability, mesh excitation, energy attenuation and modelling of air loading for a more accurate real-world drum reproduction. Therefore, we chose to base the implementation of our project upon the Digital Waveguide method.

A reference implementation of a Digital Waveguide mesh is already available in an early version (0.62)¹ of the Sound Design Toolkit [Baldan, 2017]. The topology of the mesh available in the toolkit is rectilinear; however the authors describe how using a triangular mesh optimises the dispersion error for most geometries, and especially for the circular membranes of drums [Fontana, 1998]. We will implement a triangular mesh from first principles following this suggestion.

2.2 Theory

The following summary of the theory behind digital waveguides in a triangular mesh structure is based on [Fontana, 1998]; some ideas and concepts are taken from the implementation of triangular membranes for room acoustics in [Murphy, 2000]. A more complete review of the whole process applied to virtual drums can also be found in [Laird, 2001].

¹http://www.soundobject.org/SDT/downloads/SDT_src-062.zip

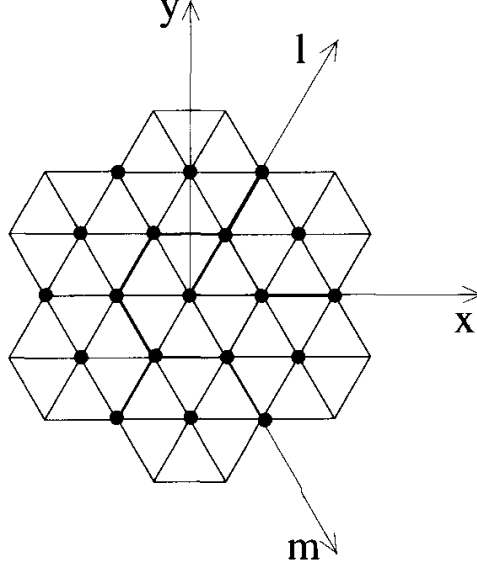


Figure 1: Triangular mesh coordinates [Fontana, 1998].

Waveguide basics. The principle of the one-dimensional waveguide comes from the well-known D'Alembert solution of the wave equation for the transverse velocity $v(t, x)$, indicating two velocity waves travelling in opposite directions:

$$v(t, x) = v_+(x - ct) + v_-(x + ct) \quad (1)$$

We can assume that waves propagate following the equation above in ideal strings. When a number N of identical strings is joined together, we can write an equation for the transmission and scattering of the waves among those strings at their junction point i :

$$v_{i-}(t) = \frac{2}{N} \sum_{k=1}^N v_{k+}(t) - v_{i+}(t) \quad (2)$$

v_{i-} being the outgoing wave from the junction, as a function of the incoming waves v_{k+} from the strings joined together.

Both equations can be discretised by sampling the space into Δs spatial samples and time as $t = nT$ periods:

$$v(nT, x) = v_+(x - cnT) + v_-(x + cnT) \quad (3)$$

$$v_{i-}(nT) = \frac{2}{N} \sum_{k=1}^N v_{k+}(nT) - v_{i+}(nT) \quad (4)$$

We are going to assume that the mesh is perfectly homogeneous (no difference in impedance in the interfaces) and the boundaries are perfectly rigid, that is, the wave is fully reflected back ($r = 1$) and any point beyond the boundary will have ($v = 0$).

Triangular mesh. Joining six waveguides we have a structure that can effectively sample a two-dimensional space along directions x , $l = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$ and $m = \frac{1}{2}x - \frac{\sqrt{3}}{2}y$. Figure 1 depicts this structure and the related coordinates.

We can then rewrite equation 2.2 in two parts and restrict the number of waveguides to a maximum of 6, assuming impedance is either homogeneous or infinite (no transmission). We can then divide the computation into a *scattering equation* and a *junction output* (after [Murphy, 2000]):

$$v_i(nT) = \frac{2}{N} \sum_{k=1}^N v_{k+}(nT) \quad (5)$$

$$v_{k-}(nT) = v_i(nT) - v_{k+}(nT) \quad \text{for } k = 1 \dots N \quad (6)$$

Junctions having fewer than six waveguides will just ignore the directions that aren't connected to another junction point. A good convention to enumerate the coordinates of the junction points comes from [Murphy, 2000]. We will count clockwise from the first point at top-right: North-East, East, South-East, South-West, West and North-West.

