1 discrete actions

Let $m : [0,1] \to [0,1]$ be the sender's message function. Let the receiver's **discrete** action $A : [0,1] \to \{0,1\}$ be given by

$$A(\tilde{m}) = \begin{cases} 1 & \text{if} \quad m^{-1}(\tilde{m}) > V \\ 0 & \text{otherwise} \end{cases}$$

where V is distributed according to G. The expected utility—with respect to V—for $e > 0$ is

$$\begin{aligned} U_{+}(q, m(q), e) &= \underbrace{G(q)}_{A=0 \text{ and } V<q} + \underbrace{(1 - G(m^{-1}(m(q)+e)))}_{A=1 \text{ and } V \geq q} \\ &\approx G(q) + (1 - G(q + m^{-1\prime}(m(q))e)) \\ &= G(q) + (1 - G(q + e/m'(q))) \\ &\approx G(q) + (1 - G(q) - g(q)(e/m'(q))) \\ &= 1 - g(q)(e/m'(q)) \end{aligned}$$

while the expected utility—again, with respect to V—for $e \leq 0$ is

$$\begin{aligned} U_{-}(q, m(q), e) &= \underbrace{G(m^{-1}(m(q)+e))}_{A=0 \text{ and } V<q} + \underbrace{(1 - G(q))}_{A=1 \text{ and } V \geq q} \\ &\approx G(q + m^{-1\prime}(m(q))e) + (1 - G(q)) \\ &= G(q + e/m'(q)) + (1 - G(q)) \\ &\approx G(q) + g(q)(e/m'(q)) + (1 - G(q)) \\ &= 1 + g(q)(e/m'(q)). \end{aligned}$$

The sender chooses m to maximize total expected utility:

$$\begin{aligned} \min_{m} &= \int_0^1 \left\{ \int_{-\bar{\epsilon}}^{0} U_{-}(q, m(q), e) \left(\frac{de}{2\bar{\epsilon}}\right) + \int_0^{\bar{\epsilon}} U_{+}(q, m(q), e) \left(\frac{de}{2\bar{\epsilon}}\right) \right\} I(q)dq \\ &\approx \int_0^1 \left\{ \int_{-\bar{\epsilon}}^{0} (1 + g(q)(e/m'(q))) \left(\frac{de}{2\bar{\epsilon}}\right) + \int_0^{\bar{\epsilon}} (1 - g(q)(e/m'(q))) \left(\frac{de}{2\bar{\epsilon}}\right) \right\} I(q)dq \\ &= \int_0^1 \left\{ 1 + \int_{-\bar{\epsilon}}^{0} g(q)(e/m'(q)) \left(\frac{de}{2\bar{\epsilon}}\right) - \int_0^{\bar{\epsilon}} g(q)(e/m'(q)) \left(\frac{de}{2\bar{\epsilon}}\right) \right\} I(q)dq \\ &= \int_0^1 \left\{ 1 - \frac{\bar{\epsilon} g(q)}{2m'(q)} \right\} I(q)dq. \end{aligned}$$

The Euler-Lagrange equation reads

$$\left\{ 1 + \frac{\bar{\epsilon} g(q)}{2m'(q)^2} \right\} I(q) = K$$

for some constant K. Equivalently,

$$\left\{ 2m'(q)^2 + \bar{\epsilon} g(q) \right\} I(q) = 2Km'(q)^2$$

or

$$\bar{\epsilon} g(q) I(q) = 2(K - I(q))m'(q)^2$$

or

$$\frac{\bar{\epsilon}g(q)I(q)}{2(K - I(q))} = m'(q)^2$$

which implies that $K > I(q)$ for all $q \in [0, 1]$.