$\boxed{1}$

(a) $q = \begin{bmatrix} x \\ \theta \end{bmatrix}$

Generalized forces:

(b) $r = \begin{bmatrix} x \\ \frac{l}{2}\sin\theta \end{bmatrix}$

$V = \begin{bmatrix} \dot{x} \\ \frac{l}{2}\cos\theta\,\dot{\theta} \end{bmatrix}$

$\omega = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$

(c) $T = \frac{1}{2} m v \cdot v + \frac{1}{2}\omega \cdot M \cdot \omega$

$T = \frac{1}{2} m (\dot{x}^2 + \frac{l^2}{4}\cos^2\theta\,\dot{\theta}^2) + \frac{1}{2} I_z \dot{\theta}^2$

$U = mg\frac{l}{2}\sin\theta$

(d) $L = T - U = \frac{1}{2} m (\dot{x}^2 + \frac{l^2}{4}\cos^2\theta\,\dot{\theta}^2) + \frac{1}{2} I_z \dot{\theta}^2 - \frac{1}{2} mgl\sin\theta$

$\frac{\partial L}{\partial x} = 0, \quad \frac{\partial L}{\partial \theta} = \frac{1}{4} m l^2 \dot{\theta}^2 \cos\theta \cdot -\sin\theta - \frac{1}{2} mgl\cos\theta$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x}, \quad \frac{\partial L}{\partial \dot{\theta}} = \frac{1}{4} m l^2 \cos^2\theta\,\dot{\theta} + I_z \dot{\theta}$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x}, \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{4} m l^2 (\cos^2\theta\,\ddot{\theta} - 2\cos\theta\sin\theta\,\dot{\theta}^2) + I_z\ddot{\theta}$

1) $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = \underline{m\ddot{x} = 0}$

2) $\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \frac{1}{4} m l^2 (\cos^2\theta\,\ddot{\theta} - 2\cos\theta\sin\theta\,\dot{\theta}^2) + I_z\ddot{\theta} + \frac{1}{2} mgl\cos\theta$

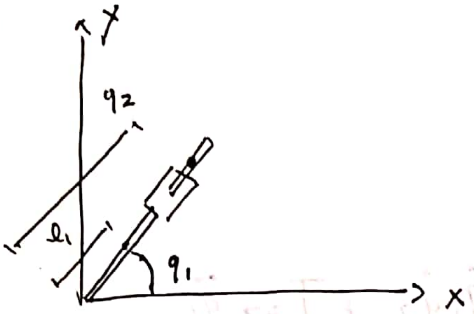
$+ \frac{1}{4} m l^2 \dot{\theta}^2 \cos\theta\sin\theta = \underline{\frac{1}{4} m l^2 (\cos^2\theta\,\ddot{\theta} - \cos\theta\sin\theta\,\dot{\theta}^2) + I_z\ddot{\theta} + \frac{1}{2} mgl\cos\theta}$

$= 0$

So the equations of motion are:

$$\begin{cases} m\ddot{x} = 0\,, \\ \left(\frac{1}{4}ml^2\cos^2\theta + I_z\right)\ddot{\theta} - \frac{1}{4}ml^2\cos\theta\sin\theta\,\dot{\theta} + \frac{1}{2}mgl\cos\theta = 0 \end{cases}$$

2



$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i$$

Let the actuator torque for q_1 be τ and the actuator force for q_2 be F.

ⓐ We have:

$$T_1 = \frac{1}{2} m_i V_{ci}^{i\,T} V_{ci}^{i} + \frac{1}{2} \omega_{0i}^{i\,T} M_{ci}^{i} \omega_{0i}^{i}$$

$$T_1 = \frac{1}{2} m_1 (l_1 \dot{q}_1)^2 + \frac{1}{2} I_{zz_1} \dot{q}_1^2$$

$$T_2 = \frac{1}{2} m_2 (q_2 \dot{q}_1)^2 + \frac{1}{2} I_{zz_2} \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2$$

$$\Rightarrow T = \frac{1}{2}(m_1 l_1^2 \dot{q}_1^2 + m_2 q_2^2 \dot{q}_1^2 + m_2 \dot{q}_2^2) + \frac{1}{2}(I_{zz_1} + I_{zz_2})\dot{q}_1^2$$

$$T = \frac{1}{2} [\dot{q}_1 \; \dot{q}_2] \begin{bmatrix} m_1 l_1^2 + m_2 q_2^2 + I_{zz_1} + I_{zz_2} & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$T = \frac{1}{2} \dot{q}^T M(q) \dot{q} \quad \square$$

