1 Two Asset Case

Consider two stocks, A and B. You plan to invest a fraction x in A and $1-x$ in B. A and B have mean returns μ_A and μ_B.

$$\mu_P = x\mu_A + (1-x)\mu_B \tag{1}$$
$$\sigma_P^2 = x^2\sigma_A^2 + (1-x)^2\sigma_B^2 + 2x(1-x)\sigma_{AB} \tag{2}$$

and hence

$$\sigma_P^2 = \left[\frac{\mu_P - \mu_B}{\mu_A - \mu_B}\right]^2 \sigma_A^2 + \left[\frac{\mu_A - \mu_P}{\mu_A - \mu_B}\right]^2 \sigma_B^2 + 2\left[\frac{\mu_P - \mu_B}{\mu_A - \mu_B}\right]\left[\frac{\mu_A - \mu_P}{\mu_A - \mu_B}\right]\sigma_{AB} \tag{3}$$
$$\tag{4}$$

2 N-Asset Case

Suppose the returns of $N \geq 1$ assets are normally distributed with mean $\mu \in \mathbb{R}^N$ and covariance $\Sigma \in \mathbb{R}^{N\times N}$. Fix, for the moment, the desired return μ_P. For each portfolio $x \in \mathbb{R}^N$, we require that $\mu' x = \mu_P$ and $\mathbf{1}' x = w_P$. Let $\sigma_P^2(x)$ denote the variance of its returns: $\sigma_P^2(x) = x'\Sigma x$. The investor's problem is to choose the minimum-variance portfolio subject to it earning a return of μ_P.

$$\begin{array}{rl} & \min_{x\in\mathbb{R}} \sigma_P^2(x) \\ \text{s.t.} & x'\mu = \mu_P \\ & x'\mathbf{1} = w_P \end{array} \tag{5}$$

Let λ_μ and $\lambda_{\mathbf{1}}$ denote the Lagrange multipliers of the constraints $\mu' x = \mu_P$ and $\mathbf{1}' x = w_P$ respectively. Applying standard results from quadratic programming, we obtain the solution

$$\begin{bmatrix} x \\ \lambda_\mu \\ \lambda_{\mathbf{1}} \end{bmatrix} = \begin{bmatrix} \Sigma & \mu & \mathbf{1} \\ \mu' & 0 & 0 \\ \mathbf{1}' & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \mu_\mu \\ w_{\mathbf{1}} \end{bmatrix} \tag{6}$$