



Sèrie 2

Exercici 1

Q1 b

Q2 c

Q3 d

Q4 b

Q5 a

Exercici 2

a)

$$U_{\text{an}} = \frac{U}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230,94 \text{ V}$$

b)

$$I_{\text{a}} = \frac{U_{\text{an}}}{Z} = \frac{U_{\text{an}}}{|Z|} = \frac{U_{\text{an}}}{\sqrt{R^2 + X^2}} = \frac{230,94}{\sqrt{12^2 + 7^2}} = 16,623 \text{ A}$$

c)

$$P = 3 R I_{\text{a}}^2 = 3 \cdot 12 \cdot 16,623^2 = 9,948 \text{ kW}$$

$$Q = 3 X I_{\text{a}}^2 = 3 \cdot 7 \cdot 16,623^2 = 5,803 \text{ kvar}$$

Alternativament,

$$\varphi = \tan^{-1} \left(\frac{X}{R} \right) = \tan^{-1} \left(\frac{7}{12} \right) = 30,2564^\circ$$

$$P = \sqrt{3} U I_{\text{a}} \cos \varphi = \sqrt{3} \cdot 400 \cdot 16,623 \cdot \cos(30,2564^\circ) = 9,948 \text{ kW}$$

$$Q = \sqrt{3} U I_{\text{a}} \sin \varphi = \sqrt{3} \cdot 400 \cdot 16,623 \cdot \sin(30,2564^\circ) = 5,803 \text{ kvar}$$

Alternativament, en cas que se sàpiga operar amb números complexos,

$$\underline{S} = P + j Q = \frac{U^2}{\underline{Z}^*} = \frac{400^2}{12 - j 7} = \underbrace{9,948}_{9,948 \text{ kW}} + j \underbrace{5,803}_{5,803 \text{ kvar}} \text{ kVA}$$



d)

$$I = 0 \text{ A}$$

Exercici 3

a)

$$\omega_N = \frac{P_N}{I_N} = \frac{7500}{50} = 150 \frac{\text{rad}}{\text{s}} \quad \rightarrow \quad n_N = \omega_N \frac{60}{2\pi} = 150 \frac{60}{2\pi} = 1432,4 \text{ min}^{-1}$$

b)

$$\eta(\%) = 100 \frac{P_N}{\sqrt{3} U_N I_N \cos \varphi_N} = 100 \frac{7500}{\sqrt{3} \cdot 400 \cdot 14,5 \cdot 0,82} = 91,05 \%$$

c)

$$\omega_s = \frac{\omega}{p} = \frac{2\pi f_N}{p} = \frac{2\pi \cdot 50}{2} = 157,08 \frac{\text{rad}}{\text{s}}$$
$$s_N(\%) = 100 \frac{\omega_s - \omega_N}{\omega_s} = 100 \frac{157,08 - 150}{157,08} = 4,51 \%$$

d)

$$U_{\text{triangle}} = \frac{U_{\text{estrella}}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230,94 \text{ V} \quad \text{Si indiquen 230 V, també és correcte.}$$

$$I_{\text{triangle}} = \sqrt{3} I_{\text{estrella}} = \sqrt{3} \cdot 14,5 = 25,11 \text{ A}$$

e)

$$I = I_N = 14,5 \text{ A}$$



Exercici 4

a)

$$I_1 = \frac{U_1}{R_1 + R_3} = \frac{12}{5 + 5} = 1,2 \text{ A}$$

$$P_{R1} = R_1 I_1^2 = 5 \cdot 1,2^2 = 7,2 \text{ W}$$

b)

$$P_{R1} = 0 \text{ W} \quad \rightarrow \quad I_1 = 0 \text{ A} \quad \rightarrow \quad U_{R3} = U_1 = 12 \text{ V}$$

$$I_{R3} = \frac{U_{R3}}{R_3} = \frac{12}{5} = \frac{U_1 + U_2}{R_2 + R_3} = \frac{12 + 12}{R_2 + 5} = \frac{24}{R_2 + 5} = 2,4 \text{ A}$$

$$2,4 (R_2 + 5) = 24 \quad \rightarrow \quad R_2 = \frac{24}{2,4} - 5 = 5 \Omega$$

c)

$$P_{U1} = U_1 I_{R3} = 12 \cdot 2,4 = 28,8 \text{ W}$$

d)

$$P_{R3} = R_3 I_{R3}^2 = \frac{U_{R3}^2}{R_3} = 5 \cdot 2,4^2 = \frac{12^2}{5} = 28,8 \text{ W}$$



