$\boxed{1}$

$0 \leq z \leq h\left(1 - \max\left\{\frac{|x|}{r}, \frac{|y|}{s}\right\}\right)$,

$-r \leq x \leq r$, $-s \leq y \leq s$, $\rho = ax + by + cz + d$,

s.t. $ch + d > 0$, $d > r|a| + s|b|$

(a) 2b ensures that $\rho(x, y, z)$ is always positive. The point with the lowest density is $(-r, -s, 0)$, so

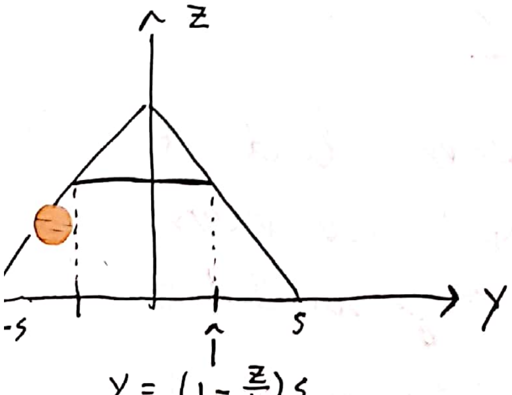
$\rho(-r, -s, 0) = -ra - sb + d > 0$

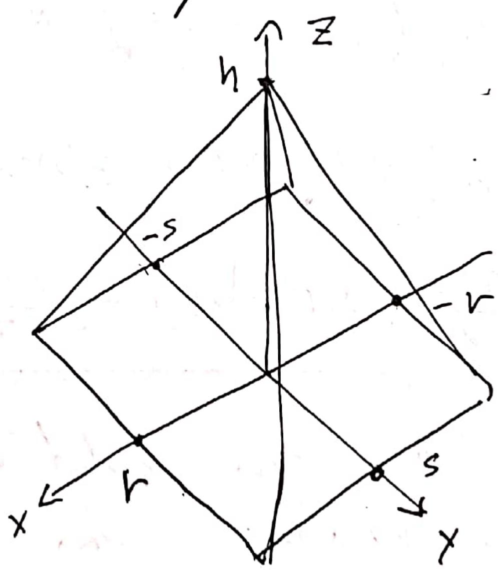
$\Rightarrow d > ra + sb$

In the case where a and b are both negative instead we get the opposite, therefore the absolute values.

2a ensures that the point on the top of the pyramid has positive density (?)

(b) $m = \int_b dm = \int_b \rho dV$





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$$m = \int_0^h \int_{-(1-\frac{z}{h})s}^{(1-\frac{z}{h})s} \int_{-(1-\frac{z}{h})r}^{(1-\frac{z}{h})r} (ax+by+cz+d)\,dx\,dy\,dz$$

Since the contribution of the extra mass from the ax-term for positive ax is removed for negative ax, by symmetry, the term can be removed. Simmilarly for the by-term.

Let $w = (1-\frac{z}{h}) \Rightarrow dz = -h\,dw,\ z = (1-w)h$

$$\Rightarrow m = h \int_0^1 \int_{-ws}^{ws} \int_{-wr}^{wr} (ch(1-w)+d)\,dx\,dy\,dw$$

$$= h \int_0^1 \int_{-ws}^{ws} (d+ch(1-w))\,2wr\,dy\,dw$$

$$= h \int_0^1 (d+ch(1-w))\,4w^2 rs\,dw$$

$$= 4rsh \int_0^1 ((d+ch)w^2 - chw^3)\,dw$$

$$= 4rsh \left(\tfrac{1}{3}(d+ch)w^3 - \tfrac{1}{4}chw^4\right)\Big|_0^1$$

$$= 4rsh\left(\tfrac{1}{3}d + \tfrac{1}{12}ch\right)$$

$$\underline{\underline{m = \frac{rsh}{3}(4d+ch)}} \quad \square$$

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ⓒ $\int_b r_p \, dm = \int_b \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rho \, dV = \iiint_{z\,y\,x} \begin{bmatrix} ax^2 + (by+cz+d)x \\ by^2 + (ax+cz+d)y \\ cz^2 + (ax+by+d)z \end{bmatrix} dx\,dy\,dz$

