TTK4190 Guidance and Control of Vehicles

Assignment 2 Part 1

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Problem 1 - Open-loop analysis

(a) In the absence of wind the ground speed and air speed are equal, i.e.

$$V_w = 0 \implies V_a = V_g = 580\text{km/h} \approx 161,11\text{m/s}. \tag{1}$$

(b) In the absence of wind the crab angle β_c and the sideslip angle β are equal. Then the two expressions for sideslip are:

$$\beta = \arcsin \frac{v_r}{V_a}, \tag{2a}$$

$$\beta = \chi - \psi. \tag{2b}$$

(c) The system dynamics are:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}, \tag{3}$$

with

$$\mathbf{A} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -7.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7.5 \end{bmatrix}. \tag{4}$$

Using the system matrix $\mathbf{A}$ the following model parameters were found:

$$N_r = A_{44} = -0.32, \tag{5a}$$

$$L_r = A_{34} = 0.46, \tag{5b}$$

$$L_p = A_{33} = -2.87, \tag{5c}$$

$$Y_v = A_{11} = -0.322, \tag{5d}$$

$$L_v = \frac{A_{31}}{V_a} = -0.0183, \tag{5e}$$

$$N_v = \frac{A_{41}}{V_a} = 0.0118, \tag{5f}$$

$$Y_r = A_{14} V_a = -650, \tag{5g}$$

$$\tag{5h}$$

The approximate dutch-roll mode is per [1]:

$$\lambda_{\text{dutch roll}} = \frac{Y_v + N_r}{2} \pm \sqrt{\left(\frac{Y_v + N_r}{2}\right)^2 - (Y_v N_r - N_v Y_r)} \tag{6}$$

If we input the computed parameters we get:

$$\lambda_{\text{dutch roll}} = \alpha \pm \beta i \approx -0.321 \pm 2.7739i \tag{7}$$

Then the natural frequency and relative damping ratio is:

$$\omega_0 = \sqrt{\alpha^2 + \beta^2} \approx 2.79 \tag{8}$$

$$\zeta = -\frac{\alpha}{\omega_0} \approx 0.115 \tag{9}$$

Observe that this is a complex conjugated pole with a mostly imaginary part i.e. a low relative damping ratio. The dutch-roll mode then induces under-damped oscillations in the yaw and roll of the aircraft. A larger damping ratio will make the oscillations more damped.

(d) The approximate spiral-divergence mode is given by

$$\lambda_{\text{spiral}} = \frac{N_r L_v - N_v L_r}{L_v} \approx -0.0219. \tag{10}$$

The mode is in the left-hand plane and therefore stable.

(e) The approximate roll mode is

$$\lambda_{\text{rolling}} = L_p = -2.87 \tag{11}$$

The roll mode is therefore faster than the spiral-divergence mode.

Problem 2 - Autopilot for course hold using aileron and successive loop closure

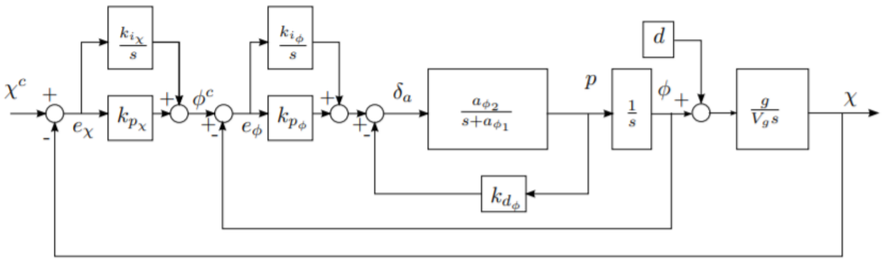


Figure 1: Successive loop closure for course-hold autopilot.

(a) From fig. 1, the transfer-function from δ_a to p is given by the dynamics of p

$$\dot{p} = -10.6\beta - 2.87p + 0.46r - 0.65\delta_a,$$

where the numerical values are given by eq. (4). Laplace transforming the above equation we get

$$\begin{aligned} sp &= -10.6\beta - 2.87p + 0.46r - 0.65\delta_a \\ \Rightarrow p(s+2.87) &= -10.6\beta + 0.46r - 0.65\delta_a \\ \Rightarrow p &= \frac{(-10.6\beta + 0.46r - 0.65\delta_a)}{(s+2.87)} \\ &= \frac{-0.65}{s+2.87}\delta_a. \end{aligned} \tag{12}$$

Where we have used $\beta = r = 0$. Comparing the transfer-function given by eq. (12) to that of fig. 1, we see that $a_{\phi_2} = -0.65$ and $a_{\phi_1} = 2.87$.

(b) We will now calculate appropriate gains k_{p_ϕ}, k_{i_ϕ}, and k_{d_ϕ} for the inner loop and k_{p_χ} and k_{i_χ} for the outer loop of fig. 1. For k_{p_ϕ} we have

$$\begin{aligned} k_{p_\phi} &= \frac{\delta_a^{max}}{e_\phi^{max}}\operatorname{sign}(a_{\phi_2}) \\ &= -2 \end{aligned} \tag{13}$$

where δ_a^{max} and e_ϕ^{max} denote the maximum aileron input angle and corresponding maximum error. These values were provided to be 30° and 15° for input and error respectively.

