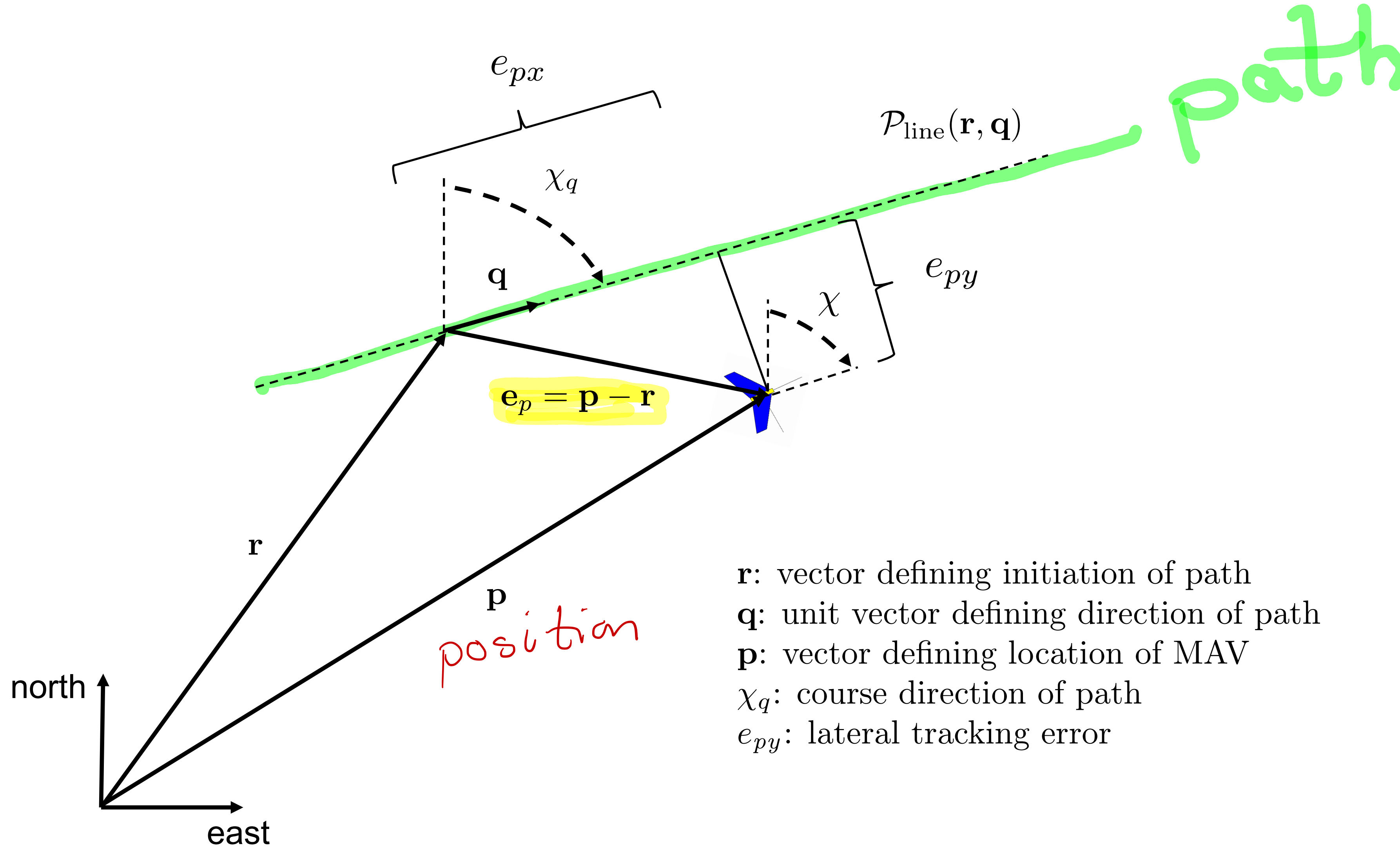
Chapter 10

Path Following

Path Following

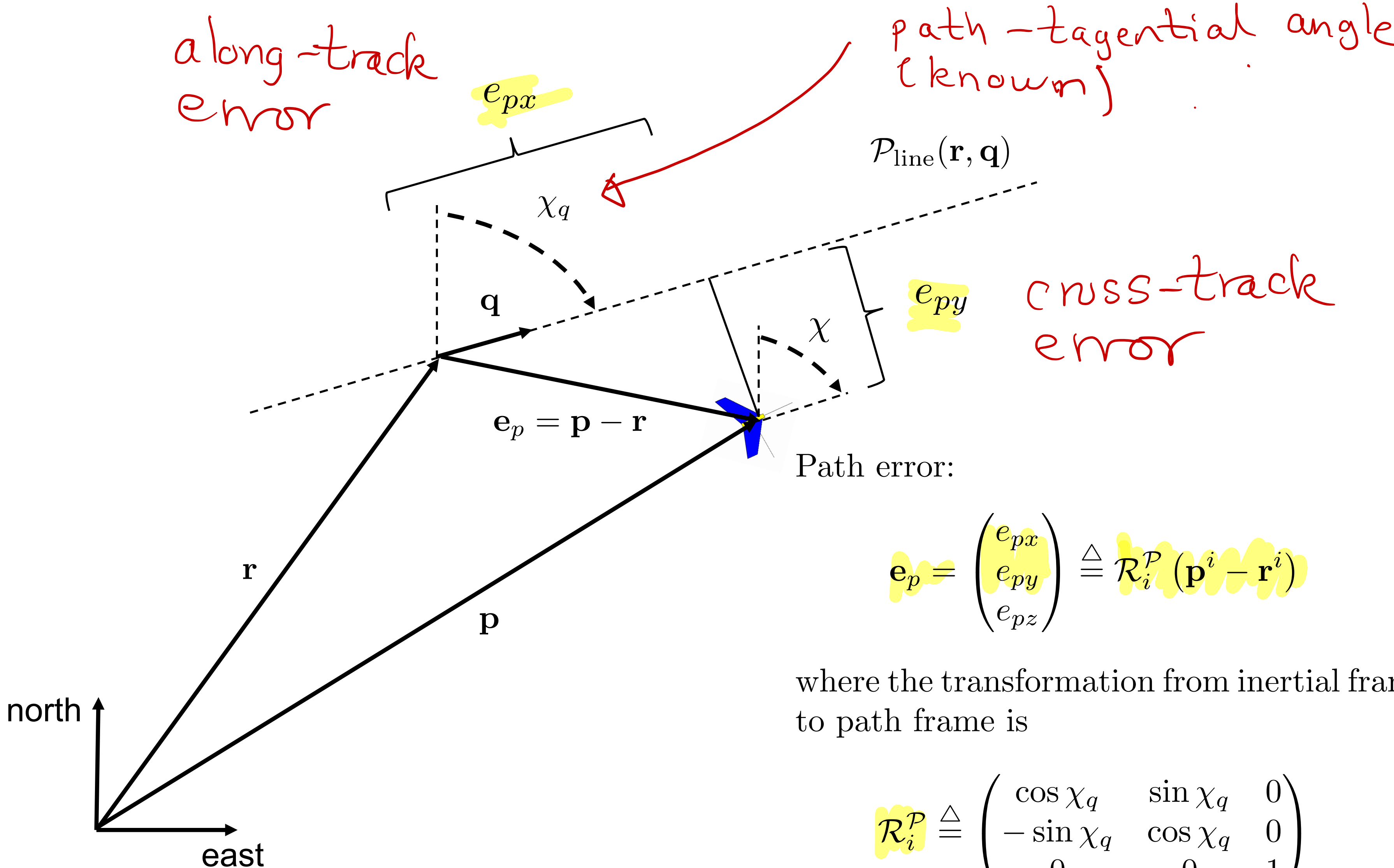
- For small UAVs, a major issue is wind
  - Always present to some degree
  - Usually significant with respect to commanded airspeed
- Wind makes traditional trajectory tracking approaches difficult, if not infeasible
  - Have to know the wind precisely at every instant to determine desired airspeed
- Better approach: path following
- Rather than "follow this trajectory", we control UAV to "stay on this path"