Exercici 5

a)

$$X_L = \omega L = 2 \pi f L = 2 \pi 50 \cdot 100 \cdot 10^{-3} = 31,416 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2 \pi f C} = \frac{1}{2 \pi 50 \cdot 65 \cdot 10^{-6}} = 48,971 \Omega$$

$$A = I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{100}{\sqrt{12^2 + (31,416 - 48,971)^2}} = 4,703 \text{ A}$$

b)

$$W = P_R = R I^2 = 12 \cdot 4,703^2 = 265,42 \text{ W}$$

c)

Com $X_L < X_C$, el factor de potència és **capacitiu**.

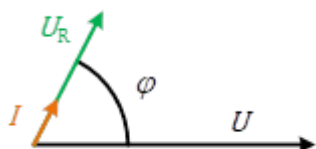
$$\text{fdp} = \cos \varphi = \frac{R}{Z} = \frac{R}{Z} = \frac{12}{\sqrt{12^2 + (31,416 - 48,971)^2}} = 0,5643$$

Alternativament,

$$\varphi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right) = \tan^{-1} \left(\frac{31,416 - 48,971}{12} \right) = -55,645^\circ$$

$$\text{fdp} = \cos \varphi = \cos(-55,645^\circ) = 0,5643$$

d)



Tot i que no es demanava a l'enunciat del problema, els valors corresponents són:

$$U = 100 \text{ V} \quad I = 4,703 \text{ A} \quad \varphi = 55,645^\circ \quad U_R = R I = 56,436 \text{ V}$$



Exercici 6

a)

$$\Delta U_{\text{màx}} = \frac{3}{100} U_{\text{N Línia}} = \frac{3}{100} 230 = 6,9 \text{ V}$$

Amb un 3% de caiguda de tensió, la tensió en borns del transformador és:

$$U_{\text{trafo}} = U_{\text{N Línia}} - \Delta U_{\text{màx}} = 230 - 6,9 = 223,1 \text{ V}$$

El corrent que circula pel primari del transformador:

$$R_{\text{Eq.}} = \frac{U_{\text{primari N}}^2}{P_{\text{Lluminàries}}} = \frac{230^2}{10 \cdot 100} = 52,9 \Omega$$

Alternativament,

$$R_{1 \text{ Lluminària}} = \frac{U_{\text{N Lluminària}}^2}{P_{\text{Lluminària}}} = \frac{6^2}{100} = 0,36 \Omega$$

$$R_{\text{Eq. Secundari}} = \frac{1}{\sum_{i=1}^{10} \frac{1}{R_i}} = \frac{1}{10 \cdot \frac{1}{0,36}} = 0,036 \Omega$$

$$R_{\text{Eq.}} = R_{\text{Eq. Secundari}} r_t^2 = 0,036 \left(\frac{230}{6} \right)^2 = 52,9 \Omega$$

$$I = \frac{U_{\text{trafo}}}{R_{\text{Eq.}}} = \frac{223,1}{52,9} = 4,217 \text{ A}$$

$$2 R_{\text{màx}} = \frac{\Delta U_{\text{màx}}}{I} \rightarrow R_{\text{màx}} = \frac{\Delta U_{\text{màx}}}{2 I} = \frac{6,9}{2 \cdot 4,217} = 0,8181 \Omega$$

$$R_{\text{màx}} = \rho \frac{L}{S_{\text{mín}}} \rightarrow S_{\text{mín}} = \frac{\rho L}{R_{\text{màx}}} = \frac{0,01786 \cdot 10^{-6} \cdot 150}{0,8181} = 3,275 \cdot 10^{-6} \text{ m}^2 = 3,275 \text{ mm}^2$$

b)