Constraints. The digital waveguide method imposes a relationship between spatial and temporal sampling. When designing the mesh or even running an offline simulation, one can leave the speed of the medium c unspecified. However, in a real-time system, the temporal sampling period T is defined by the system, and so must be the medium speed. The stability of the mesh is enforced by the Courant condition, which in this case takes the following form [Fontana, 1998]:

$$\Delta x = \Delta l = \Delta m = \sqrt{2}cT \quad (7)$$

Absorption, air loading, excitation. The sources referenced provide a good breakdown of all the more advanced problems around lossy junctions, air loading in a cylindrical drum as a spring-mass model, excitation models. We will limit ourselves to the simplest case of a lossless stable membrane, excited directly with an arbitrary audio signal $y(nT)$, using the excitation model outlined in [Murphy, 2000]. The source signal is injected as incoming waves into a designated junction:

$$v_{k+}(nT) = \frac{y(nT)}{2} \quad (8)$$

3 Design

The equations laid out in the previous section allow, in principle, the numerical implementation of any kind of topology. However, in addition to the memory constraints and computational constraints imposed by the Bela target, the triangular mesh poses non-obvious problems down to the memory layout and the change in coordinates between cartesian and (x, l, m) .

The first stage of the implementation, in practice, requires access to tools such as off-line processing, debugging, plotting and the generation of mathematically complex audio test signals. We found especially important to be able to visualise the mesh at every iteration in a simulation. For this reason, most of the code has been written within an environment comprised of some C++ core classes, a C++ to Python wrapper based on Boost.Python² and a set of validation notebooks written in Jupyter.

3.1 Mesh Topology

Rectilinear grids have an obvious mapping onto two-dimensional arrays, natively supported in most programming language. More complicated topologies require a function that maps a C++ array structure to the mesh's geometry coordinates, which we will call c, k . The idea of using a flattened array structure, albeit valid and quite well-established, came to mind a bit too late in the development.

We present a method that maps a regular C-style 2D array structure to a rectangular mesh topology, seen in figure 2. If we impose the constraints that the column dimension of a matrix must be even, and the rows must be odd, we can interleave two meshes of identical size in the even and odd indexes of this matrix, respectively. Even rows will have an odd number of mesh points, whereas odd rows will have an even number of them. What makes this structure work is that the third element of the second mesh in even rows is wrapped around to the next line, therefore the indexing stays within the required memory boundary.

The following set of equations can be used to convert back and forth from c, k indexing and Cartesian coordinates:

$$c = \frac{y}{\frac{\sqrt{3}}{2}\Delta s} \quad (9)$$

$$y = c \frac{\sqrt{3}}{2}\Delta s \quad (10)$$

$$k = \frac{x - \frac{\Delta s}{2}(c \bmod 2)}{\Delta s} \quad (11)$$

²This has been the theme of a past AIL workshop at QMUL, the slides for which can be found here: https://docs.google.com/presentation/d/1FK514MeOesJG-2egqraF3FT9DIQkZYGH_I2nMmFxFxUXI

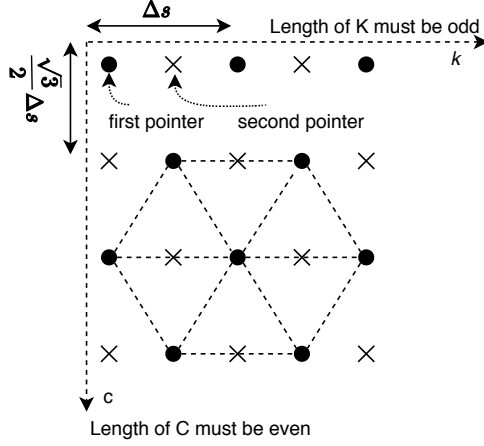


Figure 2: Mapping between two interleaved meshes and 2D array.

