A Complex-Valued Strategy Operator for Robotics: Coherence, Coupling, and Orientation Features in Real Time

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Abstract

We formulate a complex-valued strategy operator Ξ for multi-link robots that aggregates windowed evidence from actuation and state data $(q, \dot{q}, \ddot{q}, \tau_{\text{cmd}}, \tau_{\text{meas}})$. $\Re\{\Xi\}$ captures *feasibility* evidence (coherence, reactivity, efficiency, latency), $\Im\{\Xi\}$ encodes *orientation/chirality* and directed coupling. A projection onto fixed decision axes (`execute/guard/stop/pivot`) yields robust, ROS-friendly commands. We provide precise definitions, an evidence-sensitive gate, causality-compatible surrogates for phases, as well as practical tuning recipes and safety checks, without disclosing production IP.

1 System model and objective

For a J-joint robot, let $q \in \mathbb{R}^J$, $\dot{q}$, $\ddot{q}$ be state variables, and $\tau_{\text{cmd}}, \tau_{\text{meas}} \in \mathbb{R}^J$ be command and measured/estimated torques (incl. friction/contact components), respectively. The objective is a compact carrier Ξ for robust *real-time* decisions between discrete actions.

Windowing and notation. We consider windows of length W over samples $t = 1, \ldots, W$ with taper $w(t) \geq 0$. Unless otherwise stated, $\sum_t w(t) = 1$. Joint weights $\alpha_j \geq 0$ with $\sum_j \alpha_j = 1$.

2 Analytic phases and coherence

For a real-valued series $x(t)$, the analytic signal is $z_x(t) = x(t) + i\,\mathcal{H}\{x\}(t)$, phase $\phi_x(t) = \arg z_x(t)$.

Joint PLV (motion coherence).

$$\text{PLV}_v \;=\; \frac{1}{W} \sum_{t=1}^{W} \left| \sum_{j=1}^{J} \alpha_j \, e^{i\phi_{\dot{q},j}(t)} \right| \;\in [0, 1]. \tag{1}$$

Pairwise PLV and directed component. For scalar series $a(t), b(t)$ and $\Delta\phi(t) = \phi_a(t) - \phi_b(t)$ we define

$$\mathrm{PLV}(a,b) = \left| \frac{\sum_t w(t)\, e^{i\Delta\phi(t)}}{\sum_t w(t)} \right| \in [0,1], \tag{2}$$

$$\mathrm{IAI}(a,b) = \Im \left\{ \frac{\sum_t w(t)\, e^{i\Delta\phi(t)}}{\sum_t w(t)} \right\} \in [-1,1], \tag{3}$$

where IAI yields an *imaginary-antisymmetric* (directed) coupling.

Weighted correlation. With weighted means $\mu_x = \sum_t w(t)x(t)$ we have

$$\rho_w(x,y) = \frac{\sum_t w(t)\,(x(t)-\mu_x)(y(t)-\mu_y)}{\sqrt{\sum_t w(t)(x(t)-\mu_x)^2}\,\sqrt{\sum_t w(t)(y(t)-\mu_y)^2}} \in [-1,1]. \tag{4}$$

Oddness/chirality (time-reversal sensitive). With $x^{\mathrm{rev}}(t) = x(W+1-t)$, $x_{\mathrm{even}} = \frac{1}{2}(x + x^{\mathrm{rev}})$, $x_{\mathrm{odd}} = \frac{1}{2}(x - x^{\mathrm{rev}})$, weighted energies $E_{\mathrm{even}}, E_{\mathrm{odd}}$ and symplectic flux relative to y:

$$\mathrm{Odd}(x) = \frac{E_{\mathrm{odd}}}{E_{\mathrm{even}} + E_{\mathrm{odd}}} \in [0,1], \tag{5}$$

$$\mathrm{Chi}(x;y) = \frac{\sum_t w(t)\big(x(t)\dot{y}(t) - y(t)\dot{x}(t)\big)}{\sum_t w(t)\big(x^2(t) + y^2(t)\big) + \varepsilon} \in [-1,1], \tag{6}$$

$$g_P(x;y) = \mathrm{Odd}(x)\cdot \mathrm{Chi}(x;y). \tag{7}$$

3 Robotics-specific features

Aggregated scalar series: $u(t) = \sum_j \alpha_j\, \tau_{\mathrm{cmd},j}(t)$, $\tau_\Sigma(t) = \sum_j \alpha_j\, \tau_{\mathrm{meas},j}(t)$, $\ddot{q}_\Sigma(t) = \sum_j \alpha_j\, \ddot{q}_j(t)$.
(i) Motion coherence: $f_{\mathrm{coh}} = \mathrm{PLV}_v$ (1).
(ii) Reactivity (cause→effect):

$$f_{\mathrm{react}} = \big|\rho_w\big(u, \ddot{q}_\Sigma\big)\big| \quad \text{(normalized per (4))}. \tag{8}$$

