

SIMD Acceleration for Index Structures

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Abstract—

• summary:

Give short an overview of SIMD and modern index structures

Explain what are the problems of the “old” index structures made for disk-based database systems

Explain which approaches were made to adapt index structures to modern systems and what they have in common and what are differences

• Why is this work important:

Give a state of current development of the index structures

Collect common approaches to adapt other index structures **TODO: ReThink**

• K-ary search trees, FAST, VAST and ART compared

• Contribution: What are important approaches used by different implementations to adapt index structures to modern systems

Index Terms—SIMD, index

I. INTRODUCTION

After decades of creating and improving index structures for disk-based database systems, nowadays even large databases fit into the main memory. Since index structures like the B^+ -tree or the radix tree have an important part in database systems to realise scan or range-based search operations, these index structures experienced many adaptations to fulfill the needs of modern database systems. Instead of overcoming the bottleneck of IO-operations from disk to RAM, the target of modern index structures is to improve the usage of CPU cache and processor architectures.

Several index structures have already shown that the bottleneck from RAM to CPU can be overcome using Single Instruction Multiple Data (SIMD) [1] operations. These index structures include: the K-ary Search Tree (Seg-tree) [3], Adapted Radix Tree (ART) [4], Fast Architecture Sensitive Tree (FAST) [6], and Vector-Advanced and Compressed Structure Tree (VAST) [5]. As the authors of VAST-Tree show, important causes for increased runtime are cache misses and branch mispredictions. To overcome branch mispredictions and to decrease CPU cycles, SIMD is used in modern index structures for tree traversal [2]. The authors of the k-ary search show how to use SIMD to compare multiple keys in one CPU cycle. To decrease cache misses, the authors of FAST and ART show how to adapt index structures to the cache line size.

All approaches use SIMD only for key comparison within tree traversal and try to decrease the key size to fit more keys into one SIMD register. Therefore FAST and Seg-tree only

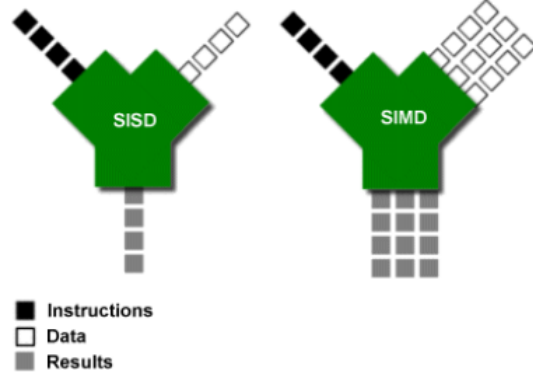


Fig. 1. Coherence between data, instructions, and the results of SISD and SIMD.

provide implementations for search algorithms. We consider the design approaches of VAST and ART to implement operations like update and insert and name ideas to use SIMD for them. Consequently, with this work we make the following contributions:

- We compare different adaptations of index structures to fulfill requirements of modern database systems
- We highlight the usage of SIMD and the cache line adaptations in all approaches
- We show opportunities for adaption for other approaches to use SIMD

We organized the rest of the paper as follows. In Section 2, we give the preliminaries for SIMD in general and for the use in index structures. In Section 3, we analyse the different approaches of adapted index structures and evaluate the comparison in Section 4. In Section 5, we name related work. In Section 6, we present our conclusion and describe future work in Section 7.

II. SIMD-STYLE PROCESSING

A common approach of decreasing CPU cycles for algorithms is to adapt the algorithm to pipelining. While one instruction is executed, the next instruction is already fetched. This approach executes one instruction on one data item, called Single Instruction Single Data (SISD). In contrast to execute one operation on one data item after another, the idea of SIMD is to execute a single instruction on multiple data. In Figure 1 we show the coherence between data, instructions, and the results of SIMD and SISD. Modern CPUs have additional

TABLE I
SIMD INSTRUCTIONS FROM STREAMING SIMD EXTENSIONS 2 (SSE2) TODO: MAYBE ADD SOME MORE :)

SIMD instruction	Explanation
<code>_mm128i _mm_load_si128 (_mm128i *p)</code>	Loads a 128-bit value. Returns the value loaded into a variable representing a register.
<code>_mm128i _mm_cmpgt_epi32 (_mm128i a, _mm128i b)</code>	Compares 4 signed 32-bit integers in a and 4 signed 32-bit integers in b for greater-than.

