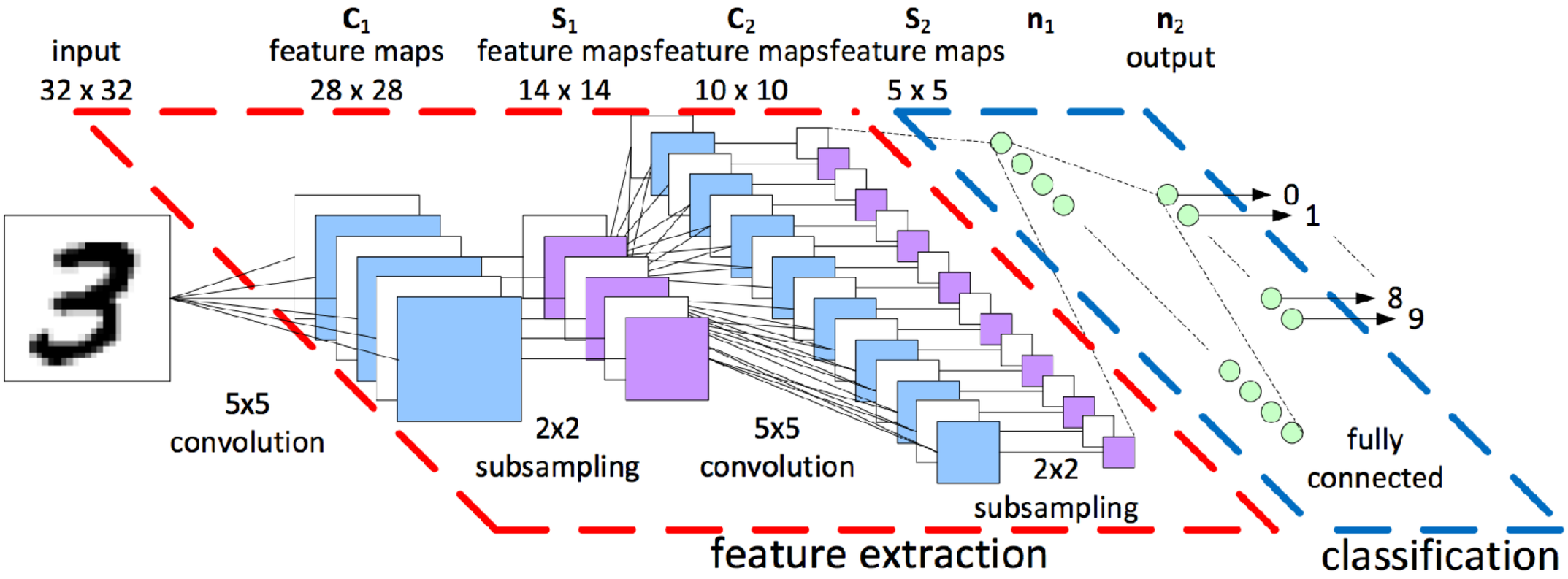
Part 2
Intro to DL: CNNs & ImageClass

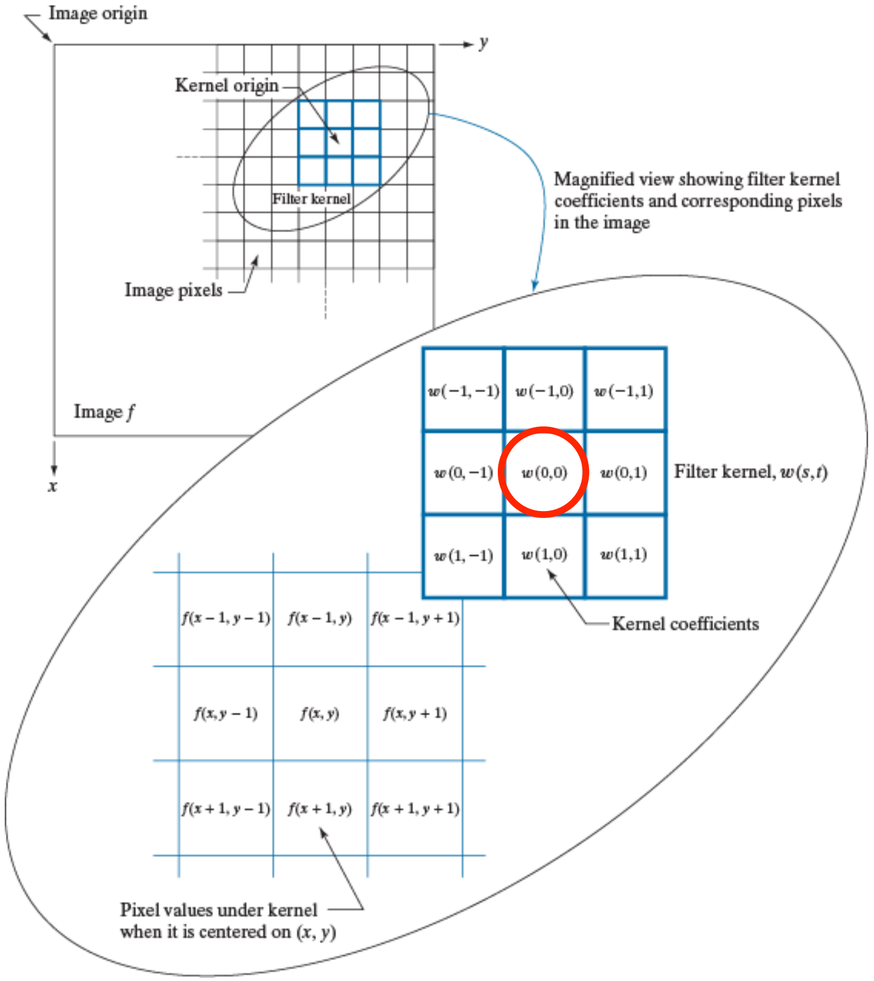
Frank Lindseth, IDI, NTNU

CNNs & ImageClass



Motivation: Base model for advanced CV

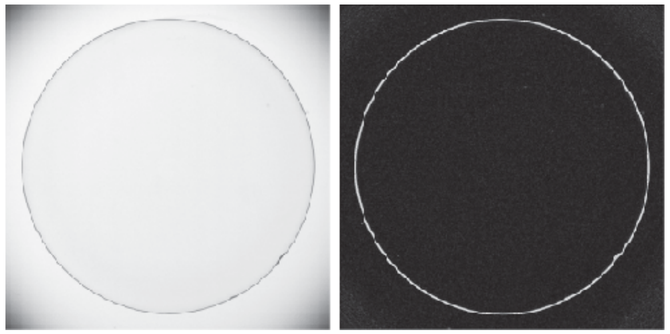
Motivation: Old school

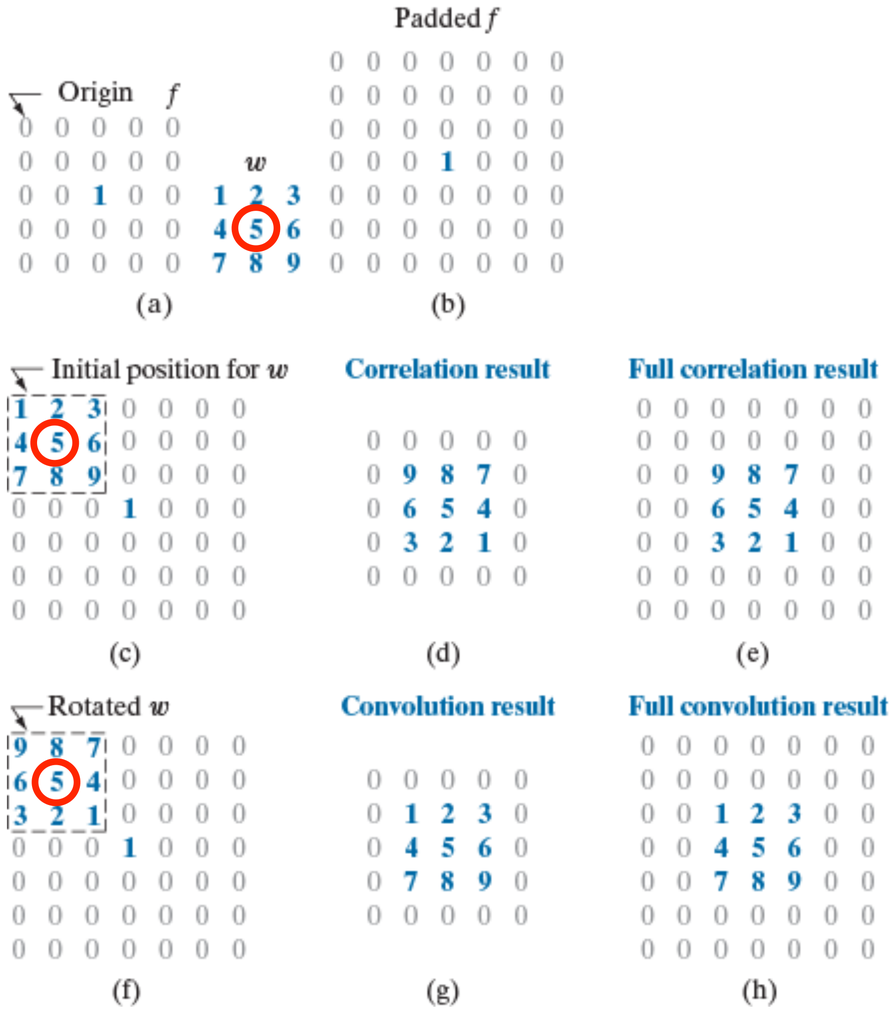