(iii) Coupling/orientation between actuation and torque:

$$g_{\mathrm{cs}} = \mathrm{IAI}\big(u, \tau_\Sigma\big), \qquad \mathrm{PLV}_{u\tau} = \mathrm{PLV}\big(u, \tau_\Sigma\big). \tag{9}$$

(iv) Energy efficiency (dimensionally consistent, bounded): Let $\bar{P}$ be a robust power scale (e.g. the 95th percentile of $|\boldsymbol{\tau}_{\mathrm{meas}}^\top \dot{\boldsymbol{q}}|$ in the window). Then

$$f_{\mathrm{eff}} = \tanh\!\left(\frac{\sum_t w(t)\, \boldsymbol{\tau}_{\mathrm{meas}}^\top(t)\, \dot{\boldsymbol{q}}(t)}{\bar{P} + \varepsilon} \right) \in (-1,1). \tag{10}$$

(v) Latency attenuation:

$$f_{\mathrm{lat}} = \exp\!\left(-\frac{\tau_{\mathrm{e2e}}}{\tau_{\mathrm{budget}}} \right), \tag{11}$$

where τ_{e2e} is the end-to-end latency (sensor→decision→actuation; PTP/TimeSync + buffer age) and τ_{budget} is task-dependent (e.g. 30–50 ms for compliant, 5–10 ms for highly dynamic).

Acceleration estimation. Observers are used instead of raw differencing (e.g. Savitzky–Golay 3rd–5th order or alpha–beta–gamma/Kalman with white-noise model), since f_{react} is otherwise noise-sensitive.

Frequency-direction backup (optional). For broadband excitation, the *Phase Slope Index* (PSI) is robust:

$$\mathrm{PSI}(a \to b) = \sum_{f \in \mathcal{B}} \omega_f \, \sin\big(\phi_{ab}(f{+}\Delta f) - \phi_{ab}(f)\big),$$

with complex coherence phase $\phi_{ab}(f)$ and task band $\mathcal{B}$. PSI can complement g_{cs}.

4 Strategy operator, gate, and projection

Feature sets $\mathcal{F}_R = \{f_{\mathrm{coh}}, f_{\mathrm{react}}, f_{\mathrm{eff}}, f_{\mathrm{lat}}, \dots\}$, $\mathcal{F}_I = \{g_{\mathrm{cs}}, g_P, \dots\}$. With weights $\boldsymbol{w}, \boldsymbol{v}$ (learned or rule-based) we define

$$\Re\{\Xi\} = \sum_i w_i f_i, \quad \Im\{\Xi\} = \sum_j v_j g_j, \quad \Xi = \Re\{\Xi\} + i\,\Im\{\Xi\}, \qquad \text{(both parts subsequently scaled to } [-1,1]). \tag{12}$$

Evidence gate (log-additive, less brittle). Set $f^{+}_{\mathrm{react}} = \max(0, f_{\mathrm{react}})$ and

$$G' = \lambda_1 \log(\mathrm{PLV}_v + \varepsilon) + \lambda_2 \log(\mathrm{PLV}_{u\tau} + \varepsilon) + \lambda_3 \log(f^{+}_{\mathrm{react}} + \varepsilon), \qquad \tilde{G} = \tanh(\kappa G') \in (0,1), \tag{13}$$

with $\lambda_k > 0$, $\kappa \in [1,5]$. A *gentle rotation enrichment* rotates Ξ towards the sign of the coupling,

$$\tilde{\Xi} = \Xi \, \exp\Big(i\,\beta\,\tilde{G}\,\mathrm{sign}(g_{\mathrm{cs}})\Big), \qquad \beta \in \big[0, \tfrac{\pi}{12}\big]. \tag{14}$$

Projection onto decision axes. With $\theta_k \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ and mapping $0{=}$`execute`, $\pi/2{=}$`guard`, $\pi{=}$`stop`, $3\pi/2{=}$`pivot`:

$$u_k = \Re\{\tilde{\Xi}\, e^{-i\theta_k}\}, \qquad p_k = \frac{\exp(u_k/T_{\mathrm{eff}})}{\sum_\ell \exp(u_\ell/T_{\mathrm{eff}})}, \tag{15}$$

$$T_{\mathrm{eff}} = \max\big(T_{\min},\, T_0\,(1 - \gamma\,\tanh|\tilde{\Xi}|)\big), \quad \gamma \in [0,1],\ T_{\min} > 0 \tag{16}$$

(*self-confidence-based lowering* of the softmax temperature for large $|\tilde{\Xi}|$). Alternatively: $s_k = \tanh(\alpha|\tilde{\Xi}|)\,\mathrm{sign}(u_k)$.

