## SÈRIE 1

### Exercici 1

**Q1** b **Q2** c **Q3** b **Q4** a **Q5** c

### Exercici 2

a) $I_1 = \frac{U_{ab}}{R} = \frac{400}{25} = 16\ \text{A}$

$I_2 = \frac{U_{bc}}{X_L} = \frac{400}{50} = 8\ \text{A}$

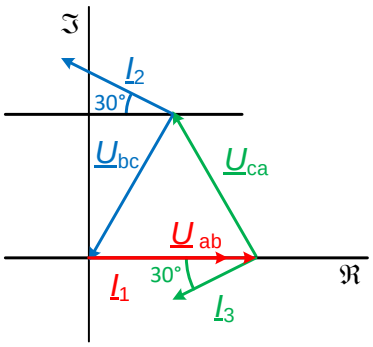
$I_3 = \frac{U_{ca}}{X_C} = \frac{400}{80} = 5\ \text{A}$

b)

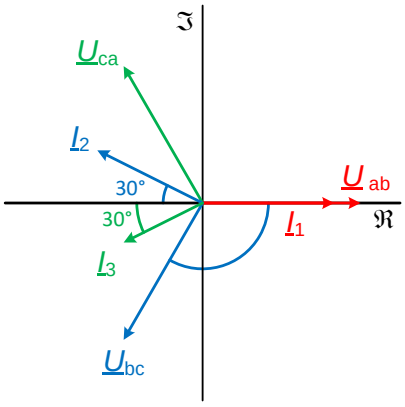
$P = R\, I_1^2 = \frac{U^2}{R} = 25 \cdot 16^2 = \frac{400^2}{25} = 6{,}4\ \text{kW}$

$Q = X_L\, I_2^2 - X_C\, I_3^2 = \frac{U^2}{X_L} - \frac{U^2}{X_C} = 50 \cdot 8^2 - 80 \cdot 5^2 = \frac{400^2}{50} - \frac{400^2}{80} = 1{,}2\ \text{kvar}$

c)

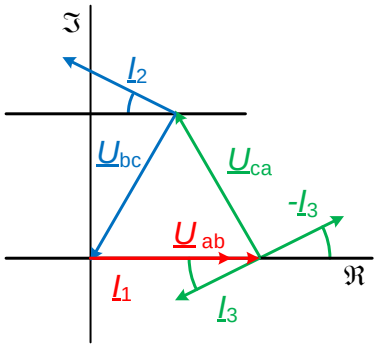



Alternativament, recordant que els fasors no tenen punt d'aplicació,



**Informació addicional** (que no es demanava en la resolució de l'exercici):

Si s'hagués de calcular el valor del corrent $I_a$ de la fase $a$ consumit per la càrrega, llavors:




$$\underline{I}_a = \underline{I}_1 - \underline{I}_3 = \underline{I}_1 + (-\underline{I}_3) \quad \rightarrow \quad I_a = \sqrt{(I_1 + I_3\ \cos 30°)^2 + (I_3\ \sin 30°)^2}$$

$$I_a = \sqrt{\left(16 + 5\ \frac{\sqrt{3}}{2}\right)^2 + \left(5\ \frac{1}{2}\right)^2} = 20{,}48\ \text{A}$$

## Exercici 3

a)

$$\Gamma = \frac{P_N}{\omega_N} = \frac{P_N}{n_N \frac{2\pi}{60}} = \frac{7500}{1755 \frac{2\pi}{60}} = 40{,}81\ \text{N m}$$

b)

$$U_{\text{estrella}} = \sqrt{3}\ U_{\text{triangle}} = \sqrt{3} \cdot 400 = 692{,}8\ \text{V}$$

$$I_{\text{estrella}} = \frac{I_{\text{triangle}}}{\sqrt{3}} = \frac{13{,}5}{\sqrt{3}} = 7{,}79\ \text{A}$$

c)

$$\eta(\%) = 100 \frac{P_N}{\sqrt{3}\ U_N\ I_N\ \cos\varphi_N} = 100 \frac{7500}{\sqrt{3} \cdot 400 \cdot 13{,}5 \cdot 0{,}86} = 93{,}24\ \%$$

d)

$$s_N = \frac{\omega_s - \omega_{mec}}{\omega_s} = \frac{n_s - n_N}{n_s} \quad \rightarrow \quad n_s = \frac{n_N}{1 - s_N} = \frac{1755}{1 - 0{,}025} = 1800\ \text{min}^{-1}$$

