NTNU – Trondheim
Norwegian University of
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Department of Engineering Cybernetics

Examination paper for TTK4190 Guidance and Control of Vehicles

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Examination date: Tuesday 1 December 2015

Examination time (from-to): 09:00–13:00

Permitted examination support material (Code C):

- Textbooks by Fossen (2011) and Beard & McLain (2012).
- Printed lecture notes/slides, assignments and problems/exams.
- All handwritten materials are allowed. All type of calculators is approved*.

* For *all type of calculators* the following applies:
- Calculators must not communicate with other electronic units/computers.
- The calculator must not be attached to the power outlet.
- The calculator must not make noise.
- The unit's display must be the only printing device.
- The calculator must only be one unit.
- The calculator must be in pocket size.

Language: English

Number of pages (front page excluded): 5

Number of pages enclosed:

You may answer in Norwegian or English

Checked by:

Date Signature

Problem 1: Ship Speed and Propeller Control Systems (30 %)

Consider the shaft dynamics (see Figure 1):

$$I_s \dot{n} + D_f n = Q_m - Q$$

where I_s is the propeller shaft moment of inertia and D_f is the propeller friction coefficient. The propeller torque is denoted Q and n is the propeller revolution. The motor torque Q_m is the control input.

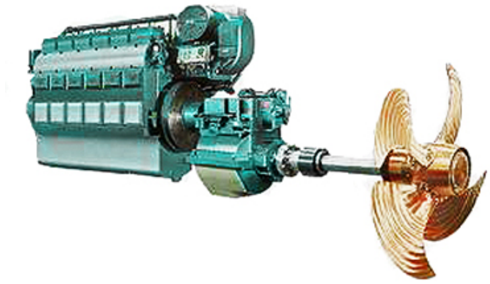

Fig 1. Marine diesel engine with propeller and shaft.

A. (8 %) Assume that the propeller load moment can be modelled as:

$$Q = \rho D^5 K_Q |n| n$$

where ρ is the density of water, D is the propeller diameter and K_Q is the propeller torque coefficient. Design a motor torque <u>feedback linearizing controller</u> Q_m with integral action for tracking of a time-varying reference signal n_d representing the desired propeller revolution. All parameters in the model can be assumed constant and known. Derive explicit formulae for the controller gains as a function of the desired natural frequency ω_n and relative damping ratio ζ_n.

B. (2 %) Find an expression for the control bandwidth ω_b.

C. (6 %) Let the tracking error be denoted $\tilde{n} = n - n_d$. Explain what kind of stability can be proven for the closed-loop system under (1A). A mathematical proof is required.

D. (10 %) Consider the ship model:

$$(m - X_{\dot{u}})\dot{u} - X_{|u|u}|u|u = (1 - t)T$$

where $X_{\dot{u}}$ and $X_{|u|u}$ are hydrodynamic coefficients, u is the surge velocity and t is a constant. The propeller generates a force:

$$T = \rho D^4 K_T |n| n$$

where K_T is the propeller force coefficient. Assume that $n/n_d \approx 1$ and use successive loop closures to design a speed controller for tracking of the time-varying desired velocity u_d. The controller gains should be a function of the natural frequency ω_u and relative damping ratio ζ_u. In addition, the two control loops should satisfy $\omega_n/\omega_u = 10$.

E. (4 %) Draw a block diagram showing the feedback interconnected system and explain why the ratio ω_n/ω_u is chosen as 10.

Problem 2: UAV Autopilot Design (40 %)

Consider the linearized UAV model:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{g}{U_0} & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ 0 \\ 0 \end{bmatrix} \delta_A$$

where a_{ij} and b_i are constants, *g* is the acceleration of gravity and U_0 is the forward speed. The control input:

$$\delta_A = 1/2(\delta_{A_L} + \delta_{A_R})$$

weights the left and right aileron angles equally. The state *v* is the sway velocity, φ and *p* are the roll angle and roll rate, respectively and, ψ and *r* are the yaw angle and yaw rate, respectively.

Fig 2. Unmanned aerial vehicle (UAV).

A. (4 %) Assume that there is no wind such that $\psi = \chi$. Show how the two last equations in the state-space model are obtained from the nonlinear kinematic model of a UAV under the assumption coordinated turn. State all assumptions for this to be valid.

B. (4 %) Explain the physical meaning of the parameter a_{14}. Explain how the parameter can be computed.

C. (3 %) The state-space model has five states and it depends of the air speed. Which one of these quantities can be measured using standard sensors and which one are estimated using a UAV state estimator/observer? Make a bullet list of the instruments/sensors you need for this.

D. (12 %) Design a sliding mode control for tracking of the heading reference signal ψ_d using the eigenvalue decomposition. Write down all equations needed to implement the controller and write a Matlab script using pseudocode (high-level description of the computer program) explaining how you compute all the terms in the controller by explicit formulae.