```
kernel = [1  0  -1]
```

−1	−2	−1
0	0	0
1	2	1

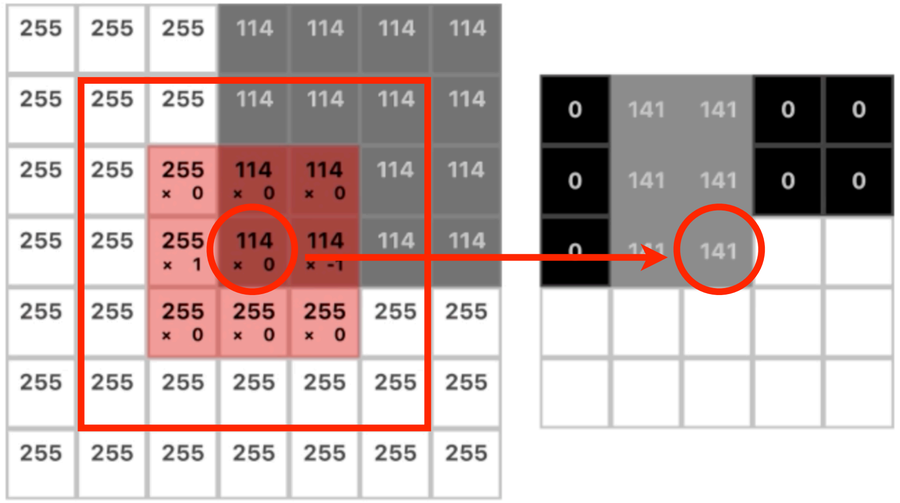
−1	0	1
−2	0	2
−1	0	1

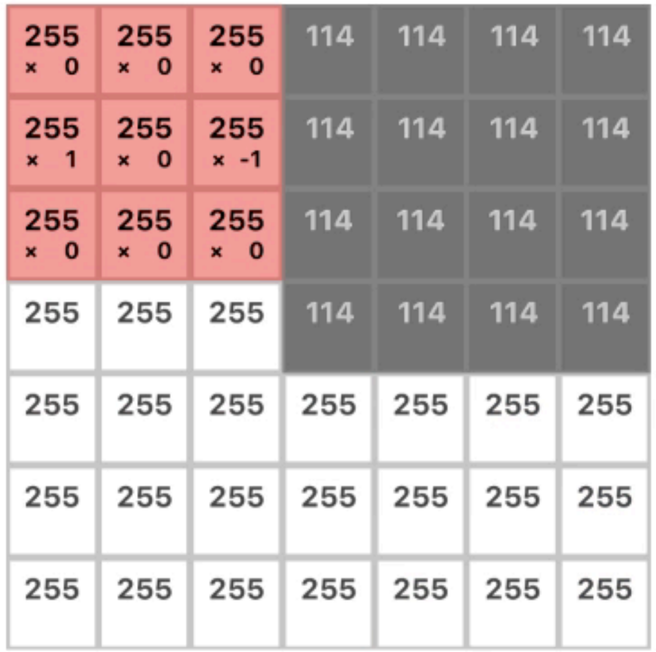


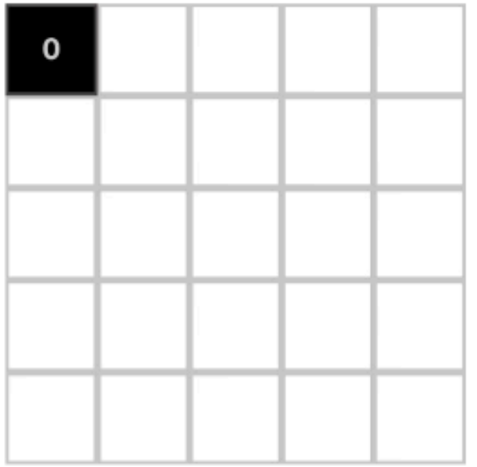


Feature **engineering**

Motivation: Old school





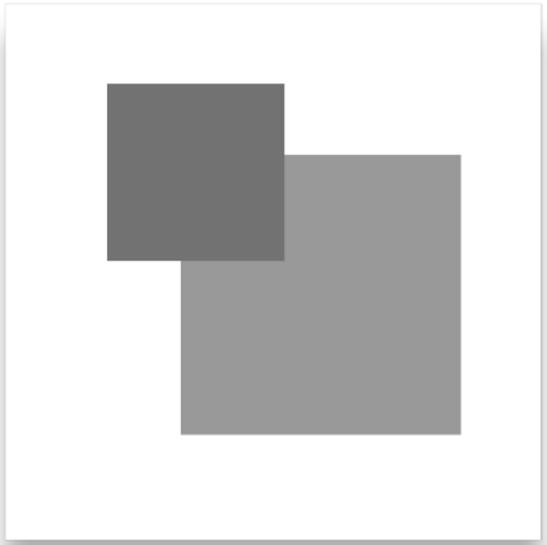