Straight Line Path Description



$\mathbf{r}$: vector defining initiation of path
$\mathbf{q}$: unit vector defining direction of path
$\mathbf{p}$: vector defining location of MAV
χ_q: course direction of path
e_{py}: lateral tracking error

Lateral Tracking Problem



Path error:

$$\mathbf{e}_p = \begin{pmatrix} e_{px} \\ e_{py} \\ e_{pz} \end{pmatrix} \triangleq \mathcal{R}_i^{\mathcal{P}} \left(\mathbf{p}^i - \mathbf{r}^i \right)$$

where the transformation from inertial frame to path frame is

$$\mathcal{R}_i^{\mathcal{P}} \triangleq \begin{pmatrix} \cos\chi_q & \sin\chi_q & 0 \\ -\sin\chi_q & \cos\chi_q & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Lateral Tracking Problem

autopilot is designed such that $\chi \to \chi^c$

Relative error dynamics in path frame:

$$\begin{pmatrix} \dot{e}_{px} \\ \dot{e}_{py} \end{pmatrix} = \begin{pmatrix} \cos\chi_q & \sin\chi_q \\ -\sin\chi_q & \cos\chi_q \end{pmatrix} \begin{pmatrix} V_g\cos\chi \\ V_g\sin\chi \end{pmatrix}$$

$$= V_g \begin{pmatrix} \cos(\chi - \chi_q) \\ \sin(\chi - \chi_q) \end{pmatrix}$$

Regulate the cross-track error e_{py} to zero by commanding the course angle:

$$\dot{e}_{py} = V_g \sin(\chi - \chi_q)$$

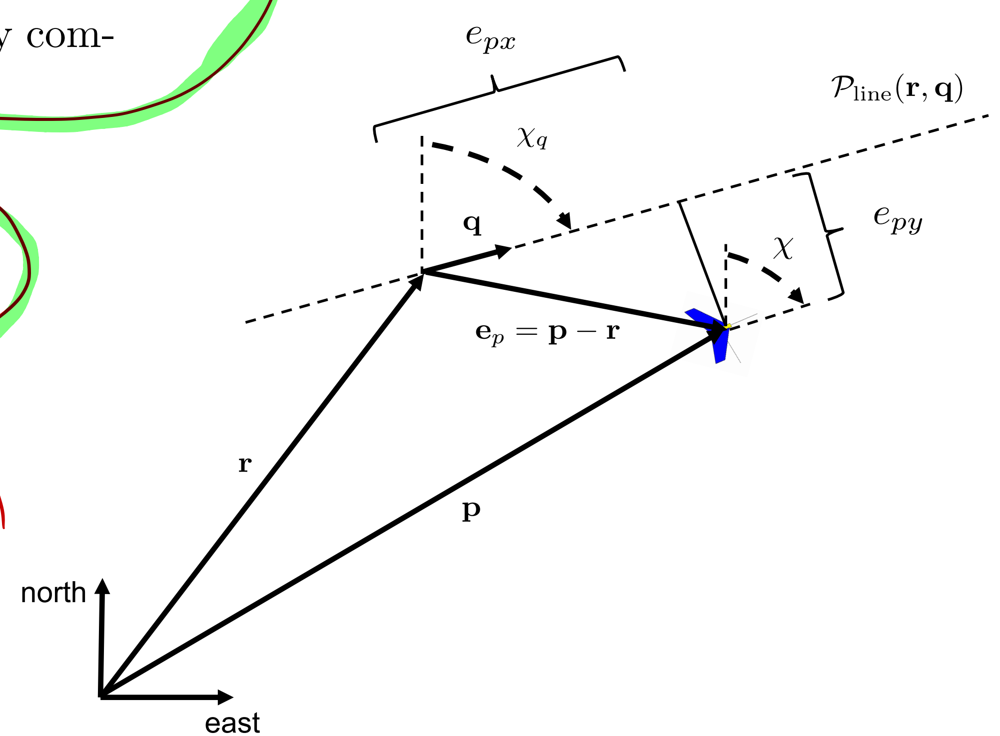
$$\ddot{\chi} = b_{\dot{\chi}}(\dot{\chi}^c - \dot{\chi}) + b_{\chi}(\chi^c - \chi)$$

