



## SÈRIE 1

### Primera part

#### Exercici 1

Q1 a      Q2 c      Q3 a      Q4 b      Q5 b

#### Exercici 2

$$a) A_1 = \frac{V_{an}}{R} = \frac{\frac{U}{\sqrt{3}}}{R} = \frac{\frac{400}{\sqrt{3}}}{100} = 2,31 \text{ A}$$

b)

$$A_2 = \frac{V_{bn}}{X_L} = \frac{\frac{400}{\sqrt{3}}}{50} = 4,62 \text{ A}$$

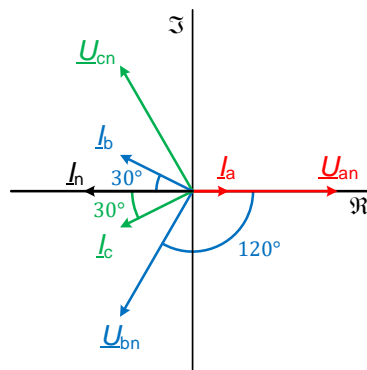
$$A_3 = \frac{V_{cn}}{X_C} = \frac{\frac{400}{\sqrt{3}}}{50} = 4,62 \text{ A}$$

c)

$$P = R A_1^2 = 100 \cdot 2,31^2 = 533,6 \text{ W}$$

$$Q = X_L A_2^2 - X_C A_3^2 = 50 \cdot 4,62^2 - 50 \cdot 4,62^2 = 0 \text{ var}$$

d)



$$A_4 = I_n = |I_a - I_b \cos 30^\circ - I_c \cos 30^\circ| = \left| 2,31 - 4,62 \frac{\sqrt{3}}{2} - 4,62 \frac{\sqrt{3}}{2} \right| = 5,69 \text{ A}$$



## Segona part

### OPCIÓ A

#### Exercici 3

$$a) \eta_N = \frac{P_N}{\sqrt{3} U_N I_N \cos \varphi_N} = \frac{7500}{\sqrt{3} \cdot 208 \cdot 28 \cdot 0,86} = 86,45 \%$$

Alternativament,

$$\eta_N = \frac{P_N}{\sqrt{3} U_N I_N \cos \varphi_N} = \frac{7500}{\sqrt{3} \cdot 120 \cdot 48,5 \cdot 0,86} = 86,51 \%$$

$$b) \frac{60 f}{n_N} = \frac{60 \cdot 400}{5850} = 4,1 \rightarrow p = 4$$

$$c) \Gamma = \frac{P}{\omega} = \frac{7500}{5850 \frac{2\pi}{60}} = 12,24 \text{ N m}$$

$$d) s = \frac{n_s - n_m}{n_s} = \frac{\frac{60 f}{p} - n_m}{\frac{60 f}{p}} = \frac{\frac{60 \cdot 400}{4} - 5850}{\frac{60 \cdot 400}{4}} = 0,025$$

$$e) Q = \sqrt{3} \cdot 208 \cdot 28 \cdot \sqrt{1 - 0,86^2} = 5147,6 \text{ var}$$

Alternativament,

$$Q = \sqrt{3} \cdot 120 \cdot 48,5 \cdot \sqrt{1 - 0,86^2} = 5144 \text{ var}$$

#### Exercici 4

$$a) \begin{cases} U_1 = (R_1 + R_3) I_1 + R_4 (I_1 + I_2) \\ U_2 = R_2 I_2 + R_4 (I_1 + I_2) \end{cases} \rightarrow \begin{cases} 48 = (6 + 6) I_1 + 15 (I_1 + I_2) \\ 12 = 6 I_2 + 15 (I_1 + I_2) \end{cases}$$

$$\begin{cases} 48 = 27 I_1 + 15 I_2 \\ 12 = 15 I_1 + 21 I_2 \end{cases} \rightarrow \begin{bmatrix} 27 & 15 \\ 15 & 21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 48 \\ 12 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2,42 \text{ A} \\ -1,16 \text{ A} \end{bmatrix}$$

b)

$$P_{R_1} = P_{R_3} = R_1 I_1^2 = R_3 I_1^2 = 6 \cdot 2,42^2 = 35,14 \text{ W}$$

$$P_{R_2} = R_2 I_2^2 = 6 \cdot (-1,16)^2 = 8,07 \text{ W}$$

$$P_{R_4} = R_4 (I_1 + I_2)^2 = 15 \cdot (2,42 - 1,16)^2 = 23,81 \text{ W}$$

$$P_{R_{1a4}} = P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} = 35,14 + 8,07 + 35,14 + 23,81 = 102,16 \text{ W}$$



