Evaluation 2: Quadcopter Control

Symbols Used:

Symbol	Description	Unit	Observability
θ	Pitch Euler Angle	rad	Stereo Vision
$\dot{\theta}$	Pitch Euler Angular Velocity	rad/s	Gyroscope
$\ddot{\theta}$	Pitch Euler Angular Accel.	$rad\ /s^2$	-
ϕ	Roll Euler Angle	rad	Stereo Vision
$\dot{\phi}$	Roll Euler Angular Velocity	rad/s	Gyroscope
$\ddot{\phi}$	Roll Euler Angular Accel.	$rad\ /s^2$	-
ψ	Yaw Euler Angle	rad	Stereo Vision
$\dot{\psi}$	Yaw Euler Angular Velocity	rad/s	Gyroscope
$\ddot{\psi}$	Yaw Euler Angular Accel.	$rad\ /s^2$	-
X	Position in X	m	Stereo Vision
$\dot{X}$	Velocity in X	m/s	-
$\ddot{X}$	Accel. in X	$m\ /s^2$	Accelerometer
Y	Position in Y	m	Stereo Vision
$\dot{Y}$	Velocity in Y	m/s	-
$\ddot{Y}$	Accel. in Y	$m\ /s^2$	Accelerometer
Z	Position in Z	m	Stereo Vision
$\dot{Z}$	Velocity in Z	m/s	-
$\ddot{Z}$	Accel. in Z	$m\ /s^2$	Accelerometer
Ω_1	Motor 1 Angular Velocity	rad/s	Voltage/Current/Optical
Ω_2	Motor 2 Angular Velocity	rad/s	Voltage/Current/Optical
Ω_3	Motor 3 Angular Velocity	rad/s	Voltage/Current/Optical
Ω_4	Motor 4 Angular Velocity	rad/s	Voltage/Current/Optical
θ-Theta (/they-tuh/), ϕ-Phi (/fi/), ψ-Psi (/sai/), Ω-Omega (/oh-mega/)			

Mathematical Equations

Euler angles are the three angles used to describe the orientation of a rigid body with respect to a fixed body coordinate system.

Before generating the plant model equations of motion, the parameters need to be defined. The first parameters that are to be determined are the moments of inertia in each axis. The moment of inertia is the amount of torque needed for a desired angular acceleration around a rotation axis. These rely heavily on the physical design and components of the quadcopter. These are generated through various methods which mostly include estimates from lengths/diameters and masses of the quadcopter. The rotor inertia, which is the total rotational moment of inertia around the propeller axis is another parameter used in the equations of motion. The distance from the rotor axis to the centre of the quadcopter also can be defined, along with the thrust and drag coefficients.

Symbol	**Description**
I_{xx}, I_{yy}, I_{zz}	Moment of Inertia
J_r	Rotor Inertia
l	Rotor axis to quadcopter centre distance
b	Thrust coefficient
d	Drag coefficient

Thrust and drag coefficients are found experimentally. We have the following relations:

$$\text{Thrust} = b\Omega^2$$
$$\text{Torque} = d\Omega^2$$
$$\Omega = 2\pi(\text{RPM})/60$$

- In the real world, quadcopters experience aerodynamic effects, namely from air resistance and wind. Air resistance can be thought of as a drag on the quadcopter, opposing the motion of the quadcopter. This is what makes quadcopters slow down when no actuator force is acting on them. Since air resistance affects the quadcopter in all three axial directions, a coefficient is needed for each. Measuring the air resistance is extremely complex, so in most cases estimates suffice.

A_x, A_y, A_z	Air resistance in each axis

Plant Model Equations

To generate mathematical equations, the following assumptions are made:

- The structure of the quadcopter is rigid.
- Thrust and drag are proportional to the square of a propeller speed.
- There are no external disturbances, such as wind or temperature.

Next, the physical effects on the quadcopter need to be considered. These include aerodynamic effects from the propellers, inertial counter-torques from changes in propeller speed, gravity, and gyroscopic effects from changes in body orientation. These all play a role in the equations of motion of the quadcopter.

