Master in Computer Vision Barcelona

Module: Optimization and Inference Techniques in CV
Project: Segmentation

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Image Segmentation

To decompose an image using active contours without edges The student will learn:

- Level sets.
- Mumford-Shah Energy Functional.
- Active Contours without Edges segmentation.
- The relationship between the Mumford-Shah Model and the ROF Denoising Model (Optional).
- Anisotropic Mumford Shah Energy Functional (Optional).
- The relationship between the Anisotropic Mumford-Shah Model and the ROF Denoising Model (Optional).

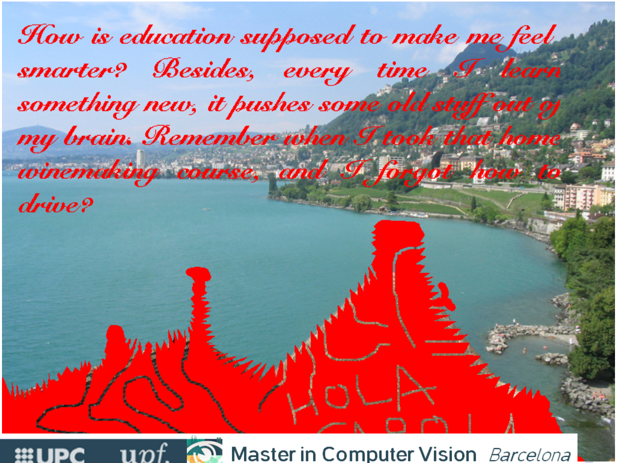
Goal

GOAL

To smooth the boundary of the region to inapaint on the roof.

How

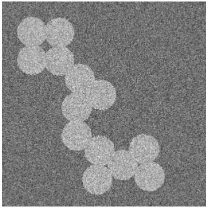
Using the Chan-Vese segmentation model.



Segmentation

Mumford and Shah Model

Let $f \in L^\infty(\Omega)$ be the given image.



D. Mumford and J. Shah
Optimal approximations by piecewise smooth functions and associated variational problems.
Commun. Pure Appl. Math. 42 (1989) 577-685

Segmentation

Mumford and Shah Model

Let $f \in L^\infty(\Omega)$ be the given image. We search for a pair (u, Γ) where $\Gamma \subset \Omega$ is the set of discontinuities.

$$J_{ms}(u,\Gamma) = \mu \int_{\Omega \setminus \Gamma} |\nabla u|^2 dx + \nu \text{length}(\Gamma) + \int_\Omega |u - f|^2 dx$$

where μ and ν are nonnegative constants and length(Γ) is defined as $\int_\Gamma d\sigma$

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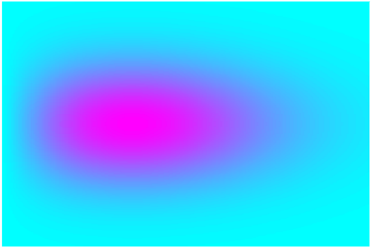
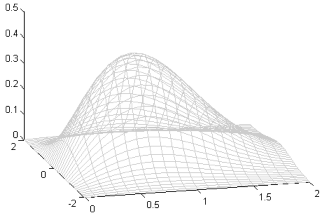
Coarea formula

Suppose $u \in BV(\Omega)$, then

$$\int_\Omega |Du| = \int_{-\infty}^{\infty} \mathrm{Per}(E_s)ds \qquad \text{where} \quad E_s = \{(x,y) \in \Omega | u > s\}$$





For smooth images, the co-area formula shows that $TV[u]$ is to sum up the lenghts of all level curves (level sets) weighted by the Lebesgue element ds.

