Let q denote the state of the world, which is normally distributed with mean zero and variance one, and ϵ the noise, which is normally distribution with mean zero and precision $\tau > 0$. The sender chooses a function $m : \mathbb{R} \to \mathbb{R}$ that maps states of the world to messages. Messages are corrupted by noise:

$$\tilde{m} = m(q) + \epsilon. \tag{1}$$

Errors are more costly for some states. Let $I : \mathbb{R} \to [0, 1]$ be differentiable. The sender and receiver incur a cost

$$(q - A(\tilde{m}))^2 dI(q). \tag{2}$$

Senders face a *message budget*, in the sense that the average rate of change of the message must be finite:

$$E[m'(q)] \leq 1 \tag{3}$$

where we normalize the budget to unity for simplicity. The sender chooses a message m to minimize the cost

$$C[m] \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (q - m^{-1}(m(q) + \epsilon))^2 I(q) d\Phi(q) d\Phi(\tau\epsilon). \tag{4}$$

subject to $E[m'(q)] \leq 1$ (Equation 3).