Same symmetry simplification as before: (all terms linear in x and y dissappear)

$$\int_b r_p \, dm = \iiint_{z\,y\,x} \begin{bmatrix} ax^2 \\ by^2 \\ cz^2 + dz \end{bmatrix} dx\,dy\,dz$$

Same substitution with $w = (1 - \frac{z}{h})$:

$$\int_b r_p \, dm = h \int_0^1 \int_{-ws}^{ws} \int_{-wr}^{wr} \begin{bmatrix} ax^2 \\ by^2 \\ ch^2(1-w)^2 + dh(1-w) \end{bmatrix} dx\,dy\,dw$$

$$= h \int_0^1 \int_{-ws}^{ws} \begin{bmatrix} \frac{2}{3} a w^3 r^3 \\ 2 b y^2 w r \\ (2ch^2(1-w)^2 + 2dh(1-w))\, wr \end{bmatrix} dy\,dw$$

$$= h \int_0^1 \begin{bmatrix} \frac{4}{3} a r^3 s w^4 \\ \frac{4}{3} b r s^3 w^4 \\ 4ch^2(1-w)^2 w^2 rs + 4dh(1-w) w^2 rs \end{bmatrix} dw$$

$$= 4rsh \begin{bmatrix} \frac{1}{15} a r^2 w^5 \\ \frac{1}{15} b s^2 w^5 \\ ch^2(\frac{1}{3} w^3 - \frac{1}{2} w^4 + \frac{1}{5} w^5) + dh(\frac{1}{3} w^3 - \frac{1}{4} w^4) \end{bmatrix}_{w=1}$$

$$= 4rsh \begin{bmatrix} \frac{1}{15} a r^2 \\ \frac{1}{15} b s^2 \\ \frac{1}{30} ch^2 + \frac{1}{12} dh \end{bmatrix}$$

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$$r_c^b = \frac{1}{m}\int_b r_p \, dm = \frac{3}{rsh} \cdot \frac{4rsh}{4d+ch} \begin{bmatrix} ar^2/15 \\ bs^2/15 \\ ch^2/30 + dh/12 \end{bmatrix}$$

$$\underline{\underline{r_c^b = \frac{12}{4d+ch} \begin{bmatrix} ar^2/15 \\ bs^2/15 \\ ch^2/30 + dh/12 \end{bmatrix}}}$$

(d) $M_{b/o}^b = \int_b (r^{bT} r^b \mathbb{I} - r^b r^{bT})\, dm$

$$= \int_b (x^2+y^2+z^2)\mathbb{I} - \begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{bmatrix} dm = \int_b \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} dm$$

This is actually ridiculous...
We simplify by removing all linear and cubic terms in x and y:

$$M_{b/o}^b = \iiint_{z\,y\,x} \begin{bmatrix} (y^2+z^2)(cz+d) & -xy\cdot 0 & -xz\cdot ax \\ -xy\cdot 0 & (x^2+z^2)(cz+d) & -yz\cdot by \\ -xz\cdot ax & -yz\cdot by & (x^2+y^2)(cz+d) \end{bmatrix}$$

$$= \iiint_{z\,y\,x} \begin{bmatrix} y^2cz+y^2d+cz^3+dz^2 & 0 & -ax^2z \\ 0 & cx^2z+dx^2+cz^3+dz^2 & -by^2z \\ -ax^2z & -by^2z & cx^2z+dx^2+cy^2z+dy^2 \end{bmatrix}$$

$$= \iint_{z\,y} \begin{bmatrix} (cy^2z+dy^2+cz^3+dz^2)2wr & 0 & -\frac{2a}{3}w^3r^3z \\ 0 & \begin{pmatrix} (cz^3+dz^2)2wr \\ +(cz+d)\frac{2}{3}w^3r^3 \end{pmatrix} & -by^2z2wr \\ -\frac{2a}{3}w^3r^3z & -by^2z2wr & \begin{pmatrix} (cy^2z+dy^2)2wr+ \\ (cz+d)\frac{2}{3}w^3r^3 \end{pmatrix} \end{bmatrix}$$