$$\omega_s = \frac{\omega}{p} = \frac{2\pi f_N}{p} \quad \rightarrow \quad f_N = \frac{\omega_s\ p}{2\pi} = \frac{n_s \frac{2\pi}{60} p}{2\pi} = \frac{n_s\ p}{60} = \frac{1800 \cdot 2}{60} = 60\ \text{Hz}$$

e)

$$Q = \sqrt{3}\ U_N\ I_N\ \sin\varphi_N = \sqrt{3} \cdot 400 \cdot 13{,}5 \sqrt{1 - 0{,}86^2} = 4773\ \text{var}$$

**Exercici 4**

a)

$$P_{R3} = \frac{U_2^2}{R_3} \quad \rightarrow \quad U_2 = \sqrt{P_{R3}\, R_3} = \sqrt{28{,}8 \cdot 5} = 12\ \text{V}$$

b)

$$P_{R3} = R_3\, I_{R3}^2 \quad \rightarrow \quad I_{R3} = \sqrt{\frac{P_{R3}}{R_3}} = \sqrt{\frac{28{,}8}{5}} = 2{,}4\ \text{A}$$

$$\frac{1}{2}\, P_{R3} = U_2\, I_2 \quad \rightarrow \quad I_2 = \frac{\frac{1}{2}\, P_{R3}}{U_2} = \frac{\frac{1}{2}\, 28{,}8}{12} = 1{,}2\ \text{A}$$

$$I_1 + I_2 = I_{R3} \quad \rightarrow \quad I_1 = I_{R3} - I_2 = 2{,}4 - 1{,}2 = 1{,}2\ \text{A}$$

$$I_1 = \frac{U_1 - U_2}{R_1 + R_2} \quad \rightarrow \quad U_1 - U_2 = R_1\, I_1 + R_2\, I_1 \quad \rightarrow \quad R_2 = \frac{U_1 - U_2 - R_1\, I_1}{I_1}$$

$$R_2 = \frac{24 - 12 - 6{,}5 \cdot 1{,}2}{1{,}2} = 3{,}5\ \Omega$$

c)

$$P_{U1} = U_1\, I_1 = 24 \cdot 1{,}2 = 28{,}8\ \text{W}$$

d)

$P_{U2} = U_2\, I_2 = 0$ vol dir $I_2 = 0$ i, per tant, $I_1 + I_2 = I_{R3} = I_1$

$$I_1 = \frac{U_1 - U_2}{R_1 + R_2} = \frac{24 - 12}{6{,}5 + 3{,}5} = 1{,}2\ \text{A} = I_{R3}$$

$$U_2 = R_3\, I_1 = R_3\, I_{R3} \quad \rightarrow \quad R_3 = \frac{U_2}{I_1} = \frac{U_2}{I_{R3}} = \frac{12}{1{,}2} = 10\ \Omega$$

**Exercici 5**

a)

$$X_1 = \omega\, L_1 = 2\,\pi\, f\, L_1 = 2\,\pi\, 50 \cdot 250 \cdot 10^{-3} = 78{,}54\ \Omega$$

$$X_\mathrm{C} = \frac{1}{\omega\, C} = \frac{1}{2\,\pi\, f\, C} = \frac{1}{2\,\pi\, 50 \cdot 100 \cdot 10^{-6}} = 31{,}83\ \Omega$$

$$A = I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (X_1 + X_2 - X_\mathrm{C})^2}} = \frac{100}{\sqrt{12^2 + (78{,}54 + 25 - 31{,}83)^2}} = 1{,}375\ \mathrm{A}$$

b)

$$W = P_\mathrm{R} = R\, I^2 = 12 \cdot 1{,}375^2 = 22{,}69\ \mathrm{W}$$

c)