c)

$$P_{U_2} = U_2 I_{U_2} = 12 \text{ W} \quad \rightarrow \quad I_{U_2} = \frac{P_{U_2}}{U_2} = \frac{12}{12} = 1 \text{ A}$$

$$I_2 + I_{U_2} + I_{R_5} = 0 \quad \rightarrow \quad I_{R_5} = -I_2 - I_{U_2} = 1,16 - 1 = 0,16 \text{ A}$$

$$R_5 = \frac{U_2}{I_{R_5}} = \frac{12}{0,16} = 75 \Omega$$

d)

$$P_{U_1} = U_1 I_1 = 48 \cdot 2,42 = 116,16 \text{ W}$$

Alternativament,

$$P_{U_1} = P_{R_{1a4}} + P_{R_5} + P_{U_2} = P_{R_{1a4}} + U_2 I_{R_5} + P_{U_2}$$

$$P_{U_1} = 102,16 + 12 \cdot 0,16 + 12 = 116,08 \text{ W}$$

Alternativament,

$$P_{U_1} = P_{R_{1a4}} - U_2 I_2$$

$$P_{U_1} = 102,16 - 12 \cdot (-1,16) = 116,08 \text{ W}$$



OPCIÓ B

Exercici 3

$$a) W_2 = W_1 - P_{R_1} = W_1 - R_1 A_1^2 = 860 - 50 \cdot 3,7^2 = 175,5 \text{ W}$$

$$b) W_2 = R_2 I_C^2 \rightarrow I_C^2 = \frac{W_2}{R_2} \quad I_C = \frac{V_1}{\sqrt{R_2^2 + X_C^2}} \rightarrow I_C^2 = \frac{V_1^2}{R_2^2 + X_C^2}$$

$$\frac{W_2}{R_2} = \frac{V_1^2}{R_2^2 + X_C^2} \rightarrow R_2^2 + X_C^2 = \frac{V_1^2}{W_2} R_2 \rightarrow R_2^2 - \frac{V_1^2}{W_2} R_2 + X_C^2 = 0$$

$$R_2^2 - \frac{73,4^2}{175,5} R_2 + \left( \frac{1}{2 \pi 50 \cdot 320 \cdot 10^{-6}} \right)^2 = 0 \rightarrow R_2^2 - 30,7 R_2 + 98,95 = 0$$

$$R_2 = \frac{30,7 \pm \sqrt{30,7^2 - 4 \cdot 98,95}}{2} = \begin{cases} 27,04 \, \Omega & \text{solució buscada} \\ 3,66 \, \Omega < 15 \, \Omega, & \text{per tant, descartada} \end{cases}$$

$$c) Q = Q_L - Q_C = \frac{V_1^2}{X_L} - X_C I_C^2 \rightarrow Q + X_C I_C^2 = \frac{V_1^2}{X_L} \rightarrow X_L = \frac{V_1^2}{Q + X_C I_C^2}$$

$$X_L = \frac{V_1^2}{Q + X_C I_C^2} = \frac{73,4^2}{195 + \frac{1}{2 \pi 50 \cdot 320 \cdot 10^{-6}} \frac{175,5}{27,04}} = 20,76 \, \Omega \rightarrow L = \frac{X_L}{\omega} = \frac{20,76}{2 \pi 50} = 66,08 \text{ mH}$$

Exercici 4

a)

$$R_{\text{estufa}} = \frac{U^2}{P} = \frac{230^2}{1500} = 35,27 \, \Omega$$

$$R_{\text{Línia}} = \rho \frac{l}{S} = 0,01786 \cdot 10^{-6} \frac{70}{2,5 \cdot 10^{-6}} = 0,5 \, \Omega$$

$$I_{\text{estufa}} = \frac{U}{2 R_{\text{Línia}} + 2 R_{\text{Contacte Endoll}} + R_{\text{estufa}}} = \frac{230}{2 \cdot 0,5 + 2 \cdot 0,1 + 35,27} = 6,31 \text{ A}$$

$$P = R_{\text{estufa}} I_{\text{estufa}}^2 = 35,27 \cdot 6,31^2 = 1404 \text{ W}$$

b)

$$\Delta U_L(\%) = 100 \frac{U - U_{\text{Final Línia}}}{U} = 100 \frac{U - (U - 2 R_{\text{Línia}} I_{\text{estufa}})}{U} = 100 \frac{2 R_{\text{Línia}} I_{\text{estufa}}}{U}$$

$$\Delta U_L(\%) = 100 \frac{2 \cdot 0,5 \cdot 6,31}{230} = 2,74 \%$$



c)

