# 1 discrete actions

Let $m : [0,1] \to [0,1]$ be the sender's message function. Let the receiver's **discrete** action $A : [0,1] \to \{0,1\}$ be given by

$$A(\tilde{m}) = \begin{cases} 1 & \text{if} \quad m^{-1}(\tilde{m}) > V \\ 0 & \text{otherwise} \end{cases}$$

where $V$ is distributed according to $G$. The expected utility—with respect to $V$—for $e > 0$ is

$$\begin{aligned} U_+(q, m(q), e) &= \underbrace{G(q)}_{A=0 \text{ and } V<q} + \underbrace{(1 - G(m^{-1}(m(q) + e)))}_{A=1 \text{ and } V \geq q} \\ &\approx G(q) + (1 - G(q + m^{-1'}(m(q))e)) \\ &= G(q) + (1 - G(q + e/m'(q))) \end{aligned}$$

while the expected utility—again, with respect to $V$—for $e \leq 0$ is

$$\begin{aligned} U_-(q, m(q), e) &= \underbrace{G(m^{-1}(m(q) + e))}_{A=0 \text{ and } V<q} + \underbrace{(1 - G(q))}_{A=1 \text{ and } V \geq q} \\ &\approx G(q + m^{-1'}(m(q))e) + (1 - G(q)) \\ &= G(q + e/m'(q)) + (1 - G(q)). \end{aligned}$$

The sender chooses $m$ to maximize total expected utility:

$$\begin{aligned} \min_m &= \int_0^1 \left\{ \int_{-\bar{\epsilon}}^0 U_+(q, m(q), e) \left( \frac{de}{2\bar{\epsilon}} \right) + \int_0^{\bar{\epsilon}} U_-(q, m(q), e) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) dq \\ &\approx \int_0^1 \left\{ \int_{-\bar{\epsilon}}^0 (G(q) + (1 - G(q + e/m'(q)))) \left( \frac{de}{2\bar{\epsilon}} \right) + \int_0^{\bar{\epsilon}} (G(q + e/m'(q)) + (1 - G(q))) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) dq \\ &= \int_0^1 \left\{ 1 - \int_{-\bar{\epsilon}}^0 G(q + e/m'(q)) \left( \frac{de}{2\bar{\epsilon}} \right) + \int_0^{\bar{\epsilon}} G(q + e/m'(q)) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) dq. \end{aligned}$$

The Euler-Lagrange formula reads:

$$-\left\{ -\int_{-\bar{\epsilon}}^0 g(q + e/m'(q)) \left( -\frac{e}{m'(q)^2} \right) \left( \frac{de}{2\bar{\epsilon}} \right) + \int_0^{\bar{\epsilon}} g(q + e/m'(q)) \left( -\frac{e}{m'(q)^2} \right) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) = K$$

for some constant $K$. Equivalently,

$$-\int_{\bar{\epsilon}}^0 e \cdot g(q + e/m'(q)) de + \int_0^{\bar{\epsilon}} e \cdot g(q + e/m'(q)) de = \frac{2\bar{\epsilon} K m'(q)^2}{I(q)}.$$

Next step, compute

$$\frac{dm'}{dI} = \frac{m''(q)}{I'(q)} \ldots$$