Select χ^c so that $e_{py} \to 0$

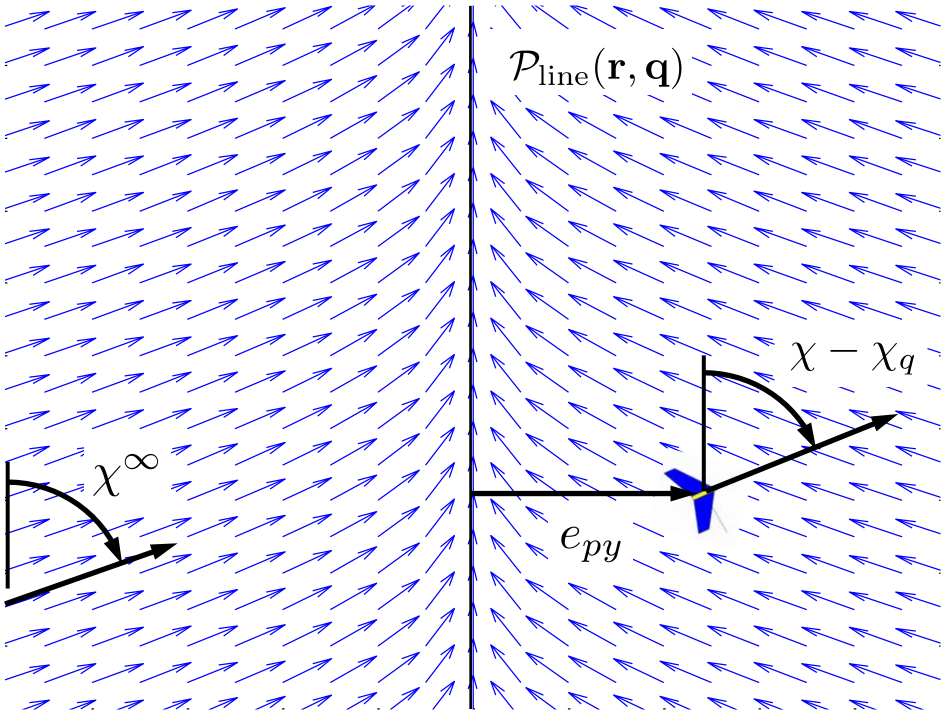
$\dot{e}_{py} = V_g \sin(\chi^c - \chi_q)$