Per què el corrent (potència) sigui màxim (màxima), el circuit ha d'estar en resonància sèrie:

$$L_2 = \frac{X_2}{2\,\pi\, f} = \frac{25}{2\,\pi\, 50} = 79{,}58\ \mathrm{mH} \quad \rightarrow \quad L = L_1 + L_2 = 250 + 79{,}58 = 329{,}58\ \mathrm{mH}$$

$$f = \frac{1}{2\,\pi\,\sqrt{L\, C}} \quad \rightarrow \quad C = \frac{1}{(\,2\,\pi\,\mathrm{f}\,)^2\, L} = \frac{1}{(2\,\pi\, 50)^2\, 329{,}58 \cdot 10^{-3}} = 30{,}74\ \mu\mathrm{F}$$

d)

$$W = P_\mathrm{R} = \frac{U^2}{R} = \frac{100^2}{12} = 833{,}3\ \mathrm{W}$$

**Exercici 6**

a)

$$\Delta U_{\text{màx}} = \frac{5}{100}\, U_{\text{N Línia}} = \frac{5}{100}\, 230 = 11{,}5 \text{ V}$$

Amb un 5% de caiguda de tensió, la tensió en els borns del motor és:

$$U_{\text{motor}} = U_{\text{N Línia}} - \Delta U_{\text{màx}} = 230 - 11{,}5 = 218{,}5 \text{ V}$$

El corrent que circula pel motor:

$$I = \frac{U_{\text{motor}}}{Z} = \frac{218{,}5}{\sqrt{7{,}5^2 + 4{,}8^2}} = 24{,}54 \text{ A}$$

$$2\, R_{\text{màx}} = \frac{\Delta U_{\text{màx}}}{I} \quad \rightarrow \quad R_{\text{màx}} = \frac{\Delta U_{\text{màx}}}{2\, I} = \frac{11{,}5}{2 \cdot 24{,}54} = 0{,}2343\ \Omega$$

$$R_{\text{màx}} = \rho\, \frac{L}{S_{\text{mín}}} \quad \rightarrow \quad S_{\text{mín}} = \frac{\rho\, L}{R_{\text{màx}}} = \frac{0{,}01786 \cdot 10^{-6} \cdot 100}{0{,}2343} = 7{,}62 \cdot 10^{-6} \text{ m}^2 = 7{,}62 \text{ mm}^2$$

b) La secció escollida, és, doncs, $S = 10 \text{ mm}^2$

c)

$$R_{\text{màx}} = \rho\, \frac{L}{S_{\text{mín}}} = \frac{0{,}01786 \cdot 10^{-6} \cdot 100}{10 \cdot 10^{-6}} = 0{,}1786\ \Omega$$

$$I = \frac{U_{\text{N Línia}}}{Z_{\text{Total}}} = \frac{230}{\sqrt{(2 \cdot 0{,}1786 + 7{,}5)^2 + 4{,}8^2}} = 24{,}98 \text{ A}$$

$$\Delta U(\%) = 100\, \frac{U_{\text{N Línia}} - U_{\text{motor}}}{U_{\text{N Línia}}} = 100\, \frac{U_{\text{N Línia}} - Z\, I}{U_{\text{N Línia}}}$$

$$\Delta U(\%) = 100\, \frac{230 - 24{,}98\sqrt{7{,}5^2 + 4{,}8^2}}{230} = 3{,}29\ \%$$

## SÈRIE 3

### Exercici 1

**Q1** d **Q2** b **Q3** a **Q4** b **Q5** c

### Exercici 2

b) $I_1 = \frac{U_2 - U_1}{R_1 + R_4} = \frac{12-10}{5+5} = 0{,}2\ \text{A}$

$I_3 = \frac{U_3 - U_2}{R_2 + R_3} = \frac{14-12}{5+5} = 0{,}2\ \text{A}$

$I_3 = I_1 + I_2 \quad \rightarrow \quad I_2 = I_3 - I_1 = 0{,}2 - 0{,}2 = 0\ \text{A}$

b)

Com $I_1 = I_3$,

$$P_\text{R} = (R_1 + R_2 + R_3 + R_4)\, I_1^2 = (5 + 5 + 5 + 5)\, 0{,}2^2 = 0{,}8\ \text{W}$$

c)

$P_{\text{U1}} = -U_1\, I_1 = -10 \cdot 0{,}2 = -2\ \text{W}$

$P_{\text{U2}} = -U_2\, I_2 = -12 \cdot 0 = 0\ \text{W}$

$P_{\text{U3}} = U_3\, I_3 = 14 \cdot 0{,}2 = 2{,}8\ \text{W}$

d)

$I_3 = \frac{U_3 - U_2}{R_2 + R_3}$ i, per tant, si $U_2 = U_3 = 14\ \text{V}$, llavors $I_3 = 0\ \text{A}$

