MIRROR, MIRROR: A RANDOM WALK WITH A TWIST

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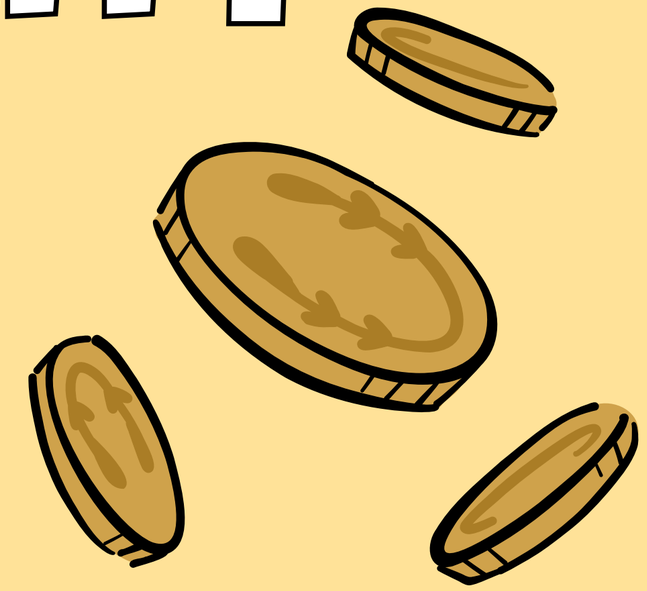
ABOUT ME

- Grade: 22
- Math: Games and puzzles

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

PROBABILITY



PROBABILITY

Concerns events and numerical descriptions of how likely they are to occur

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The probability of an event is a number between zero and one

TYPES OF PROBABILITY

Empirical/Experimental

Estimates by using experience and observation

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Empirical/Experimental

Estimates by using experience and observation

Theoretical

Calculates the likeliness of an event happening based on reasoning and mathematics

EXAMPLE 1

Empirical Probability: You toss a coin 8 times and you record the following outcomes

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Experiment	1	2	3	4	5	6	7	8
Outcome	heads	heads	tails	heads	tails	tails	heads	heads

EXAMPLE 1

Empirical Probability

Experiment	1	2	3	4	5	6	7	8
Outcome	heads	heads	tails	heads	tails	tails	heads	heads

What is the empirical probability of getting tails?

EXAMPLE 1

Empirical Probability

Experiment	1	2	3	4	5	6	7	8
Outcome	heads	heads	tails	heads	tails	tails	heads	heads

What is the empirical probability of getting tails? 3/8 = 37.5%

EXAMPLE 2

Theoretical Probability:

$$\frac{\text{number of desired outcomes}}{\text{total number of all outcomes}}$$

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Probability of getting tails? P(tails) =

EXAMPLE 2

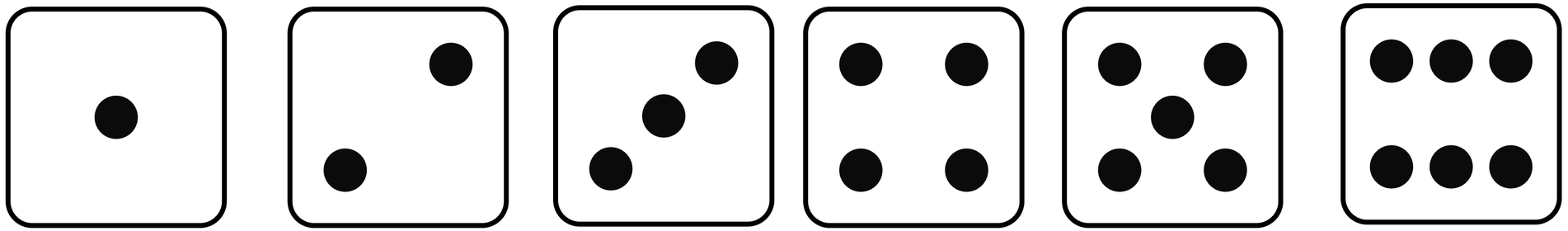
Theoretical Probability:

$$\frac{\text{number of desired outcomes}}{\text{total number of all outcomes}}$$