ⓑ $U(q) = m_1 g^T r_{c1} + m_2 g^T r_{c2}$

$$U(q) = m_1 g l_1 \sin q_1 + m_2 g q_2 \sin q_1$$

ⓒ $\mathcal{L}(\dot{q}, q) = T(\dot{q}, q) - U(q)$

$\boxed{\text{let } I_{zz} = I_{zz_1} + I_{zz_2}}$

$$= \frac{1}{2}(m_1 l_1^2 \dot{q}_1^2 + m_2 q_2^2 \dot{q}_1^2 + m_2 \dot{q}_2^2) + \frac{1}{2} I_{zz} \dot{q}_1^2$$
$$- m_1 g l_1 \sin q_1 - m_2 g q_2 \sin q_1$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m_1 l_1^2 \dot{q}_1 + m_2 q_2^2 \dot{q}_1 + I_{zz} \dot{q}_1, \quad \frac{\partial \mathcal{L}}{\partial \dot{q}_2} = m_2 \dot{q}_2$$

$$\frac{\partial \mathcal{L}}{\partial q_1} = -m_1 g l_1 \cos q_1 - m_2 g q_2 \cos q_1$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = m_2 q_2 \dot{q}_1^2 - m_2 g \sin q_1$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m_1 l_1^2 \ddot{q}_1 + m_2 q_2^2 \ddot{q}_1 + 2 m_2 q_2 \dot{q}_1 \dot{q}_2 + I_{zz} \ddot{q}_1$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_2} = m_2 \ddot{q}_2$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i$$

1. $m_1 l_1^2 \ddot{q}_1 + m_2 q_2^2 \ddot{q}_1 + 2 m_2 q_2 \dot{q}_1 \dot{q}_2 + I_{zz} \ddot{q}_1 + m_1 g l_1 \cos q_1 + m_2 g q_2 \cos q_1 = \tau$
2. $m_2 \ddot{q}_2 + m_2 g \sin q_1 - m_2 q_2 \dot{q}_1^2 = F$

(d) Writing the EOM in matrix form we get:

$$\begin{bmatrix} m_1 l_1^2 + m_2 q_2^2 + I_{zz} & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 2 m_2 q_2 \dot{q}_1 \dot{q}_2 \\ - m_2 q_2 \dot{q}_1^2 \end{bmatrix} + \begin{bmatrix} m_1 g l_1 \cos q_1 + m_2 g q_2 \cos q_1 \\ m_2 g \sin q_2 \end{bmatrix}$$

$$= \begin{bmatrix} \tau \\ F \end{bmatrix} \iff M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

Where $\tau = \begin{bmatrix} \tau \\ F \end{bmatrix}$, $g(q) = \begin{bmatrix} m_1 g l_1 \cos q_1 + m_2 g q_2 \cos q_1 \\ m_2 g \sin q_2 \end{bmatrix}$,

and $C(q,\dot{q})$ can be defined in multiple ways, because of the $\dot{q}_1 \dot{q}_2$ term that allows for splitting the term arbitrarily between the two elements in the first row.

The Christoffel symbol representation is:

$$C(q,\dot q) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \quad c_{kj} = \sum_{i=1}^{2} \frac{\dot q_i}{2}\left(\frac{\partial m_{kj}}{\partial q_i} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{ij}}{\partial q_k}\right)$$

$$c_{11} = \frac{\dot q_1}{2}\left(\frac{\partial m_{11}}{\partial q_1} + \frac{\partial m_{11}}{\partial q_1} - \frac{\partial m_{11}}{\partial q_1}\right) + \frac{\dot q_2}{2}\left(\frac{\partial m_{11}}{\partial q_2} + \frac{\partial m_{21}}{\partial q_1} - \frac{\partial m_{21}}{\partial q_1}\right)$$

$$= \frac{\dot q_1}{2}\frac{\partial m_{11}}{\partial q_1} + \frac{\dot q_2}{2}\frac{\partial m_{11}}{\partial q_2} = \underline{m_2 q_2 \dot q_2}$$

