Clarifying by Discretizing

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Research Question

Why do people discretize their communication?

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Examples of cheap, discrete messages:

- Bond ratings (AAA,...,D)
- Analysts' recommendations (Strong Buy, Buy, Hold, Sell, Strong Sell)
- Course grades (A,...,F)
- Weather advisories (Tornado Watch, Tornado Warning)
- Film and restaurant reviews (★★★, ..., ★)

Our Result

- Crawford and Sobel (1982):

$$\text{conflict of interest} \;\Rightarrow\; \text{discrete messages.}$$

- We show that the converse is false:

$$\text{conflict of interest} \;\not\Leftarrow\; \text{discrete messages.}$$

- How? If (1) the message space is bounded, and (2) messages are received with noise, then

$$\underline{\text{no}}\ \text{conflict of interest} \;\Rightarrow\; \text{discrete messages.}$$

- **Discrete messages less precise, but easier to interpret.**

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In practice, both discrete and continuous (different receivers?)

Literature

- Cheap talk: Large
- Interests aligned, but messaging is discrete by assumption: Sobel (2012), Cremer, Garicano, and Prat (2007), Garicano and Prat (2011)
- Benevolent sender, optimal messages discrete, but receivers' interests are not aligned: Agranov and Schotter (2012) and Crawford, Gneezy and Rottenstreich (2008)
- Electrical Engineering

The Model

There are two players, sender (*Sally*), and receiver (*Robert*).

- t_0 The sender privately observes the state of the world, $q \sim U[0,1]$.
- t_1 The sender sends the receiver a costless message, $m(q)$.
- t_2 The receiver receives

$$\widetilde{m}(q) \equiv m(q) + \epsilon \tag{1}$$

  where $\epsilon \sim U[-\bar{\epsilon}, +\bar{\epsilon}]$, $\bar{\epsilon} > 0$, and (for ease of expo) $\mathrm{mod}(1, \bar{\epsilon}) = 0$.
- t_3 The receiver takes an action $a(\widetilde{m}) \in \mathbb{R}$, and payoffs are realized.

The sender and receiver have preferences over the receiver's action

$$U^S(a, q, b) \equiv -(a - (q + b))^2 \tag{2}$$

$$U^R(a, q) \equiv -(a - q)^2 \tag{3}$$

where $b \geq 0$ measures the degree of conflict.

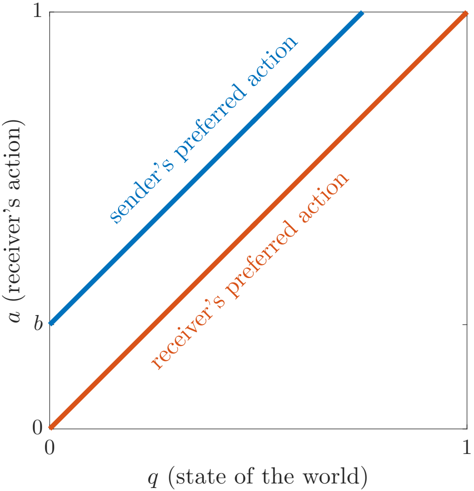
Comparison

	Interests	Noise	Messages
Crawford and Sobel	$b > 0$	$\bar{\epsilon} = 0$	$\mathbb{R}$
Martel and 2×Van Wesep	$b = 0$	$\bar{\epsilon} > 0$	$[0, 1]$

- Crawford and Sobel: conflicts of interests, no noise, unbounded message space.
- Martel and 2×Van Wesep: no conflicts of interests, noise, bounded message space.

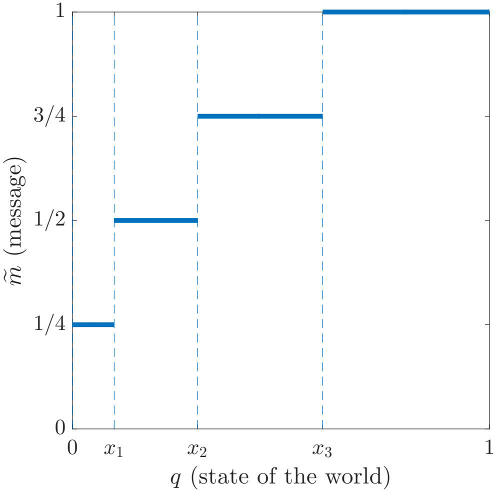
CRAWFORD AND SOBEL

Preferences



For each q, the sender prefers that the receiver take a larger a.

Crawford and Sobel



Theorem. Unique equilibrium features discrete messages.