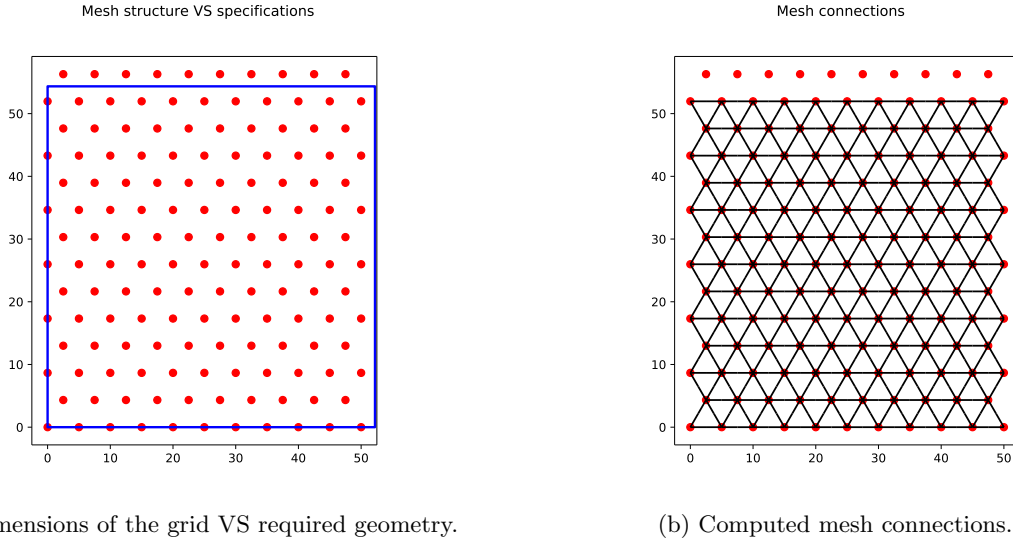


Figure 3: Generated mesh for a rectangle of width 52.21 mm, height 54.35 mm, spatial resolution 5 mm.

$$x = k \Delta s + \frac{\Delta s}{2} (c \bmod 2) \quad (12)$$

Using this coordinate system, we can then assess whether each point in the mesh lies inside or outside any geometry defined in Cartesian coordinates. We can then choose to create a waveguide between two points if both of them lie within the given geometry. An example made whilst evaluating our mesh in the Jupyter notebook is shown in figure 3.

3.2 Processing

The last step needed before implementing the processing routine is to calculate how many values we need to store for each mesh point and/or for each waveguide. Following the reference implementation in the Sound Design Toolkit [Baldan, 2017], we are going to use two values for each waveguide, respectively an outgoing and an incoming velocity value. The outgoing value will be reused as an incoming value in the following time iteration (delay step). Therefore we need a total of 13 meshes for each mesh point, one for each waveguide plus a record of the junction values that is going to be used for plotting.

A fourteenth mesh is going to be created to store a bitmask containing the connections between a given point and its neighbouring point. We're going to observe Murphy's convention of clockwise numbering from NE to NW.

Finally, we need to choose a source and a pickup point. At startup, the source is allocated at the centre of the membrane, whereas the pickup lies close to the edge of the default rectangular structure, as no mode is supposed to have a node at the corners of a shoebox shape.

All these actions are implemented in the constructor for the object `Triangular2DMesh`: it also provides a method to externally allocate the memory required, which is then passed as an argument to the constructor.

Optionally, the API allows to re-draw the mesh connections based upon a user-defined lambda expression evaluating the x, y coordinates of each mesh point. Source and pickup points can also be moved before or during processing.

`ProcessSample()` implements equations 5, 6 and 8 for every point c, k in the mesh and swaps the two mesh pointers for each iteration. The outgoing values are therefore consumed by the neighbouring points of each junction.

3.3 Evaluation Framework

The simulation environment is based on an Ubuntu 20.04 Docker container. This allows the C++ and the Python bindings to be built in a reliable environment without installing a development toolkit system-wide.

The Bash script `docker_host_run.sh` will build, create and run the appropriate Docker container; an SSH port is created so that a remote debug session can be created via `gdb`. The source code is automatically mounted into the Docker container's file system, therefore any change will be picked up by both the host machine and the container.