Probability of getting tails? P(tails) = $\frac{1}{2}$ = 50%

EXAMPLE 3

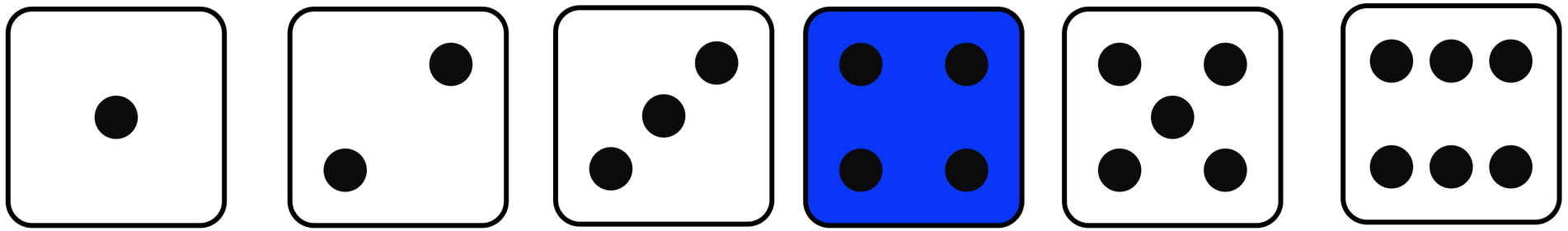
Example: Consider a six-sided die.



What is the probability of getting a 4? How likely do we get a 4 if you throw the dice?

EXAMPLE 3

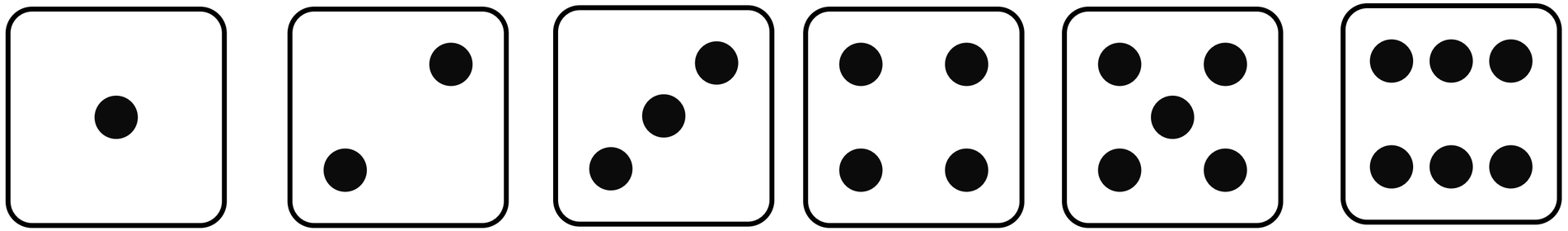
Example: Consider a six-sided die.



What is the probability of getting a 4? How likely do we get a 4 if you throw the dice? Answer: $\frac{1}{6}$

EXAMPLE 4

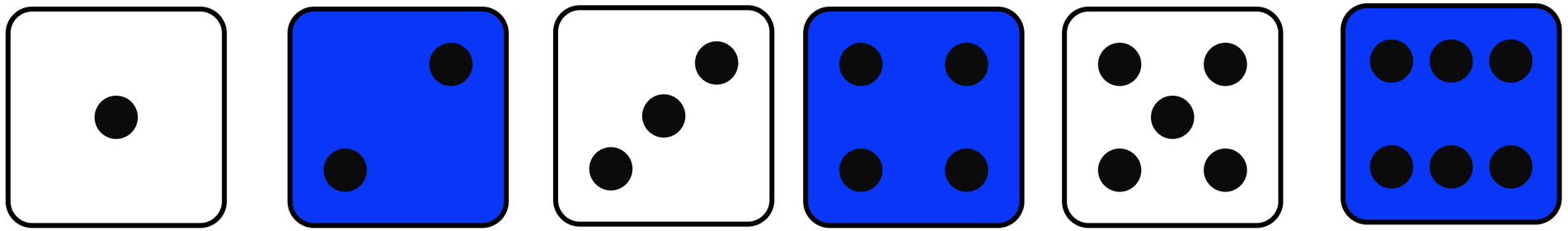
Example: Consider a six-sided die.



What is the probability of getting an even number?

EXAMPLE 4

Example: Consider a six-sided die.