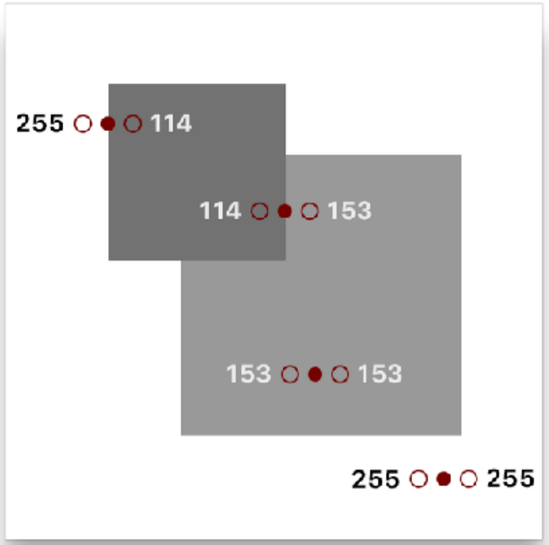
Motivation: Old school

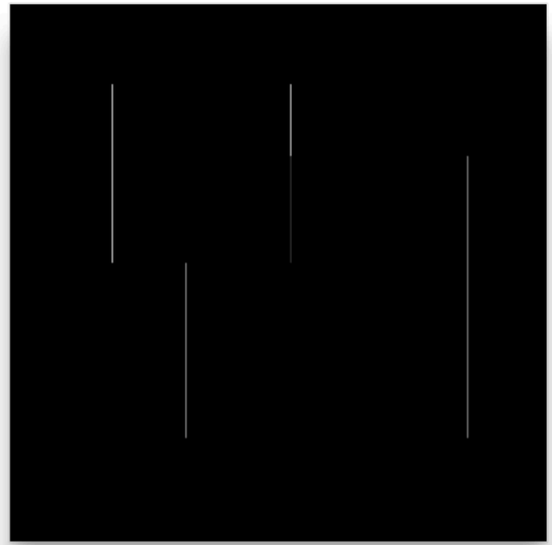
```
kernel = [1  0 -1]
```

```
kernel = [
  [0  0  0]
  [1  0 -1]
  [0  0  0]
]
```

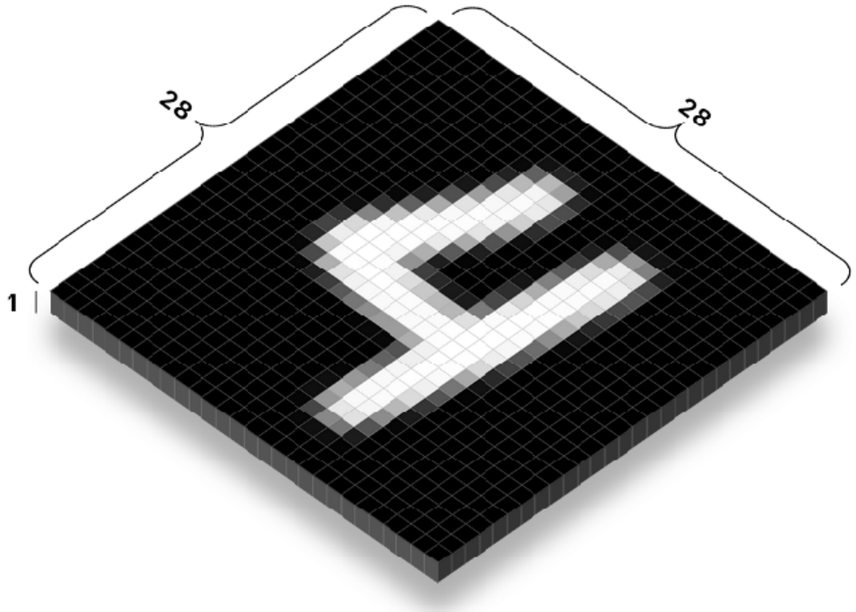
```
kernel = [
  [0  0  0]
  [1  0 -1]
  [0  0  0]
]
```