5 Causal quadrature surrogate (redacted)

Offline, $\mathcal{H}\{\cdot\}$ (FFT-Hilbert) is used. For real time, we use a *causal* surrogate $\mathcal{Q}\{\cdot\}$ with properties $\langle x, \mathcal{Q}\{x\}\rangle \approx 0$, $\mathcal{Q}\{\mathcal{Q}\{x\}\} \approx -x$, LTI with finite group delay Δ. Construction/parameters of $\mathcal{Q}$ are not disclosed. The resulting delay Δ is compensated in the state machine (timestamps, FIFO age).

6 Safety, hysteresis, and state machine

Safety first: action `execute` only if predicate $\mathcal{S}$ is true (torque/power/speed below limits, no limit poses/faults). Chatter is prevented via minimum hold time and an evidence criterion: $\max_k p_k - \max_{\ell \neq k} p_\ell > \delta$ ($\delta \approx 0.15$) and hold $\geq$ 100ms.

7 Streaming, complexity, and real time

All metrics are $\mathcal{O}(JW)$ per window (stride S). FFT-Hilbert $\mathcal{O}(W \log W)$. Typical practice: W=256…512, S=32…64, Hann, sampling rate 500 to 1000 Hz (actuators) $\Rightarrow$ total latency $\approx$ 30–60 ms incl. filters.

8 Tuning recipe (practice)

- **Bands:** Periodic $\Rightarrow$ band-limited quadrature; otherwise full-band with prefilter (e.g. avoid gearbox resonances 20–80Hz).
- **Initial weights:** $\boldsymbol{w}:\{f_{\text{react}}:0.4,\ f_{\text{coh}}:0.3,\ f_{\text{eff}}:0.2,\ f_{\text{lat}}:0.1\}$; $\boldsymbol{v}:\{\text{IAI}:0.7,\ g_P:0.3\}$. Then Bayesian/mission cost-based fine-tuning.
- **Hysteresis:** Hold $\geq$ 100ms, refractory $\approx$ 50ms, $\delta \approx 0.15$.
- **Aggregation α_j:** Energetic or task-Jacobian-weighted, not static.

9 Implementation pitfalls (and countermeasures)

- **Time sync:** Hardware timestamps/TimeReference; do not mix local now-times.
- **Torque bias/friction:** compensate drift in τ_{meas} (thermal), otherwise g_P is driven.
- **Saturation/clipping:** mark events; $f_{\text{eff}}/f_{\text{react}}$ invalid there $\Rightarrow$ lower the gate.
- **$\ddot{q}$:** Use observers rather than finite differences; otherwise $\rho_w(u, \ddot{q}_\Sigma)$ collapses.

10 Diagnostics & reporting (ROS-friendly)

Publish $|\tilde{\Xi}|$, $\arg\tilde{\Xi}$, PLV_v, $\text{PLV}_{u\tau}$, IAI, f_{eff}, f_{react}, $\tilde{G}$, T_{eff}, action, hold time. Offline metrics: $\|\mathcal{H}\{\mathcal{H}\{x\}\} + x\|_2/\|x\|_2$, $\text{corr}(x, \mathcal{H}\{x\})$, $\text{corr}(\mathcal{H}\{x\}, \mathcal{Q}\{x\})$, relative L_2 deviation, effective Δ.

11 Test scenarios (robust validation)

- **Backlash/deadzone sweep:** slow sine drivers, varying load $\Rightarrow$ stability of IAI/PLV.
- **Stick–slip/disturbance torques:** noise sensitivity of f_{react}, gate robustness.
- **Time offset:** inject $\pm$1–2 sample offset between u and $\ddot{q}_\Sigma$ $\Rightarrow$ decision must not flip.
- **Contact transition:** ramp-to-contact, variable impedance $\Rightarrow$ g_{cs} sign change, gate down, `pivot` up.