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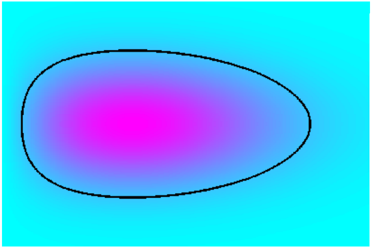

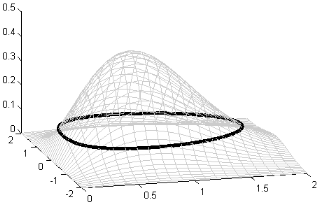

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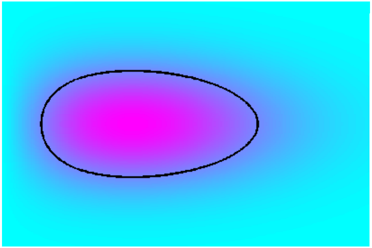

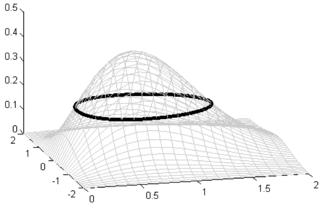

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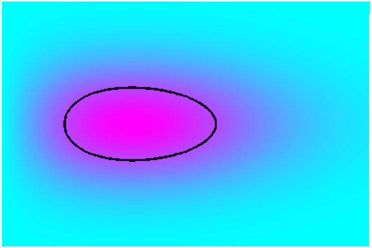

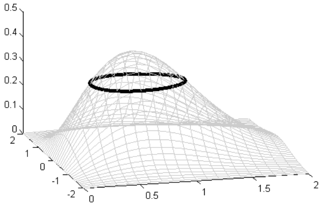

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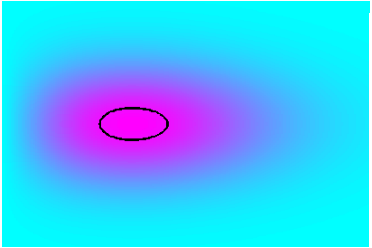

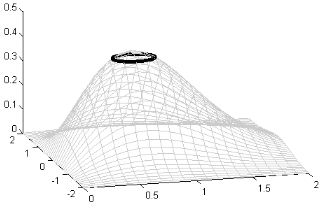

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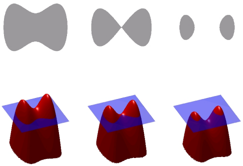
Level Sets and Chan-Vese Model

T.F.Chan L.Vese
Active Contours without edges
IEEE Trans. Image Process. 10 (2) (2001) 266-277

Level Set Dictionary



$$\Gamma \equiv \{\phi = 0\} \qquad \text{length}(\Gamma) = \int_\Omega |DH(\phi)|$$

$$H(\phi) = \begin{cases} 1 & \textit{if} \quad \phi \geq 0 \\ 0 & \textit{if} \quad \phi < 0 \end{cases}$$

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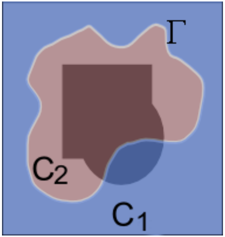

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$$J_{cv}(\bar{c}, E) = \text{Per}(E) + \frac{1}{2\lambda}\int_E |c_1 - f|^2 dx + \frac{1}{2\lambda}\int_{\mathcal{C}E} |c_2 - f|^2 dx$$

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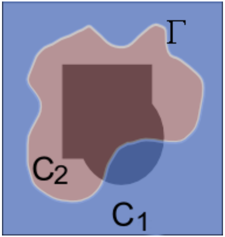

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$$J_{cv}(\bar{c}, \phi) = \int_\Omega |DH(\phi)| + \frac{1}{2\lambda_{cv}} \int_\Omega |c_2 - f|^2 H(\phi) dx + \frac{1}{2\lambda_{cv}} \int_\Omega |c_1 - f|^2 (1 - H(\phi)) dx$$

$$c_1 = \frac{\int_\Omega f\,(1 - H(\phi))\,dx}{\int_\Omega 1 - H(\phi) dx} \qquad c_2 = \frac{\int_\Omega f H(\phi) dx}{\int_\Omega H(\phi) dx}$$

Active Contours without Edges: Chan-Vese Model

IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 10, NO. 2, FEBRUARY 2001

Active Contours Without Edges

Tony F. Chan, *Member, IEEE*, and Luminita A. Vese

$$
\begin{aligned}
F(c_1, c_2, C) = &\mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(inside(C)) \\
&+ \lambda_1 \int_{inside(C)} |u_0(x,y) - c_1|^2 \, dx \, dy \\
&+ \lambda_2 \int_{outside(C)} |u_0(x,y) - c_2|^2 \, dx \, dy,
\end{aligned}
$$

where $\mu \geq 0, \nu \geq 0, \lambda_1, \lambda_2 > 0$ are fixed parameters. In almost all our numerical calculations (see further), we fix $\lambda_1 = \lambda_2 = 1$ and $\nu = 0$.