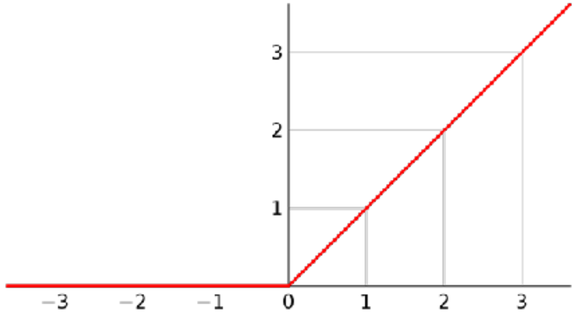


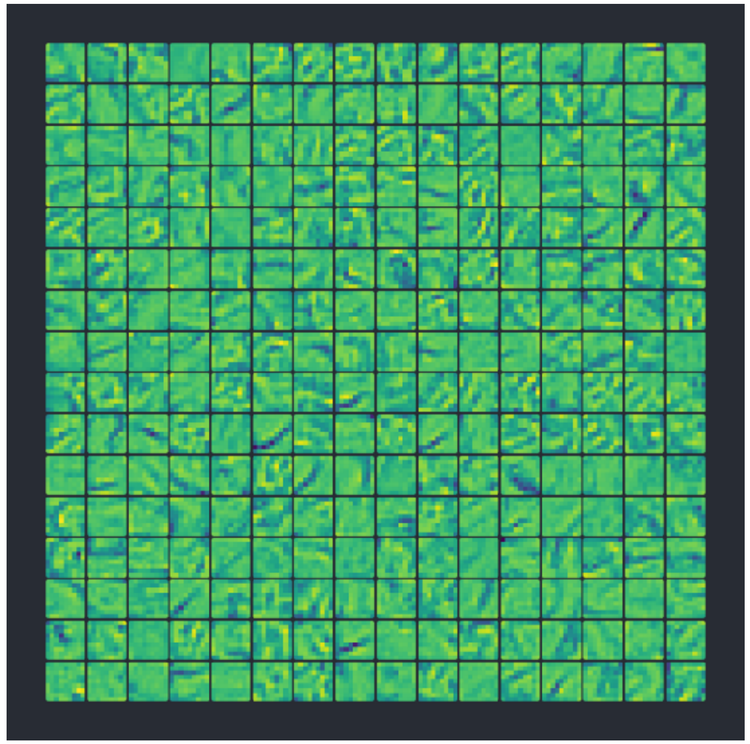
Motivation: New: Learned Kernels



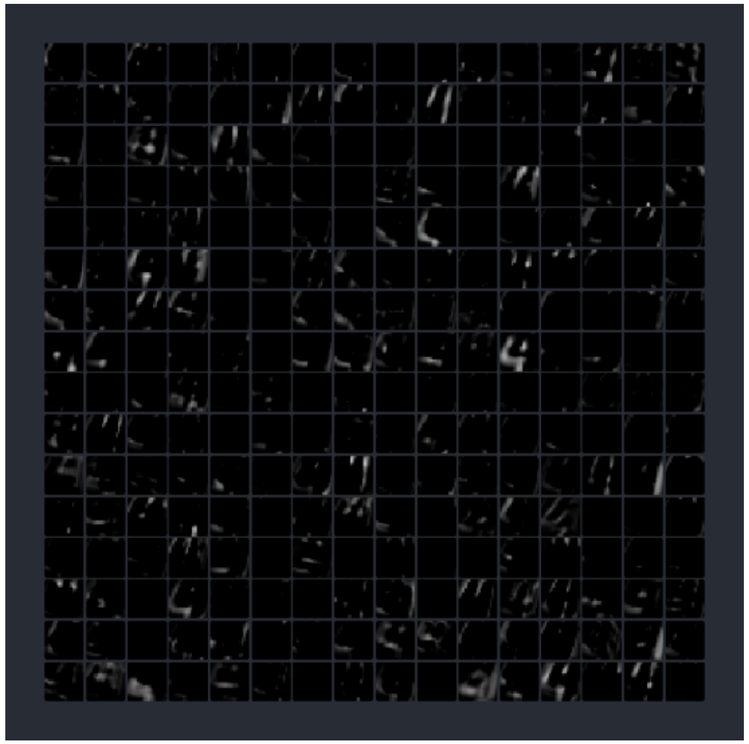
Feature **Learning**

```
kernel = [
  [ 0.02 -0.01  0.01 -0.05 -0.08 -0.14 -0.16 -0.22 -0.02]
  [ 0.01  0.02  0.03  0.02  0.00 -0.06 -0.14 -0.28  0.03]
  [ 0.03  0.01  0.02  0.01  0.03  0.01 -0.11 -0.22 -0.08]
  [ 0.03 -0.01 -0.02  0.01  0.04  0.07 -0.11 -0.24 -0.05]
  [-0.01 -0.02 -0.02  0.01  0.06  0.12 -0.13 -0.31  0.04]
  [-0.05 -0.02  0.00  0.05  0.08  0.14 -0.17 -0.29  0.08]
  [-0.06  0.02  0.00  0.07  0.07  0.04 -0.18 -0.10  0.05]
  [-0.06  0.01  0.04  0.05  0.03 -0.01 -0.10 -0.07  0.00]
  [-0.04  0.00  0.04  0.05  0.02 -0.04 -0.02 -0.05  0.04]
]
```