La secció escollida, és, doncs, $S = 4 \text{ mm}^2$



c)

$$R_{\text{Línia}} = \rho \frac{2L}{S} = \frac{0,01786 \cdot 10^{-6} \cdot 2 \cdot 150}{4 \cdot 10^{-6}} = 1,3395 \, \Omega$$

$$I = \frac{U_{\text{N Línia}}}{R_{\text{Total}}} = \frac{230}{1,3395 + 52,9} = 4,24 \, \text{A}$$

$$U_{\text{Luminàries}} = \frac{U_{\text{trafo}}}{r_t} = \frac{R_{\text{Eq.}} I}{r_t} = \frac{52,9 \cdot 4,24}{\frac{230}{6}} = 5,851 \, \text{V}$$



SÈRIE 5

Exercici 1

Q1 b

Q2 a

Q3 d

Q4 b

Q5 b

Exercici 2

a)

$$R_{\text{EqO}} = \frac{(R + 3R)(2R + 4R)}{(R + 3R) + (2R + 4R)} = \frac{24}{10} R \quad \rightarrow \quad U = R_{\text{EqO}} I_O = 2,4 R \cdot 3 = 7,2 R$$

$$R_{\text{EqT}} = \frac{(R)(2R)}{(R) + (2R)} + \frac{(3R)(4R)}{(3R) + (4R)} = \left(\frac{2}{3} + \frac{12}{7} \right) R = \frac{50}{21} R \quad P_T = \frac{U^2}{R_{\text{EqT}}}$$

$$413,68 = \frac{(7,2 R)^2}{\frac{50}{21} R} \quad \rightarrow \quad R = \frac{413,68 \cdot \frac{50}{21}}{7,2^2} = 19 \Omega$$

$$U = 7,2 R = 7,2 \cdot 19 = 136,8 \text{ V}$$

b)

$$I_{\text{SW}} = I_R - I_{3R}$$

$$I_R = \frac{U_R}{R} = \frac{\frac{\frac{(R)(2R)}{(R) + (2R)}}{\frac{(R)(2R)}{(R) + (2R)} + \frac{(3R)(4R)}{(3R) + (4R)}} U}{R} = \frac{\frac{2}{3}}{\frac{50}{21}} \cdot \frac{136,8}{19} = 2,016 \text{ A}$$

$$I_{3R} = \frac{U_{3R}}{3R} = \frac{\frac{\frac{(3R)(4R)}{(3R) + (4R)}}{\frac{(R)(2R)}{(R) + (2R)} + \frac{(3R)(4R)}{(3R) + (4R)}} U}{3R} = \frac{\frac{12}{7}}{3 \cdot \frac{50}{21}} \cdot \frac{136,8}{19} = 1,728 \text{ A}$$

$$I_{\text{SW}} = I_R - I_{3R} = 2,016 - 1,728 = 0,288 \text{ A}$$



c)

$$P_0 = U I_0 = 136,8 \cdot 3 = 410,4 \text{ W}$$

Exercici 3

a)

$$W_1 = R A_1^2 \rightarrow R = \frac{W_1}{A_1^2} = \frac{400}{4^2} = 25 \Omega$$

b)

$$A_1 = \frac{\frac{U}{\sqrt{3}}}{\sqrt{R^2 + X_L^2}} \rightarrow R^2 + X_L^2 = \left(\frac{\frac{U}{\sqrt{3}}}{A_1} \right)^2 \rightarrow X_L = \sqrt{\left(\frac{\frac{U}{\sqrt{3}}}{A_1} \right)^2 - R^2}$$

$$X_L = \sqrt{\left(\frac{\frac{400}{\sqrt{3}}}{4} \right)^2 - 25^2} = 52,04 \Omega \quad L = \frac{X_L}{\omega} = \frac{X_L}{2 \pi f} = \frac{52,04}{2 \pi 50} = 165,6 \text{ mH}$$