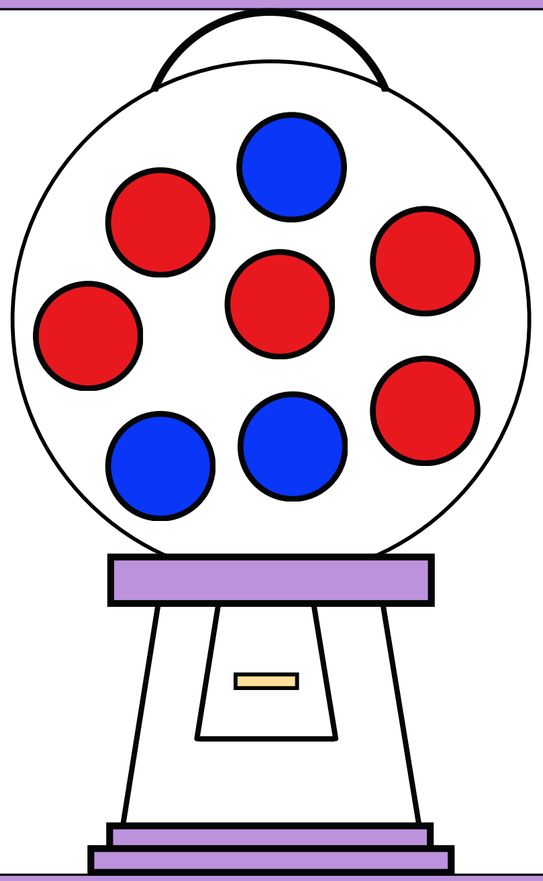
What is the probability of getting an even number? Answer:

$$\frac{3}{6} = \frac{1}{2} = 50\%$$

EXAMPLE 5

How likely is it you will get a blue gumball?

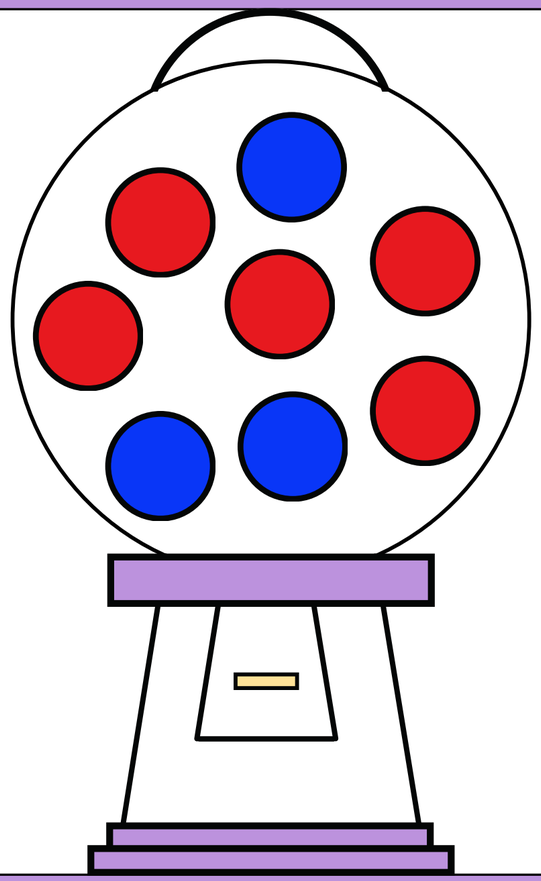
How likely is it you will get a red gumball?