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$$= \int_z \begin{bmatrix} (cz^3+dz^2)4w^2rs + (cz+d)2wr\cdot\frac{2}{3}w^3s^3 & 0 & - \\ 0 & (cz^3+dz^2)4w^2rs + (cz+d)\frac{4}{3}w^4r^3s & - \\ -\frac{4}{3}aw^4r^3sz & -\frac{4}{3}bw^4rs^3z & (cz+d)\frac{4}{3}w^4r^3s + (cz+d)\frac{4}{3}w^4rs^3 \end{bmatrix}$$

Change of variables: $z = (1-w)h$

$$\Rightarrow M^b_{b10} = h\int_0^1 \begin{bmatrix} (ch+d-chw)4w^2rs\cdot(1-w^2)h^2 + \frac{4}{3}(ch+d-chw)\cdot w^4rs^3 & 0 & - \\ 0 & (ch+d-chw)\cdot(1-w)^2h^2 + \frac{4}{3}(ch+d-chw)\cdot w^4r^3s & - \\ -\frac{4}{3}ar^3sh(1-w)w^4 & -\frac{4}{3}brs^3h(1-w)w^4 & \frac{4}{3}(ch+d-chw)w^4rs(r^2+s^2) \end{bmatrix}$$

$$= 4rsh\int_0^1 \begin{bmatrix} (ch+d-chw)\cdot\left(w^2(1-w)^2h^2 + \frac{1}{3}w^4s^2\right) & 0 & - \\ 0 & (ch+d-chw)\cdot\left(w^2(1-w)^2h^2 + \frac{1}{3}w^4r^2\right) & - \\ -\frac{1}{3}aw^4r^2(1-w)h & -\frac{1}{3}bw^4s^2(1-w)h & \frac{1}{3}(ch+d-chw)\cdot w^4(r^2+s^2) \end{bmatrix}$$

$$= 4rsh \begin{bmatrix} (ch+d)\left(\frac{h^2}{30}+\frac{s^2}{15}\right) - ch\left(\frac{h^2}{60}+\frac{s^2}{18}\right) & 0 & -\frac{1}{90}ar^2h \\ 0 & (ch+d)\left(\frac{h^2}{30}+\frac{r^2}{15}\right) - ch\left(\frac{h^2}{60}+\frac{s^2}{18}\right) & -\frac{1}{90}bs^2h \\ -\frac{1}{90}ar^2h & -\frac{1}{90}bs^2h & \frac{1}{3}(r^2+s^2)\left(\frac{ch}{30}+\frac{d}{5}\right) \end{bmatrix}$$

$$= 4rsh \begin{bmatrix} d\left(\frac{h^2}{30}+\frac{s^2}{15}\right) + ch\left(\frac{h^2}{60}+\frac{s^2}{90}\right) & 0 & -\frac{1}{90}ar^2h \\ 0 & d\left(\frac{h^2}{30}+\frac{r^2}{15}\right) + ch\left(\frac{h^2}{60}+\frac{r^2}{90}\right) & -\frac{1}{90}bs^2h \\ -\frac{1}{90}ar^2h & -\frac{1}{90}bs^2h & \frac{1}{3}(r^2+s^2)\left(\frac{ch}{30}+\frac{d}{5}\right) \end{bmatrix}$$

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$$M_{b/o}^{b} = \frac{2}{45} rsh \begin{bmatrix} d(3h^2+6s^2) + ch(\frac{3}{2}h^2+s^2) & 0 & -ar^2h \\ 0 & d(3h^2+6r^2) + ch(\frac{3}{2}h^2+r^2) & -bs^2h \\ -ar^2h & -bs^2h & (r^2+s^2)(ch+6d) \end{bmatrix}$$