c)

$$V_1 = X_L A_1 = 52,04 \cdot 4 = 208,16 \text{ V}$$

d)

$$Q = X_L A_1^2 = 52,04 \cdot 4^2 = 832,64 \text{ var}$$

$$X_C = \frac{U^2}{Q} = \frac{400^2}{832,64} = 192,16 \Omega \quad C = \frac{1}{\omega X_C} = \frac{1}{2 \pi f X_C} = \frac{1}{2 \pi 50 \cdot 192,16} = 16,56 \mu\text{F}$$



Exercici 4

a)

$$f = \frac{1}{T} = \frac{1}{4 \text{ div} \cdot 10 \frac{\text{ms}}{\text{div}} \cdot \frac{1 \text{ s}}{1000 \text{ ms}}} = 25 \text{ Hz}$$

b)

Quan condueixen D_2 i D_4 :

$$I_{G_1 \text{ Min}} = I_{R_1 \text{ Min}} = I_{R_2 \text{ Min}} = I_{R_3} = \frac{-1 \text{ div} \cdot 5 \frac{\text{V}}{\text{div}}}{R_1} = \frac{-5}{5} = -1 \text{ A}$$

$$I_{G_1 \text{ Min}} = \frac{U_{G_1 \text{ Min}}}{R_1 + R_2 + R_3} \rightarrow R_3 = \frac{U_{G_1 \text{ Min}}}{I_{G_1 \text{ Min}}} - R_1 - R_2 = \frac{-100}{-1} - 5 - 75 = 20 \Omega$$

c)

Quan condueixen D_1 i D_3 :

$$I_{G_1 \text{ Max}} = I_{R_1 \text{ Max}} = I_{R_4} = \frac{2,5 \text{ div} \cdot 5 \frac{\text{V}}{\text{div}}}{R_1} = \frac{12,5}{5} = 2,5 \text{ A}$$

Alternativament,

$$I_{G_1 \text{ Max}} = I_{R_1 \text{ Max}} = I_{R_4} = \frac{U_{G_1 \text{ Max}} - U_2}{R_1 + R_4} = \frac{100 - 25}{5 + 25} = 2,5 \text{ A}$$

$$\bar{P}_{\text{Total R}} = \bar{P}_{R_1} + \bar{P}_{R_2} + \bar{P}_{R_3} + \bar{P}_{R_4} + \bar{P}_{R_5}$$

$$\bar{P}_{R_1} = R_1 I_{G_1 \text{ Max}}^2 \frac{2,5 \text{ div}}{4 \text{ div}} + R_1 I_{G_1 \text{ Min}}^2 \frac{1,5 \text{ div}}{4 \text{ div}} = R_1 \left(I_{G_1 \text{ Max}}^2 \frac{2,5}{4} + I_{G_1 \text{ Min}}^2 \frac{1,5}{4} \right)$$

$$\bar{P}_{R_1} = 5 \left(2,5^2 \frac{2,5}{4} + 1^2 \frac{1,5}{4} \right) = 21,406 \text{ W}$$

$$\bar{P}_{R_2} = R_2 I_{G_1 \text{ Min}}^2 \frac{1,5 \text{ div}}{4 \text{ div}} = 75 \cdot 1^2 \frac{1,5}{4} = 28,125 \text{ W}$$

$$\bar{P}_{R_3} = R_3 I_{G_1 \text{ Min}}^2 \frac{1,5 \text{ div}}{4 \text{ div}} = 20 \cdot 1^2 \frac{1,5}{4} = 7,5 \text{ W}$$

$$\bar{P}_{R_4} = R_4 I_{G_1 \text{ Max}}^2 \frac{2,5 \text{ div}}{4 \text{ div}} = 25 \cdot 2,5^2 \frac{2,5}{4} = 97,656 \text{ W}$$



$$\bar{P}_{R5} = \frac{U_2^2}{R_5} = \frac{25^2}{50} = 12,5 \text{ W}$$

$$\bar{P}_{\text{Total R}} = 21,406 + 28,125 + 7,5 + 97,656 + 12,5 = 167,19 \text{ W}$$

