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Exam TTK4190
Guidance and Control of Vehicles

Monday May 21, 2012

Hours: 09:00-13:00

Code A: All printed and handwritten materials are allowed. All type of calculators is approved*.

* For *all type of calculators* the following applies:
- Calculators must not communicate with other electronic units/computers.
- The calculator must not be attached to the power outlet.
- The calculator must not make noise.
- The unit's display must be the only printing device.
- The calculator must only be one unit.
- The calculator must be in pocket size.

Language: English

Number of Pages: 4

Sensor Deadline: 3 weeks

You may write your answers in *Norwegian* or *English.*

Perfect scores amounts to 100 %.

Problem 1: Autonomous Underwater Vehicles (AUV) Modeling (40 %)

Consider the AUV REMUS 100 in Figure 1. REMUS 100 is an autonomous underwater vehicle (AUV) manufactured by Kongsberg. We will approximate the hull form as a cylinder with diameter 19 cm and length 160 cm. When floating, the vehicle displacement is 85 % of the cylinder volume and its dry weight is 46 kg. Assume that the CO is located in the volume center of the cylinder, that is CB = CO, with CG located 0.04 m below the CO. The density of water is 1025 kg/m^3.

Figure 1. The REMUS 100 a compact lightweight Autonomous Underwater Vehicle. Courtesy to Kongsberg.

A. (2 %) What is the weight and buoyancy of the vehicle?

B. (2 %) What are the locations of the transverse metacenter when 1) the vehicle is submerged and 2) the vehicle is floating?

C. (1 %) Will the uncontrolled vehicle sink or rise when submerged? Explain why.

D. (3 %) The AUV loses all power at 100 m depth. How long time will it take for the vehicle to reach the surface (assume that the attitude is constant during the vertical motion and that there is no drag)?

E. (3 %) The moments of inertia about CG is approximated by **I** = *diag*(0.18, 0.18, 8.41) kgm^2. Assume that the added moment of inertia in pitch is in the range of 50-100 % of the vehicle mass and compute the minimum and maximum pitch periods (assume that the pitching motion is decoupled).

F. (3 %) Assume that the linear damping in pitch reduce the frequency of oscillation with 0.05 % such that frequency of oscillation is $\omega = 0.995\omega_n$ where ω_n is the natural frequency. Use this relationship to compute the linear damping coefficients M_q. Use I_y and the maximum value for $M_{\dot{q}}$ given by problem 1E.

G. (6 %) Show that the depth (positive downwards) satisfies

$$\dot{d} = -U(\theta - \alpha)$$

for constant speed *U* and state the necessary assumptions for this to be true.

H. (10 %) Design a sliding mode controller for automatic depth control using $\dot{d} = U(\theta - \alpha)$. Assume that the angle of attack is measured together with depth, speed, pitch angle and pitch rate and that the desired depth d_d is constant. The pitching motion is decoupled and given by:

$$(I_y - M_{\dot{q}})\ddot{\theta} - M_q\dot{\theta} + W\overline{BG}_z\theta = \tau$$

where τ is the control input. The sliding surface is given by the following equation:

$$s = \mathbf{h}^T(\mathbf{x} - \mathbf{x}_d),$$

where $\mathbf{x} = [d, \theta, q]^T$ and $\mathbf{x}_d = [d_d, 0, 0]^T$. Propose a Lyapunov function candidate for the system and explain how the vector **h** must computed in order to ensure that the system is globally convergent. Include the mathematical proof in your answer.

I. (10 %) The AUV is used in a search operation under water. The search path is parameterized as an Archimedean spiral given by:

$$r = 1 + \theta$$

where (r,θ) are the *polar coordinates* describing the path. Find the tangent line: $y = y_0 + \left.\frac{dy}{dx}\right|_{x=x_0}(x - x_0)$

to the Archimedean spiral for $\theta_0 = 10$ rad. The tangent line goes through the point (r_0, θ_0). Assume that the lookahead distance Δ is positive and that the vehicle location is given by the triangle in Figure 1. The control objective is path following. Make a sketch showing how the desired course angle χ_d can be computed for the vehicle location in Figure 1. Write down the formulae need to compute the desired course angle as a function of (r_0, θ_0). Hint: $\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