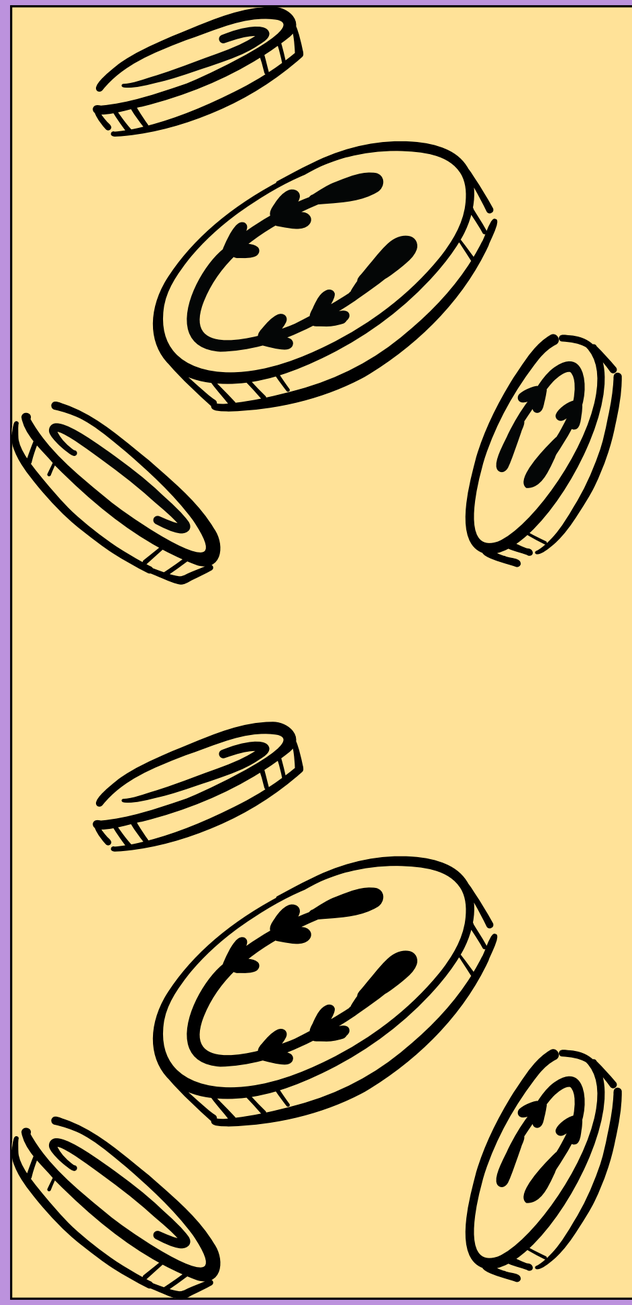
EXAMPLE 5

How likely is it you will get a blue gumball? Answer: $\frac{3}{8}$

How likely is it you will get a red gumball? Answer: $\frac{5}{8}$



WALKS



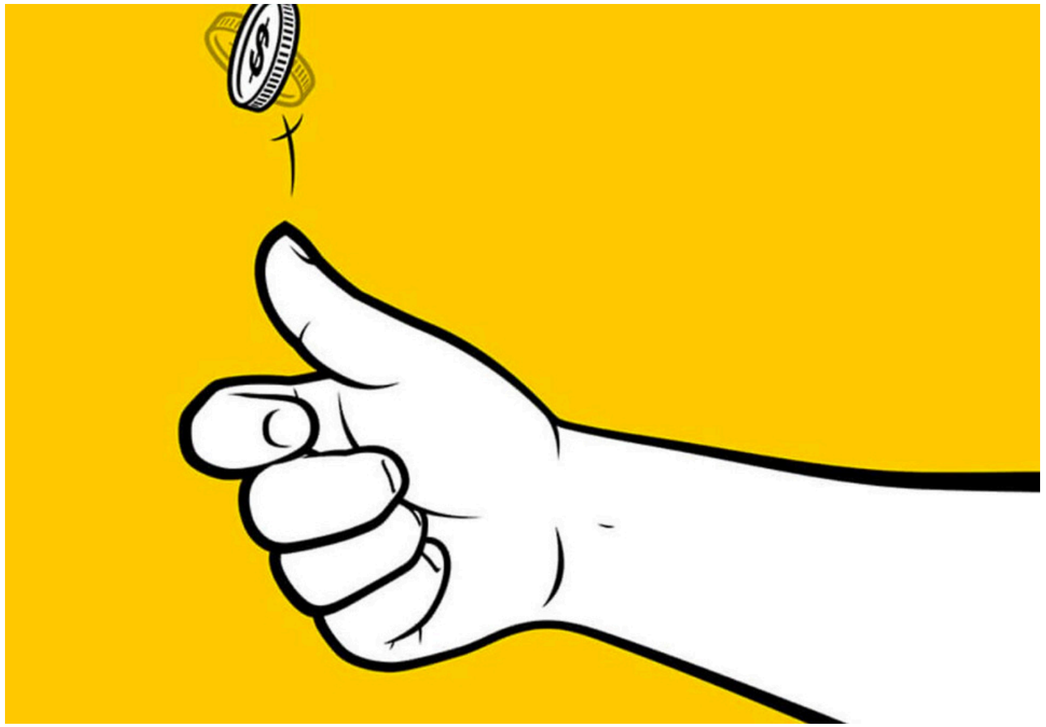
ACTIVITY 1

Flip a coin:

Heads: take one step forward

Tails: take one step back

Record the location



ACTIVITY 1

Share your thoughts: If you take a walk with coin flips as your guide like our volunteer just did, what would you wonder about?

