Put

$$g(t) = \left[\int_{\alpha(t)}^{\beta(t)} f(s)ds\right]^{-1} \left[\int_{\alpha(t)}^{\beta(t)} sf(s)ds\right]. \tag{1}$$

Observe that

$$g'(t) = -\left(\beta'(t)f(\beta(t)) - \alpha'(t)f(\alpha(t))\right)\left[\int_{\alpha(t)}^{\beta(t)} f(s)ds\right]^{-2}\left[\int_{\alpha(t)}^{\beta(t)} sf(s)ds\right] + \left(\beta'(t)\beta(t)f(\beta(t)) - \alpha'(t)\alpha(t)f(\alpha(t))\right)\left[\int_{\alpha(t)}^{\beta(t)} f(s)ds\right]^{-1} \tag{2}$$

$$= \left[\int_{\alpha(t)}^{\beta(t)} f(s)ds\right]^{-2}\left\{-\left(\beta'(t)f(\beta(t)) - \alpha'(t)f(\alpha(t))\right)\int_{\alpha(t)}^{\beta(t)} sf(s)ds + \left(\beta'(t)\beta(t)f(\beta(t)) - \alpha'(t)\alpha(t)f(\alpha(t))\right)\int_{\alpha(t)}^{\beta(t)} f(s)ds\right\} \tag{3}$$

$$\tag{4}$$