Exam TTK4190, Fall 2015

Draft Solution

December 1, 2015

Problem 1

1.A

Choose $Q_m = I_S \dot{n}_d + Q + D_f n - K_P(n - n_d) - K_I \int_0^t (n - n_d)\mathrm{d}t$ to obtain

$$I_S \dot{\tilde{n}} + (D_f + K_P)\tilde{n} + K_I \int_0^t \tilde{n}\mathrm{d}t = 0 \tag{1}$$

$$\dot{\tilde{n}} + I_S^{-1}(D_f + K_P)\tilde{n} + I_S^{-1} K_I \int_0^t \tilde{n}\mathrm{d}t = 0 \tag{2}$$

which can be rewritten in the Laplace domain as

$$S^2 + \frac{1}{I_S}(D_f + K_P)S + \frac{1}{I_S}K_I = 0 \tag{3}$$

The gains K_P and K_I are obtain by solving $\frac{1}{I_S}(D_f + K_P) = 2\zeta_n\omega_n$ and $\frac{1}{I_S}K_I = \omega_n^2$ for a given ζ_n and ω_n.

1.B

$$\omega_b = \omega_n \sqrt{1 - 2\zeta_n^2 + \sqrt{4\zeta_n^4 - 4\zeta_n^2 + 2}} \tag{4}$$

1.C

The dynamics of the error is linear, as in 1.A. For some chosen $\zeta_n > 0$ and $\omega_n > 0$, the system is GES if the gains K_P and K_I are chosen such that $\frac{1}{I_S}(D_f + K_P) = 2\zeta_n\omega_n$ and $\frac{1}{I_S}K_I = \omega_n^2$.

1.D

Same method as roll and course control for UAVs, but the model is nonlinear.
The input T is non-linear, but it can be redefined as $\delta = |n|n$ and it would become linear in the new input δ. The real n will be then calculated as $n = \mathrm{sgn}(\delta)\sqrt{|\delta|}$. The ship model with the new input is then

$$(m - X_{\dot{u}})\dot{u} - X_{|u|u}|u|u = (1 - t)\rho D^4 K_T \delta \tag{5}$$

and the dynamics of the error for a time-varying reference input is

$$(m - X_{\dot{u}})\dot{\tilde{u}} - X_{|u|u}|\tilde{u}|\tilde{u} = (1 - t)\rho D^4 K_T \delta \tag{6}$$

A feedback-linearizing controller similar to the previous one can again be used, for instance

$$\delta = \frac{1}{(1-t)\rho D^4 K_T}(-X_{|\tilde{u}|\tilde{u}} - K_P\tilde{u} - K_I\int_0^t \tilde{u}\mathrm{d}t) \tag{7}$$

which leads to

$$(m - X_{\dot{u}})\dot{\tilde{u}} + \frac{K_P}{(1-t)\rho D^4 K_T}\tilde{u} + \frac{K_I}{(1-t)\rho D^4 K_T}\int_0^t \tilde{u}\mathrm{d}t = 0 \tag{8}$$

$$\dot{\tilde{u}} + 2\zeta_u\omega_u\tilde{u} + \omega_u^2\int_0^t \tilde{u}\mathrm{d}t = 0 \tag{9}$$

and K_P and K_I can be found as functions of ζ_u and $\omega_u = 0.1\omega_n$.

1.E

The ratio between the frequencies means that the inner loop has to be faster than the outer loop (10 is a typical value for it), otherwise it will not be able to follow the reference signals generated by the outer loop.
The block diagram is similar to the one on page 97 of the UAV book, but with $r_3 = u_d$, $y_3 = u$, and $u_3 = \delta = |n|n$.

Problem 2

2.A

Kinematic equation

$$\dot{\eta} = J(\eta)\omega \tag{10}$$

with $\eta = (\phi, \theta, \psi)^T$ the vector of Euler angles, ω angular velocity. Linearization of the equation about 0, with the assumption of small Euler angles, gives $\dot{\phi} = p$. Regarding ψ, the present model uses the coordinated turn equation to write ψ as a function of ϕ instead of r.
The coordinated turn equation is

$$\dot{\chi} = \frac{g}{V_g}\tan\phi\cos(\chi - \psi) \tag{11}$$

and by setting $V_a = V_g$ and $\psi = \chi$ it becomes

$$\dot{\psi} = g/V_a \tan\phi \tag{12}$$

. with V_a the velocity relative to the air. With no wind and no sideslip it is true that $v = 0$, and for a small angle ϕ and $U_0 = V_a$ the equation becomes

$$\dot{\psi} = g/U_0\phi \tag{13}$$

2.B

Lateral acceleration component given by deviation of roll from trim. Roll deviation must still be very small, or linearization won't be valid anymore.
Calculated by linearization of the non-linear equation about the trim condition. The value a_{14}, at the end, depends on g, $\phi^\star$, and $\theta^\star$ according to

$$a_{14} = g\cos\theta^\star\cos\theta^\star \tag{14}$$

2.C

Sensors:

- rate gyros for p, r;
- inclinometer for ϕ, but not very accurate;
- compass for ψ;
- airspeed sensor for V_a.

Estimated:

- v;
- ϕ if the inclinometer is ruled not sufficiently accurate;
- V_a if the sensor is not available.

Additional sensors for estimation:

- INS (accelerometers, rate gyros) and GNSS to estimate v.