---

e) $h = r = s,\ c = 0,\ d = 3ah,\ a = b > 0$

$$\Rightarrow \underline{M_{b/o}^{b}} = \underline{\frac{2}{45} h^6 a \begin{bmatrix} 27 & 0 & -1 \\ 0 & 27 & -1 \\ -1 & -1 & 36 \end{bmatrix}}$$

$$\underline{m} = \frac{h^3}{3} \cdot 12ah = \underline{4ah^4}$$

$$\underline{r_c^b} = \begin{bmatrix} \frac{r^2a}{5d} & \frac{r^2a}{5d} & \frac{h}{4} \end{bmatrix}^T = \underline{\begin{bmatrix} \frac{h}{15} & \frac{h}{15} & \frac{h}{4} \end{bmatrix}^T}$$

Parallel axis theorem:

$$M_{b/c}^{b} = M_{b/o}^{b} - m(r_c^{bT} r_c^b \mathbb{I} - r_c^b r_c^{bT})$$

$$r_c^{bT} r_c^b \mathbb{I} - r_c^b r_c^{bT} = \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} = \begin{bmatrix} \frac{241h^2}{3600} & * & * \\ -h^2/225 & \frac{241h^2}{3600} & * \\ -h^2/60 & -h^2/60 & \frac{2h^2}{225} \end{bmatrix}$$

$$\Rightarrow M_{b/c}^{b} = \begin{bmatrix} 27 & 0 & -1 \\ 0 & 27 & -1 \\ -1 & -1 & 36 \end{bmatrix} \frac{2}{45} h^6 a - \begin{bmatrix} \frac{241}{3600} & * & * \\ -\frac{1}{225} & \frac{241}{3600} & * \\ -\frac{1}{60} & -\frac{1}{60} & \frac{2}{225} \end{bmatrix} 4ah^6$$

$$\underline{\underline{M_{b/c}^{b} = \frac{h^6 a}{900} \begin{bmatrix} 839 & 16 & 20 \\ 16 & 839 & 20 \\ 20 & 20 & 1408 \end{bmatrix}}} \quad \square$$

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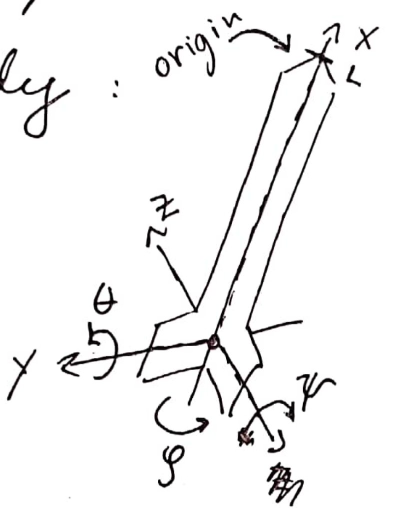
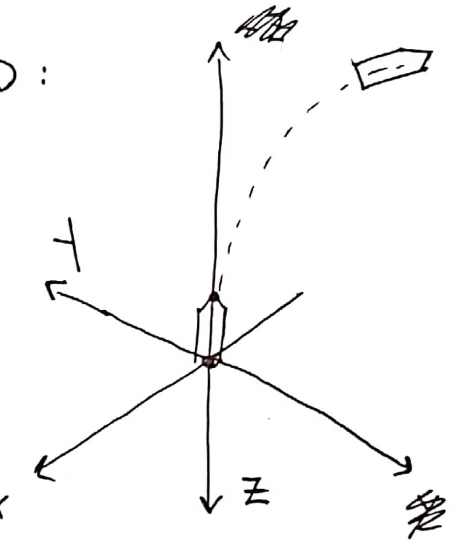
2 (a) The normal force creates a torque around the CoM. If the CoP is behind the CoM the torque will "straighten" the rocket. Otherwise it will flip the rocket.