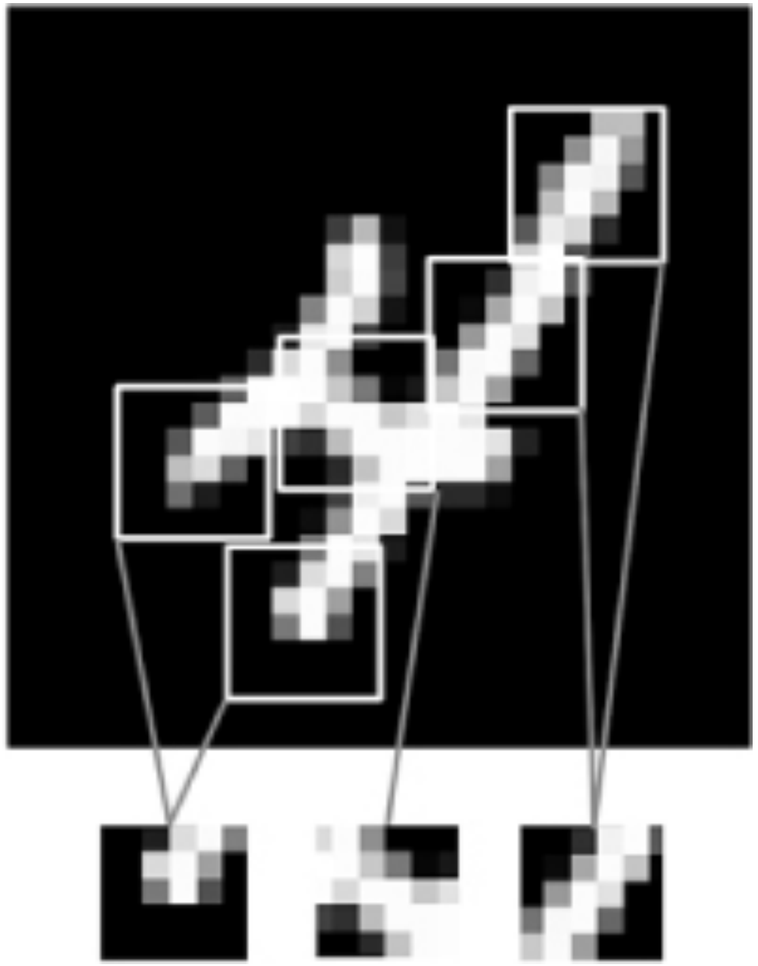


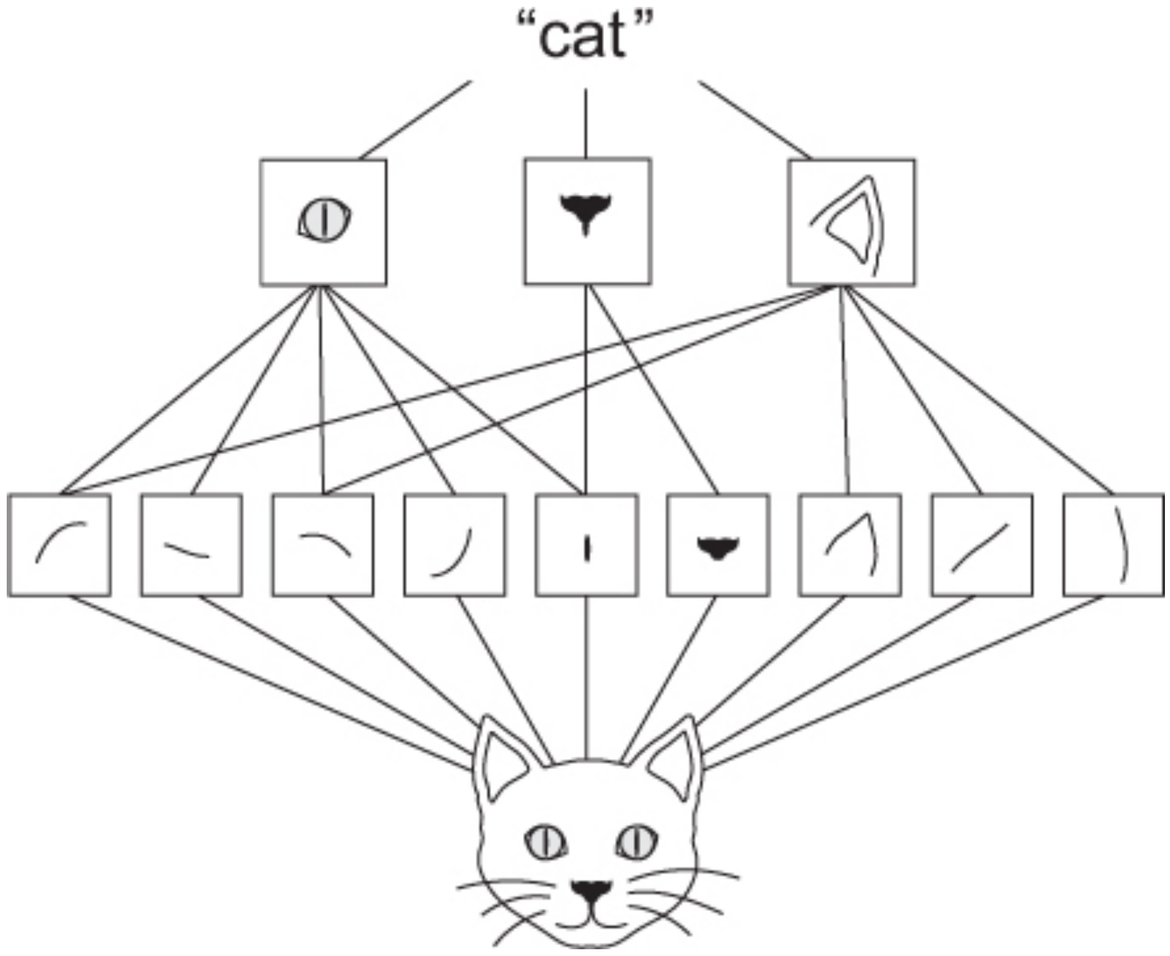
Kernels / filters (after training)