SIMD registers along with an additional instruction set adapted to process multiple data items in parallel. In Table 1, we show some SIMD instructions from Streaming SIMD Extensions 2 (SSE2). We consider `_mm_cmpgt_epi32` to show how SIMD works. After loading 4 different signed 32-bit integers in a and 4 equal signed 32-bit integers in b, we compare them using `_mm_cmpgt_epi32`. The comparison returns a bitmask showing which of the search keys in a is greater than the one in b. This example increases the performance of comparison by times of four.

Consequently, the main advantage of SIMD is to process multiple data parallel in contrast to pipelining and SISD. The main restriction of SIMD instructions is that a sequential load of data is required. To load data into a SIMD register, the data has to be stored consecutively in main memory. Additionally, the size of SIMD registers is limited. Therefore processing data types of the common size of 64-bit and more lead to a small performance increase since only few data items are processed with a single instruction.

Polychroniou et al. [7] show two general approaches to use SIMD in in-memory databases, horizontal and vertical vector processing. They name the comparison of one search key to multiple other keys horizontal vectorization, whereas processing a different input key per vector lane is named vertical vectorization.

Since FAST, Seg-Tree, ART and VAST only use horizontal vectorization, we focus on this approach. For example, Zeuch et al. [3] use 128-bit SIMD registers and adjusted SIMD operations to load data into a register and to compare the data of one SIMD register with another. A 128-bit SIMD register processes sixteen 8-bit or eight 16-bit data items with one instruction. In Table 2 TODO: Insert Table! we show a comparison of key size and the number of keys that can be processed parallel with one SIMD instruction.

III. ADAPTED TREE STRUCTURES

In this section we present the previously mentioned index structures Seg-Tree, FAST, ART, and VAST. We consider the adaptations made compared to the base index structure, the usage of SIMD, and the performance gain presented by the authors of the certain index structures.

A. Seg-Tree

Zeuch et al. adapted the B^+ -Tree by having a k-ary search tree as each inner node, called segment, and perform a k-ary search on each segment. In Figure 2, we show the adaption of nodes made by Zeuch et al. for Seg-Tree. The k-ary search bases on the binary search but divides the search space into k partitions with k-1 separators. Compared to binary search



Fig. 2. Inner node format of Seg-Tree

the k-ary search reduces the complexity from $O(\log 2n)$ to $O(\log kn)$. They consider m as the most bits to represent a data type and $|SIMD|$ as the size of SIMD-register, called SIMD bandwidth. Then, $k = \frac{|SIMD|}{m}$ defines the number of partitions for the k-ary search.

As mentioned before, each segment of the Seg-Tree is a k-ary search tree. To perform a k-ary search on a segment, Zeuch et al. linearise the elements of the segment. They show two algorithms for linearisation, breadth-first search and depth-first search. Because of the condition $k = \frac{|SIMD|}{m}$, each partition of the k-ary search fits into a SIMD register and is compared to the search key. A perfect k-ary search tree contains $S_{max} = k^h - 1$ keys for an integer $h > 0$. The considered search algorithm only works for sequences with a multiple of $k - 1$ keys. In case of sequences with less than a multiple of $k - 1$ keys, they replenish the sequence with elements having the value $k_{max} + 1$ for the maximal key value k_{max} in the sequence. Consequently, the adapted search algorithm works for sequences with less than a multiple of $k - 1$ keys.

The performance of Seg-Tree depends on k-ary search. The smaller a key the more keys are compared parallel. According to the relevance of 32 and 64-bit data types in modern systems, the k-ary search performance increases only by the factor of four for 32-bit types and two for 64-bit types.

Zeuch et al. also show the k-ary search on an adapted prefix tree (*trie for short*) called Seg-Trie. A trie is a search tree where each node stores a part of the key. Each node is again designed as a k-ary search tree. Complete keys are



Fig. 3. (a) Node indices (=memory locations) of the binary tree (b) Rearranged nodes with SIMD blocking (c) Index tree blocked in three-level hierarchy first-level page blocking, second-level cache line blocking, third-level SIMD blocking. Adapted from Zeuch et al. [3].

stored in leaf nodes or are build by concatenating partial keys from the root node to a leaf node. This approach benefits of the separation of the keys in different levels of the tree. Consequently, the compare keys are smaller and more keys can be compared in parallel. The Seg-Trie_L is defined as a balanced trie where each node on each level contains one part of the key with L Bits. The tree has $r = \frac{m}{L}$ levels (E_0, E_1, \dots, E_r), where m is the number of most bits to represent the data type.