$$c_{12} = \frac{\dot q_1}{2}\left(\frac{\partial m_{12}}{\partial q_1} + \frac{\partial m_{11}}{\partial q_2} - \frac{\partial m_{12}}{\partial q_1}\right) + \frac{\dot q_2}{2}\left(\frac{\partial m_{12}}{\partial q_2} + \frac{\partial m_{21}}{\partial q_2} - \frac{\partial m_{22}}{\partial q_1}\right)$$

$$= \frac{\dot q_1}{2}\frac{\partial m_{11}}{\partial q_2} - \frac{\dot q_2}{2}\frac{\partial m_{22}}{\partial q_1} = \underline{m_2 q_2 \dot q_1}$$

$$c_{21} = \frac{\dot q_1}{2}\left(\frac{\partial m_{21}}{\partial q_1} + \frac{\partial m_{12}}{\partial q_1} - \frac{\partial m_{11}}{\partial q_2}\right) + \frac{\dot q_2}{2}\left(\frac{\partial m_{21}}{\partial q_2} + \frac{\partial m_{22}}{\partial q_1} - \frac{\partial m_{21}}{\partial q_2}\right)$$

$$= \frac{\dot q_2}{2}\frac{\partial m_{22}}{\partial q_1} - \frac{\dot q_1}{2}\frac{\partial m_{11}}{\partial q_2} = \underline{-m_2 q_2 \dot q_1}$$

$$c_{22} = \frac{\dot q_1}{2}\left(\frac{\partial m_{22}}{\partial q_1} + \frac{\partial m_{12}}{\partial q_2} - \frac{\partial m_{12}}{\partial q_2}\right) + \frac{\dot q_2}{2}\left(\frac{\partial m_{22}}{\partial q_2} + \frac{\partial m_{22}}{\partial q_2} - \frac{\partial m_{22}}{\partial q_2}\right)$$

$$\underline{= 0}$$

$$\Rightarrow \underline{C(q,\dot q) = \begin{bmatrix} m_2 q_2 \dot q_2 & m_2 q_2 \dot q_1 \\ -m_2 q_2 \dot q_1 & 0 \end{bmatrix}} \quad \square$$

(e) ~~It is trivial to see that C =~~
C is clearly not symmetric or skew-symmetric ($C \neq C^T, C \neq -C^T$).
From Sylvester's criterion we see that it is also not positive ~~semi~~ definite, as $m_2 q_2 \dot q_2 > 0$ is not always satisfied.

M is symmetric and positive definite, as $M = M^T$ and $m_{11} > 0$, $m_{22} > 0$.

(f) $\dot{M} - 2C = \begin{bmatrix} 2m_2 q_2 \dot{q}_2 & 0 \\ 0 & 0 \end{bmatrix} - 2 \begin{bmatrix} m_2 q_2 \dot{q}_2 & m_2 q_2 \dot{q}_1 \\ -m_2 q_2 \dot{q}_1 & 0 \end{bmatrix}$

$\dot{M} - 2C = \begin{bmatrix} 0 & -2m_2 q_2 \dot{q}_1 \\ 2m_2 q_2 \dot{q}_1 & 0 \end{bmatrix}$

It is evident that $(\dot{M} - 2C) = -(\dot{M} - 2C)^T$. Wow!

(g) $\dot{E}(q, \dot{q}) = \dot{T}(q, \dot{q}) + \dot{U}$

$= \dot{q}^T M(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \frac{\partial U}{\partial q} \dot{q}$

$= \dot{q}^T M(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + g^T \dot{q}$

$\overset{(4)}{=} \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \dot{q}^T \tau - \dot{q}^T C(q, \dot{q}) \dot{q}$

$= \dot{q}^T \tau + \frac{1}{2} \underbrace{\dot{q}^T (\dot{M}(q) - 2C(q, \dot{q})) \dot{q}}_{=0}$

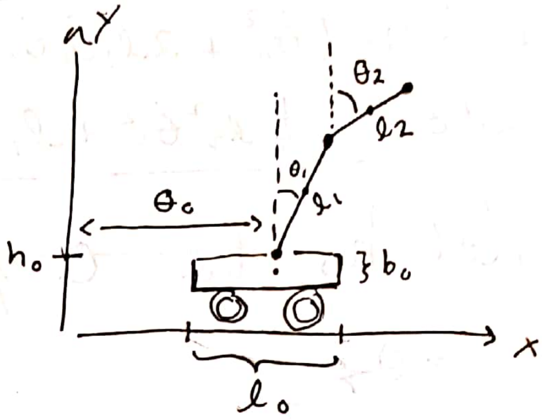
since $\dot{M} - 2C$ is skew-symmetric

$\dot{E}(q, \dot{q}) = \dot{q}^T \tau$ $\square$

3 ⓐ $q = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}$

CoM of the rods + cart:

~~Assume~~



$$r_{i0} = \begin{bmatrix} \theta_0 \\ h_0 - \frac{1}{2}b_0 \end{bmatrix}$$

$$r_{i1} = \begin{bmatrix} \frac{l_1}{2}\sin\theta_1 \\ \frac{l_1}{2}\cos\theta_1 + \frac{1}{2}b_0 \end{bmatrix} + r_{i0} = \begin{bmatrix} \theta_0 + \frac{l_1}{2}\sin\theta_1 \\ h_0 + \frac{l_1}{2}\cos\theta_1 \end{bmatrix}$$

$$r_{i2} = \begin{bmatrix} \frac{l_2}{2}\sin\theta_2 \\ \frac{l_2}{2}\cos\theta_2 \end{bmatrix} + \begin{bmatrix} \theta_0 + l_1\sin\theta_1 \\ h_0 + l_1\cos\theta_1 \end{bmatrix}$$

$$r_{i2} = \begin{bmatrix} \theta_0 + l_1\sin\theta_1 + \frac{l_2}{2}\sin\theta_2 \\ h_0 + l_1\cos\theta_1 + \frac{l_2}{2}\cos\theta_2 \end{bmatrix}$$

ⓑ $\boxed{T_1 = \frac{1}{2}m_0\dot\theta_0^2}$

$$T_2 = \frac{1}{2}m_1\left(\left(\dot\theta_0 + \frac{l_1}{2}\cos\theta_1\dot\theta_1\right)^2 + \left(\frac{l_1}{2}\dot\theta_1\sin\theta_1\right)^2\right) + \frac{1}{2}I_1\dot\theta_1^2$$

$$= \frac{1}{2}m_1\left(\dot\theta_0^2 + l_1\dot\theta_0\dot\theta_1\cos\theta_1 + \frac{l_1^2}{4}\dot\theta_1^2\cos^2\theta_1 + \frac{l_1^2}{4}\dot\theta_1^2\sin^2\theta_1\right) + \frac{1}{2}I_1\dot\theta_1^2$$

$$\boxed{T_2 = \frac{1}{2}m_1\left(\dot\theta_0^2 + l_1\dot\theta_0\dot\theta_1\cos\theta_1 + \frac{l_1^2}{4}\dot\theta_1^2\right) + \frac{1}{2}I_1\dot\theta_1^2}$$

$$T_3 = \frac{1}{2}m_2\left(\left(\dot\theta_0 + l_1\dot\theta_1\cos\theta_1 + \frac{l_2}{2}\dot\theta_2\cos\theta_2\right)^2 + \left(l_1\dot\theta_1\sin\theta_1 + \frac{l_2}{2}\dot\theta_2\sin\theta_2\right)^2\right) + \frac{1}{2}I_2\dot\theta_2^2$$

$$= \frac{1}{2}m_2\left(\dot\theta_0^2 + 2\dot\theta_0\left(l_1\dot\theta_1\cos\theta_1 + \frac{l_2}{2}\dot\theta_2\cos\theta_2\right) + l_1^2\dot\theta_1^2\cos^2\theta_1 + l_1l_2\dot\theta_1\dot\theta_2\cos\theta_1\cos\theta_2 + \frac{l_2^2}{4}\dot\theta_2^2\cos^2\theta_2 + l_1^2\dot\theta_1^2\sin^2\theta_1 + l_1l_2\dot\theta_1\dot\theta_2\sin\theta_1\sin\theta_2 + \frac{l_2^2}{4}\dot\theta_2^2\sin^2\theta_2\right) + \frac{1}{2}I_2\dot\theta_2^2$$

$$\boxed{T_3 = \frac{1}{2} m_2 \left(\dot\theta_0^2 + 2 l_1 \dot\theta_0 \dot\theta_1 \cos\theta_1 + l_2 \dot\theta_0 \dot\theta_1 \cos\theta_2 + l_1^2 \dot\theta_1^2 + \frac{1}{4} l_2^2 \dot\theta_2^2 + l_1 l_2 \dot\theta_1 \dot\theta_2 \cos(\theta_1 - \theta_2) \right) + \frac{1}{2} I_2 \dot\theta_2^2}$$