Therefore, we consider the minimization problem:

$$
\inf_{c_1, c_2, C} F(c_1, c_2, C).
$$

Active Contours without Edges: Chan-Vese Model

Level Set Dictionary

$$\text{Length}\{\phi = 0\} = \int_\Omega |\nabla H(\phi(x,y))|\, dx\, dy$$
$$= \int_\Omega \delta_0(\phi(x,y))|\nabla\phi(x,y)|\, dx\, dy,$$
$$\text{Area}\{\phi \geq 0\} = \int_\Omega H(\phi(x,y))\, dx\, dy,$$

Active Contours without Edges: Chan-Vese Model

Level Set Dictionary

$$\text{Length}\{\phi = 0\} = \int_\Omega |\nabla H(\phi(x, y))| \, dx \, dy = \int_\Omega \delta_0(\phi(x, y)) |\nabla \phi(x, y)| \, dx \, dy,$$

$$\text{Area}\{\phi \geq 0\} = \int_\Omega H(\phi(x, y)) \, dx \, dy,$$

$$\int_{\phi>0} |u_0(x, y) - c_1|^2 \, dx \, dy = \int_\Omega |u_0(x, y) - c_1|^2 H(\phi(x, y)) \, dx \, dy,$$

$$\int_{\phi<0} |u_0(x, y) - c_2|^2 \, dx \, dy = \int_\Omega |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) \, dx \, dy.$$

Then, the energy $F(c_1, c_2, \phi)$ can be written as

$$\begin{aligned} F(c_1, c_2, \phi) &= \mu \int_\Omega \delta(\phi(x, y)) |\nabla \phi(x, y)| \, dx \, dy \\ &+ \nu \int_\Omega H(\phi(x, y)) \, dx \, dy \\ &+ \lambda_1 \int_\Omega |u_0(x, y) - c_1|^2 H(\phi(x, y)) \, dx \, dy \\ &+ \lambda_2 \int_\Omega |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) \, dx \, dy. \end{aligned}$$

Active Contours without Edges: Chan-Vese Model

Gradient Descent

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left[\mu \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \nu - \lambda_1(u_0 - c_1)^2 + \lambda_2(u_0 - c_2)^2\right] = 0 \text{ in } (0, \infty) \times \Omega,$$

$$\phi(0, x, y) = \phi_0(x, y) \text{ in } \Omega,$$

$$\frac{\delta_\varepsilon(\phi)}{|\nabla \phi|}\frac{\partial \phi}{\partial \vec{n}} = 0 \text{ on } \partial\Omega$$

Active Contours without Edges: Chan-Vese Model

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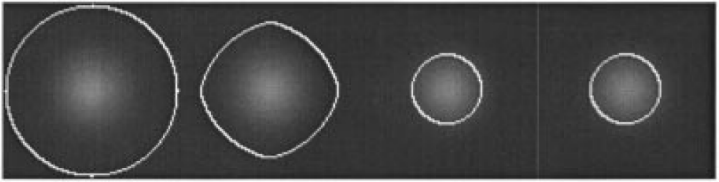
Discretization