The Makefile supports target `test` to build a pure C++ unit test suite based on Catch2³, and a `python` target building a shared library `DSPPythonWrapper.so` that can be imported onto Python, and will be the basis for our validation in Jupyter.

4 Bela Implementation

The `Triangular2DMesh` object has been embedded into a Bela project that interfaces with the same hardware prototype built for the drum machine in Assignment 2. The accelerometer has been reused as a transducer for low-frequency, rumble-like displacements that are characteristic of percussive interaction. Only the Z-axis (up-down) acceleration is needed.

Firstly, the signal from the accelerometer has been observed from Bela's internal oscilloscope when the board was lying on a desk, and someone was tapping on the same desk next to the board. The resulting signal was well above the noise floor and a clean impulse was clearly detectable.

Minimal signal conditioning has been implemented in the `DetectHit` class: a high-pass filter brings the accelerometer signal's mean to 0, then full-wave rectification and a low-pass filter create a positive impulse that would inject a controlled amount of energy onto the mesh.

5 Evaluation

5.1 Unit Tests

`make test` builds a set of unit tests implemented in `main.cpp`: the purpose of those tests is to verify the correct allocation of memory and the correct implementation of the indexing routines. It is easier to debug and inspect the routines in a pure C++ environment: a short Gaussian impulse is passed to the processing function to ensure that the processing happens within the memory bounds and the mesh does not become unstable and return `Inf` or `NaN`.

This is a really limited evaluation and it does not allow the physical validation of the formulae's implementation; two evaluation cases have been run in a Jupyter notebook to visualise the junction's pressure and analyse the resulting plots.

5.2 Simulation

Perhaps the simplest case of 2D membrane is the degenerate case of a one-dimensional string. By setting the Y-axis to an amount that is lower than a full Δs we essentially obtain this structure. We can excite the structure with a unit impulse and we obtain the periodic reflection of that impulse across twelve samples for a 11-point mesh size. The impulse travels back and forth with no apparent loss of energy (figure 4).

The next step taken was to simulate a larger rectangular ideal membrane excited by a Gaussian impulse, with the expectation that the impulse would propagate radially to the edge of the membrane and it would scatter at the edges. Figure 5 outlines the result: the impulse seems to create a circular shape around the source. The Jupyter animation follows with what looks like a potentially correct evolution, however it seems that most of the energy is dissipated really early on after the event's onset. This is confirmed when examining the pickup signal in detail 5d: the signal recorded is a thousand times quieter than the source, it decays very rapidly and the spectrum also shows a certain degree of aliasing.

³<https://github.com/catchorg/Catch2>

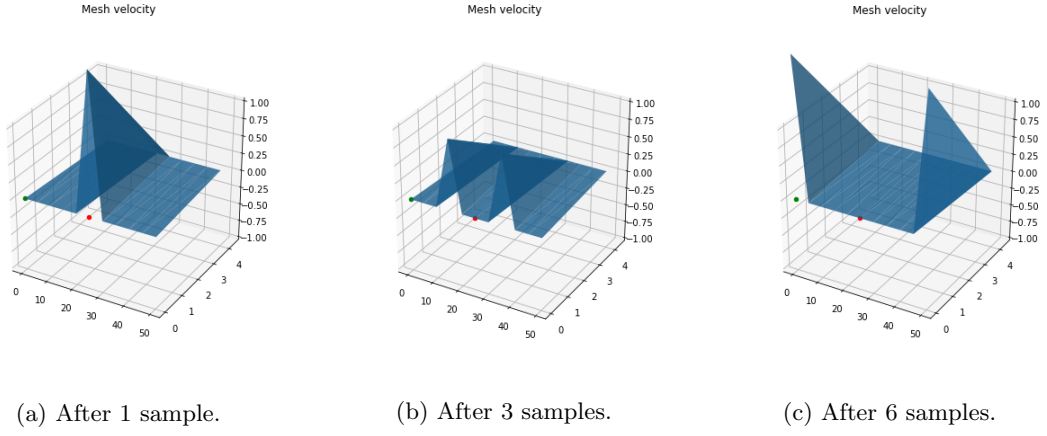


Figure 4: Simulation for a mesh of width 52.21 mm, height 2.35 mm, spatial resolution 5 mm.