12 Core formulas (compact)

$$
\begin{aligned}
&\text{PLV}_v = \frac{1}{W}\sum_t \Big|\sum_j \alpha_j e^{i\phi_{\dot{q},j}(t)}\Big|, \quad \text{PLV}(u,\tau_\Sigma) = \left|\frac{\sum_t w e^{i(\phi_u - \phi_{\tau_\Sigma})}}{\sum_t w}\right|, \\
&\text{IAI} = \Im\left\{\frac{\sum_t w e^{i(\phi_u - \phi_{\tau_\Sigma})}}{\sum_t w}\right\}, \quad f_{\text{react}} = \big|\rho_w(u, \ddot{q}_\Sigma)\big|, \\
&f_{\text{eff}} = \tanh\!\left(\frac{\sum_t w\, \boldsymbol{\tau}_{\text{meas}}^\top \dot{\boldsymbol{q}}}{\bar{P} + \varepsilon}\right), \quad g_P = \text{Odd}(u)\cdot \text{Chi}(u;\tau_\Sigma), \\
&\Re\{\Xi\} = \sum_i w_i f_i, \quad \Im\{\Xi\} = \sum_j v_j g_j, \quad \tilde{G} = \tanh(\kappa G'), \quad \tilde{\Xi} = \Xi e^{i\beta\tilde{G}\,\text{sign}(g_{\text{cs}})}, \\
&u_k = \Re\{\tilde{\Xi}e^{-i\theta_k}\}, \quad p_k = \frac{e^{u_k/T_{\text{eff}}}}{\sum_\ell e^{u_\ell/T_{\text{eff}}}}, \quad \theta_k \in \{0, \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}\}.
\end{aligned}
$$

13 Conclusion

Ξ serves as a compact “context operator”: $\Re\{\Xi\}$ answers “may I execute?”, $\Im\{\Xi\}$ “in which orientation/directed coupling?” The log-additive evidence, the gentle rotation enrichment, and an explicit projection onto decision axes build a clean bridge from the signal to the policy level — real-time capable, robust, and without disclosing production details.

References

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[2] B. Siciliano, O. Khatib (eds.): *Springer Handbook of Robotics*. Springer, 2016.

[3] A. Pikovsky, M. Rosenblum, J. Kurths: *Synchronization: A Universal Concept*. Cambridge, 2003.


```
python codez/xi_xi_trigger_test.py --fs 100 --duration 40 --window 256 --step 8 --plot --save out.png

Baseline: dom. Aktion = stop  | Verteilung = {'stop': 469}

================================================================================
Szenario: Orientierungs-Flip (_meas Segment * -1)
================================================================================
Baseline-Aktion      : stop
Ziel-Aktionen        : ['guard', 'pivot', 'stop']
Umschalt-Latenz      : 3 Frames  (~0.240 s)
Recovery nach Ende   : 4 Frames  (~0.320 s)
  p_execute = +0.023
  p_guard   = +0.090
  p_stop    = -0.073
  p_pivot   = -0.040
Gate                 : +0.000
PLV_v                : +0.004
PLV(u,)            : +0.362
||                  : -0.052
IAI (vor→nach)       : -0.004 → -0.182
PASS Redirect?       : True
PASS Recovery?       : True

================================================================================
Szenario: Latenz/Desync (_cmd um 5 Samples verzögert)
================================================================================
Baseline-Aktion      : stop
Ziel-Aktionen        : ['guard', 'stop']
Umschalt-Latenz      : KEIN Wechsel erkannt
Recovery nach Ende   : 0 Frames  (~0.000 s)
  p_execute = n/a
  p_guard   = n/a
  p_stop    = n/a
  p_pivot   = n/a
Gate                 : n/a
PLV_v                : n/a
PLV(u,)            : n/a
||                  : n/a
IAI (vor→nach)       : n/a → +0.303
PASS Redirect?       : False
PASS Recovery?       : True

================================================================================
```

```
Szenario: Kohärenz-Drop (qd Noise-Burst)
================================================================================
Baseline-Aktion      : stop
Ziel-Aktionen        : ['guard', 'pivot']
Umschalt-Latenz      : KEIN Wechsel erkannt
Recovery nach Ende   : 0 Frames  (~0.000 s)
  p_execute = +0.043
  p_guard   = +0.055
  p_stop    = -0.053
  p_pivot   = -0.044
Gate                 : +0.000
PLV_v                : -0.002
PLV(u,)            : +0.190
||                  : -0.185
IAI (vor→nach)        : +0.122 → +0.198
PASS Redirect?        : False
PASS Recovery?        : True

--------------------------------------------------------------------------------
Zusammenfassung (PASS Redirect / PASS Recovery):
A: True / True
B: False / True
C: False / True
--------------------------------------------------------------------------------
```


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