To perform a tree traversal on the Seg-Trie, the search key is split into r segments and each segment r_i is compared to the level E_i . If a matching partial key is found in one node of E_i , the search continues at the referenced node for the partial key. If no match of the partial key is found, the Seg-Trie does not contain the search key and the search is finished. Consequently, the advantage of Seg-Trie against tree structures is the reduced comparison effort for non-existing key segments.

B. FAST

Changkyu et al. adapted a binary tree to optimize for architecture features like page size, cache line size, and SIMD bandwidth called Fast Architecture Sensitive Tree. In contrast to Seg-Tree, FAST is also adapted to disk-based database systems. Changkyu et al. show the performance increase because of decreasing cache misses and better cache line usage. In order to optimize for architectural features, tree nodes are rearranged for hierarchical blocking. In Figure 3, we show an index tree blocked in three-level hierarchy introduced by Changkyu et al.

According to Figure 3, we define $d_K = 2$ as the depth of a SIMD block, $d_L = 5$ as the depth of a page block, $N_K = 2^{d_K} - 1$ as the number of keys that fit into a SIMD register, and $N_L = 2^{d_L} - 1$ as the number of keys that fit into a cache line. To rearrange the nodes, Changkyu et al. sorted the nodes, as shown in Figure 3 (a). Afterwards, they combine the first N_K keys and lay them out in breadth-first fashion (the first key as root, the next $N_K - 1$ keys as children on depth 2). These first N_K represent a SIMD block. They continue to lay out the next keys the same fashion, building the SIMD blocks with root keys 3, 6, 9, and 12, as shown in Figure 3 (b).

This process continues for all sub-trees that are completely within the first d_L levels from the root. If a sub-tree does not completely fit in the first d_L levels, the keys are laid out according to the appropriate number of levels ($d_L \% d_K$) as shown in depth 4 in Figure 3 (b). The first N_L keys represent a cache line block. We define $d_P = 31$ as the depth of a page block, $N_P = 2^{d_P} - 1$ as the number of keys that fit into a page. The procedure of hierarchical blocking continues analogue until the N_P keys are laid out, representing a page block. After building one page block, the next page block is build up the same way. In Figure 3 (c) we show the different blocks in a hierarchical blocked index tree.

Changkyu et al. present implementations for building and traversing the tree adapted for CPU and also for GPU. Building up the tree, SIMD is used to computing the index for N_K keys within the SIMD level block in parallel, achieving around 2X SIMD scaling as compared to the scalar code. With a Core i7 processor, the runtime of building a FAST tree with 64M tuples is less than 0.1 seconds. Traversing the tree, they compare one search key to multiple keys of the index structure. To use the complete bandwidth of cache and main memory within the search, blocks are loaded completely into associated memories from large blocks to small blocks. For a page block, at first the page is loaded into main memory. Then cache line blocks are loaded one after another in the cache and for each cache line block, the included SIMD blocks are loaded into the SIMD register. All keys of this SIMD block are compared with one SIMD instruction. After looking up the resulting bitmask, the corresponding SIMD block is loaded (including the load of the surrounding larger blocks) until the key is found or the last level of the index structure is reached.

Changkyu et al. consider the search performance as queries per second. The CPU search performance on the Core i7 with 64M 32-bit (key, rowID) pairs is 5X faster than the best reported number [8 TODO ref], resulting in a throughput of 50M search queries per second. The GPU search on the GTX 280 is around 1.7X faster than the best reported number, resulting in 85M search queries per second. Considering larger index structures, the GPU performance increase exceeds the

CPU performance increase, because TLB and cache misses grow up and the CPU search becomes memory bandwidth bound.

C. ART

Leis et al. adapted a radix tree for efficient indexing in main-memory database systems called Adaptive Radix Tree [4]. The height of a radix tree depends on the chunk size of the keys stored in each node, ART divides keys into 8-bit chunks. They differentiate between inner nodes and leaf nodes and adapt each of them in a different way.

Instead of using a constant node size for each inner node, they present four types of nodes with different numbers of keys and children. The types of nodes, sorted ascending by their size, are *Node4*, *Node16*, *Node48*, and *Node256*. When the capacity of a node is exhausted due to insertion, it is replaced by a larger node type. When a node becomes underfull due to key removal, it is replaced by a smaller node type. In Figure 4, we show these node types containing keys that are mapped to subtrees.

