Benchmark case

I think it is worthwhile thinking about the following benchmark case. Consider a situation in which there is a *single* agent who observes the state with noise; that is, the agent observes $\tilde{m} = q + \varepsilon$, where $\varepsilon \in [-\bar{\varepsilon}, \bar{\varepsilon}]$. Let $h(q \mid \tilde{m})$ be the conditional distribution of q given m. Assuming q is sufficiently away from the corners 0 and 1, the support of this distribution is $[\tilde{m} - \bar{\varepsilon}, \tilde{m} + \bar{\varepsilon}]$. The optimal action of the agent is then

$$A^b(\tilde{m}, \bar{\varepsilon}) \quad = \quad \arg\min_A - \int_{\tilde{m}-\bar{\varepsilon}}^{\tilde{m}+\bar{\varepsilon}} I(q)L(A-q)h(q \mid \tilde{m})dq. \tag{1}$$

(The subscript ^{b} refers to "benchmark".) The strategy A^b, which depends on $\tilde{m}$, maps to an induced distribution of actions given a true state q. Let's call this induced distribution $\alpha(q, \bar{\varepsilon})$. The first-best action is $A^f(q) = q$ for all q. Let δ_q be the distribution that has mass 1 at q and zero mass everywhere else. Now, as $\bar{\varepsilon} \to 0$, in the limit the support of $h(q \mid \tilde{m})$ collapses to q. It seems therefore that it must be that as $\bar{\varepsilon} \to 0$, $\alpha(q) \to \delta_q$ for all q, so that we achieve the first-best outcome.

Now, let's turn to the case in the paper, in which agent 1 (sender) observes the true state and sends a message to agent 2 (receiver). Receiver observes $\tilde{m} = m + \varepsilon$. Let A^r be the receiver's action in some equilibrium of the game. The first question that comes up is: Does there exist an equilibrium in which the expected utility of the receiver and sender is *strictly greater* than in the benchmark case above?

My conjecture is no, but if this conjecture is incorrect, that would be a very interesting point to make.

Assume the conjecture is correct, and let's call the outcome induced by the strategy A^b the "second-best" outcome. The next question is: Does there exist an equilibrium in which the second-best outcome is induced? It seems to me that there are at least two possibilities to consider, and indeed that thought process in general may (or may not) lead one to a continuum of such equilibria.

1. Fix the receiver's strategy at $A(\tilde{m}) = \tilde{m}$. Given this strategy, determine what reporting strategy $m(q)$ the sender must follow so that the induced distribution over actions corresponds to $\alpha(q, \bar{\varepsilon})$ (if this problem admits a feasible solution). On the surface, it seems possible that either this problem does not have a feasible solution, or that the conjectured $A(\cdot)$ strategy is not a best response to the derived reporting strategy $m(\cdot)$.

2. Fix the sender's strategy at $m(q) = q$. Then, the receiver is essentially solving the single-agent problem above, so trivially must be playing A^b, leading to a distribution over actions given by $\alpha(q, \bar{\varepsilon})$. Here, it is hard to see why this pair of strategies cannot be an equilibrium. If it is not, it would suggest that in the single-agent problem, the agent basically wants to lie to themselves about what their signal is.

The authors consider a class of strategies in which $A(\tilde{m}) = m^{-1}(\tilde{m})$. That's a more sophisticated receiver strategy than in case 1 above. However, note that in case 2, it follows that $m^{-1}(\tilde{m}) = \tilde{m}$, whereas it is not generally the case that $A^b = \tilde{m}$. If the class of strategies considered by the authors leads to greater welfare than in case 2 above, that's absolutely worth showing, and indeed the paper should be re-written to make that the main point of the paper. In this case, it is also worth commenting on whether the equilibrium is unique in the class of equilibria that yield that level of welfare.

