

# 1 discrete actions

Let  $m : [0, 1] \rightarrow [0, 1]$  be the sender's message function. Let the receiver's **discrete** action  $A : [0, 1] \rightarrow \{0, 1\}$  be given by

$$A(\tilde{m}) = \begin{cases} 1 & \text{if } m^{-1}(\tilde{m}) > V \\ 0 & \text{otherwise} \end{cases}$$

where  $V$  is distributed according to  $G$ . The expected utility—with respect to  $V$ —for  $e > 0$  is

$$\begin{aligned} U_+(q, m(q), e) &= \underbrace{G(q)}_{A=0 \text{ and } V < q} + \underbrace{(1 - G(m^{-1}(m(q) + e)))}_{A=1 \text{ and } V \geq q} \\ &\approx G(q) + (1 - G(q + m^{-1'}(m(q))e)) \\ &= G(q) + (1 - G(q + e/m'(q))) \\ &\approx G(q) + (1 - G(q) - g(q)(e/m'(q))) \\ &= 1 - g(q)(e/m'(q)) \end{aligned}$$

while the expected utility—again, with respect to  $V$ —for  $e \leq 0$  is

$$\begin{aligned} U_-(q, m(q), e) &= \underbrace{G(m^{-1}(m(q) + e))}_{A=0 \text{ and } V < q} + \underbrace{(1 - G(q))}_{A=1 \text{ and } V \geq q} \\ &\approx G(q + m^{-1'}(m(q))e) + (1 - G(q)) \\ &= G(q + e/m'(q)) + (1 - G(q)) \\ &\approx G(q) + g(q)(e/m'(q)) + (1 - G(q)) \\ &= 1 + g(q)(e/m'(q)). \end{aligned}$$

The sender chooses  $m$  to maximize total expected utility:

$$\begin{aligned} \min_m &= \int_0^1 \left\{ \int_{-\bar{\epsilon}}^0 U_-(q, m(q), e) \left( \frac{de}{2\bar{\epsilon}} \right) + \int_0^{\bar{\epsilon}} U_+(q, m(q), e) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) dq \\ &\approx \int_0^1 \left\{ \int_{-\bar{\epsilon}}^0 (1 + g(q)(e/m'(q))) \left( \frac{de}{2\bar{\epsilon}} \right) + \int_0^{\bar{\epsilon}} (1 - g(q)(e/m'(q))) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) dq \\ &= \int_0^1 \left\{ 1 + \int_{-\bar{\epsilon}}^0 g(q)(e/m'(q)) \left( \frac{de}{2\bar{\epsilon}} \right) - \int_0^{\bar{\epsilon}} g(q)(e/m'(q)) \left( \frac{de}{2\bar{\epsilon}} \right) \right\} I(q) dq \\ &= \int_0^1 \left\{ 1 - \frac{\bar{\epsilon}g(q)}{2m'(q)} \right\} I(q) dq. \end{aligned}$$

The Euler-Lagrange equation reads

$$\left\{ 1 + \frac{\bar{\epsilon}g(q)}{2m'(q)^2} \right\} I(q) = K$$

for some constant  $K$ . Equivalently,

$$\{2m'(q)^2 + \bar{\epsilon}g(q)\} I(q) = 2Km'(q)^2$$

or

$$\bar{\epsilon}g(q)I(q) = 2(K - I(q))m'(q)^2$$

or

$$\frac{\bar{\epsilon}g(q)I(q)}{2(K-I(q))} = m'(q)^2$$