$$\begin{aligned}\Delta U_E(\%) &= 100 \frac{U_{\text{Final Línia}} - U_{\text{després contactes}}}{U} = \\ &= 100 \frac{U_{\text{Final Línia}} - (U_{\text{Final Línia}} - 2 R_{\text{Contace Endoll}} I_{\text{estufa}})}{U} = 100 \frac{2 R_{\text{Contace Endoll}} I_{\text{estufa}}}{U} \\ \Delta U_E(\%) &= 100 \frac{2 \cdot 0,1 \cdot 6,31}{230} = 0,55 \%\end{aligned}$$

d)

$$\begin{aligned}\eta_L(\%) &= 100 \frac{P_{\text{Final Línia}}}{P_{\text{Inici Línia}}} = 100 \frac{U I_{\text{estufa}} - 2 R_{\text{Línia}} I_{\text{estufa}}^2}{P_{\text{Inici Línia}}} \\ \eta_L(\%) &= 100 \frac{230 \cdot 6,31 - 2 \cdot 0,5 \cdot 6,31^2}{230 \cdot 6,31} = 97,26 \%\end{aligned}$$

e)

$$\begin{aligned}\eta_E(\%) &= 100 \frac{P}{P_{\text{Final Línia}}} = 100 \frac{P}{U I_{\text{estufa}} - 2 R_{\text{Línia}} I_{\text{estufa}}^2} \\ \eta_E(\%) &= 100 \frac{1404}{230 \cdot 6,31 - 2 \cdot 0,5 \cdot 6,31^2} = 99,47 \%\end{aligned}$$



## SÈRIE 4

### Primera part

#### Exercici 1

Q1 c      Q2 a      Q3 d      Q4 b      Q5 c

#### Exercici 2

$$a) \quad W_2 = R_3 I_{R_3}^2 \rightarrow I_{R_3} = \sqrt{\frac{W_2}{R_3}} = \sqrt{\frac{3920}{5}} = 28 \text{ A}$$

$$U = (R_2 + R_3) I_{R_3} = (5 + 5) 28 = 280 \text{ V}$$

$$b) \quad I_{R_1} = \frac{U}{\sqrt{R_1^2 + (X_L - X_C)^2}} = \frac{280}{\sqrt{5^2 + (35 - 15)^2}} = 13,582 \text{ A}$$

$$W_1 = (R_2 + R_3) I_{R_3}^2 + R_1 I_{R_1}^2 = (5 + 5) 28^2 + 5 \cdot 13,582^2 = 8,762 \text{ kW}$$

$$c) \quad Q = (X_L - X_C) I_{R_1}^2 = (35 - 15) 13,582^2 = 3689,4 \text{ var}$$

$$\cos \varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{8762}{\sqrt{8762^2 + 3689,4^2}} = 0,9216 \rightarrow \varphi = 0,3986 \text{ rad} = 22,84^\circ$$

$$d) \quad P = U A_1 \cos \varphi \rightarrow A_1 = \frac{P}{U \cos \varphi} = \frac{8762}{280 \cdot 0,9216} = 33,95 \text{ A}$$

Alternativament,

$$Q = U A_1 \sin \varphi \rightarrow A_1 = \frac{Q}{U \sin \varphi} = \frac{3689,4}{280 \sqrt{1 - 0,9216^2}} = 33,95 \text{ A}$$



## Segona part

### OPCIÓ A

#### Exercici 3

$$a) A_2 = \frac{U}{\sqrt{R_2^2 + X_L^2}} = \frac{230}{\sqrt{20^2 + 60^2}} = 3,637 \text{ A}$$

$$b) W = R_1 A_1^2 + R_2 A_2^2 \rightarrow A_1 = \sqrt{\frac{W - R_2 A_2^2}{R_1}} = \sqrt{\frac{830 - 20 \cdot 3,637^2}{35}} = 4,02 \text{ A}$$

$$c) A_1 = \frac{U}{\sqrt{R_1^2 + X_{C2}^2}} \rightarrow X_{C2} = \sqrt{\frac{U^2}{A_1^2} - R_1^2} = \sqrt{\frac{230^2}{4,02^2} - 35^2} = 45,26 \Omega$$

$$d) Q = X_L A_2^2 - X_{C2} A_1^2 = 60 \cdot 3,637^2 - 45,26 \cdot 4,02^2 = 62,25 \text{ var}$$

$$X_{C1} = \frac{U^2}{Q} = \frac{230^2}{62,25} = 849,8 \Omega \rightarrow C_1 = \frac{1}{\omega X_{C1}} = \frac{1}{2 \pi 50 \cdot 849,8} = 3,75 \mu\text{F}$$