If not (i.e., if the welfare in the authors' equilibrium is the same as in case 2), it seems that the authors have selected a particular equilibrium out of a set of equilibria that achieve the same welfare. Then, there is more work to be done to argue that the authors' case is of interest. The authors' equilibrium says "Fix the action strategy as if the message was received without error, and modify the reporting strategy accordingly". Case 2 says "Fix the reporting strategy to be truthful, assume the message was received with error, and modify the action strategy accordingly". Potentially, there than be a continuum of equilibria in which both reporting and action strategies are suitably modified. The question then turns to "Why is the authors' equilibrium the more interesting case?" One may be able to make an argument based on relative computational costs for sender and receiver (e.g., in the Yelp example, the senders have voluntarily chosen to put in the time and effort to provide a review; the receiver quickly looks at outcomes and chooses a restaurant). But an argument does need to be made here.

My comments below are restricted to the continuous signal—continuous action case (Section 2.1). I agree with both referees that there are too many models in the paper, and it is best to focus on one.

Lemma 1

Lemma 1 is imprecise in several ways. First, is this a statement about equilibrium or about the receiver's best response function? It seems to me it has already fixed some reporting strategy $m(\cdot)$ for the sender. Otherwise, it's hard to understand the statement after the lemma that "the distribution of q around $m^{-1}(\tilde{m})$ is symmetric". In fact, there is already an assumption that $m(q)$ is invertible, so presumably strictly monotone. Similarly, requiring that the conditional distribution of q given m be uniform requires a specific m. But then, we haven't been told what $m(\cdot)$ strategy has been fixed, other than the generic statement at the top of page 16 that "$m'(q)$ should be high." Therefore, it's difficult to follow the Lemma at this stage of the paper.

Second, as R1 mentions, it is worth being precise about what "approximately" means. My guess is that here you want to say something like the following. Let (m^*, A^*) be a pair of equilibrium strategies for sender and receiver (where it seems that m^* is a specific equlibrium strategy that you have in mind). Let $A^r(\tilde{m}) = m^{-1}(m)$ as in the statement of the proposition. Then, $E(U \mid (m^*, A^r)) \to E(U \mid (m^*, A^*)$ as $\bar{\varepsilon} \to 0$.

Rest of Section 2.1

The approximation idea shows up throughout, so the results obtain in the limiting case that $\bar{\varepsilon} \to 0$. R1's comment (Model, point 2) is very pertinent here. The limiting case is a little uninteresting, so to the extent that you can show something in the case in which noise remains positive, the paper becomes stronger. One possibility is to show in a numerical example or two the welfare gain from your equilibrium relative to the case in which (i) the message is $m(q) = q$ (ii) the receiver understands they have a garbled message, and adjust actions to be a best response.

Again, there appear to be some conditions imposed on $m(q)$ (e.g., in Lemma 2, $m(q)$ is assumed to be differentiable).

My suggestion for this section is as follows. Your main proposition here should be that the optimal message function characterized in the current Proposition 1 and the message strategy $A(\tilde{m}) = m^{-1}(\tilde{m})$ constitute an equilibrium or perhaps an approximate equilibrium (the formal game theoretic notion for approximate equilibrium is, of course, epsilon-equilibrium; given that you have defined ε to be the error in the transmission, it's not clear you can consistently use that language without causing some confusion). To show that, you need to show that each player's strategy is a best response (or an epsilon-best response) to the other player's strategy. I think you may be able to write this section in a more clean way by saying at the outset that you will look for an equilibrium (or approximate equilibrium) in which the receiver plays $A(\tilde{m}) = m^{-1}(\tilde{m})$, and determine the best response strategy of the sender. Then, at the end, you can show that indeed $A(\tilde{m}) = m^{-1}(\tilde{m})$ is a best response (or approximate best response) of the receiver.

Other Comments

R1 comments that for a finance journal, it's worth having an interesting finance application upfront. R2 adds that the link to work in information theory should be clarified. I agree with these points.