**Exercici 3**

b)

$$X_{\mathrm{L}} = \omega\, L = 2\,\pi\, f\, L = 2\,\pi\, 50 \cdot 160 \cdot 10^{-3} = 50{,}27\ \Omega$$

$$X_{\mathrm{C}} = \frac{1}{\omega\, C} = \frac{1}{2\,\pi\, f\, C} = \frac{1}{2\,\pi\, 50 \cdot 130 \cdot 10^{-6}} = 24{,}49\ \Omega$$

$$U_{\mathrm{an}} = I_{R_2}\sqrt{R_2^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2} = A_3\sqrt{R_2^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2} = 3{,}6\sqrt{50^2 + (50{,}27 - 24{,}49)^2}$$

$$U_{\mathrm{an}} = 202{,}5\ \mathrm{V}$$

$$A_2 = I_{R_1} = \frac{U_{\mathrm{an}}}{R_1} = \frac{202{,}5}{100} = 2{,}025\ \mathrm{A}$$

b)

$$P = 3\, R_1\, I_{R_1}^2 + 3\, R_2\, I_{R_2}^2 = 3 \cdot 100 \cdot 2{,}025^2 + 3 \cdot 50 \cdot 3{,}6^2 = 3174{,}2\ \mathrm{W}$$

$$Q = 3\, X_{\mathrm{L}}\, I_{R_2}^2 - 3\, X_{\mathrm{C}}\, I_{R_2}^2 = 3 \cdot 50{,}27 \cdot 3{,}6^2 - 3 \cdot 24{,}49 \cdot 3{,}6^2 = 1002{,}3\ \mathrm{var}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{3174{,}2^2 + 1002{,}3^2} = 3328{,}7\ \mathrm{VA}$$

c)

$$S = \sqrt{3}\, U\, I = \sqrt{3}\,\sqrt{3}\, U_{\mathrm{an}}\, I \quad \rightarrow \quad A_1 = I = \frac{S}{3\, U_{\mathrm{an}}} = \frac{3328{,}7}{3 \cdot 202{,}5} = 5{,}48\ \mathrm{A}$$

**Exercici 4**

e)

$$f = \frac{1}{T} = \frac{1}{5\ \mathrm{div} \cdot 2\ \frac{\mathrm{ms}}{\mathrm{div}} \cdot \frac{1\ \mathrm{s}}{10^3\ \mathrm{ms}}} = 100\ \mathrm{Hz}$$

f) En el semiperiode negatiu condueix el díode i, per tant, no circula corrent per $R_2$; i, en canvi, si que circula corrent per $R_1$. Llavors,

$$U_{\mathrm{secundari\ pic}} = 5\ \mathrm{div} \cdot 10\ \frac{\mathrm{V}}{\mathrm{div}} = 50\ \mathrm{V}$$

En el semiperiode positiu no condueix el díode i el corrent circula per $R_1$ i $R_2$:

$$U_{\mathrm{R1\ pic}} = 2\ \mathrm{div} \cdot 10\ \frac{\mathrm{V}}{\mathrm{div}} = 20\ \mathrm{V}$$

$$U_{\mathrm{secundari\ pic}} = U_{\mathrm{R1\ pic}} + U_{\mathrm{R2\ pic}} \quad \rightarrow \quad U_{\mathrm{R2\ pic}} = 50 - 20 = 30\ \mathrm{V}$$

$$I_{\mathrm{pic}} = \frac{U_{\mathrm{R1\ pic}}}{R_1} = \frac{U_{\mathrm{R2\ pic}}}{R_2} = \frac{20}{10} = 2\ \mathrm{A} \quad \rightarrow \quad R_2 = \frac{U_{\mathrm{R2\ pic}}}{I_{\mathrm{pic}}} = \frac{30}{2} = 15\ \Omega$$

c)

$$r_\mathrm{t} = \frac{U_{\text{Nominal primari}}}{U_{\text{Nominal secundari}}} = \frac{80}{40} = 2$$

$$U = U_\text{primari} = r_\mathrm{t}\, U_\text{secundari} = r_\mathrm{t}\, \frac{U_{\text{secundari pic}}}{\sqrt{2}} = 2\, \frac{50}{\sqrt{2}} = 70{,}71\ \mathrm{V}$$

