Observe that

$$C[m] = \frac{1}{2\epsilon}\int_{-\bar{\epsilon}}^{1+\bar{\epsilon}}\int_{q_-(\tilde{m})}^{q_+(\tilde{m})}(q - A(\tilde{m}))^2 dG(q)d\tilde{m} \tag{1}$$

$$= \frac{1+2\bar{\epsilon}}{2\epsilon}\int_{-\bar{\epsilon}}^{1+\bar{\epsilon}}\left[\int_{q_-(\tilde{m})}^{q_+(\tilde{m})}(q-a)^2 dG(q)\right]\frac{d\tilde{m}}{1+2\bar{\epsilon}} \tag{2}$$

The optimal discrete message has $N+1$ messages given by knots

$$0 = x_0 < x_1 < \cdots < x_N < x_{N+1} = 1. \tag{3}$$

For $k = 0, \ldots, N$, put $y_k \equiv G^{-1}(k/N)$. The x_k satisfy

$$x_k = \frac{a_k + a_{k+1}}{2} \tag{4}$$

and for $k = 0, \ldots, N$, the a_k are given by

$$a_k = A(x_k, x_{k+1}) \equiv \frac{\int_{x_k}^{x_{k+1}} q dG(q)}{\int_{x_k}^{x_{k+1}} dG(q)} = \frac{(x_{k+1}G(x_{k+1}) - x_k G(x_k)) - (G(x_{k+1}) - G(x_k))}{G(x_{k+1}) - G(x_k)} \tag{5}$$

From the sender's first-order condition

$$x_k = \frac{A(x_{k-1}, x_k) + A(x_k, x_{k+1})}{2} \tag{6}$$

Main result: $x_2 - x_1$ is decreasing in $(g(x_2) - g(x_1))/(x_2 - x_1)$.