$$\boxed{\begin{aligned} T = {} & \frac{1}{2}(m_0 + m_1 + m_2)\dot\theta_0^2 + \frac{1}{2}(m_1 + 2m_2) l_1 \dot\theta_0 \dot\theta_1 \cos\theta_1 \\ & + \frac{1}{2}\left(\frac{1}{4} m_1 + m_2\right) l_1^2 \dot\theta_1^2 + \frac{1}{2} m_2 \Big(l_2 \dot\theta_0 \dot\theta_1 \cos\theta_2 \\ & + \frac{1}{4} l_2^2 \dot\theta_2^2 + l_1 l_2 \dot\theta_1 \dot\theta_2 \cos(\theta_1 - \theta_2) \Big) + \frac{1}{2} I_1 \dot\theta_1^2 + \frac{1}{2} I_2 \dot\theta_2^2 \end{aligned}}$$

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ⓒ Let $U_0 = 0$.

$$\Rightarrow U_1 = \frac{l_1}{2}\cos\theta_1 m_1 g \quad , \quad U_2 = (l_1\cos\theta_1 + \frac{l_2}{2}\cos\theta_2) m_2 g$$

$$\Rightarrow U = l_1 g\cos\theta_1 (\tfrac{1}{2}m_1 + m_2) + \tfrac{1}{2} l_2 m_2 g\cos\theta_2$$

ⓓ $\mathcal{L} = T - U$

$$\frac{\partial\mathcal{L}}{\partial\dot\theta_0} = (m_0+m_1+m_2)\dot\theta_0 + \tfrac{1}{2}(m_1+2m_2) l_1\dot\theta_1\cos\theta_1$$
$$+ \tfrac{1}{2} m_2 l_2 \dot\theta_1 \cos\theta_2$$

$$\frac{\partial\mathcal{L}}{\partial\dot\theta_1} = \tfrac{1}{2}(m_1+2m_2) l_1\dot\theta_0\cos\theta_1 + (\tfrac{1}{4}m_1+m_2) l_1^2\dot\theta_1$$
$$+ \tfrac{1}{2}m_2 (l_2\dot\theta_0\cos\theta_2 + l_1 l_2\dot\theta_2\cos(\theta_1-\theta_2)) + I_1\dot\theta_1$$

$$\frac{\partial\mathcal{L}}{\partial\dot\theta_2} = \tfrac{1}{2}m_2(\tfrac{1}{2}l_2^2\dot\theta_2 + l_1 l_2\dot\theta_1\cos(\theta_1-\theta_2)) + I_2\dot\theta_2$$

$$\frac{\partial\mathcal{L}}{\partial\theta_0} = 0$$

$$\frac{\partial\mathcal{L}}{\partial\theta_1} = -\tfrac{1}{2}(m_1+2m_2) l_1\dot\theta_0\dot\theta_1\sin\theta_1$$
$$- \tfrac{1}{2}m_2 l_1 l_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2) + l_1 g(\tfrac{1}{2}m_1+m_2)\sin\theta_1$$

$$\frac{\partial\mathcal{L}}{\partial\theta_2} = \tfrac{1}{2}m_2(-l_2\dot\theta_0\dot\theta_1\sin\theta_2 + l_1 l_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2))$$
$$+ \tfrac{1}{2} l_2 m_2 g\sin\theta_2$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_0} = (m_0+m_1+m_2)\ddot\theta_0 + \frac{1}{2}(m_1+2m_2)l_1(\ddot\theta_1\cos\theta_1 - \dot\theta_1^2\sin\theta_1)$$

$$+\frac{1}{2}m_2 l_2(\ddot\theta_1\cos\theta_2 - \dot\theta_1\sin\theta_2\dot\theta_2)$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_1} = \frac{1}{2}(m_1+2m_2)l_1(\ddot\theta_0\cos\theta_1 - \dot\theta_0\dot\theta_1\sin\theta_1)$$

$$+\left(\frac{1}{4}m_1+m_2\right)l_1^2\ddot\theta_1 + \frac{1}{2}m_2\Big(l_2\ddot\theta_0\cos\theta_2 - l_2\dot\theta_0\dot\theta_2\sin\theta_2$$

$$+l_1l_2\ddot\theta_2\cos(\theta_1-\theta_2) - l_1l_2\dot\theta_2\sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\Big) + I_1\ddot\theta_1$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_2} = \frac{1}{2}m_2\Big(\frac{1}{2}l_2^2\ddot\theta_2 + l_1l_2\ddot\theta_1\cos(\theta_1-\theta_2)$$