What are you curious about? Write it down!

ACTIVITY 2

Let's investigate

Starting at zero on the number line:

if heads: step forward one unit

if tails: step back one unit

Where will you arrive on the number line after six flips?

ACTIVITY 2

	Flip 1	Flip 2	Flip 3	Flip 4
	heads	tails	tails	heads
Location				
0				

ACTIVITY 2

Let's all share our thoughts

ACTIVITY 2

Let's all share our thoughts

Now let's all do the activity!

OBSERVATIONS

After first step: Is this what you expected?

OBSERVATIONS

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How did you get to your location?

OBSERVATIONS

After first step: Is this what you expected?

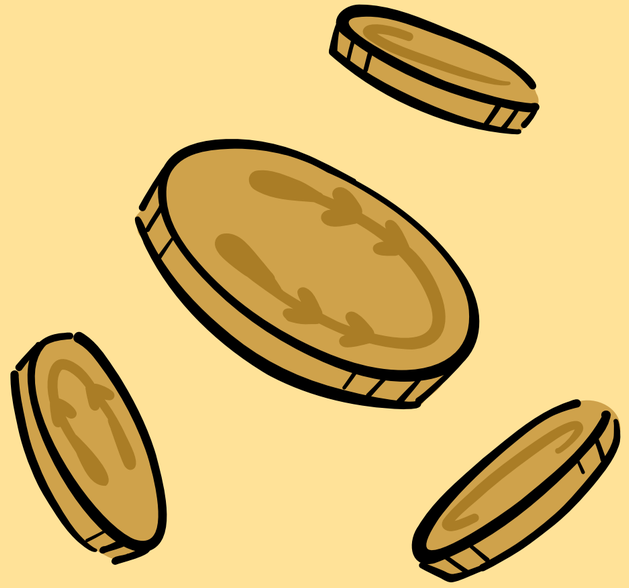
How did you get to your location?

Final step: Single columns

More observations

What do we notice in the bar graph we have created?

RANDOM WALKS!



More observations

Question: What were some of your noticings and wonderings about the human bar graph we created? Let's record them on our handout and then share with someone!

LET'S ANALYZE

Let's predict what the human bar graph would like if we only did 2-flips, 3-flips, 4-flips, or 5-flips with the coin.

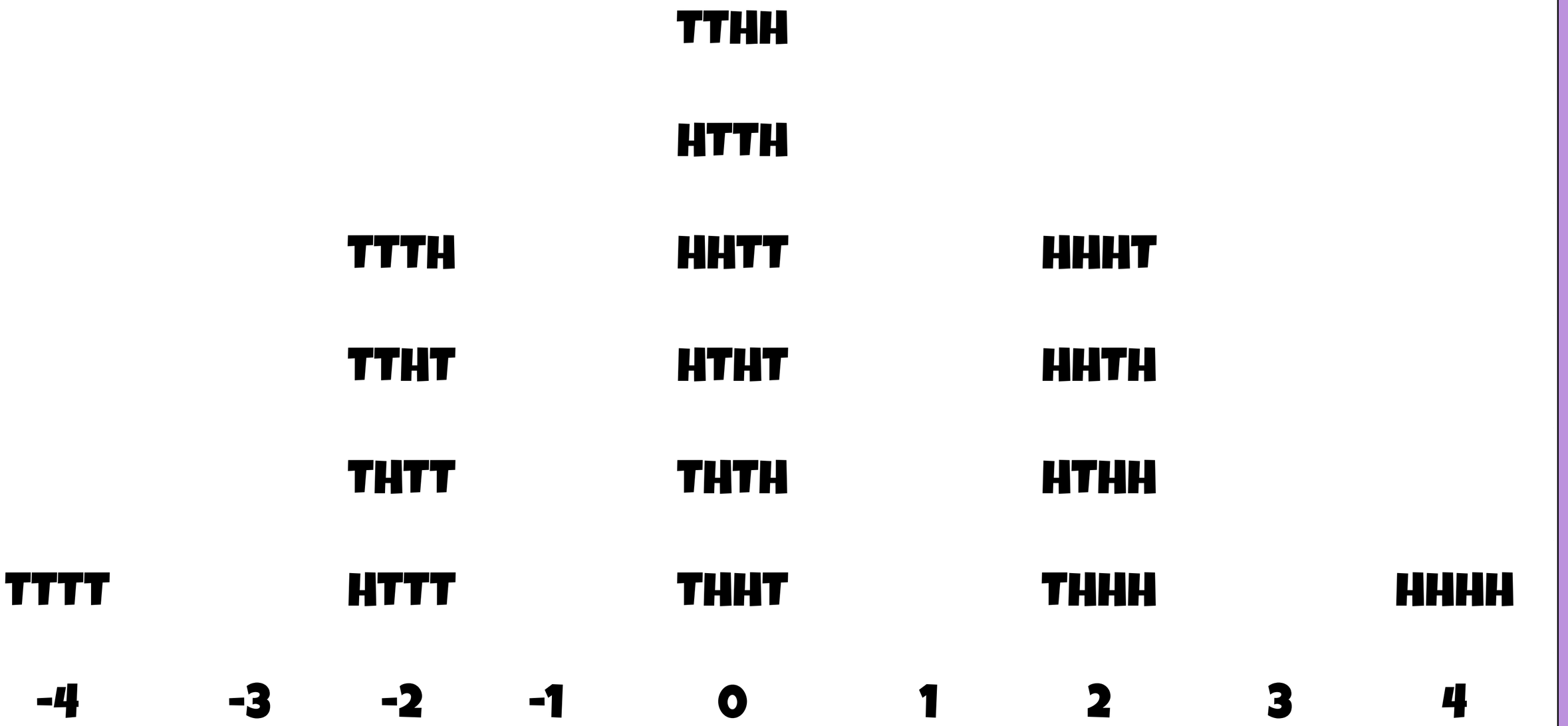
Be specific: Where could people arrive after 4-flips?
What fraction or percentage of the people would arrive in each column of the bar graph?