MARTEL AND $2\times$VAN WESEP

Martel, 2×Van Wesep

1. *The sender and receivers' preferences are the same.*
   - Teams
   - Partnerships
2. *The message space is bounded* (w.l.o.g. to $[0, 1]$).
   - Hitler $\leq$ CEO $\leq$ Mother Teresa
3. *Messages are received with error.*
   - *The panda eats, shoots and leaves.*
   - *The new manager believes in collaborative problem-solving.*
   - *Congress is unable to address budget problems.*
   - *Ann wrote poignan essays in my class.*

Utility

Given a message and action

$$m(q) : [0,1] \to [0,1] \tag{4}$$

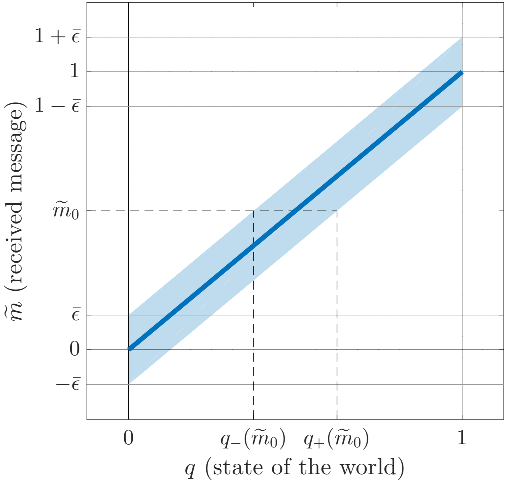
$$a(\widetilde{m}) : [-\bar{\epsilon}, 1+\bar{\epsilon}] \to \mathbb{R} \tag{5}$$

The sender and receivers' expected utility is

$$E[U] = -\int_0^1 \left[\int_{-\bar{\epsilon}}^{\bar{\epsilon}} |a(m(q)+e) - q|^2 \cdot \frac{de}{2\bar{\epsilon}} \right] dq. \tag{6}$$

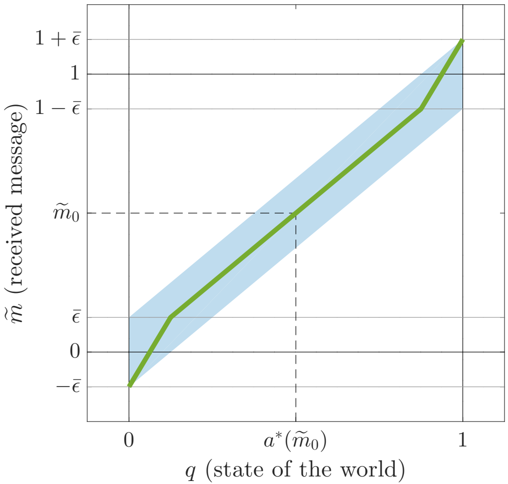
- $|a(m(q)+e) - q|^2$ is the quadratic loss given error e, and state q.
- Integrate over errors, $e \sim U[-\bar{\epsilon}, \bar{\epsilon}]$.
- Integrate over states, $q \sim U[0,1]$.

Identity Message



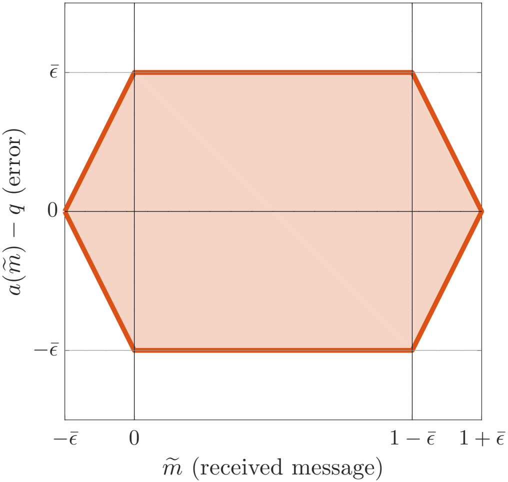
Having received $\widetilde{m}_0$, the receiver infers that $q \in [q_-(\widetilde{m}_0), q_+(\widetilde{m}_0)]$.

Identity Action



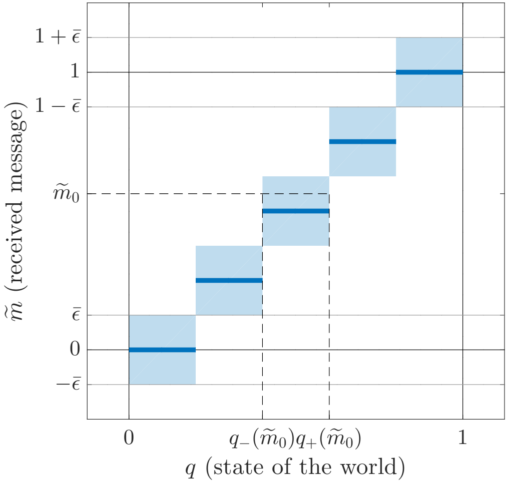
Lemma 1. $a^*(\widetilde{m}) = (q_-(\widetilde{m}_0) + q_+(\widetilde{m}_0))/2.$

Identity Error



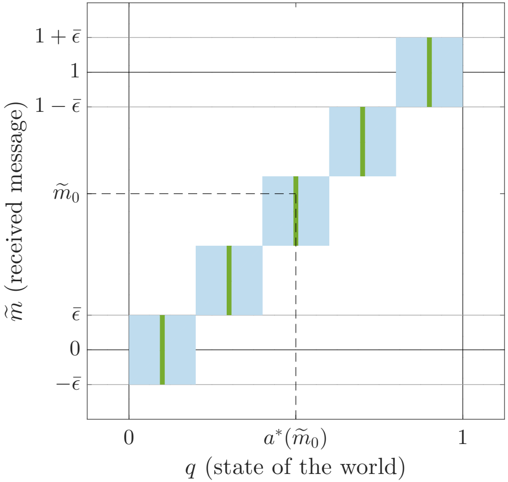
The error, $|a(\widetilde{m}) - q|$, is uniformly distributed on this hexagon.