**Exercici 5**

b)

$$\Gamma_\mathrm{N} = \frac{P_\mathrm{N}}{\omega_\mathrm{N}} = \frac{P_\mathrm{N}}{n_\mathrm{N}\, \frac{2\,\pi}{60}} = \frac{30000}{1750\, \frac{2\,\pi}{60}} = 163{,}7\ \mathrm{N\,m}$$

b)

$$E_\mathrm{N} = \frac{P_\mathrm{N}}{I_\mathrm{N}} = \frac{30000}{69} = 434{,}8\ \mathrm{V} \quad \rightarrow \quad R_\mathrm{i} = \frac{U_\mathrm{N} - E_\mathrm{N}}{I_\mathrm{N}} = \frac{500 - 434{,}8}{69} = 0{,}945\ \Omega$$

c)

L'alternador és de dos parells de pols ($p = 2$) i, per tant, ha de girar a la velocitat de sincronisme $n_\mathrm{s} = \frac{60\, f}{p} = \frac{60 \cdot 50}{2} = 1500\ \mathrm{min^{-1}}$ per tal de generar a $50$ Hz.

$$E' = \frac{n_\mathrm{s}}{n_\mathrm{N}}\, E_\mathrm{N} = \frac{1500}{1750}\, 434{,}8 = 372{,}7\ \mathrm{V}$$

$$P = \Gamma\, \omega_\mathrm{s} = 100 \cdot 1500\, \frac{2\,\pi}{60} = 15708\ \mathrm{W}$$

$$I = \frac{P}{E'} = \frac{15708}{372{,}7} = 42{,}15\ \mathrm{A}$$

$$U = R_\mathrm{i}\, I + E' = 0{,}945 \cdot 42{,}15 + 372{,}7 = 412{,}5\ \mathrm{V}$$

**Exercici 6**

a)

$$f = \frac{1}{2\,\pi\sqrt{L_1\,C_1}} = \frac{1}{2\,\pi\sqrt{120\cdot10^{-3}\cdot50\cdot10^{-6}}} = 64{,}97\ \text{Hz}$$

b)

$$X_{\text{L1}} = \omega\,L_1 = 2\,\pi\,f\,L_1 = 2\,\pi\,64{,}97\cdot120\cdot10^{-3} = 48{,}99\ \Omega$$

$$X_{\text{C1}} = \frac{1}{\omega\,C_1} = \frac{1}{2\,\pi\,f\,C_1} = \frac{1}{2\,\pi\,64{,}97\cdot50\cdot10^{-6}} = 48{,}99\ \Omega$$

$$I_{\text{R1}} = \frac{V_2}{\sqrt{R_1^2 + (X_{\text{L1}} - X_{\text{C1}})^2}} = \frac{V_2}{R_1} = \frac{85}{100} = 0{,}85\ \text{A}$$

$$P_{\text{R1}} = R_1\,I_{\text{R1}}^2 = 100\cdot0{,}85^2 = 72{,}25\ \text{W}$$

$$X_{\text{L2}} = \omega\,L_2 = 2\,\pi\,f\,L_2 = 2\,\pi\,64{,}97\cdot35\cdot10^{-3} = 14{,}29\ \Omega$$

$$I_{\text{R2}} = \frac{V_2}{\sqrt{R_2^2 + X_{\text{L2}}^2}} = \frac{85}{\sqrt{40^2 + 14{,}29^2}} = 2\ \text{A}$$

$$P_{\text{R2}} = R_2\,I_{\text{R2}}^2 = 40\cdot2^2 = 160\ \text{W}$$

c)

$$W = P = P_{\text{R1}} + P_{\text{R2}} + P_{\text{R3}} = P_{\text{R1}} + P_{\text{R2}} + R_3\,I_{\text{R3}}^2 \quad \rightarrow \quad I_{\text{R3}} = \sqrt{\frac{W - P_{\text{R1}} - P_{\text{R2}}}{R_3}}$$

$$I_{\text{R3}} = \sqrt{\frac{351{,}21 - 72{,}25 - 160}{15}} = 2{,}82\ \text{A}$$

d)

$$Q = X_{\text{L2}}\,I_{\text{R2}}^2 = 14{,}29\cdot2^2 = 57{,}16\ \text{var}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{351{,}21^2 + 57{,}16^2} = 355{,}8\ \text{VA}$$

$$S = U\,I_{\text{R3}} \quad \rightarrow \quad U = \frac{S}{I_{\text{R3}}} = \frac{355{,}8}{2{,}82} = 126{,}2\ \text{V}$$