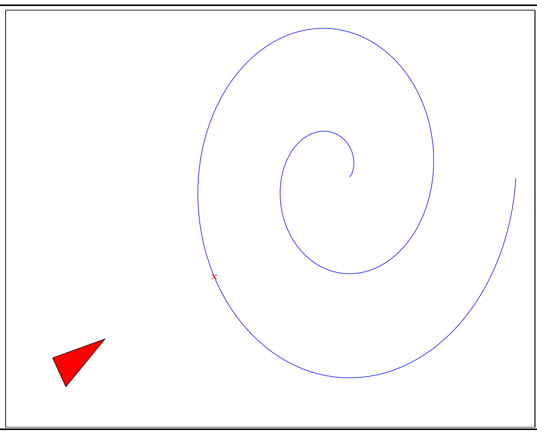

Figure 1: Archimedean spiral: It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

Problem 2: Navigation Systems (30 %)

Consider the following models describing two navigation systems:

$$\Sigma_1 : \begin{cases} \dot{\mathbf{p}}^n = \mathbf{R}_b^n(\Theta)\mathbf{v}^b \\ m\dot{\mathbf{v}}^b + d\mathbf{v}^b = \tau^b \end{cases}$$

$$\Sigma_2 : \begin{cases} \dot{\mathbf{p}}^n = \mathbf{v}^n \\ \mathbf{a}^b_{imu} = \mathbf{R}^n_b(\Theta)^T [\dot{\mathbf{v}}^n - \mathbf{g}^n] + \mathbf{b}^b \\ \dot{\mathbf{b}}^b = \mathbf{0} \end{cases}$$

where $\mathbf{p}^n$ is the position, $\mathbf{v}^n$ is the linear velocity, $\mathbf{b}^b$ is the accelerometer bias, $\mathbf{g}^n = [0,0,9.81]^T$ is the gravity vector, τ^b is the control forces and $\mathbf{R}^n_b(\Theta)$ is Euler angle rotation matrix. The accelerometer measurement is denoted $\mathbf{a}^b_{imu}$. The mass and damping coefficient are denoted *m* and *d*, respectively.

A. (8 %) Assume that $\mathbf{R}^n_b(\Theta)$ is known. Design a fixed-gain observer for the model Σ_1 using $\mathbf{p}^n$ and τ^b as measurements (write down the differential equations for the observer and error dynamics using constant gain matrices). Make a block diagram of the system and observer equations showing the states, filter gains and integrators in the system.

B. (8 %) Assume that $\mathbf{R}^n_b(\Theta)$ is known. Design a fixed-gain observer for the model Σ_2 using $\mathbf{p}^n$ and $\mathbf{a}^b_{imu}$ as measurements (write down the differential equations for the observer and error dynamics using constant gain matrices). Make a block diagram of the system and observer equations showing the states, filter gains and integrators in the system.

C. (3 %) Which model do companies selling motion sensors prefer? Explain why they prefer one model to the other one.

D. (7 %) Assume that bias $\mathbf{b}^b = \mathbf{0}$ in model Σ_2 and assume that the vehicle acceleration in NED is $\dot{\mathbf{v}}^n = [0.1000, 0, 0]^T$. The accelerometer reads $\mathbf{a}^b_{imu} = [1.8018, 2.4958, -9.3151]^T$. Compute the roll and pitch angles.

E. (4 %) In practice it is not possible to measure $\dot{\mathbf{v}}^n$ using standard instrumentation. Hence, in vertical reference units (VRUs) it is common to assume that $\dot{\mathbf{v}}^n$ is zero. This is not a good assumption for accelerated vehicles. Is it possible estimate $\dot{\mathbf{v}}^n$ from other measurements? (If yes, explain how and what kind of measurement system you need to do this. If no, explain why it is impossible).

Problem 3: Guidance Systems (30 %)

A. (5 %) What is the problem of using straight lines and inscribed circles in waypoint tracking systems? How can this problem be addressed?

B. (5 %) What is the geometrical interpretation of the guidance law described by (10.72) in Fossen (2011)? (Hint: What does each one of the angles χ_p and χ_r represent?

C. (10 %) Consider a vehicle moving on the surface of the Earth. Assume that the GNSS position measurement *P(x,y,z)* of the vehicle is *P(4 m, 5 m ,0 m)* at one time instance. The goal of the line-of-sight guidance law is 2-D path following where the desired path is a straight line given by $y = 3x + 1$. Furthermore, assume that the lookahead distance is $\Delta = 100$ m. Compute the desired course angle in degrees (only the first step of the lookahead-based steering law is required).

D. (5 %) The vehicle in Problem 3C is exposed to wind and the sideslip angle is $\beta = -5^o$. Compute the desired heading angle in degrees using the data from Problem 3C.

E. (5 %) Assume that the lookahead distance is time varying. How would a variable lookahead distance affect the behavior of a vehicle, which navigates using a lookahead-based steering law?