Node4: The smallest node type consists of one array with up to four sorted keys and another array with up to four children. The keys and pointers are stored at corresponding positions.

Node16: This node type consists of arrays of 16 keys and 16 children, storing keys and children analogue to *Node4*.

Node48: To avoid searching keys in many elements, this node type does not store the keys explicitly. Instead, an array with 256 elements is used. This array can be indexed with key bytes directly. It stores indexes into a second array with the size of 48 elements containing the pointers to child nodes.

Node256: The largest node type is simply an array of 256 pointers. Consequently, the next node can be found efficiently using a single lookup of the key byte in that array.

For leaf nodes, Leis et al. use a mix of pointer and value slots in an array. If the value fits within the slot, they store it directly in the slot. Otherwise, a pointer to the value is stored. They tag each element with an additional bit indicating if a pointer or a value is stored.

According to FAST and Seg-Tree, Leis et al. use SIMD within the tree traversal. They use horizontal vectorization, comparing the search key against multiple keys of a node. In contrast to Zeuch et al., using horizontal vectorization for each inner node, Leis et al. only compare the keys of nodes with type *Node16* in parallel. Therefore, they replicate the search key 16 times and compare these against all keys of nodes of type *Node16*.

In contrast to FAST and Seg-Tree, the goal of ART is also to reduce space consumption. Leis et al. use lazy expansion and path compression. The first technique, lazy expansion, is to create inner nodes only if they are required to distinguish at least two leaf nodes. The second technique, path compression, removes all inner nodes that have only one child.

Since SIMD is only used in the tree search, we don't consider the performance increases of ART in insert and update operations. Leis et al. show, that looking up random keys using ART is faster than Seg-Tree and FAST, because ART has

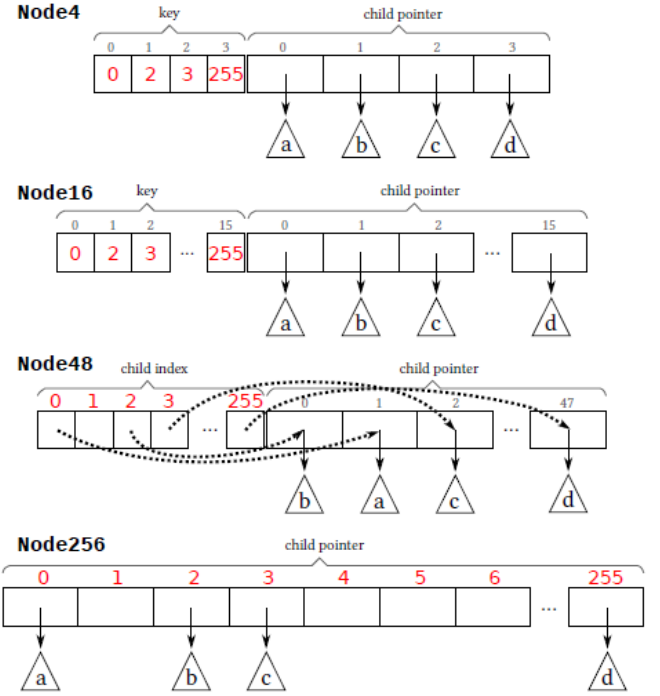


Fig. 4. Inner nodes of ART. The partial keys 0, 2, 3, and 255 are mapped to the subtrees a, b, c, and d. Adapted from Leis et al. [4].

less cache misses and less CPU cycles per comparison. They consider the performance increases for dense and sparse keys, whereas ART works better with dense keys. Also they show that a span of 8 results in better performance than a smaller span.

D. VAST

Yamamuro et al. extended FAST building an index structure called Vector-Advanced and Compressed Structure Tree (VAST) [5]. They adapt the blocking and aligning structure of FAST and add compression of nodes along with improved SIMD usage. In Figure 5, we show the structure of the VAST-Tree including layers and different blocking elements. The top layer P_{32} uses the techniques of FAST with the same SIMD, cache line and page blocking with 32 bit keys.

In order to decrease the size of the index structure to better fit into main memory, the middle and bottom layers use compressed nodes. In the layer P_{16} the 32 bit keys of each node are compressed to 16 bit keys using lossy compression. Analogue, in P_8 the keys are compressed to 8 bit with lossy compression. In the leaf nodes Yamamuro et al. decrease the node size with the lossless compression algorithm *P4Delta*, which a good balance between compression ratio and decompression speed. If an error occurs due to information loss, they present an algorithm for error correction.