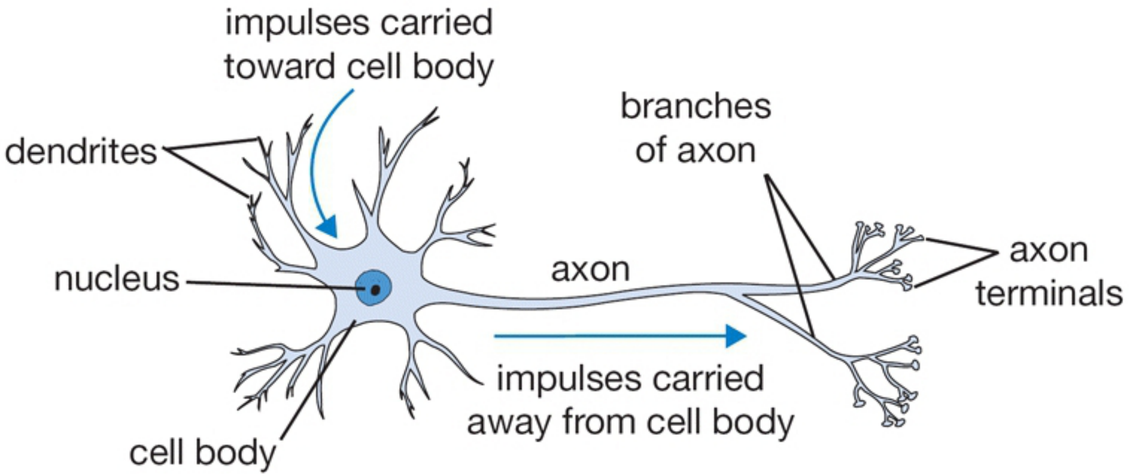
Activations (Conv + ReLU)

