Suppose $\pi > 0$, $\lambda > 0$, $r > 0$, $Y > 0$, $c > 0$, and $Q \in (0,1)$ are known parameters and that

$$g(t) = \frac{Qe^{-\lambda t}}{Qe^{-\lambda t} + 1 - Q}.$$

In addition, assume that

$$Q\left(\frac{\lambda Y}{r}\right) - c > 0.$$

Our problem (which studies optimal trading behavior when sellers learn privately over time about the quality of their assets) can then be summarized by the following system of differential equations

$$g'(t) = -\lambda g(t)(1 - g(t)) \tag{1}$$

$$S'(t) = (r + g(t)\lambda + \pi)S(t) - (1 - g(t))\lambda Y \tag{2}$$

$$q'(t) = \frac{r}{\lambda Y}\left[r\left(q(t)\frac{\lambda Y}{r} - c\right) - g(t)\lambda(Y + S(t))\right] \tag{3}$$

$$\Theta'(t) = \lambda\left(\Theta(t) + \Delta(t)\right) \tag{4}$$

$$\Delta'(t) = \pi\left[\Delta(t) - \left(\frac{g(t)(1 - q(t))}{q(t) - g(t)}\right)\Theta(t)\right], \tag{5}$$

with the following boundary conditions

$$g(0) = Q \tag{6}$$

$$S(T) = \frac{rc}{r + \pi} \tag{7}$$

$$q(\hat{t}) = Q \tag{8}$$

$$q(T) = 1 \tag{9}$$

$$\Theta(\hat{t}) = e^{\lambda\hat{t}} - 1 \tag{10}$$

$$\Delta(\hat{t}) = 1 \tag{11}$$

$$\Delta(T) = 0, \tag{12}$$

for some unknown times $\hat{t}$ and T satisfying $0 \leq \hat{t} < T$. Our problem is to show that, regardless of parameters, there always exist times $\hat{t}$ and T such that the six boundary conditions are satisfied simultaneously.