(b) Body (r):



NED:



T^n_r is the position + orientation of r relative to the NED. Which means we get

$$\underline{R^n_r} = R_x(\varphi) R_z(\psi) R_y(\theta)$$

$$= \begin{bmatrix} c\theta c\psi & s\psi & -s\theta c\psi \\ -c\theta s\psi c\varphi + s\theta s\varphi & c\psi c\varphi & s\theta s\psi c\varphi + c\theta s\varphi \\ c\theta s\psi s\varphi + s\theta c\varphi & -c\psi s\varphi & -s\theta s\psi s\varphi + c\theta c\varphi \end{bmatrix},$$

$$\underline{r^n_{nr}} = \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix},$$

$$\underline{T^n_r = \begin{bmatrix} R^n_r & r^n_{nr} \\ 0^T & 1 \end{bmatrix}}$$

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d $R_r^n = R_x R_z R_y$

$$\omega_{nr}^n = \omega_x(\dot{\varphi}) + R_x\, \omega_z(\dot{\psi}) + R_x R_z\, \omega_y(\dot{\theta})$$

$$\omega_{nr}^n = \begin{bmatrix} \dot{\varphi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -s\varphi\, \dot{\psi} \\ c\varphi\, \dot{\psi} \end{bmatrix} + \begin{bmatrix} -s\psi\, \dot{\theta} \\ c\psi\, c\varphi\, \dot{\theta} \\ c\psi\, s\varphi\, \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\varphi} - s\psi\, \dot{\theta} \\ -s\varphi\, \dot{\psi} + c\psi\, c\varphi\, \dot{\theta} \\ c\varphi\, \dot{\psi} + c\psi\, s\varphi\, \dot{\theta} \end{bmatrix}$$

$$\omega_{nr}^r = R_y^T R_z^T\, \omega_x(\dot{\varphi}) + R_y^T\, \omega_z(\dot{\psi}) + \omega_y(\dot{\theta})$$

$$= \begin{bmatrix} c\theta\, c\psi \\ -s\psi \\ s\theta\, c\psi \end{bmatrix} \dot{\varphi} + \begin{bmatrix} -s\theta \\ 0 \\ c\theta \end{bmatrix} \dot{\psi} + \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}$$

$$\omega_{nr}^r = \begin{bmatrix} c\theta\, c\psi & 0 & -s\theta \\ -s\psi & 1 & 0 \\ s\theta\, c\psi & 0 & c\theta \end{bmatrix} \dot{\Theta}$$

Please tell me if I am doing the transformation the wrong way around...

$$\det E_r(\Theta) = c\theta\, c\psi\, c\theta + s\theta\, c\psi\, s\theta = c\psi$$

So for $\psi = \pm \frac{\pi}{2}$ we get a singularity.

It would be very unfortunate if the singularity was in y instead of z, as it is in the standard z-y-x convention. In order to transform the NED frame to the r frame when the rocket is pointing straight up one would need to rotate 90 degrees about the y axis, leading to a singularity. Now, it would still be probable that we would want to rotate 90 degrees about the z-axis as well, but at least it would be a rarer situation.

After some boring algebra:

$$E_r(\theta)^{-1} = \begin{bmatrix} c\theta & 0 & s\theta \\ s\psi\, c\theta & c\psi & s\theta\, s\psi \\ -s\theta\, c\psi & 0 & c\theta\, c\psi \end{bmatrix} \frac{1}{c\psi}$$

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(e) $T^r = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix}$, $G^r = R^r_n G^n = R^r_n \cdot \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$

$$F^r_{ro} = T^r + G^r + D^r + N^r$$

$$\tau^r_{ro} = (N^r + D^r) \times r^r_p$$ (G and T does not create any torque)

Using the result in 7.3.8:

$$\begin{bmatrix} m\mathbb{I} & m(r^r_g)^{\times T} \\ m(r^r_g)^{\times} & M^r_{r/o} \end{bmatrix} \cdot \begin{bmatrix} \dot{v}^r_{nr} \\ \dot{\omega}^r_{nr} \end{bmatrix} + \begin{bmatrix} m(\omega^r_{nr}) \times ((\omega^r_{nr})^{\times} r^r_g + v^r_{nr}) \\ (\omega^r_{nr})^{\times} M^r_{r/o} \omega^r_{nr} + m(r^r_g)^{\times} (\omega^r_{nr})^{\times} v^r_o \end{bmatrix}$$

$$= \begin{bmatrix} F^r_{ro} \\ \tau^r_{ro} \end{bmatrix} = \begin{bmatrix} T^r + G^r + D^r + N^r \\ (N^r + D^r) \times r^r_p \end{bmatrix}$$