Alternativament,

$$\bar{P}_{\text{Total R}} = \bar{P}_{G1} - \bar{P}_{U2}$$

$$\bar{P}_{G1} = U_{G1 \text{ Max}} I_{G1 \text{ Max}} \frac{2,5}{4} + U_{G1 \text{ Min}} I_{G1 \text{ Min}} \frac{1,5}{4}$$

$$\bar{P}_{G1} = 100 \cdot 2,5 \frac{2,5}{4} + (-100)(-1) \frac{1,5}{4} = 193,75 \text{ W}$$

$$I_{R5} = \frac{U_2}{R_5} = \frac{25}{50} = 0,5 \text{ A} \quad I_{G1 \text{ Max}} = I_{U2} + I_{R5} \quad \rightarrow \quad I_{U2} = I_{G1 \text{ Max}} - I_{R5}$$

$$I_{U2} = \begin{cases} 2,5 - 0,5 = 2 \text{ A} & (\text{durant } 2,5 \text{ div}) \\ -0,5 \text{ A} & (\text{durant } 1,5 \text{ div}) \end{cases}$$

$$\bar{P}_{U2} = U_2 I_{U2} = 25 \left(2 \frac{2,5}{4} - 0,5 \frac{1,5}{4} \right) = 26,563 \text{ W}$$

$$\bar{P}_{\text{Total R}} = 193,75 - 26,563 = 167,19 \text{ W}$$



Exercici 5

a)

$\Gamma = 0$, doncs està en buit

b)

$$\eta(\%) = 100 \frac{P_{\text{util}}}{P_{\text{consumida}}} = 100 \frac{0}{1500} = 0 \%$$

c)

$$\cos \varphi_0 = \frac{P_0}{S_0} = \frac{P_0}{\sqrt{3} U_N I_0} = \frac{1500}{\sqrt{3} \cdot 690 \cdot 30} = \frac{1500}{35854} = 0,042$$

d)

$$p = 3$$

e)

$$Q_0 = \sqrt{S_0^2 - P_0^2} = \sqrt{35854^2 - 1500^2} = 35823 \text{ var}$$

Exercici 6

a)

$$S = \sqrt{P^2 + Q^2} \rightarrow Q = \sqrt{S^2 - P^2} = \sqrt{(U A_1)^2 - P^2}$$

$$Q = \sqrt{(200 \cdot 6,54)^2 - 1265^2} = 332,6 \text{ var}$$

b)

$$W = R_1 A_1^2 + R_2 I_{R_2}^2 \rightarrow I_{R_2} = \sqrt{\frac{W - R_1 A_1^2}{R_2}} = \sqrt{\frac{1265 - 20 \cdot 6,54^2}{15}} = 5,23 \text{ A}$$

$$U_{R_2} = R_2 I_2 = 15 \cdot 5,23 = 78,45 \text{ V}$$

$$I_C = \frac{U_{R_2}}{X_C} = \frac{U_{R_2}}{\frac{1}{\omega C}} = \frac{78,45}{\frac{1}{2 \pi 50 \cdot 160 \cdot 10^{-6}}} = \frac{78,45}{19,89} = 3,94 \text{ A}$$



c)

$$Q_C = X_C I_C^2 = 19,89 \cdot 3,94^2 = 308,8 \text{ var}$$

$$Q = Q_L - Q_C \quad \rightarrow \quad Q_L = Q + Q_C = 332,6 + 308,8 = 641,4 \text{ var}$$

$$Q_L = X_L A_1^2 \quad \rightarrow \quad X_L = \frac{Q_L}{A_1^2} = \frac{641,4}{6,54^2} = 15 \, \Omega \quad \rightarrow \quad L = \frac{X_L}{\omega} = \frac{15}{2 \pi 50} = 47,75 \text{ mH}$$