OUTCOMES

		TH		
TT		HT		HH
-2	-1	0	1	2

OUTCOMES (2)

		TTH		HHT		
		THT		HTH		
TTT		HTT		THH		HHH
-3	-2	-1	0	1	2	3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			HHTTT	TTHHH				
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		TTHTT	TTHHT	HHTTH	HHTHH			
		THTTT	TTHTH	HHTHT	HTHHH			
TTTTT		HTTTT	TTTHH	HHHTT	THHHH		HHHHH	

QUESTIONS

Is this a coincidence?

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How often will I get heads or tails?

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How often will I get heads or tails?

What happens to the total possible outcomes when flipping a coin as the number of flips grows by 1? How many more outcomes do we get from 2-flips to 3-flips?

RESULTS

Number of possible outcomes doubles as the number of flips grows by one:

- 2 flips: 4 outcomes
- 3 flips: 8 outcomes
- 4 flips: 16 outcomes

QUESTIONS

Where will I arrive after a certain number of flips?

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What do you notice about the number of ways to arrive at a location in a N-flips compared to the number of ways of arriving at the two adjacent locations in (N-1)-flips?

QUESTIONS

Where will I arrive after a certain number of flips?

What do you notice about the number of ways to arrive at a location in a N-flips compared to the number of ways of arriving at the two adjacent locations in (N-1)-flips?

Look at the number of ways to arrive at ‘zero’ and the number of ways of arriving at ‘two’ in 4-flips and compare to the number of ways of arriving at ‘one’ in 5-flips