For a quadcopter in the plus configuration, there are four main thrust equations, vertical thrust, roll thrust, pitch thrust, and yaw thrust.

Input	Thrust Equation	Description
$U1_+$	$b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$	General Thrust
$U2_+$	$b(-\Omega_2^2 + \Omega_4^2)$	Roll Thrust
$U3_+$	$b(\Omega_1^2 - \Omega_3^2)$	Pitch Thrust
$U4_+$	$d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2)$	Yaw Thrust

$U1_+$ applies a general thrust to each of the motors to increase the altitude. $U2_+$ changes thrust between motor two and motor 4 to roll the quadcopter. $U3_+$ changes thrust between motor three and motor 1 to pitch the quadcopter. $U4_+$ changes the thrust to motors 2 and 4 and equally changes the thrust to motors 1 and 3 to yaw the quadcopter.

The equations of motion can now be defined. The origin of these are in the following three equations:

$$\text{Force} = b(\Omega_n)^2$$
$$\text{Force} = ma$$
$$\text{Torque} = I\alpha$$

- The moments of the quadcopter consist of rolling moments, pitching moments, and yawing moments. Each of these consists of gyroscopic effects, propeller effects, and actuator action, and respectively sum to form the angular acceleration equations as follows:

Rolling Torque: $$I_{xx}\ddot{\phi} \quad = \quad \dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) \quad + \quad J_r\dot{\theta}\Omega_r \quad + \quad l(U2)$$

Rolling Ang. Accel: $$\ddot{\phi} \quad = \quad \frac{\dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) \quad + \quad J_r\dot{\theta}\Omega_r \quad + \quad l(U2)}{I_{xx}}$$

Pitching Torque: $$I_{yy}\dot{\theta} \quad = \quad \dot{\phi}\dot{\psi}(I_{yy} - I_{zz}) \quad - \quad J_r\dot{\phi}\Omega_r \quad + \quad l(U3)$$

Pitching Ang. Accel. $$\ddot{\theta} \quad = \quad \frac{\dot{\phi}\dot{\psi}(I_{yy} - I_{zz}) \quad - \quad J_r\dot{\phi}\Omega_r \quad + \quad l(U3)}{I_{yy}}$$

Yawing Torque: $$I_{zz}\ddot{\psi} \quad = \quad \dot{\theta}\dot{\phi}(I_{yy} - I_{zz}) \quad + \quad (U4)$$

Yaw Ang. Accel. $$\ddot{\psi} \quad = \quad \frac{\dot{\theta}\dot{\phi}(I_{yy} - I_{zz}) \quad + \quad (U4)}{I_{zz}}$$

- Next, the force along each fixed frame axis can be defined, which include actuators actions and aerodynamic effects. These forces can be seen in the above equations. To describe these equations in a physical sense, a use case can be made for the above equations.