$$- l_1l_2\dot\theta_1\sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\Big) + I_2\ddot\theta_2$$

$$1: \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_0} - \frac{\partial \mathcal{L}}{\partial \theta_0} = (m_0+m_1+m_2)\ddot\theta_0 + \frac{1}{2}(m_1+2m_2)l_1$$

$$\cdot(\ddot\theta_1\cos\theta_1 - \dot\theta_1^2\sin\theta_1) + \frac{1}{2}m_2l_2(\ddot\theta_1\cos\theta_2 - \dot\theta_1\sin\theta_2\dot\theta_2) = \tau$$

$$2: \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_1} - \frac{\partial \mathcal{L}}{\partial \theta_1} = \frac{1}{2}(m_1+2m_2)l_1(\ddot\theta_0\cos\theta_1 - \dot\theta_0\dot\theta_1\sin\theta_1)$$

$$+\left(\frac{1}{4}m_1+m_2\right)l_1^2\ddot\theta_1 + \frac{1}{2}m_2\Big(l_2\ddot\theta_0\cos\theta_2 - l_2\dot\theta_0\dot\theta_2\sin\theta_2$$

$$+l_1l_2\ddot\theta_2\cos(\theta_1-\theta_2) - l_1l_2\dot\theta_2\sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\Big) + I_1\ddot\theta_1$$

$$+\frac{1}{2}m_2l_1l_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2) + \frac{1}{2}(m_1+2m_2)l_1\dot\theta_0\dot\theta_1\sin\theta_1$$

$$-l_1g\left(\frac{1}{2}m_1+m_2\right)\sin\theta_1 = 0$$

$$3: \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_2} - \frac{\partial \mathcal{L}}{\partial \theta_2} = \frac{1}{2}m_2\Big(\frac{1}{2}l_2^2\ddot\theta_2 + l_1l_2\ddot\theta_1\cos(\theta_1-\theta_2)$$

$$-l_1l_2\dot\theta_1\sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\Big) + I_2\ddot\theta_2 + \frac{1}{2}m_2\Big($$

$$l_2\dot\theta_0\dot\theta_1\sin\theta_2 - l_1l_2\dot\theta_1\dot\theta_2\sin(\theta_1-\theta_2)\Big) - \frac{1}{2}l_2m_2g\sin\theta_2$$

$$=0$$

To sum up, the EoM of the DIPC system are:

$$(m_0+m_1+m_2)\ddot{\theta}_0 + \left(\tfrac{1}{2}(m_1+2m_2)l_1\cos\theta_1 + \tfrac{1}{2}m_2 l_2\cos\theta_2\right)\ddot{\theta}_1$$
$$- \tfrac{1}{2}(m_1+2m_2)l_1\dot{\theta}_1^2\sin\theta_1 - \tfrac{1}{2}m_2 l_2\dot{\theta}_1\dot{\theta}_2\sin\theta_2 = \tau,$$

$$\left(\tfrac{1}{2}(m_1+2m_2)l_1\cos\theta_1 + \tfrac{1}{2}m_2 l_2\cos\theta_2\right)\ddot{\theta}_0$$
$$+ \left(\left(\tfrac{1}{4}m_1+m_2\right)l_1^2+I_1\right)\ddot{\theta}_1 + l_1 l_2\cos(\theta_1-\theta_2)\ddot{\theta}_2$$
$$+ \tfrac{1}{2}m_2 l_1 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) - \tfrac{1}{2}m_2 l_2\dot{\theta}_0\dot{\theta}_2\sin\theta_2$$
$$- \tfrac{1}{2}(m_1+2m_2)l_1 g\sin\theta_1 = 0,$$

$$\tfrac{1}{2}m_2 l_1 l_2\cos(\theta_1-\theta_2)\ddot{\theta}_1 + \left(\tfrac{1}{4}m_2 l_1^2 + I_2\right)\ddot{\theta}_2$$
$$+ \tfrac{1}{2}m_2 l_2\sin\theta_2\dot{\theta}_0\dot{\theta}_1 - \tfrac{1}{2}m_2 l_1 l_2\sin(\theta_1-\theta_2)\dot{\theta}_1^2$$
$$- \tfrac{1}{2}m_2 l_2 g\sin\theta_2 = 0$$

(ish)