RESULTS

There are 6 ways to arrive at zero in 4-flips

There are 4 ways to arrive at one in 4-flips

There are 10 ways to arrive at one in 5-flips

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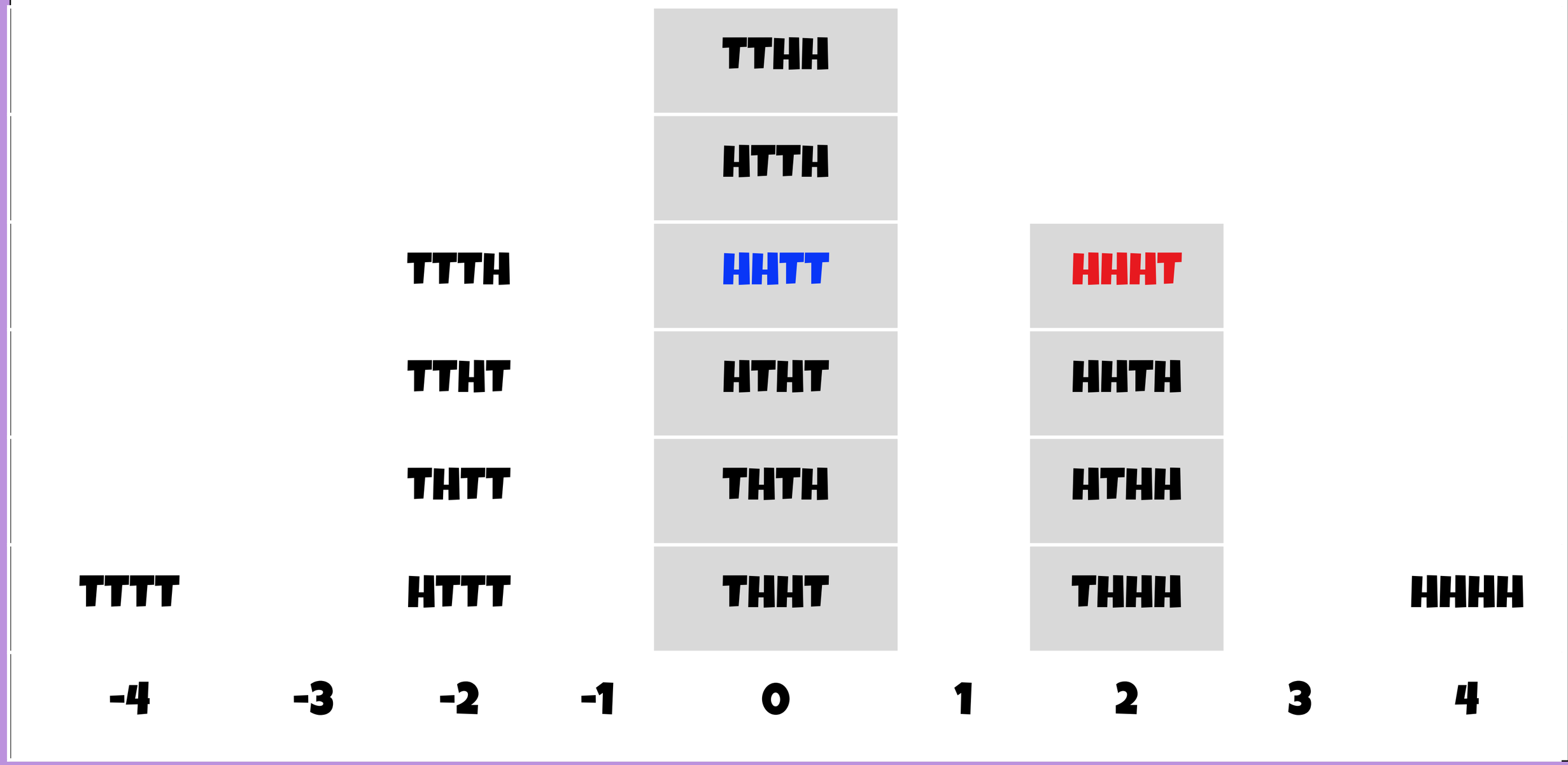
This is not a coincidence!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-1
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			HTHTT	THTHH		
			HTTHT	THHTH		
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		TTTHT	THTTH	HTHHT	HHHTH	
		TTHTT	TTHHT	HHTTH	HHTHH	
		THTTT	TTHTH	HHTHT	HTHHH	
TTTTT		HTTTT	TTTHH	HHHTT	THHHH	HHHHH

More Questions

What if we used dice and made a walk like so:

Move forward by one if you get 1,2,3, or 4

Move back by one if you get 5 or 6

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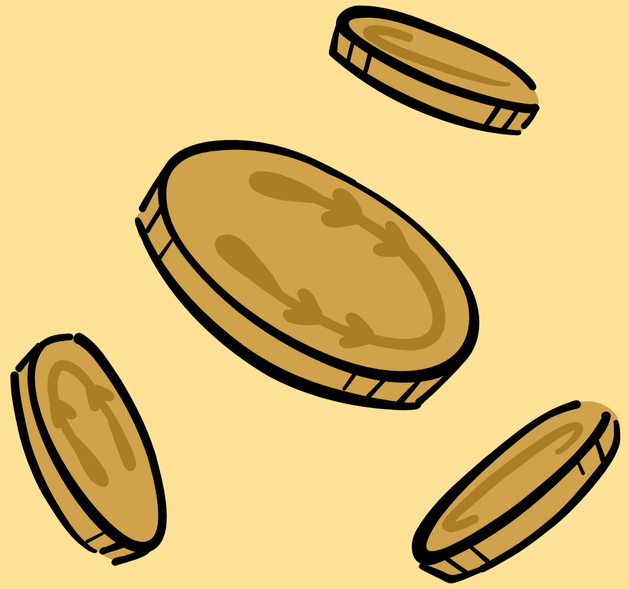
What if we used dice and made a walk like so:

Move forward by one if you get 1,2,3, or 4

Move back by one if you get 5 or 6

How would the final positions change?

