Let $x : [-\overline{e}, \overline{e}] \to [0,1]$ be given by

$$x(e) = \frac{1}{2}\left[1 + \frac{e}{\overline{e}}\right] \tag{1}$$

Define

$$I_{(\alpha,\beta,\gamma)}(\widetilde{m}) \equiv \int_{\alpha}^{\beta} q^{\gamma} f(\tilde{m} - q)dq \tag{2}$$

and

$$A(\widetilde{m}) = \begin{cases} \overline{a}(\widetilde{m}) & \text{if } 1 - \overline{e} < \widetilde{m} \leq 1 + \overline{e} \\ a(\widetilde{m}) & \text{if } \overline{e} < \widetilde{m} \leq 1 - \overline{e} \\ \underline{a}(\widetilde{m}) & \text{if } -\overline{e} \leq \widetilde{m} \leq \overline{e} \end{cases} \tag{3}$$

where

$$\overline{a}(\widetilde{m}) = \frac{I_{(\widetilde{m}-\bar{e},1,1)}(\widetilde{m})}{I_{(\widetilde{m}-\bar{e},1,0)}(\widetilde{m})} \tag{4}$$

$$a(\widetilde{m}) = \frac{I_{(\widetilde{m}-\bar{e},\widetilde{m}+\bar{e},1)}(\widetilde{m})}{I_{(\widetilde{m}-\bar{e},\widetilde{m}+\bar{e},0)}(\widetilde{m})} \tag{5}$$

$$\underline{a}(\widetilde{m}) = \frac{I_{(0,\widetilde{m}+\bar{e},1)}(\widetilde{m})}{I_{(0,\widetilde{m}+\bar{e},0)}(\widetilde{m})} \tag{6}$$

Note that the normalizing constant cancels out when computing the conditional expectation. We need three sets of weights and knots. The cost is given by

$$C[m_i] = \int_{-\bar{e}}^{1+\bar{e}} \int_{q_-(\widetilde{m})}^{q_+(\widetilde{m})} (q - A(\widetilde{m}))^2 f(\widetilde{m} - q)dqd\widetilde{m}. \tag{7}$$

where

$$q_+(\widetilde{m}) = \min\{\widetilde{m} + \bar{e}, 1\} \tag{8}$$

$$q_-(\widetilde{m}) = \max\{\widetilde{m} - \bar{e}, 0\} \tag{9}$$

Define

$$\overline{z} = \int_{1-\bar{e}}^{1+\bar{e}} \int_{\widetilde{m}-\bar{e}}^{1} (q - \overline{a}(\widetilde{m}))^2 f(\widetilde{m} - q)dqd\widetilde{m} \tag{10}$$

$$z = \int_{\bar{e}}^{1-\bar{e}} \int_{\widetilde{m}-\bar{e}}^{\widetilde{m}+\bar{e}} (q - a(\widetilde{m}))^2 f(\widetilde{m} - q)dqd\widetilde{m} \tag{11}$$

$$\underline{z} = \int_{-\bar{e}}^{\bar{e}} \int_{0}^{\widetilde{m}+\bar{e}} (q - \underline{a}(\widetilde{m}))^2 f(\widetilde{m} - q)dqd\widetilde{m} \tag{12}$$

Consider, as an example, the Beta PDF:

$$f(e) = x(e)^{a-1}(1 - x(e))^{b-1} \tag{13}$$

for constants $a > 0$ and $b > 0$.