TTK4115 LINEAR SYSTEM THEORY SUMMARY

$\mathcal{L}: \hat{y}(s) = (C(s\mathbb{I}-A)^{-1}B + D)\hat{u}(s)$

USEFUL LINEAR ALGEBRA:

DIAGONAL FORM: $\Lambda = Q^{-1}AQ$

JORDAN FORM: $A v_2 = \lambda v_2 + v_1$

CAYLEY - HAMILTON:

$\det(\lambda\mathbb{I}-A) = \Delta(\lambda) = \lambda^n + \alpha_1\lambda^{n-1} + \dots + \alpha_n = 0$

$\Rightarrow \Delta(A) = A^n + \alpha_1 A^{n-1} + \dots + \mathbb{I}\alpha_n = 0$

P.D. ① $\lambda_i \geq 0$, ② all leading principal values are positive.

③ $\alpha = v^T P v > 0 \ \forall v$

LINEARIZATION:

$\dot{x} = h(x, u)$

$A = \frac{\partial h}{\partial x}, B = \frac{\partial h}{\partial u}$

LTI SYSTEM:

$\dot{x} = Ax + Bu$

$y = Cx + Du$

REALIZATION:

$G(s) = G_{sp}(s) + G_\infty$,

$A = \begin{bmatrix} -\alpha_1\mathbb{I}_p & \cdots & -\alpha_n\mathbb{I}_p \\ \mathbb{I}_p & 0 & \cdots \\ 0 & \mathbb{I}_p & \cdots \\ \vdots & & \cdots \ 0\ \mathbb{I}_p \end{bmatrix}, B = \begin{bmatrix} \mathbb{I}_p \\ 0 \\ \vdots \\ 0 \end{bmatrix}$,

$y = [N_1 \cdots N_r]x + G_\infty u$

(A, B, C, D) is a minimal realization of $\hat{g}(s)$ if (A,B) is controllable and (A,C) is observable / $\dim A = \deg \hat{g}(s)$ / TF is coprime.

All minimal realizations of $\hat{g}(s)$ are eq.

LTI SOLUTION:

$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)\,d\tau$

STOCHASTIC PROCESSES:

$E\{X\} = \int_{-\infty}^{\infty} x f(x)\,dx$, $Var\{X\} = E\{(X - E\{X\})^2\}$

$R_X(\tau) = E\{X(t)X(t+\tau)\}$

$S_X(j\omega) = \int_{-\infty}^{\infty} R_X(\tau)e^{-j\omega\tau}\,d\tau$

$S_X(j\omega) = |G(j\omega)|^2 S_f(j\omega)$

DISCRETIZATION:

$x[k+1] = \bar{A}x[k] + \bar{B}u[k]$,

$y[k] = Cx[k] + Du[k]$,

$\bar{A} = e^{AT}$, $\bar{B} = A^{-1}(\bar{A} - \mathbb{I})B$

STABILITY:

ASYMPTOTIC: $x \xrightarrow{t\to\infty} 0 \ \forall x_0$, $Re\{\lambda_i\} < 0$

MARGINAL: $|x| < k \ \forall (x_0, t)$, $Re\{\lambda_i\} \leq 0$ (with no Jordan blocks longer than 1x1 for zero eigenvalues)

LYAPUNOV: $A^T M + MA = -N$ (*)

If for any P.D. symmetric N (*) has a unique P.D. symmetric solution M, the system is asymptotically stable.

BIBO STABLE: bounded input excites bounded outwith for $x(0) = 0$.

$\hat{G}(s)$ only tells about BIBO stability.

If every $\hat{g}(s)$ in $\hat{G}(s)$ is absolutely integrable on $[0, \infty)$, then $\hat{G}(s)$ is BIBO stable.

CONTROLLABILITY:

(A,B) is controllable if for any initial state x_0 and final state x_1 there exists an input that transfers x_0 to x_1 in finite time t, i.e. if $\mathcal{C} = [B \ AB \ \cdots \ A^{n-1}B]$ has full rank or rank $[\lambda\mathbb{I} - A \quad B] = n \ \forall \lambda \in \mathbb{C}$.

STATE FEEDBACK:

$u = K_r r - Kx$

LQR: $J = \int_{-\infty}^{\infty}(x^TQx + u^TRu)\,dt$

RICATTI: $A^TP + PA + C^TQC - PBR^{-1}B^TP = 0$

$K = R^{-1}B^TP$

OBSERVABILITY:

(A,C) is observable if for any initial state x_0 there exists a finite time $t_1 > 0$ such that knowledge of u and y in $[0, t_1)$ is sufficient to determine x_0 uniquely, i.e.

$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$ has full col rank

or rank $[A - \lambda\mathbb{I} \quad C] = n \ \forall \lambda \in \mathbb{C}$.

ESTIMATION:

LUENBERGER: $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$

KALMAN FILTER:

1 $K_k = P_k^- C^T(CP_k^-C^T + \bar{R})^{-1}$

2 $\hat{x}_k = \hat{x}_k^- + K_k(y[k] - C\hat{x}_k^-)$

3 $P_k = (\mathbb{I} - K_kC)P_k^-(\mathbb{I} - K_kC)^T + K_k\bar{R}K_k^T$

4 $\hat{x}_{k+1}^- = \bar{A}\hat{x}_k + \bar{B}u[k]$,

$P_{k+1}^- = \bar{A}P_k\bar{A}^T + \bar{Q}$