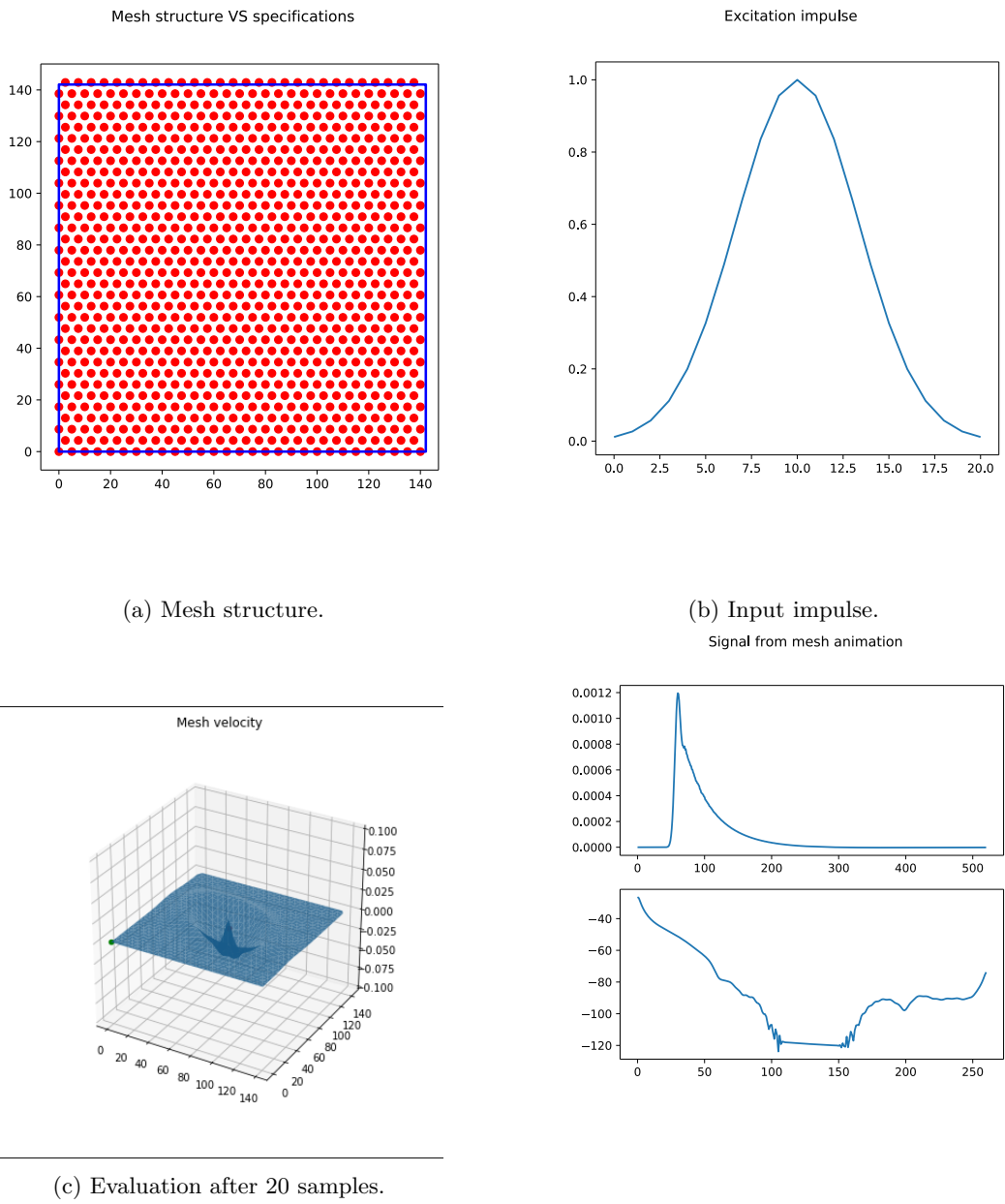


Figure 5: Simulation for 142x142 mm mesh, spatial resolution 5 mm.