The derivative gain k_{d_ϕ} is given by

$$k_{d_\phi} = \frac{2\zeta_\phi \omega_{n_\phi} - a_{\phi_1}}{a_{\phi_2}} \tag{14}$$

where ζ_ϕ and $\omega_{n\phi}$ denote the roll damping ratio and roll loop natural frequency. The damping ratio was chosen by design to be given by $\zeta_\phi = 0.707$, whereas the natural frequency is given by

$$\omega_{n_\phi} = \sqrt{|a_{\phi_2}|\,\frac{\delta_a^{\max}}{e_\phi^{\max}}}. \tag{15}$$

The numerical value for eq. (14) was thus found to be $k_{d_\phi} \approx 1.94$.

For k_{i_ϕ}, root locus analysis was used. The root locus of the closed roll loop transfer-function

$$H(s) = \frac{-a_2}{s^3 + (a_1 + a_2 k_{d_\phi})s^2 + (a_2 k_{p_\phi})s} \tag{16}$$

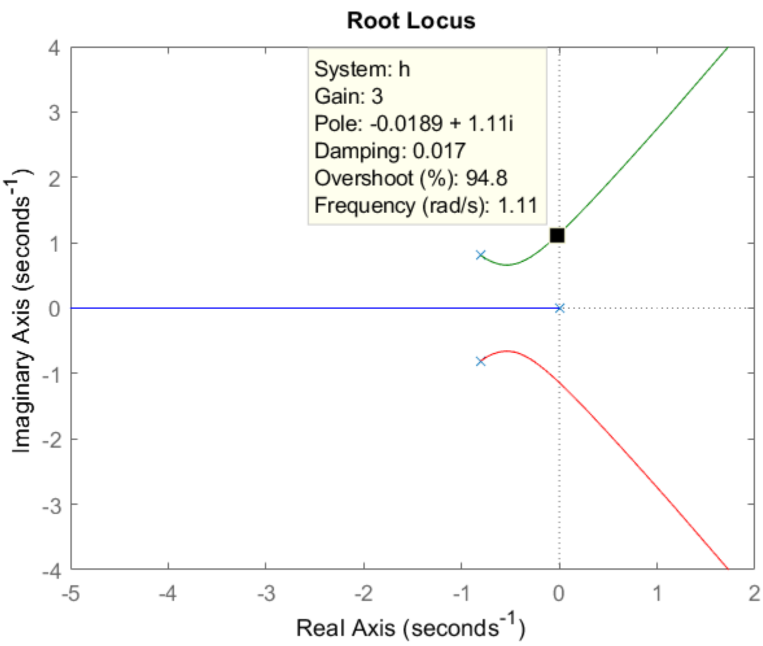


Figure 2: Root-locus of closed roll loop as a function of the integral gain.

plotted as a function of k_{i_ϕ} is shown in fig. 2, where the black square indicates the upper limit for $k_{i_\phi} = 3.0$. The lower limit is given by 0. The gain $k_{i_\phi} = 0.5$ was chosen.

For the course loop gains, k_{p_χ} was computed using

$$k_{p_\chi} = 2\zeta_\chi \omega_{n_\chi} \frac{V_g}{g}, \tag{17}$$

where g denotes the gravitational constant, and ζ_χ and ω_{n_χ} the course damping ratio and natural frequency, chosen by design to be $\frac{\omega_\chi}{20}$ and 1 respectively. Using that $V_g = V_a$, the numerical value of eq. (18) was found to be $k_{p_\chi} \approx 1.87$.

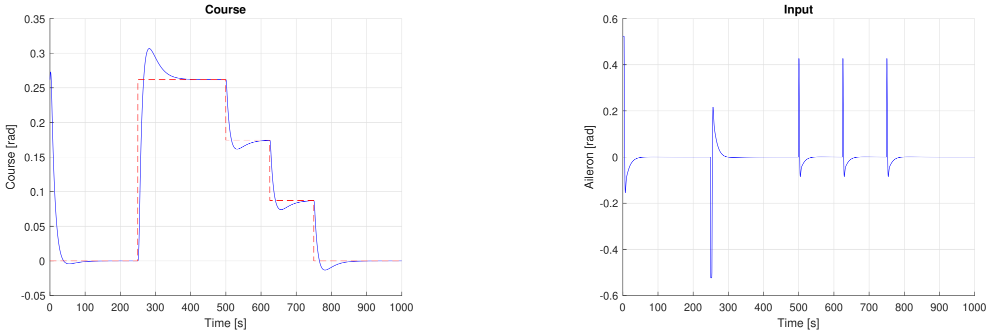
The course integral gain k_{i_χ} was computed using

$$k_{i_\chi} = \omega_{n_\chi} \frac{V_g}{g}, \tag{18}$$

for which the numerical value was found to be $k_{i_\chi} \approx 0.05$.