$$\begin{aligned}
\frac{\phi^{n+1}_{i,j} - \phi^n_{i,j}}{\Delta t} &= \delta_h(\phi^n_{i,j})\left[\frac{\mu}{h^2}\Delta^x_-\right. \\
&\cdot \left(\frac{\Delta^x_+\phi^{n+1}_{i,j}}{\sqrt{(\Delta^x_+\phi^n_{i,j})^2/(h^2) + (\phi^n_{i,j+1} - \phi^n_{i,j-1})^2/(2h)^2}}\right) \\
&+ \frac{\mu}{h^2}\Delta^y_- \\
&\cdot \left(\frac{\Delta^y_+\phi^{n+1}_{i,j}}{\sqrt{(\phi^n_{i+1,j} - \phi^n_{i-1,j})^2/(2h)^2 + (\Delta^y_+\phi^n_{i,j})^2/(h^2)}}\right) \\
&\left. -\nu - \lambda_1(u_{0,i,j} - c_1(\phi^n))^2 + \lambda_2(u_{0,i,j} - c_2(\phi^n))^2\right].
\end{aligned}$$

where

$$\Delta^x_-\phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}, \quad \Delta^x_+\phi_{i,j} = \phi_{i+1,j} - \phi_{i,j},$$
$$\Delta^y_-\phi_{i,j} = \phi_{i,j} - \phi_{i,j-1}, \quad \Delta^y_+\phi_{i,j} = \phi_{i,j+1} - \phi_{i,j}.$$

Active Contours without Edges: Results



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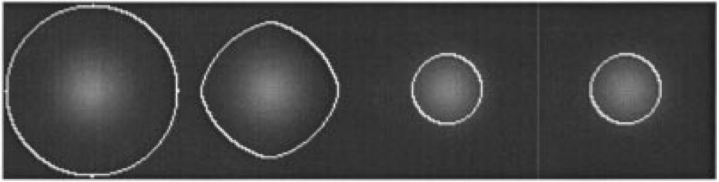

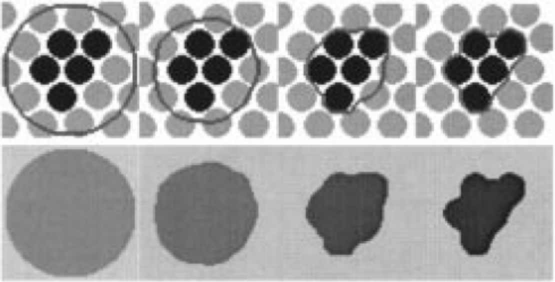

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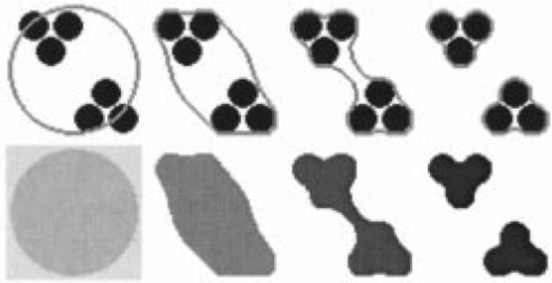

Deliverable: Mandatory

Mandatory means if there any point that it is not done, then the weekly task will FAIL.

- Implement the Chan-Vese algorithm and test it with the given images.
- Video showing the curve evolution.
- To discover under which assumptions Chan-Vese works and when it does not.

Maximum: 9 points. The evaluation will depend on the document and the code. **Deliver Thur. Nov. 1st. at 18:00 hours.**

Hints: Creating the initial function ϕ

```
[ni, nj]=size(I);
[X, Y]=meshgrid(1:ni, 1:nj);
modelParams.initialPhi=(-sqrt( ( X-round(ni/2)).^2 +
(Y-round(nj/2)).^2)+50)*h;
```

Deliverable: Optional

- O1: Test with your own real images. Up to +1 point.
- 02: Implement the Chambolle's convexification for the Chan-Vese energy functional. Up to +5 points.
- O3: Implement the Convexification for the Anisotropic Mumford-Shah proposed in "A Fast Anisotropic Mumford-Shah Functional Based. Segmentation". Up to +5 points.
- O4: Implement the minimization of a functional that works for ALL given images. Up to +5 points.
- O5: Solve the inpainting problem for the Tampa and GR images. Up to +1 points.
- O6: Implement the Coupled Pairwise scheme proposed in "Box Relaxation Schemes in Staggered Discretizations for the Dual Formulation of Total Variation Minimization". Up to +5 points.

Deliverable of mandatory and O1. Thur. Nov. 2nd. 18:00h