2.D

Page 525 of the book.
If x is the state vector and $x_d = [0, 0, r_d, 0, \psi_d]^T$, define

$$s = h^T(x - x_d) = h_1 v + h_2 p + h_3(r - r_d) + h_4 \phi + h_5(\psi - \psi_d) \tag{15}$$

Feedback from all states except ψ, i.e. $k^T = (k_1, k_2, k_3, k_4, 0)$, gives

$$A_c = A - bk^T = \begin{pmatrix} a_{11} - b_{11}k_1 & a_{12} - b_{11}k_2 & a_{13} - b_{11}k_3 & a_{14} - b_{11}k_4 & 0 \\ a_{21} - b_{12}k_1 & a_{22} - b_{12}k_2 & a_{31} - b_{12}k_3 & -b_{12}k_4 & 0 \\ a_{31} - b_{13}k_1 & a_{32} - b_{13}k_2 & a_{33} - b_{13}k_3 & -b_{13}k_4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & g/U_0 & 0 \end{pmatrix} \tag{16}$$

It has to be satisfied that $\lambda x^T h = 0$, which is possible if an eigenvalue λ of A_c is 0, and it is true (last column all zeros). The vector h is the eigenvector of A_c associated to the eigenvalue 0. The control input is then composed by

$$\delta_A = -k^T x + (h^T b)^{-1}[h^T \dot{x}_d - \eta \text{sgn}(s)] \tag{17}$$

By considering a typical disturbance as the wind, the uncertainty $f(x, t)$ will then be

$$f(x, t) = -\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ 0 \\ 0 \end{pmatrix} v_w \tag{18}$$

and if it is completely unknown then the best guess for $\hat{f}(x, t)$ is 0 and the value η can be calculated with

$$\eta > \|h\| \cdot \|(a_{11}, a_{21}, a_{31}, 0, 0)v_w^{max}\| \tag{19}$$

2.E

The two ailerons will have to be weighted differently, and this leads to a constrained optimization problem. The generic control input can be the original one modified to take into account the malfunction. Define $T = (1/2 \;\; 1/2)$ and write $\delta_A = T \begin{pmatrix} \delta_L \\ \delta_R \end{pmatrix} = Tu$, and the LS optimization problem is defined as

$$J = \min_f \{ f^T W f \} \tag{20}$$

$$\text{subject to} \quad \delta_A - Tu = 0 \tag{21}$$

The weight matrix W has a low value on the left aileron input, since it will be more expensive to use, and a high value on the right one, for example $W = \begin{pmatrix} 1 & 0 \\ 0 & 10 \end{pmatrix}$. The control input is then given by the weighted pseudoinverse

$$u = T^+ \delta_A = W^{-1} T^T (T W^{-1} T^T)^{-1} \delta_A \tag{22}$$

2.F

One can replace ψ in the model with χ, whose dynamic equation in a coordinated turn scenario is $\dot{\chi} = \frac{g}{V_a} \tan \phi$, with V_a the velocity of the UAV relative to the air.
Can use GPS to know ground speed, then inverse trigonometry to obtain the course angle.

2.G

Yes, it is, as the change in course/heading is driven by a change in roll.

2.H

No, it is not possible without a rudder, unless an underactuated control law is designed.

Problem 3

3.A

At equilibrium, weight = buoyancy, so $mg = \rho g V \Rightarrow m = 1025 \cdot (8.3/2)^2 \cdot \pi \cdot 6 = 332752$ kg.

3.B

Hydrostatic term / restoring force, corresponding to spring stiffness.
$Z_z = -\rho g A = -1025 \cdot 9.8 \cdot (8.3/2)^2 \cdot \pi = -543495$, with A the area of the water plane being displaced by movement in heave.

3.C

Calculate A_{33} and B_{33} at ω_n:

$$A_{33}(\omega_n) = 508210 \tag{23}$$

$$B_{33}(\omega_n) = 163082 \tag{24}$$

This because the dominant frequency in the heave motion is its natural frequency (the same applies to roll and pitch). The approximation at 0 frequency is valid only for surge, sway and yaw motion.

3.D

The viscous damping is the difference between total damping and potential damping, so

$$B_{33v} = B_{33}(\omega_n) - B_{33} = 163082 - 2 \cdot 0.16 \cdot 0.8 \cdot (m + A_{33}(\omega_n)) = 9146 \tag{25}$$

3.E

$$Z_{\dot{w}} = -A_{33}(\omega_n) = -508210 \tag{26}$$
$$Z_w = -B_{33} = -172229 \tag{27}$$

3.F

Taking the implicit equations for the natural frequency and evaluating it for $\omega = 1$ it results

$$1 = \sqrt{\frac{C_{33} + K_m}{m + A_{33}(1)}} \tag{28}$$

and, inverting,

$$K_m = m + m(1 + e^{-1}) - C_{33} = 244421 \tag{29}$$

3.G

The effect of the constant current is included in the dynamic equation as

$$(m - Z_{\dot{w}})\dot{w} - Z_w(w - w_c) - (Z_z - K_m)z = \tau_{wave} \tag{30}$$

The new equilibrium position is then

$$z^* = \frac{Z_w}{Z_z - K_m} = 0.21\text{m} \tag{31}$$

3.H

The frequency of the wave is the natural frequency of the structure, so there will be resonance and very large oscillations, a condition to avoid to prevent disasters.