(c) We are designing a course hold autopilot, so it is not required that the roll disturbance is handled in the inner roll loop. We only need to remove the error due to the disturbance in the outer course loop, which the course error integrator will remove for us. Therefore the inner roll error integrator is not strictly necessary. So the integrator will add an unwanted 90 degree phase shift without actually having to be there. Therefore we have chosen to remove it.

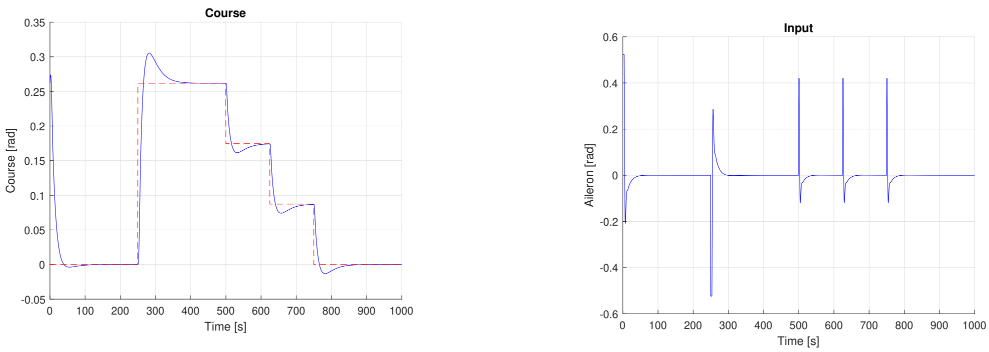
(d) The autopilot was tested with two 15 degree steps, followed by three more frequent 5 degree steps. The resulting input sequence and corresponding course response can be observed in fig. 3b and fig. 4b respectively. We observe that the input is only saturated for a couple seconds for the large inputs and not saturated at all for the smaller steps. Furthermore the course response is quite fast, but there is some overshoot. The outer integrator handles the roll disturbance well, confirming the discussion above.



(a) Course response of autopilot and simplified system to course changing maneuvers.

(b) Aileron input. Observe that the ailerons are saturated at 30 degrees.

(e) The complete linear lateral dynamics with the nonlinear bank-to-turn equation were implemented in MATLAB and simulated with the same controller and course reference. The resulting input sequence and corresponding course response can be observed in **??** and **??** respectively. Furthermore, a comparison of the response of the simplified system versus the full state-space model during the large step maneuver can be studied in fig. 5. We observe that the simplified model coincides very well with the full state-space model. Only during the overshoots do the two trajectories differ significantly, and even then it is still an acceptable error of less than a degree.



(a) Course response of autopilot and full linearized system to course changing maneuvers.

(b) Aileron input.

(f) As we observe from the input plots, the ailerons are almost not saturated as all, so lag induced by integrator wind-up is negligible. If it did turn out to be a problem, one could easily turn the integrator off in the code if the ailerons are saturated. Alternatively one could use MPC to control the entire MIMO system and add saturation in the constraints to deal with it very nicely directly in the model.

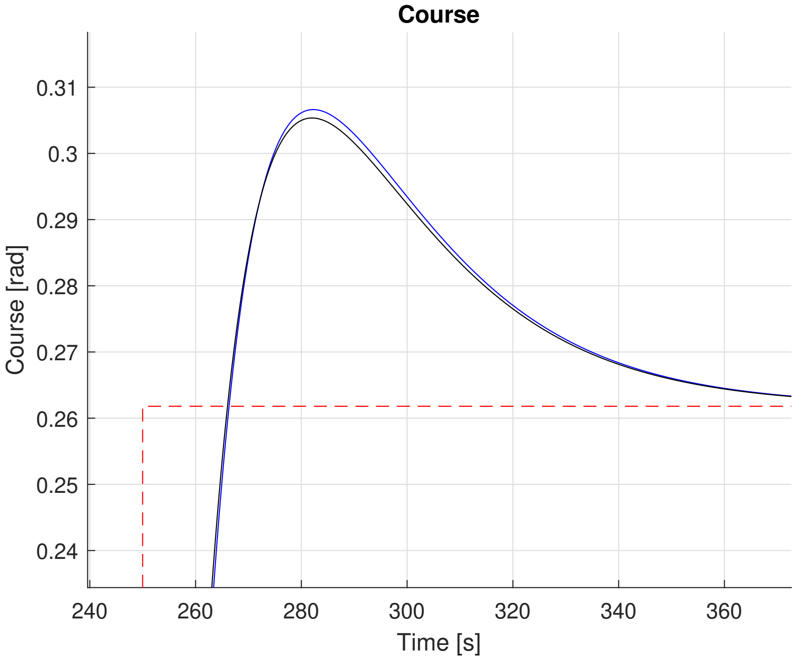


Figure 5: Comparison of course response of simplified system and full linear lateral dynamics. The systems are responding to a 15 degree step.

References

[1] R. W. Beard and T. W. McLain, *Small unmanned aircraft: Theory and practice.* Princeton university press, 2012.