Suppose the returns of $N \geq 1$ assets are normally distributed with mean $\mu \in \mathbb{R}^N$ and covariance $\Sigma \in \mathbb{R}^{N \times N}$. Fix, for the moment, the desired return μ_P. For each portfolio $x \in \mathbb{R}^N$, we require that $\mu' x = \mu_P$ and $\mathbf{1}' x = w_P$. Let $\sigma_P^2(x)$ denote the variance of its returns: $\sigma_P^2(x) = x' \Sigma x$. The investor's problem is to choose the minimum-variance portfolio subject to it earning a return of μ_P.

$$
\begin{aligned}
& \min\nolimits_{x \in \mathbb{R}} \sigma_P^2(x) \\
\text{s.t.} \quad & x' \mu = \mu_P \\
& x' \mathbf{1} = w_P
\end{aligned} \tag{1}
$$

Let λ_μ and $\lambda_{\mathbf{1}}$ denote the Lagrange multipliers of the constraints $\mu' x = \mu_P$ and $\mathbf{1}' x = w_P$ respectively. Applying standard results from quadratic programming, we obtain the solution

$$
\begin{bmatrix} x \\ \lambda_\mu \\ \lambda_{\mathbf{1}} \end{bmatrix} = \begin{bmatrix} \Sigma & \mu & \mathbf{1} \\ \mu' & 0 & 0 \\ \mathbf{1}' & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \mu_\mu \\ w_{\mathbf{1}} \end{bmatrix} \tag{2}
$$

The tangency portfolio is given by