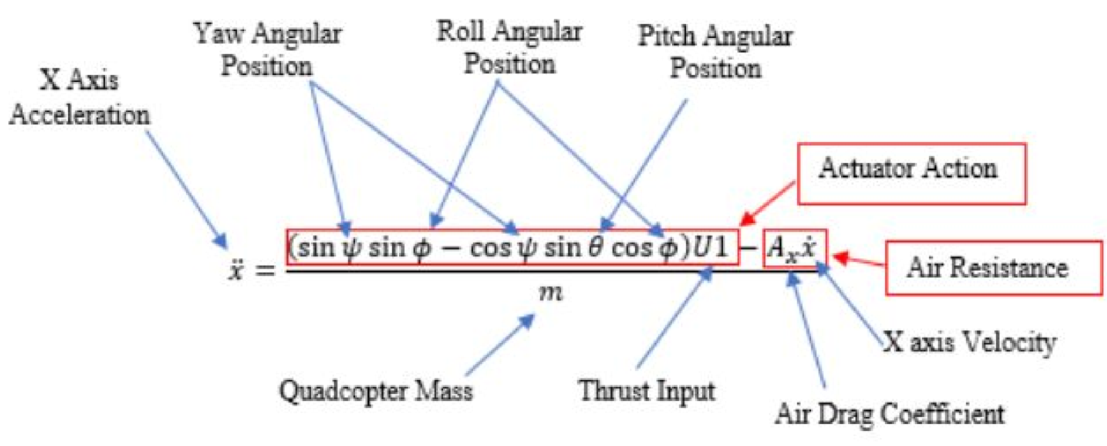


Figure 2.15-X axis acceleration equation explained

X Force: $m\ddot{X} = (Sin\psi Sin\phi - Cos\psi Cos\phi)U1 - A_z\dot{X}$

X Accel: $\ddot{X} = \frac{Sin\psi Sin\phi - Cos\psi Cos\phi)U1 - A_z\dot{X}}{m}$

Y force: $m\ddot{Y} = (Cos\psi Sin\phi + Sin\psi Sin\theta Cos\phi)U1 - A_y\dot{Y}$

Y accel: $\ddot{Y} = \frac{(Cos\psi Sin\phi + Sin\psi Sin\theta Cos\phi)U1 - A_y\dot{Y}}{m}$

Z force: $m\ddot{Z} = mg - (Cos\theta Cos\phi)U1 - A_z\dot{Z}$

Z accel: $\ddot{Z} = \frac{mg - (Cos\theta Cos\phi)U1 - A_z\dot{Z}}{m}$

- Euler angles represent a sequence of three elemental rotations, i.e. rotations about the axes of a coordinate system since any orientation can be achieved by composing three essential rotations. These rotations start from a known standard orientation. The following rotation matrices describe this combination used.

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\phi) & -s(\phi) \\ 0 & s(\phi) & c(\phi) \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} c(\theta) & 0 & s(\theta) \\ 0 & 1 & 0 \\ -s(\theta) & 0 & c(\theta) \end{bmatrix}$$

$$R_z(\psi) = \begin{bmatrix} c(\psi) & -s(\psi) & 0 \\ s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where c(φ) = cos(φ), s(φ) = sin(φ), c(θ) = cos(θ), s(θ) = sin(θ), c(ψ) = cos(ψ), s(ψ) = sin(ψ). So, the inertial position coordinates and the body reference coordinates are related by the rotation matrix Rzyx(φ, θ, ψ)

$$R_{xyz}(\phi,\theta,\psi) \quad = \quad R_z(\psi).R_y(\theta).R_z(\phi)$$

$$= \begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) \\ c(\theta)s(\psi) & s(\phi)s(\theta)s(\psi) + c(\phi)c(\psi) & c(\phi)s(\theta)s(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix}$$

- Now the inertial acceleration equation can be created. This consists of the rotational matrix, thrust matrix and gravitational matrix.

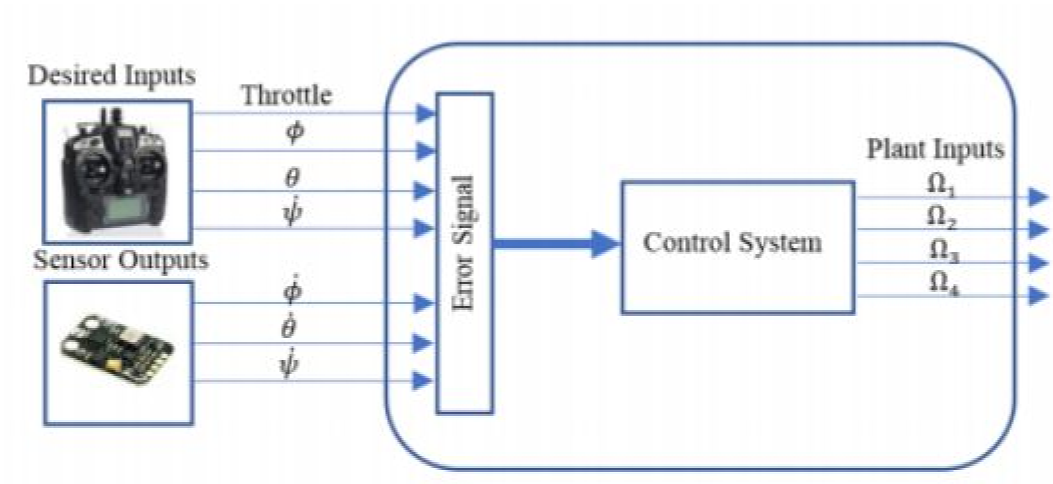
$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} - \frac{1}{m} \begin{bmatrix} C\psi C\theta & S\psi C\phi \quad + \quad C\Psi S\theta S\phi & S\psi S\phi \quad - \quad C\Psi S\theta C\phi \\ -S\psi C\theta & C\psi C\phi \quad + \quad S\Psi S\theta S\phi & C\psi S\phi \quad + \quad S\Psi S\theta C\phi \\ S\theta & -S\Psi C\theta & C\theta C\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ b({\Omega_1}^2 \quad + \quad {\Omega_2}^2 \quad + \quad {\Omega_3}^2 \quad + \quad {\Omega_4}^2) \end{bmatrix} - \frac{1}{m} \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}$$