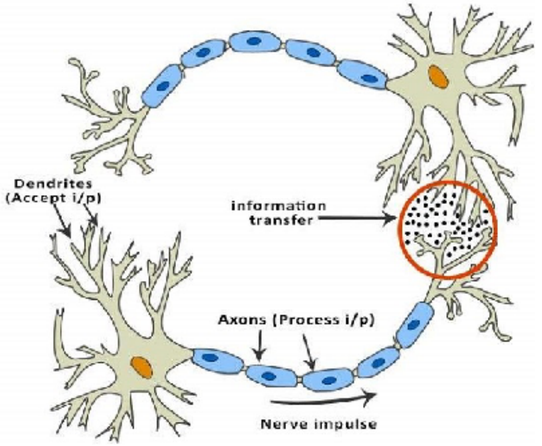
Motivation: Hierarchical



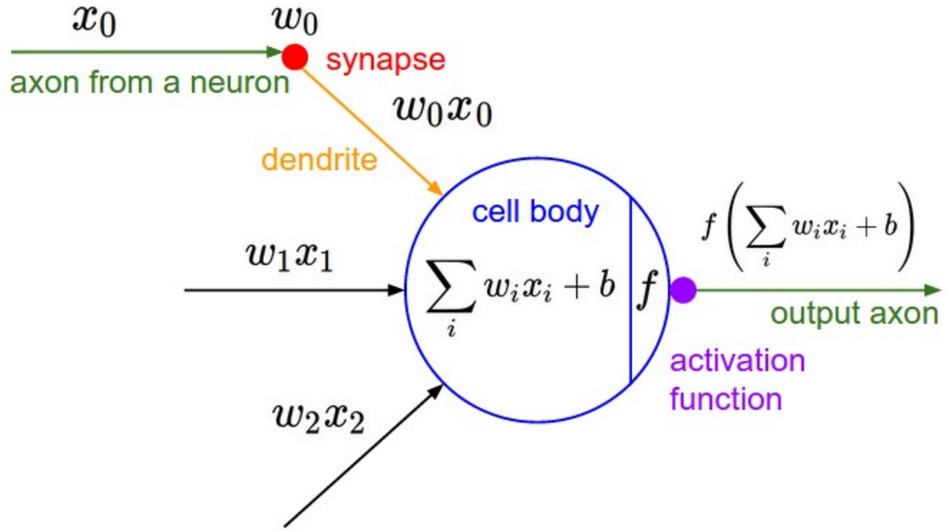


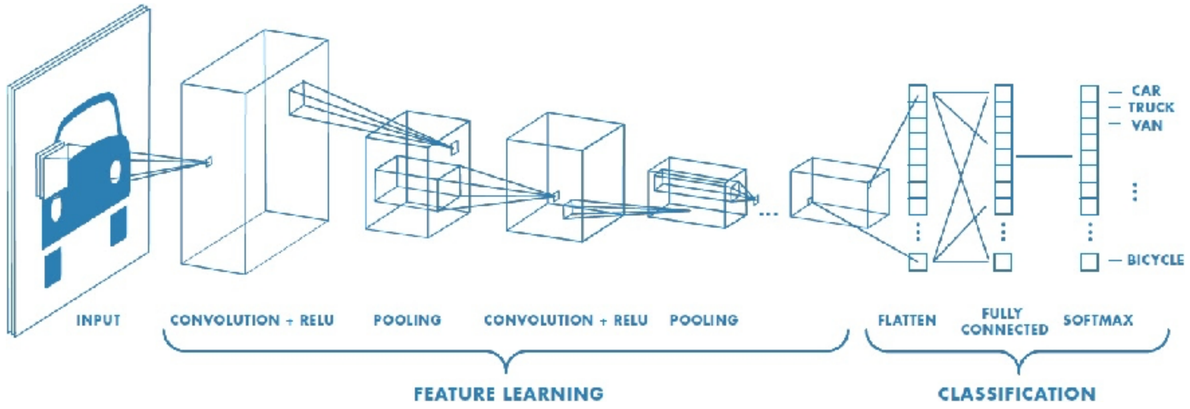
Motivation: Biological vs. Artificial





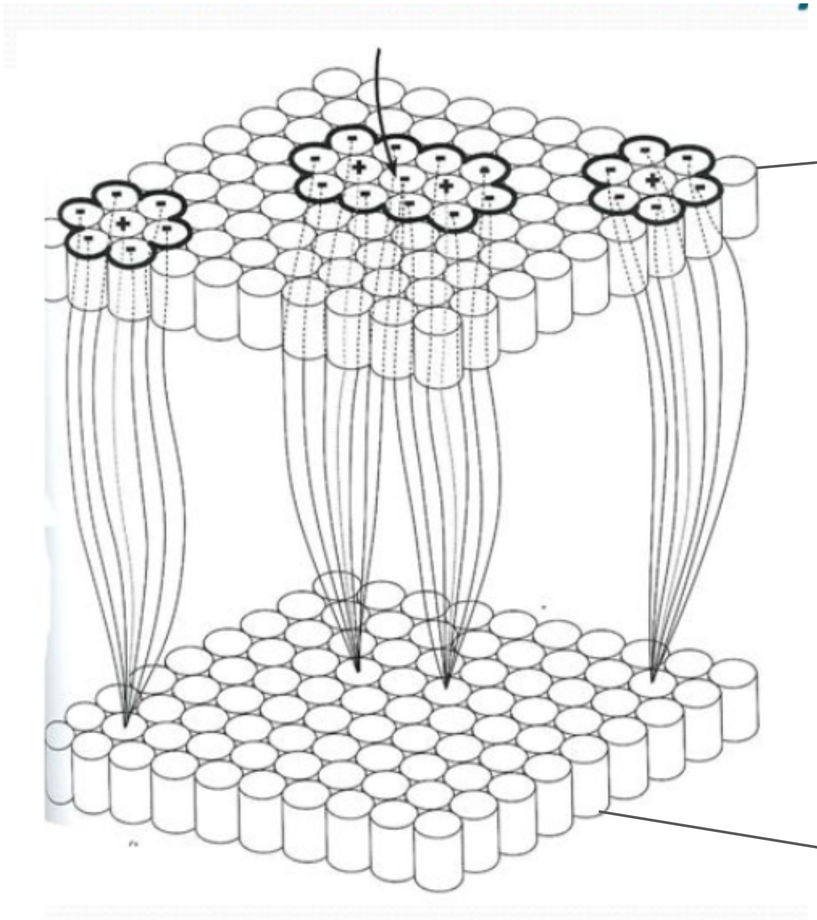
Human Visual system



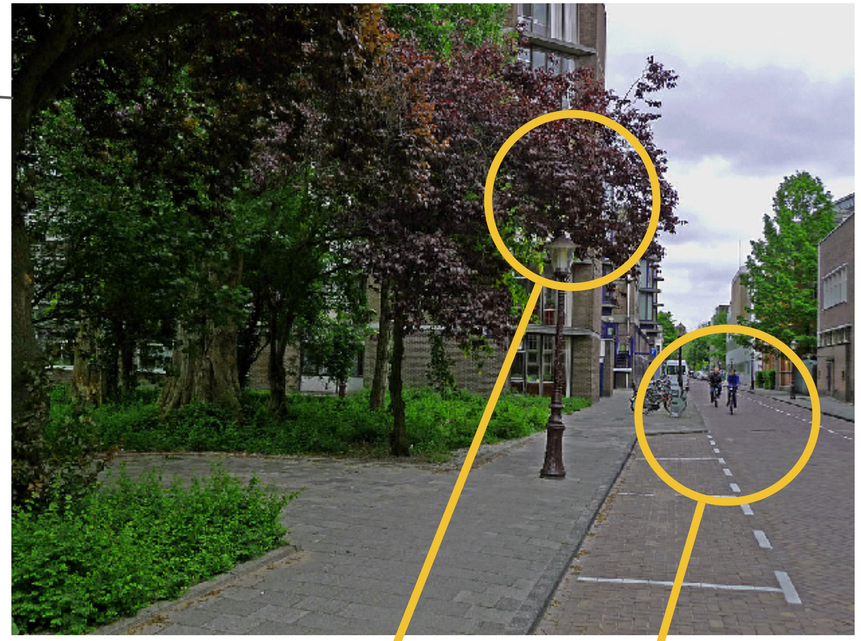


Motivation: Biological Receptive Fields

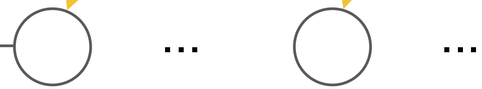
- Specific neurons in the retina cares about only a small part of the visual field