$\chi^c = \chi_q + \chi_d$

$\Rightarrow \quad \dot{e}_{py} = V_g \sin(\chi_d)$

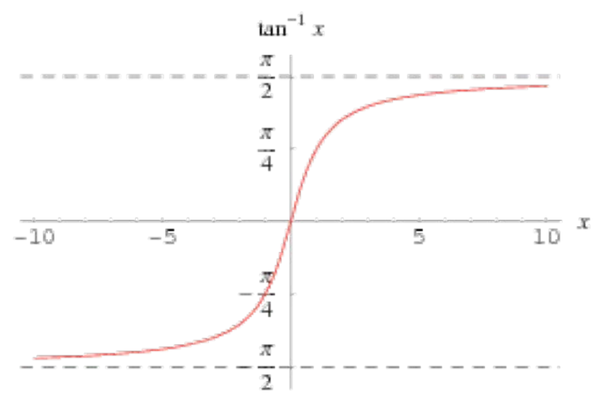


Lateral Tracking - Vector Field Concept

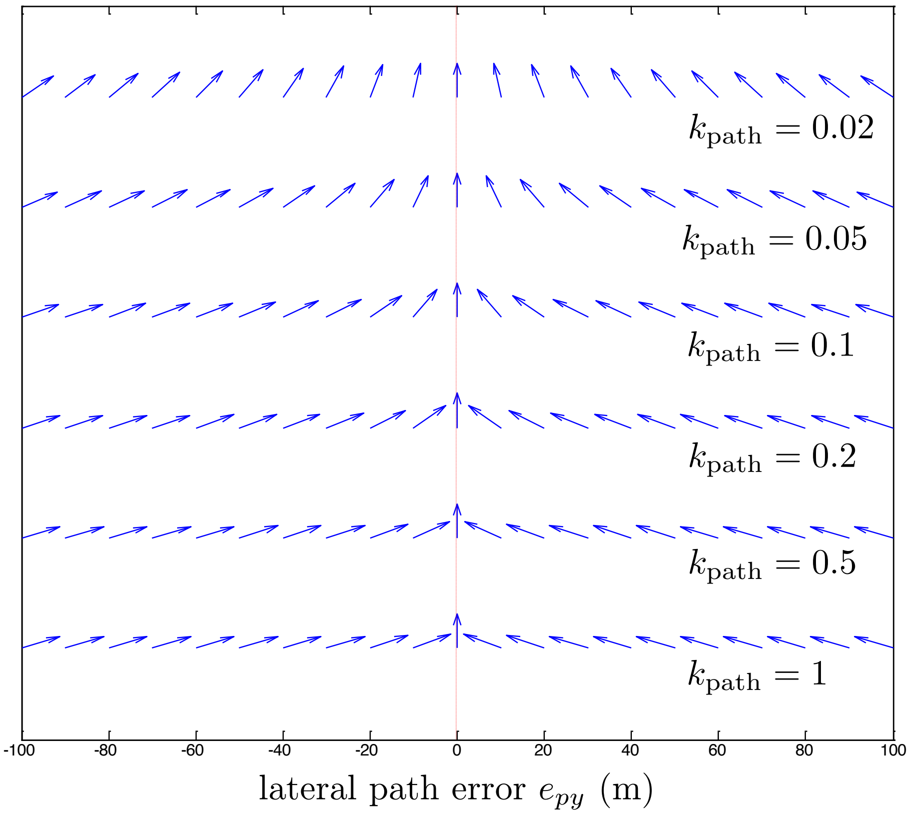


Desired course based on cross-track error:

$$\chi_d(e_{py}) = -\chi^{\infty}\frac{2}{\pi}\tan^{-1}(k_{\text{path}}e_{py})$$



Vector Field Tuning



k_{path} is a positive constant that affects the rate of transition of the desired course

- k_{path} large → short, abrupt transition
- k_{path} small → long, gradual transition

Lyapunov's 2nd Method

For a system having a state vector x, consider an energy-like function $V(x) : \mathcal{R}^n \mapsto \mathcal{R}$ such that

$$
\begin{aligned}
&V(x) \geq 0 \text{ (positive definite)} \\
&V(x) = 0 \text{ for } x = 0 \\
&\quad \text{and} \\
&\dot{V}(x) \leq 0 \text{ (negative definite)} \\
&\dot{V}(x) = 0 \text{ for } x = 0.
\end{aligned}
$$

If such a function $V(x)$ can be defined, then x goes to zero asymptotically and the system is stable.

Lateral Tracking Stability Analysis

Define the Lyapunov function $W(e_{py}) = \frac{1}{2}e_{py}^2$

Assume that course controller works and $\chi = \chi_q + \chi^d(e_{py})$

$(\chi = \chi^c)$

Since

$$\begin{aligned}\dot{W} &= e_{py}\dot{e}_{py} \\ &= -V_a e_{py} \sin\left(\chi^\infty \frac{2}{\pi}\tan^{-1}(k_{\text{path}} e_{py})\right) \\ &< 0\end{aligned}$$

for $e_{py} \neq 0$, then $e_{py} \to 0$ asymptotically