E. (8 %) Assume that the left aileron has a malfunction such that its use should be kept to a minimum. Propose a control allocation solution different from:

$$\delta_A = 1/2(\delta_{A_L} + \delta_{A_R})$$

which takes this into account.

F. (3 %) Assume that the aircraft is exposed to wind and that you do not have a compass to measure the heading angle. Explain by using equations how the state-space model and heading controller can be modified to use the course angle instead. What kind of measurement system can be used to measure the UAV course angle?

G. (3 %) Is this a bank-to-turn autopilot? Explain why/why not.

H. (3 %). Is it possible to modify the control law to minimize sideslip during turning? Explain why/why not.

Problem 3: Floating Wind Turbine (30 %)

Consider the floating wind turbine in Figure 3. The floating part of the wind turbine is a cylinder. The diameter of the cylinder is 8.3 m and the height of the submerged part of the cylinder is 6 m. The density of water is 1025 kg/m^3 and acceleration of gravity is 9.8 m/s^2.

Fig 3. Floating wind turbine (left) and mooring lines (right).

A. (2 %) Compute the mass of the whole structure.

B. (2 %) Consider the linear model:

$$(m - Z_{\dot{w}})\dot{w} - Z_w w - Z_z z = \tau_{moor} + \tau_{wave}$$

where z and w denote the heave position and velocity, respectively, τ_{moor} is the vertical mooring force and τ_{wave} is the vertical wave force. Vertical wind forces are neglected. Explain the physical meaning of the coefficient Z_z and compute its numerical value.

C. (4 %) Added mass and potential damping are computed using a hydrodynamic code. The numerical data is approximated by two functions:

$$A_{33}(\omega) = m\left(1 + e^{-\omega^2}\right)$$
$$B_{33}(\omega) = m\omega e^{-(\omega-0.1)^2}$$

where $\omega \geq 0$ (see Figure 4) and *m* is the mass. Viscous damping is modelled as:

$$B_{33v} = \text{constant}$$

The natural frequency ω_n in heave is given by the implicit equation:

$$\omega_n = \sqrt{\frac{C_{33}}{m + A_{33}(\omega_n)}} \quad \approx 0.8$$

Find constant frequency-independent values for A_{33} and B_{33} and explain the key assumption for these approximations to be valid.

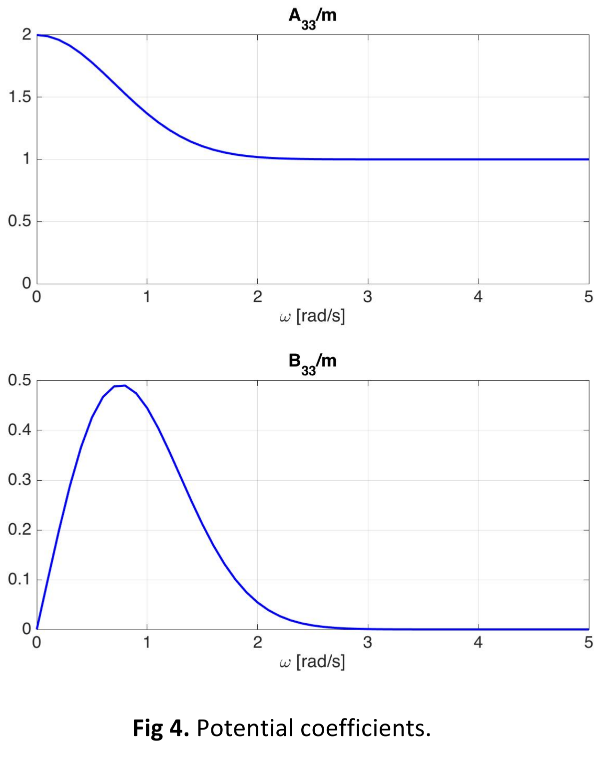

Fig 4. Potential coefficients.

D. (5 %) Compute the viscous damping term B_{33v} such that the relative damping ratio ζ in heave is 0.16 for the linear model (defined in 3B).

E. (2 %) Find numerical values for the hydrodynamic derivatives $Z_{\dot{w}}$ and Z_w.

F. (5 %) Assume that the mooring system can be approximated as a linear spring

$$\tau_{moor} = -K_m z$$

Compute the spring stiffness coefficient K_m needed to increase the natural frequency from 0.8 to 1.0.

G. (5 %) The structure is exposed to a constant vertical ocean current $w_c = 1.0$ m/s. Explain how this affects the equilibrium position of the structure and compute the new equilibrium position.

H. (5 %) Explain what happens if the wind turbine (without mooring) is exposed to a regular wave

$$\tau_{wave} = 2.1\cos(0.8t + 0.1)$$