Differential equations:

$$\begin{bmatrix} \mathbb{I} & 0 & & 0 \\ 0 & E_r & & \\ & & m\mathbb{I} & m(r^r_g)^{\times T} \\ 0 & & m(r^r_g)^{\times} & M^r_{r/o} \end{bmatrix} \cdot \begin{bmatrix} \dot{r}^n \\ \dot{\Theta} \\ \dot{v}^r_{nr} \\ \dot{\omega}^r_{nr} \end{bmatrix} + \begin{bmatrix} -R^n_r v^r_{nr} \\ -\omega^r_{nr} \\ m(\omega^r_{nr}) \times ((\omega^r_{nr})^{\times} r^r_g + v^r_{nr}) \\ (\omega^r_{nr})^{\times} M^r_{r/o} \omega^r_{nr} + m(r^r_g)^{\times} \cdot (\omega^r_{nr})^{\times} v^r_o \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ T^r + G^r + D^r + N^r \\ (N^r + D^r) \times r^r_1 \end{bmatrix}$$

12 equations.

(f) When solving with RK, start with some constraints such that $\dot{\Theta} = 0$, $\dot{\omega}^r_{nr} = 0$.

Detect event when we enter flight phase
→ remove constraints

Detection is done by interpolation during RK simulation.

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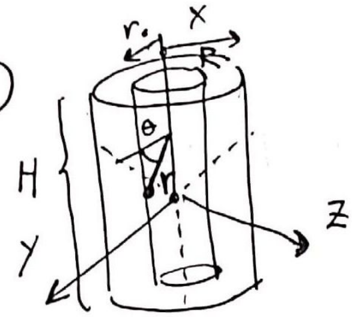
⑨



$$dm = \rho\, dV = \rho\, r\, dr\, dx\, d\theta$$

$$m_p = \rho \pi H (R^2 - r_0^2)$$

$$r_0^2 = R^2 - \frac{m_p}{\rho \pi H}$$

From symmetry we get:

$$M^r_{p,r|c} = \int_b \begin{bmatrix} x^2+z^2 & 0 & 0 \\ 0 & x^2+z^2 & 0 \\ 0 & 0 & x^2+y^2 \end{bmatrix} dm$$

$$= \int_0^{2\pi}\int_{-\frac{H}{2}}^{\frac{H}{2}}\int_{r_0}^{R} \begin{bmatrix} r^2 & 0 & 0 \\ 0 & x^2+r^2\sin^2\theta & 0 \\ 0 & 0 & x^2+r^2\cos^2\theta \end{bmatrix} \rho\, r\, dr\, dx\, d\theta$$

$$= \rho \int_0^{2\pi}\int_{-\frac{H}{2}}^{\frac{H}{2}} \begin{bmatrix} \frac{1}{4}(R^4-r_0^4) & 0 & 0 \\ 0 & \begin{pmatrix} \frac{1}{2}x^2(R^2-r_0^2) \\ +\frac{1}{4}(R^4-r_0^4)\sin^2\theta \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \frac{1}{2}x^2(R^2\pm r_0^2) \\ +\frac{1}{4}(R^4-r_0^4)\cos^2\theta \end{pmatrix} \end{bmatrix} dx\, d\theta$$

$$= \rho \int_0^{2\pi} \begin{bmatrix} \frac{1}{4}(R^4-r_0^4)H & 0 & 0 \\ 0 & \begin{pmatrix} \frac{1}{4}(R^4-r_0^4)\sin^2\theta H \\ +\frac{1}{24}(R^2-r_0^2)H^3 \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \frac{1}{4}(R^4-r_0^4)\cos^2\theta H \\ +\frac{1}{24}(R^2-r_0^2)H^3 \end{pmatrix} \end{bmatrix} d\theta$$