$$e) I = \frac{P}{U} = \frac{830}{230} = 3,61 \text{ A}$$

#### Exercici 4

a)

$$I = \frac{U}{Z} = \frac{U}{\sqrt{(2 R_{\text{conductor}} + R_{\text{consum}})^2 + (2 X_{\text{conductor}} + X_{\text{consum}})^2}}$$

$$I = \frac{230}{\sqrt{(2 \cdot 1,2 \cdot 0,2 + 7)^2 + (2 \cdot 0,3 \cdot 0,2 + 4)^2}} = 26,93 \text{ A}$$

b)

$$U = Z_{\text{consum}} I = (\sqrt{7^2 + 4^2}) 26,93 = 217,1 \text{ V}$$

c)

$$P = R_{\text{consum}} I^2 = 7 \cdot 26,93^2 = 5077 \text{ W}$$

$$Q = X_{\text{consum}} I^2 = 4 \cdot 26,93^2 = 2901 \text{ W}$$

$$S = U I = 217,1 \cdot 26,93 = 5846,5 \text{ VA}$$



Alternativament,

$$\cos \varphi = \frac{R_{\text{consum}}}{Z_{\text{consum}}} = \frac{7}{\sqrt{7^2 + 4^2}} = 0,8682$$

$$S = U I = 217,1 \cdot 26,93 = 5846,5 \text{ VA}$$

$$P = S \cos \varphi = 5846,5 \cdot 0,8682 = 5076 \text{ W}$$

$$Q = S \sin \varphi = 5846,5 \sqrt{1 - 0,8682^2} = 2901 \text{ W}$$

## OPCIÓ B

### Exercici 3

$$\text{a) } \eta_N(\%) = 100 \frac{P_N}{U_N I_N} = 100 \frac{392,7}{100 \cdot 4,68} = 83,91 \%$$

$$\text{b) } \Gamma = \frac{P_N}{\omega_N} = \frac{392,7}{2500 \frac{2\pi}{60}} = 1,5 \text{ N m}$$

$$\text{c) } E = \frac{P_N}{I_N} = \frac{392,7}{4,68} = 83,91 \text{ V} \quad R_i = \frac{U - E}{I_N} = \frac{100 - 83,91}{4,68} = 3,438 \Omega$$

$$E' = U' - R_i I = 80 - 3,438 \cdot 0,6 \cdot 4,68 = 70,35 \text{ V}$$

$$n' = n_N \frac{E'}{E} = 2500 \frac{70,35}{83,91} = 2096 \text{ min}^{-1}$$

$$\text{d) } \eta(\%) = 100 \frac{P}{U I} = 100 \frac{\Gamma \omega}{U I} = 100 \frac{0,6 \cdot 1,5 \cdot 2096 \frac{2\pi}{60}}{80 \cdot 0,6 \cdot 4,68} = 87,94 \%$$





#### Exercici 4

a)

$$I_{R_4} = 0 \text{ A} \rightarrow P_{R_4} = 0 \text{ W}$$

$$I_{R_3} = \frac{U_1}{R_3} = \frac{48}{15} = 3,2 \text{ A} \rightarrow P_{R_3} = R_3 I_{R_3}^2 = 15 \cdot 3,2^2 = 153,6 \text{ W}$$

$$I_{R_2} = \frac{U_2}{R_2} = \frac{24}{30} = 0,8 \text{ A} \rightarrow P_{R_2} = R_2 I_{R_2}^2 = 30 \cdot 0,8^2 = 19,2 \text{ W}$$

$$I_{R_1} = \frac{U_1 - U_2}{R_1} = \frac{48 - 24}{10} = 2,4 \text{ A} \rightarrow P_{R_1} = R_1 I_{R_1}^2 = 10 \cdot 2,4^2 = 57,6 \text{ W}$$

$$P_{\text{Total}} = P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} = 57,6 + 19,2 + 153,6 + 0 = 230,4 \text{ W}$$

b)

$$I_{U_1} = I_{R_1} + I_{R_3} = 2,4 + 3,2 = 5,6 \text{ A}$$

$$P_{U_1} = U_1 I_{U_1} = 48 \cdot 5,6 = 268,8 \text{ W}$$

$$P_{U_2} = P_{\text{Total}} - P_{U_1} = 230,4 - 268,8 = -38,4 \text{ W}$$

c)

$$P_{\text{Total}} = \frac{U_1^2}{R_{\text{Eq}}} = \frac{U_1^2}{\frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)}} = \frac{48^2}{\frac{(10 + 30)(15 + 5)}{10 + 30 + 15 + 5}} = 172,8 \text{ W}$$