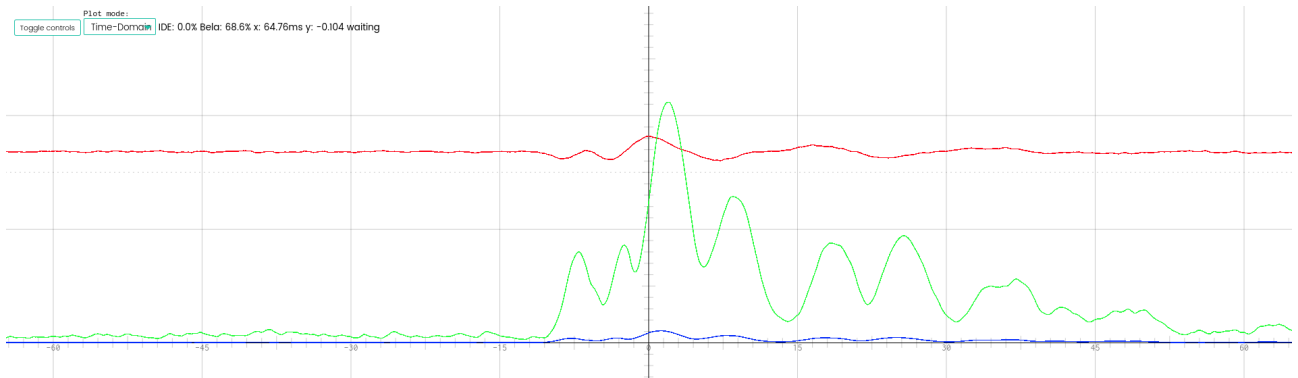


Figure 6: Representation of a tap from Bela’s oscilloscope: red line is unfiltered Z-axis signal; blue line is filtered signal; green line is mesh output.

The mesh has also been simulated with a train of Gaussian impulses hitting it every second at a sample rate of 44,100 Hz, which is supported by Bela. The simulation confirms that the sound of the membrane is anything but interesting! The processed sound is a noisy, aliased version of the input signal with no noticeable resonance, and sadly this process has very little artistic or creative value!

5.3 On-Target Evaluation

The specifications of mesh that is suitable for Bela can be found in the second section of the Jupyter notebook used for the validation. Suitable values for c and the membrane size have been found by dealing with the tight upper ceiling imposed by Bela’s computational capability: it seems that the current implementation can only perform the processing of a 7x6 point mesh at the native sampling rate.

We made a first attempt with a mesh with that number of points, to verify its stability when it is controlled by the accelerometer. The system just shows the very unimpressive feature of boosting the noise floor of the accelerometer’s signal.

One last attempt was made using the 1-d degenerate case, which showed a good amount of energy retention. Although the sonic outcome did not quite improve as much as expected, the Bela oscilloscope showed a clear modification of the input waveform, with the impulse being reflected back and forth and creating a feeble attempt at a standing wave (figure 6).

6 Conclusions and Future Work

This work has successfully implemented the initial steps to obtain a fully working implementation of a membrane on Bela. The task has turned out to be very ambitious and difficult to correctly achieve within the allocated time frame; however we managed to run a simple rectangular membrane model in real-time on Bela that satisfies stability constraints and shows some degree of physical validity in the simulation.

Further work is definitely needed in the understanding of the relationship between the time sampling rate and the medium speed: this might be key to achieving more resonating and “interesting” membranes, perhaps after implementing multi-rate processing and oversampling on Bela. In order to progress to this stage, much work must be done in the mesh’s processing function to use ARM’s SIMD intrinsics. The mesh’s topology lends itself very well to vector operations, and ARM chips usually do not penalise non-contiguous memory accesses.

Furthermore, more in-depth validation is needed to understand whether the attenuation is a result of the modelling method or a non-obvious code bug. 1D tests could be run along l - and m -axis to verify that impulse propagation happens correctly in all directions. Modal analysis, including mode shifting as a result of attenuation, is also a common method to validate the physics of a membrane model [Fontana, 1998]. After reaching this stage, attenuation coefficients and air loading can be implemented and correctly validated.

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