Along with the other considered index structures, Yamamuro et al. compare multiple keys to one search key with SIMD in the tree traversal. Due to the key compression of nodes, the VAST-Tree compares more keys in parallel

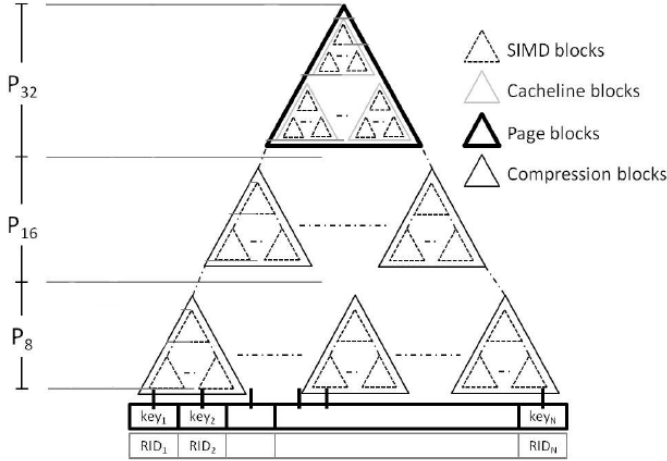


Fig. 5. An overview of VAST-Tree. The top layer (P_{32}) of VAST-Tree uses FAST techniques, and VAST-Tree compresses the middle and the bottom layers (P_{16} and P_8) of the trees. Moreover, keys in leaf nodes ($key_1, key_2, \dots, key_N$) are compressed by using lossless compression. Adapted from Yamamuro et al. [5].

than FAST. Additionally, they reduce branch misses with an adapted SIMD usage. They use addition and multiplication operations on the results of a SIMD key comparison, instead of if-then paths, to find the next node in tree traversal.

Due to lossy and lossless compression of the majority of nodes, Yamamuro et al. present a 95% less space consumption of the VAST-Tree compared to a binary tree or FAST. We consider an index with 2^{32} keys. They reach up to 6.0 and 1.24 times performance increase compared to a binary tree and FAST. Although errors occur due to lossy compression, the error correction does not take a major influence on the traversal speed.

IV. EVALUATION

In this section, we compare the adaptations made in Seg-Tree, FAST, ART, and VAST to increase performance and show differences. We focus on the usage of SIMD within the adapted index structures. All considered adaptations have the following points in common:

- Keep the key size as small as possible, to compare more keys in parallel
- Adapt node sizes and types for a better usage of SIMD registers and cache lines
- Use SIMD instructions to compare the search key with multiple keys of the index

Before looking at the differences, we go into detail for each entry of the list. At first, keeping the key size as small as possible speeds up the search performance, because more keys can be compared with a single SIMD instruction. Additionally, the smaller the keys are, the more keys fit into the cache line. Since tries or radix trees, depending on their span, have small chunks of the search key in each node, SIMD speeds up the comparison performance by a considerable factor. If keys are stored completely in one node, like in Seg-Tree and FAST,

Yamamuro et al. show that compression of keys results in better search performance [5].

Secondly, all index structures are adapted to better use SIMD and cache lines. Zeuch et al. linearise keys to a multiple of the SIMD bandwidth with the Seg-Tree, Changkyu et al. segmented FAST into SIMD, cache, and page blocks. Leis et al. use the node type *Node16* containing 16 keys that are compared in parallel with one SIMD instruction.

At least, we consider the usage of SIMD of the index structures. Segmenting the index structure is important to reach a better performance increase with SIMD instructions. Since the data for a SIMD operation has to be stored sequentially in main memory, a good segmenting and blocking strategy is necessary. In Section 2, we present the vertical vectorization as alternative to horizontal vectorization. Due to the necessary of sequential store of the data in main memory, comparing a different search key per vector lane is not applicable for SIMD instructions in tree traversal. Therefore, all considered adaptations use horizontal vectorization.

The traversal speed also depends on the evaluation of a comparison with SIMD operations. Changkyu et al. use if-then paths to evaluate the bitmask of a SIMD comparison, which leads to more branch misses compared to the approach of Yamamuro et al. They perform addition and multiplication operations on the bitmask to avoid branch misses for better search performance.

