

Two and Seven Joints

Window-Based Coupling Measures \Rightarrow Complex Strategy Operator \Rightarrow Policy

Research Demo (NumPy/Matplotlib)

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Abstract

This demo generates synthetic robot data (two velocities $q_d^{(1,2)}$, their accelerations $\dot{q}^{(1,2)}$, a command torque u , and a measured torque τ_meas). From rolling windows of length W , three quantities are estimated: (i) coherence, (ii) reactivity, and (iii) directed coupling. These are combined into a complex strategy operator $\Xi = \Re\Xi + i\Im\Xi$, which is projected along four decision axes (execute / guard / stop / pivot) using a Softmax policy. A secondary script streams a multi-joint feed, performs real-time window updates, and optionally exports a video.

CAUTION

Deterministic modeling is vulnerable to unnatural distortions and algorithmically triggered reactions. Independent safety and risk management strategies are essential.

DISCLAIMER (Research Only)

This repository contains a research prototype. It is provided for educational and research purposes only. It does **NOT** constitute financial, investment, legal, medical, or any other professional advice. No warranty is given. Use at your own risk. Before using any outputs to inform real-world decisions, obtain advice from qualified professionals and perform independent verification.

1 Setup & Notation

Discrete time with sampling rate f_s (default 100 Hz). Each analysis processes a window of length W (typically $W=256$ samples ≈ 2.56 s) with a Hann taper w , sliding forward by S samples per hop.

Analytic signal (within window). For a real series x , we use an FFT-Hilbert approximation to obtain $z = x + i\mathcal{H}[x]$ and phase $\phi = \arg z$. *Note:* This is non-causal; for real-time applications, a FIR-Hilbert implementation is recommended.

2 Window-Based Features

Define $\langle a, b \rangle_w = \sum_t w_t a_t b_t$ and $\|a\|_w = \sqrt{\langle a, a \rangle_w}$.

(1) Motion Coherence (across the two joints)

For $q_d^{(j)}$ with phases $\phi^{(j)}(t)$, the phase-locking value (PLV) is

$$\text{PLV}_v = \left| \frac{1}{W} \sum_t = 1^W \exp\left(i \frac{1}{2} \sum_j = 1^2 \phi^{(j)}(t)\right) \right| \in [0, 1].$$

(2) Reactivity (Command \rightarrow Dynamics)

Aggregate acceleration $\ddot{q}_{-\Sigma} = \frac{1}{2}(\ddot{q}^{(1)} + \ddot{q}^{(2)})$. Weighted correlation magnitude:

$$\rho_{-u, \ddot{q}} = \frac{|\langle u, \ddot{q}_{-\Sigma} \rangle_{-w}|}{\|u\|_{-w} \|\ddot{q}_{-\Sigma}\|_{-w}} \in [0, 1].$$

(3) Coupling / Orientation between u and τ_{meas}

Phase difference $\Delta\phi(t) = \arg(z_{-u}(t) \overline{z_{-\tau}(t)})$:

$$\text{PLV}(u, \tau) = \left| \sum_{-t} \tilde{w}_{-t} e^{i\Delta\phi(t)} \right|, \quad \text{IAI}(u, \tau) = \sum_{-t} \tilde{w}_{-t} \sin \Delta\phi(t),$$

with $\tilde{w} = w / \sum w$. IAI measures the *imaginary* antisymmetry (lead/lag behavior).

(4) Pseudoscalar P (Oddness \times Chirality)

Oddness under time reversal:

$$x^{\text{odd}} = \frac{1}{2}(x - x^{\text{rev}}), \quad O = \frac{\langle x^{\text{odd}}, x^{\text{odd}} \rangle_{-w}}{\langle x, x \rangle_{-w}} \in [0, 1], \quad (x = u)$$

Chirality (oriented area rate in the phase plane (x, y) with $y = \mathcal{H}[x]$):

$$C = \frac{\sum_{-t} \tilde{w}_{-t} (x_{-t} \Delta y_{-t} - y_{-t} \Delta x_{-t})}{\sum_{-t} \tilde{w}_{-t} (x_{-t}^2 + y_{-t}^2) + \varepsilon} \in [-1, 1], \quad P = O \cdot C \in [-1, 1].$$

(5) Latency Index

A small cross-correlation search over $\pm L_{\text{max}}$ samples yields the estimated lag \hat{L} between u and τ . We normalize $\text{LagIdx} = |\hat{L}| / L_{\text{max}} \in [0, 1]$.

3 Strategy Operator Ξ and Policy

The demo combines features linearly (for reproducibility):

$$\Re \Xi = 0.45 \text{PLV}_{-v} + 0.35 \rho_{-u, \ddot{q}} - 0.25 \text{LagIdx}, \quad \Im \Xi = 0.65 P + 0.35 \text{IAI}(u, \tau).$$

From $\Xi = |\Xi| e^{i\theta}$, four decision axes are defined:

$$\Theta = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\} \leftrightarrow \{\text{execute}, \text{guard}, \text{stop}, \text{pivot}\}, \quad v_{-k} = |\Xi| \cos(\theta - \Theta_{-k}).$$

A Softmax with temperature adaptation

$$T_{\text{eff}} = \text{clip}(T \cdot (1 - 0.4(|\Xi| - 0.5)), 0.35, 1.25)$$

produces policy probabilities p_{-k} and the resulting action $\arg \max_{-k} p_{-k}$.

4 Scenarios (Tests)

(A) Orientation Flip: $\tau_{\text{meas}} \mapsto -\tau_{\text{meas}}$ within an interval \Rightarrow orientation sign inversion, IAI flips, policy tends toward guard/pivot/stop.

(B) Latency / Desync: u delayed by $\hat{L} > 0 \Rightarrow \text{LagIdx}$ increases, $\Re \Xi$ decreases, policy becomes more conservative.

5 Live Streaming & Export

The second script simulates $J=7$ joints ($f_0 \approx 0.8$ Hz with small detunes), collects the last W samples in deque buffers, averages τ across joints, and calls `compute_xi` per hop. It plots $|\Xi|$, policy curves $p_k(t)$, and the dominant action; optionally, it exports an MP4 via `FFMpegWriter`.

6 Parameter Effects

With $f_s=100$ Hz, $W=256$, $S=16$, the temporal resolution is 160 ms at a lookback depth of 2.56 s. Smaller windows react faster but are noisier; larger windows are more stable but slower (especially in scenario B).

7 Limitations & Upgrades

Hilbert: FFT-Hilbert is sensitive to edges; for real-time use, prefer a causal FIR-Hilbert filter.

Significance: Optionally apply a Rayleigh test on phase clustering \Rightarrow gating for P . **T₇:** A persistence axis (run-length/stability) can weight P and $\Re\Xi$ to tune flip sensitivity.

8 Reproduction Instructions

```
# Baseline:
python two_joints.py --plot
```

```
# Streaming + MP4 export:
python seven_joints_live.py
```

Console output. Number of windows, mean $||$, action histogram. Plots show $||$, PLV, PLV(u,), $|_u, q|$, LagIdx, policy curves, and action traces.

Takeaway

The demo illustrates that (i) phase coherence and reactivity form a robust real part of , (ii) directed coupling measures (P , IAI) define the orientation axis, and (iii) a simple Softmax projection maps these onto four meaningful robotic states. Latency shifts toward guard/stop, while orientation flips rotate the imaginary component and trigger redirection.

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