Discrete Message



There is no uncertainty about which message was sent!

Discrete Action



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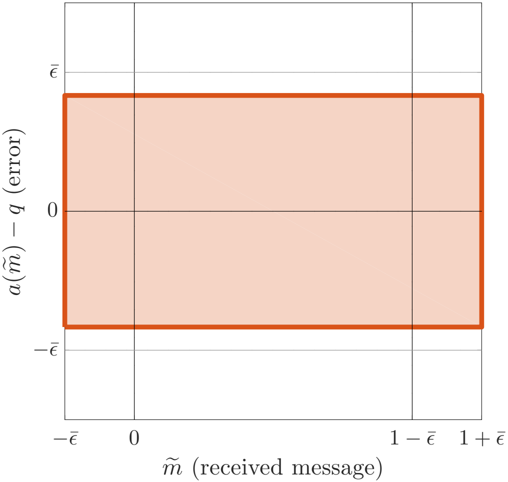
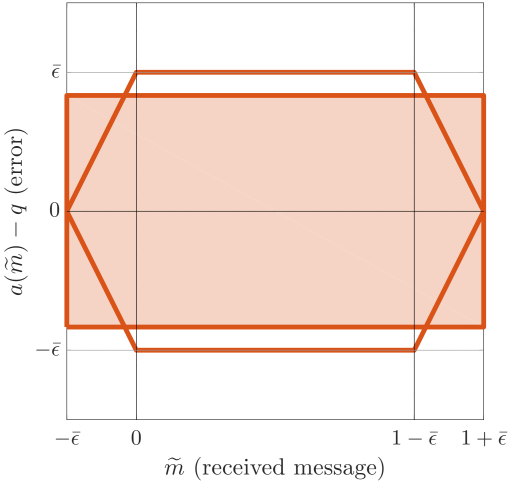
Discrete Error



The marginal loss is strictly increasing in the error

Discrete Error



The marginal loss is strictly increasing in the error

Main Result

Proposition 1 (Existence). Discrete messages are optimal.

Proof. Follows from Jensen's inequality. For any increasing message function m,

$$\begin{aligned}
C[m] &= \frac{1+2\bar{\epsilon}}{24\bar{\epsilon}} \int_{-\bar{\epsilon}}^{1+\bar{\epsilon}} (q_+(\tilde{m}) - q_-(\tilde{m}))^3 \cdot \frac{d\widetilde{m}}{1+2\bar{\epsilon}} && (7) \\
&\geq \frac{1+2\bar{\epsilon}}{24\bar{\epsilon}} \left(\int_{-\bar{\epsilon}}^{1+\bar{\epsilon}} (q_+(\tilde{m}) - q_-(\tilde{m})) \cdot \frac{d\widetilde{m}}{1+2\bar{\epsilon}} \right)^3 && (8) \\
&= \frac{1}{3}\bar{\epsilon}^2(1-\bar{\epsilon}) && (9) \\
&= C[m_d], && (10)
\end{aligned}$$

which is attained by a discrete message function with $\frac{1}{2\bar{\epsilon}} + 1$ messages. ∎

Additional Findings

- **Corollary 1.** The number of discrete messages is

$$N^* \equiv \frac{1}{2\bar{\epsilon}} + 1 \qquad (\uparrow \infty \text{ as } \bar{\epsilon} \downarrow 0). \tag{11}$$

  Empirical Implication: 21 Moody's ratings vs. 5 Yelp stars

- **Corollary 2.** The ratio

$$\frac{\text{cost of the identity message}}{\text{cost of the discrete message}} \downarrow 1 \text{ as } N^* \uparrow \infty \tag{12}$$

Intuition

$$q \xrightarrow{\text{Endogenous Noise}} m(q) \xrightarrow{\text{Exogenous Noise}} \widetilde{m}(q)$$

- Discretizing $\downarrow$ exogenous noise, but $\uparrow$ endogenous noise.
- In our setting, exogenous noise costs more than endogenous noise.
- Is our result robust?
- **Proposition 2.** If
  1. the noise is not uniformly distributed, and
  2. the noise variance is sufficiently small,

  then discrete messages are not optimal.

Conclusion

1. Discretization can be optimal, *even* when sender's and receiver's interests are aligned.
2. While coarse messages are less precise, they are easier to interpret.
3. Continuous messages are optimal when the error is "small."
4. Rethink classic problems with aligned interests and noisy communication.