$$= \rho \begin{bmatrix} \frac{1}{4}(R^4-r_0^4)H2\pi & 0 & 0 \\ 0 & \begin{pmatrix} \frac{1}{12}(R^2-r_0^2)H^3\pi \\ +\frac{1}{4}(R^4-r_0^4)H\pi \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \frac{1}{12}(R^2-r_0^2)H^3\pi \\ +\frac{1}{4}(R^4-r_0^4)H\pi \end{pmatrix} \end{bmatrix}$$

$$R^2 - r_0^2 = \frac{m_p}{\rho\pi H}, \quad R^4 - r_0^4 = 2R^2\frac{m_p}{\rho\pi H} - \frac{m_p^2}{\rho^2\pi^2H^2}$$

$$\Rightarrow M^r_{p,r|c} = \begin{bmatrix} m_p\left(R^2 - \frac{m_p}{2\rho\pi H}\right) & 0 & 0 \\ 0 & m_p\left(\frac{R^2}{2}+\frac{H^2}{12}-\frac{m_p}{4\pi\rho H}\right) & 0 \\ 0 & 0 & m_p\left(\frac{R^2}{2}+\frac{H^2}{12}-\frac{m_p}{4\pi\rho H}\right) \end{bmatrix}$$

□

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$$M^{r}_{r|o} = M^{r}_{r|c} + m_p\,(r_g^T r_g \mathbb{I} - r_g r_g^T)$$

$$r_g = \begin{bmatrix} -L + \frac{H}{2} \\ 0 \\ 0 \end{bmatrix}, \quad r_g^T r_g \mathbb{I} - r_g r_g^T = \left(L^2 - LH + \frac{\pi^2}{4}\right)\left(\mathbb{I} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & L^2 - LH + \frac{H^2}{4} & 0 \\ 0 & 0 & L^2 - LH + \frac{H^2}{4} \end{bmatrix}$$

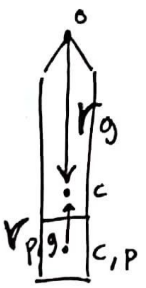
$$\Rightarrow M^{r}_{r|o} = m_p \begin{bmatrix} R^2 - \frac{m_p}{2\pi\rho H} & 0 & 0 \\ 0 & \left(\frac{H^2}{12} + \frac{R^2}{2} - \frac{m_p}{4\pi\rho H} + L^2 - LH + \frac{H^2}{4}\right) & 0 \\ 0 & 0 & \left(\frac{H^2}{12} + \frac{R^2}{2} - \frac{m_p}{4\pi\rho H} + L^2 - LH + H^2/4\right) \end{bmatrix}$$

$$M^{r}_{r|o} = m_p \begin{bmatrix} R^2 - \frac{m_p}{2\pi\rho H} & 0 & 0 \\ 0 & \left(\frac{H^2}{3} + \frac{R^2}{2} - \frac{m_p}{4\pi\rho H} + L^2 - LH\right) & 0 \\ 0 & 0 & \left(\frac{H^2}{3} + \frac{R^2}{2} - \frac{m_p}{4\pi\rho H} + L^2 - LH\right) \end{bmatrix}$$

(h) $m(t) - m_p(t) = m(0) - m_p(0)$

$\cancel{\pm 8\, m(t) = m(0) + m_p(t)}$

$$\Rightarrow m(t) = m(0) + \underbrace{m_p(t) - m_p(0)}_{-\text{ propellant used}}$$



<— This is always constant eventhough COM changes.

$$\Rightarrow x_{p,g} - x_g = x_{p,g}(0) - x_g(0)$$

$$\Rightarrow x_g(t) = x_{p,g}(t) + x_g(0) - x_{p,g}(0)$$

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By the same principle, the inertia of the rocket body without the propellant should be constant, i.e

$$M^b_{b/o}(t) - M^b_{p,b/o}(t) = M^b_{b/o}(0) - M^b_{p,b/o}(0)$$

$$\Rightarrow \underline{M^b_{b/o}(t) = M^b_{b/o}(0) + M^b_{p,b/o}(t) - M^b_{p,b/o}(0)}$$

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