Two and Seven Joints

Window-Based Coupling Measures ⇒ Complex Strategy Operator ⇒ Policy

Research Demo (NumPy/Matplotlib)

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Abstract

This demo generates synthetic robot data (two velocities $q\_d^{(1,2)}$, their accelerations $\dot{q}^{(1,2)}$, a command torque u, and a measured torque τ_meas). From rolling windows of length W, three quantities are estimated: (i) coherence, (ii) reactivity, and (iii) directed coupling. These are combined into a complex strategy operator $\Xi = \Re\Xi + i\,\Im\Xi$, which is projected along four decision axes (execute / guard / stop / pivot) using a Softmax policy. A secondary script streams a multi-joint feed, performs real-time window updates, and optionally exports a video.

CAUTION

Deterministic modeling is vulnerable to unnatural distortions and algorithmically triggered reactions. Independent safety and risk management strategies are essential.

DISCLAIMER (Research Only)

This repository contains a research prototype. It is provided for educational and research purposes only. It does **NOT** constitute financial, investment, legal, medical, or any other professional advice. No warranty is given. Use at your own risk. Before using any outputs to inform real-world decisions, obtain advice from qualified professionals and perform independent verification.

1 Setup & Notation

Discrete time with sampling rate f_s (default 100 Hz). Each analysis processes a window of length W (typically W=256 samples ≈ 2.56 s) with a Hann taper w, sliding forward by S samples per hop.

Analytic signal (within window). For a real series x, we use an FFT-Hilbert approximation to obtain $z = x + i\,\mathcal{H}[x]$ and phase $\phi = \arg z$. *Note:* This is non-causal; for real-time applications, a FIR-Hilbert implementation is recommended.

2 Window-Based Features

Define $\langle a, b\rangle\_w = \sum\_tw\_ta\_tb\_t$ and $\|a\|\_w = \sqrt{\langle a, a\rangle\_w}$.

(1) Motion Coherence (across the two joints)

For $q\_d^{(j)}$ with phases $\phi^{(j)}(t)$, the phase-locking value (PLV) is

$$\mathrm{PLV}\_v \;=\; \left|\frac{1}{W}\sum\_t = 1^W \exp\Big(i\,\tfrac{1}{2}\sum\_j = 1^2\phi^{(j)}(t)\Big)\right| \in [0,1].$$

(2) Reactivity (Command $\to$ Dynamics)

Aggregate acceleration $\ddot{q}\_\Sigma = \frac{1}{2}(\dot{q}^{(1)} + \dot{q}^{(2)})$. Weighted correlation magnitude:

$$\rho\_u, \ddot{q} \;=\; \frac{|\langle u, \ddot{q}\_\Sigma\rangle\_w|}{\|u\|\_w\,\|\ddot{q}\_\Sigma\|\_w} \in [0,1].$$

(3) Coupling / Orientation between u and $\tau_$meas

Phase difference $\Delta\phi(t) = \arg\big(z\_u(t)\,\overline{z\_\tau(t)}\big)$:

$$\mathrm{PLV}(u,\tau) \;=\; \Big|\sum\_t\tilde{w}\_t\, e^{i\,\Delta\phi(t)}\Big|, \qquad \mathrm{IAI}(u,\tau) \;=\; \sum\_t\tilde{w}\_t\,\sin\Delta\phi(t),$$

with $\tilde{w} = w/\sum w$. IAI measures the *imaginary* antisymmetry (lead/lag behavior).

(4) Pseudoscalar P (Oddness $\times$ Chirality)

Oddness under time reversal:

$$x^{\mathrm{odd}} = \tfrac{1}{2}(x - x^{\mathrm{rev}}), \quad O \;=\; \frac{\langle x^{\mathrm{odd}}, x^{\mathrm{odd}}\rangle\_w}{\langle x, x\rangle\_w} \in [0,1], \quad (x = u)$$

Chirality (oriented area rate in the phase plane (x,y) with $y = \mathcal{H}[x]$):

$$C \;=\; \frac{\sum\_tw\_t\,(x\_t\,\Delta y\_t - y\_t\,\Delta x\_t)}{\sum\_tw\_t\,(x\_t^2 + y\_t^2) + \varepsilon} \in [-1,1], \qquad P \;=\; O\cdot C \in [-1,1].$$

(5) Latency Index

A small cross-correlation search over $\pm L_$max samples yields the estimated lag $\hat{L}$ between u and τ. We normalize $\mathrm{LagIdx} = |\hat{L}|/L\_\mathrm{max} \in [0,1]$.

3 Strategy Operator Ξ and Policy

The demo combines features linearly (for reproducibility):

$$\Re\Xi \;=\; 0.45\,\mathrm{PLV}\_v \;+\; 0.35\,\rho\_u, \ddot{q} \;-\; 0.25\,\mathrm{LagIdx}, \qquad \Im\Xi \;=\; 0.65\,P \;+\; 0.35\,\mathrm{IAI}(u,\tau).$$

From $\Xi = |\Xi|e^{i\theta}$, four decision axes are defined:

$$\Theta = \{0, \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}\} \;\leftrightarrow\; \{\mathrm{execute}, \mathrm{guard}, \mathrm{stop}, \mathrm{pivot}\}, \quad v\_k = |\Xi|\cos(\theta - \Theta\_k).$$

A Softmax with temperature adaptation

$$T\_\mathrm{eff} = \mathrm{clip}\big(T\cdot(1 - 0.4(|\Xi| - 0.5)),\, 0.35,\, 1.25\big)$$

produces policy probabilities p_k and the resulting action $\arg\max\_kp\_k$.

4 Scenarios (Tests)

(A) Orientation Flip: $\tau_$meas$\mapsto -\tau_$meas within an interval $\Rightarrow$ orientation sign inversion, IAI flips, policy tends toward guard/pivot/stop.
(B) Latency / Desync: u delayed by $\hat{L} > 0 \Rightarrow$ LagIdx increases, $\Re\Xi$ decreases, policy becomes more conservative.

5 Live Streaming & Export

The second script simulates J=7 joints ($f\_0 \approx 0.8$ Hz with small detunes), collects the last W samples in `deque` buffers, averages τ across joints, and calls `compute_xi` per hop. It plots $|\Xi|$, policy curves $p\_k(t)$, and the dominant action; optionally, it exports an MP4 via `FFMpegWriter`.

6 Parameter Effects

With f_s=100 Hz, W=256, S=16, the temporal resolution is 160 ms at a lookback depth of 2.56 s. Smaller windows react faster but are noisier; larger windows are more stable but slower (especially in scenario B).

7 Limitations & Upgrades

Hilbert: FFT-Hilbert is sensitive to edges; for real-time use, prefer a causal FIR-Hilbert filter. **Significance:** Optionally apply a Rayleigh test on phase clustering $\Rightarrow$ gating for P. $\mathbf{T_7}$**:** A persistence axis (run-length/stability) can weight P and $\Re\Xi$ to tune flip sensitivity.

8 Reproduction Instructions

```
# Baseline:
python two_joints.py --plot

# Streaming + MP4 export:
python seven_joints_live.py
```

Console output. Number of windows, mean ||, action histogram. Plots show ||, PLV, PLV(u,), |_u, q|, LagIdx, policy curves, and action traces.

Takeaway

The demo illustrates that (i) phase coherence and reactivity form a robust real part of , (ii) directed coupling measures (P, IAI) define the orientation axis, and (iii) a simple Softmax projection maps these onto four meaningful robotic states. Latency shifts toward guard/stop, while orientation flips rotate the imaginary component and trigger redirection.

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