

Now consider an alternative utility function. The receiver has a threshold for quality  $V$  so that she receives a payoff of 1 if  $q > V$  and  $A = 1$  or if  $q < V$  and  $A = 0$ . She receives a payoff of 0 if  $q < V$  and  $A = 1$  or if  $q > V$  and  $A = 0$ . The threshold for any given receiver is unknown, and distributed according to  $G$ , with density  $g$ . An alternative interpretation is that  $V$  is known, but there are many receivers receiving the same signal, and their values of  $V$  are distributed according to  $G$ . An additional alternative interpretation is that  $q$  is itself a signal of true quality.  $V$  is known, and there are multiple reviewers, whose reviews are correlated with the signal  $q$ . This means that the threshold value of  $q$  such that the actor chooses  $A = 1$  or  $A = 0$  is a random variable with distribution  $G$ .

Suppose that the optimal response function is  $A(\tilde{m}) = 1$  if  $m^{-1}(\tilde{m}) > V$  and  $A(\tilde{m}) = 0$  if  $m^{-1}(\tilde{m}) < V$ . We can calculate the expected utility given  $q$  as comprised of three components. First, if  $\tilde{m} < m(V) - \bar{\varepsilon}$ , then the receiver will always take the action  $A = 0$ . If  $\tilde{m} > m(V) + \bar{\varepsilon}$ , then she will always take action  $A = 1$ . Only if  $m \in [m(V) - \bar{\varepsilon}, m(V) + \bar{\varepsilon}]$  is there a question of which action she will take.

Because  $\varepsilon$  is uniform over  $[-\bar{\varepsilon}, \bar{\varepsilon}]$ , and because  $m(q)$  is locally linear, the probability that she takes the correct action is  $pr(A = 0 \mid V > q) = pr(A = 1 \mid V < q) = \frac{(V-q)m'(q)+\bar{\varepsilon}}{2\bar{\varepsilon}}$  for small  $\varepsilon$  and  $V \in [q - \frac{\bar{\varepsilon}}{m'(q)}, q + \frac{\bar{\varepsilon}}{m'(q)}]$ . For a given message function  $m'(q)$ , and a given  $q$ , the expected utility is:

$$\begin{aligned} E(U \mid q, m'(q)) &= I(q) \int_0^{q - \frac{\bar{\varepsilon}}{m'(q)}} g(V) dV \\ &\quad + I(q) \int_{q - \frac{\bar{\varepsilon}}{m'(q)}}^q \frac{(q - V) m'(q) + \bar{\varepsilon}}{2\bar{\varepsilon}} g(V) dV \\ &\quad + I(q) \int_q^{q + \frac{\bar{\varepsilon}}{m'(q)}} \frac{(V - q) m'(q) + \bar{\varepsilon}}{2\bar{\varepsilon}} g(V) dV \\ &\quad + I(q) \int_{q + \frac{\bar{\varepsilon}}{m'(q)}}^1 g(V) dV \\ &= I(q) \left[ \left( G \left( q + \frac{\bar{\varepsilon}}{m'(q)} \right) - G \left( q - \frac{\bar{\varepsilon}}{m'(q)} \right) \right) + 2 \int_q^{q + \frac{\bar{\varepsilon}}{m'(q)}} \frac{(V - q) m'(q) + \bar{\varepsilon}}{2\bar{\varepsilon}} g(V) dV \right] \end{aligned}$$

We used a first-order approximation for  $m(q)$ , so we can also use one for  $g(V)$  so that  $G(x) = \alpha G(x - V)$  in the neighborhood of  $V$ . Then  $g(x) = g(V)$  in the neighborhood of  $V$  and we have

$$E(U \mid q, m'(q)) = I(q) \left[ \left( G \left( q + \frac{\bar{\varepsilon}}{m'(q)} \right) - G \left( q - \frac{\bar{\varepsilon}}{m'(q)} \right) \right) + \frac{1}{2} g(q) dx \right]$$