- The final matrix required to build the plant model is the angular acceleration matrix. This matrix consists of the moments of inertia, previous angular velocity, and torque matrix.

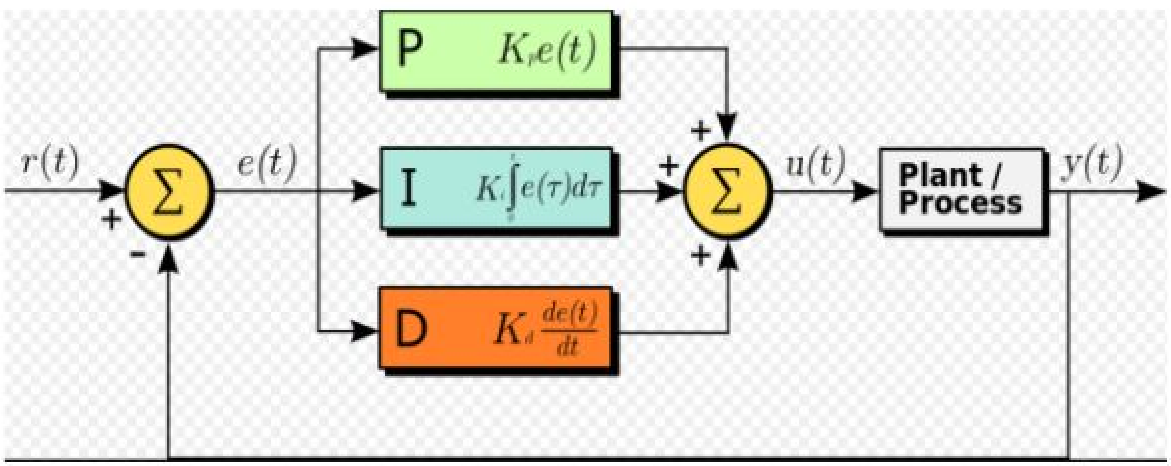
$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} (I_{yy} - I_{zz})\dot{\theta}\dot{\phi}/I_{xx} \\ (I_{zz} - I_{xx})\dot{\psi}\dot{\phi}/I_{yy} \\ (I_{xx} - I_{yy})\dot{\theta}\dot{\phi}/I_{zz} \end{bmatrix} - J_r \begin{bmatrix} \frac{\dot{\theta}}{I_{xx}} \\ \frac{-\dot{\phi}}{I_{yy}} \\ 0 \end{bmatrix} \Omega_r + \begin{bmatrix} \tau_\phi/I_{xx} \\ \tau_\theta/I_{yy} \\ \tau_\psi/I_{zz} \end{bmatrix}$$

Control Design Methodology:

The controller is a significant portion to any system as, without it, the system would inherently be unstable. There are many different control methods, but here PID control is used. A high-level model of a control system can be seen in Figure below.