MY WORK



WHAT DO I DO?

Use a die with 3 sides: one, two and three on the sides

if it lands on ‘one’, move forward by one step

if it lands on ‘two’, move back by one step

if it lands on ‘three’ move to the ‘mirror’ side

WHAT DO I DO?

Use a die with 3 sides: one, two and three on the sides

if it lands on ‘one’, move forward by one step

move from x to $x+1$

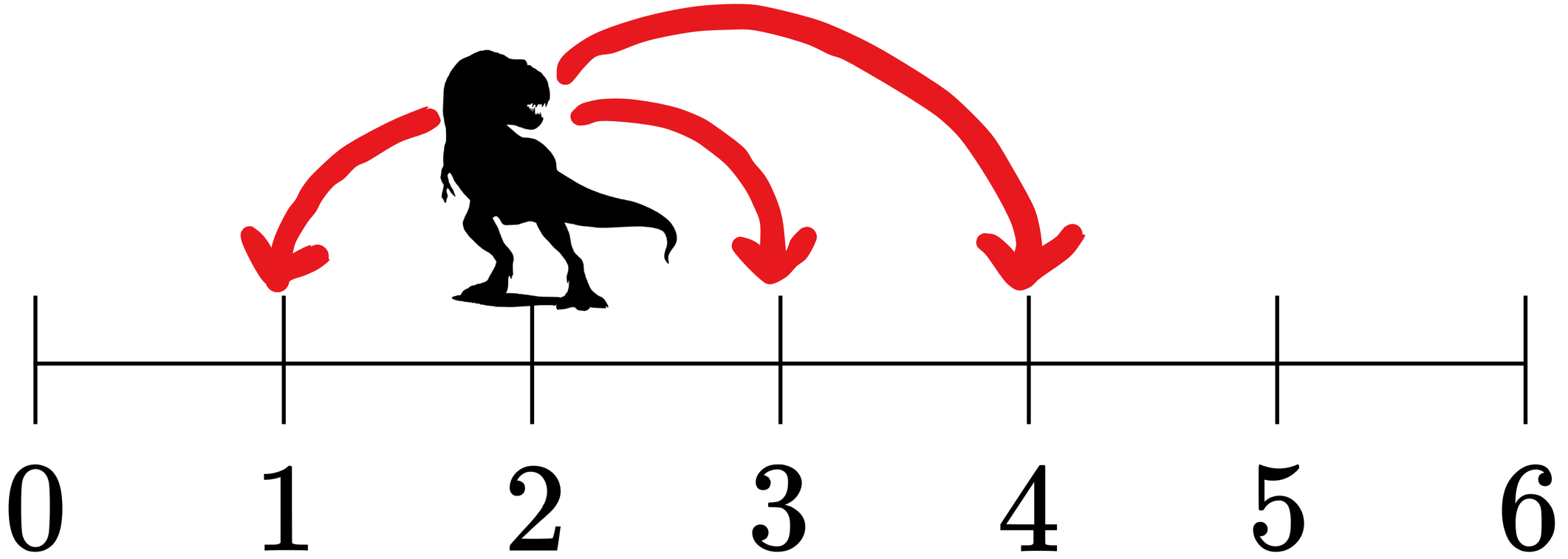
if it lands on ‘two’, move back by one step

move from x to $x-1$

if it lands on ‘three’ move to the ‘mirror’ side

move from x to $N-x$

RANDOM WALK



My Work

The kind of questions I studied:

- Probability of getting all the way to the right

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- Probability of getting all the way to the right
- How long (on average) does it take to get to either of the ends?

My Work

The kind of questions I studied:

- Probability of getting all the way to the right
- How long (on average) does it take to get to either of the ends?
- End behavior

QUESTIONS?

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