Additionally, we consider the performance of the adapted index structures. The performance increases highly depend on the underlying hardware, the data types chosen for the keys, and the amount of keys inserted in the index structure. Therefore, comparing all structures directly does not lead to reasonable results. Although, Yamamuro et al. and Leis et al. compare the VAST-Tree [5] and ART [4] to FAST, they only consider specific key sizes and data types.

Furthermore, we present the following differences in the adaptations of the index structures:

- Usage and type of compression
- FAST and Seg-Tree are read-only, ART and FAST implement update and delete operations
- Usage of GPU instead of CPU calculation

We presented above that compression leads to better search performance and less space consumption. Anyway, different compression approaches are used by the index structures. We consider two compression techniques, block and path compression. Yamamuro et al. use lossy block compression to decrease the key size of blocks. Consequently, they compare more keys with one SIMD instruction, whereas performance decrease of the occurring errors is smaller than the performance increase of the compression. Instead of compressing blocks, Seg-Trie and ART use path compression to decrease the tree height. Consequently, less nodes are passed to reach the matching leaf node, if it exists.

Since all considered index structures use SIMD only for tree traversal, Zeuch et al. and Changkyu et al. present the Seg-Tree/Trie and FAST as read-only, which means the don't show adapted update or delete operations. For statistical usage or

data warehouses, update and delete operations are not required and read-only index structures serve the purpose. To fulfill the requirements of general index structures, Leis et al. and Yamamuro et al. also present update and delete operations for ART and VAST. Both using the performance increase of tree traversal with SIMD for these operations, because the matching node for the search key has to be found in the tree.

At least, we consider the usage of SIMD with the GPU. Zeuch et al. and Changkyu et al. present algorithms to use GPU calculation for the Seg-Tree/Trie [3]. They compare one search key against multiple other keys with SIMD registers in the GPU, analogue to CPU usage. Since passing data to GPU registers is completely different compared to CPU registers and the other index structures only use the CPU, we don't go further into details of CPU calculation with SIMD. **TODO: References for SIMD with GPU?**

V. RELATED WORK

In main-memory database systems, a B^+ -Tree does not fulfill the requirements of efficient cache line usage. Rao et al. show improved cache line usage for B^+ -Trees [8]. Compression is used to decrease the index size to fill more keys into the cache line. Zukowski et al. show the performance increase of better cache line usage with compression [9].

Graefe et al. present and summarize several techniques for improving cache performance for B-Trees [10]. Furthermore, Bender et al. show a cache oblivious B-Tree [11] and a cache oblivious B-Tree for string data types [12]. Other adapted index structures are shown by Rao et al., the Cache-Sensitive Search Trees (CSS-Tree) [8], and the Cache Sensitive B^+ -Tree (CSB+-Trees) [14]. They construct the tree such that the keys are placed with the best cache optimization as possible in terms of spatial or temporal locality.

During the traversal of an index structure in modern database systems, a huge number of keys are compared. Consequently, using SIMD instructions leads to better search performance. Polychroniou et al. consider different techniques to process multiple data items with one SIMD instruction [7]. They differentiate between horizontal and vertical vector processing and present benefits of each approach. Morrison et al. show the implementations of database operations with SIMD and compare SIMD and scalar processing of data items [10]. Zhou et al. examine the application of SIMD instruction to implement database operations [2]. They address sequential scans, aggregations, index operations, and joins.

To compare as less keys as possible within one node to find the matching children, Zeuch et al. show the performance increase using k-ary search with SIMD instructions [3]. Schlegel et al. present the general benefits of k-ary search on modern processors [15].

Comparing adapted index structures for modern database systems leads to find important points to increase the performance of index structures. Yamamuro et al. and Leis et al. compare VAST [5] and ART [4] to FAST and show the performance increase.

VI. CONCLUSION

- Which points are important to speed up index structures for modern systems:
- Decrease Branch/TLB/Cache misses
- Fully use cache/SIMD lines, aligning blocks?
- Save space with compression, compare more keys
- Use SIMD for tree traverse, k-ary search
- Table of entries that improve performance and space consumption and mark which structure implement the entry
- entries:
- Weighting of important points

VII. FUTURE WORK

- Using k-ary search in FAST/VAST
- Looking in detail to the decrease of branch misses using addition/multiplication operations and SIMD
- Another use of SIMD in index structures than search, other DB operations

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