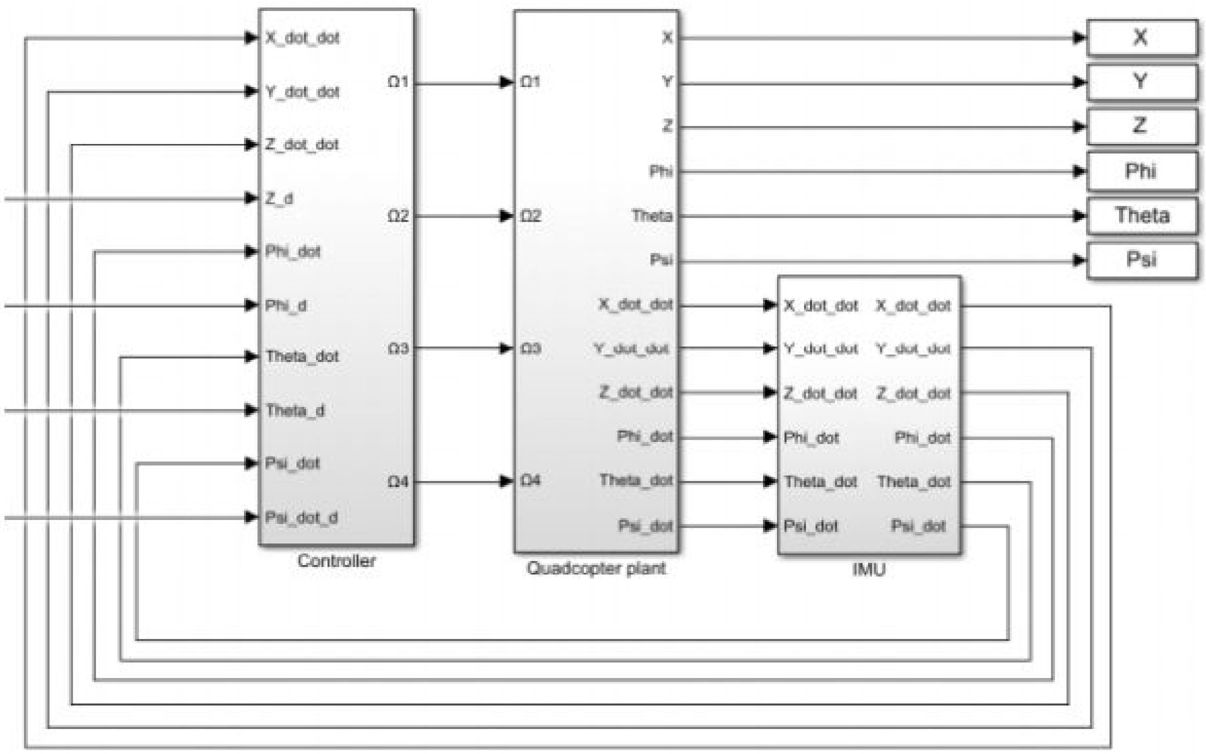


The aspects that need to be controlled are altitude, roll angle, pitch angle and yaw velocity. Feedback from the plant model that was created is used to close the feedback loop. It is challenging to maintain a constant altitude in a quadcopter without a controller, so a PID controller will also be used for height.



The Proportional part of the controller produces a general gain for the system. The Integral portion adjusts the gain by accumulating the error signal. The Derivative amount adjusts the gain by looking at the slope of the error signal, greater slope, greater contribution.

The PID controller for quadcopter contains four feedback loops altitude, pitch angle, yaw angle, roll angle which feeds the sensor data back into the input to calculate the error for the next cycle; thus PID comes in action in every loop and improves the signal output and takes it closer to the desired result. The time characteristics like Transient response, including rising time, overshoot, and settling time can be optimally tuned for an excellent performance by varying the gains. The combined loops figure of the quadcopter controller looks like this.



How will we be tuning the PID controller?

We will start with one parameter at a time.

- For P gain, we will fist start with low and work our way up, until we notice it is producing oscillations. We will Fine-tune it until we get to a point it is not sluggish & there is no oscillation.
- For I gain, we again start with low and increase slowly. We will try to get to a point where it stabilises very quickly as we will be releasing the stick. We might also try to test it under the windy conditions to get a reliable value.
- For D gain, we will be getting into a complicated interaction with P and I values. When using D gain, we need to go back and fine-tune P and I to keep the plant well stabilized.