Visual field

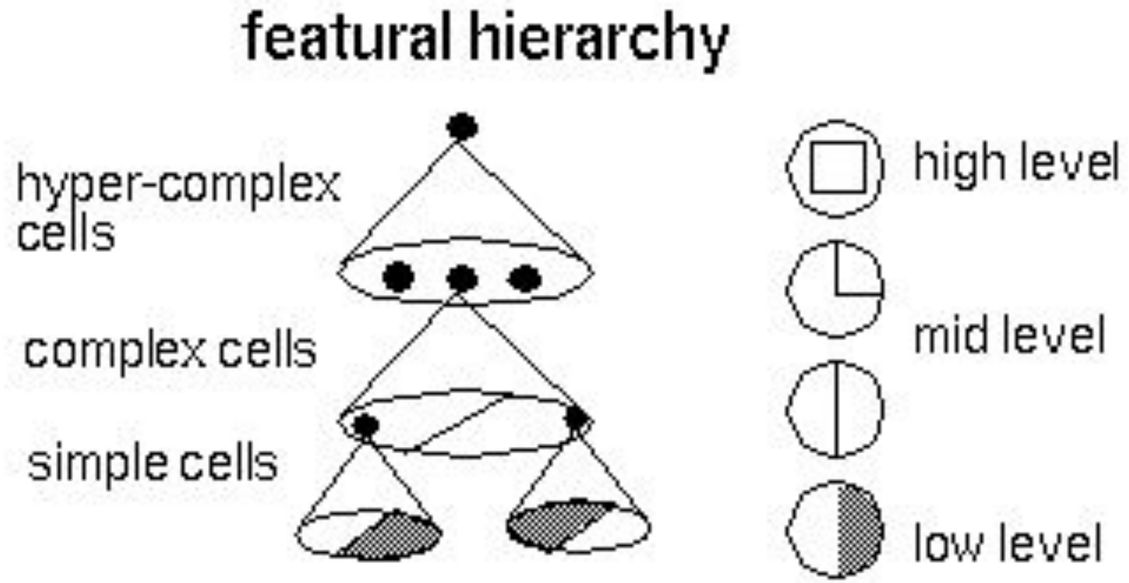


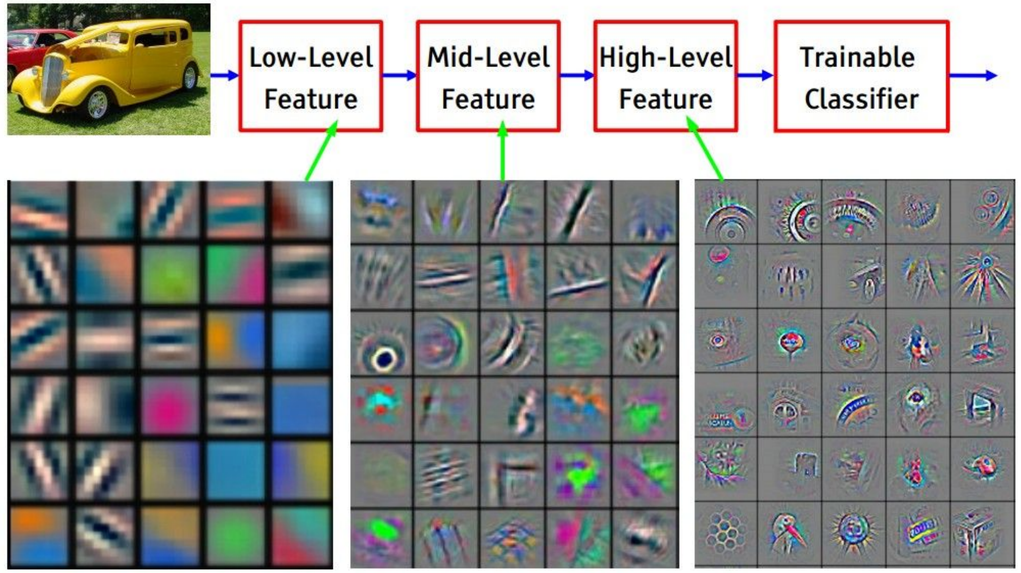
Neurons in retina



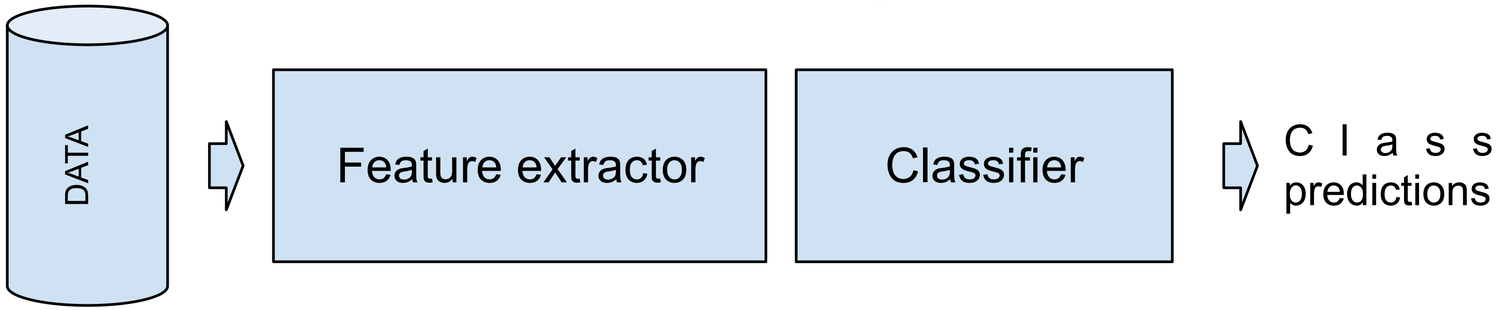
Motivation: Biological Feature Hierarchies

- Neurons later in the processing “pipeline” are able to extract more complex patterns

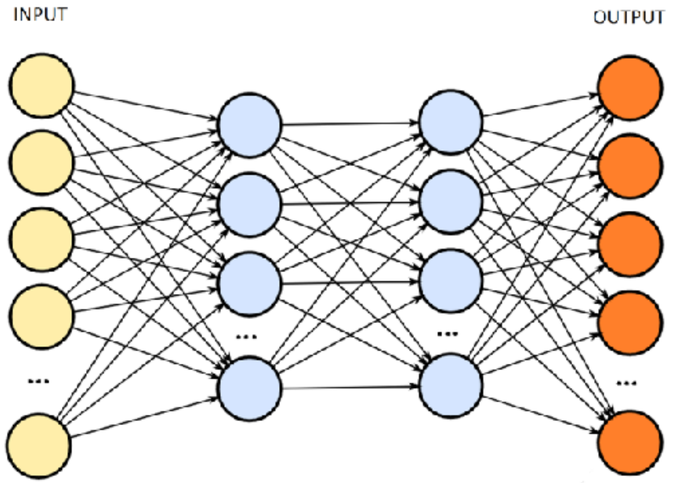




Motivation: image properties



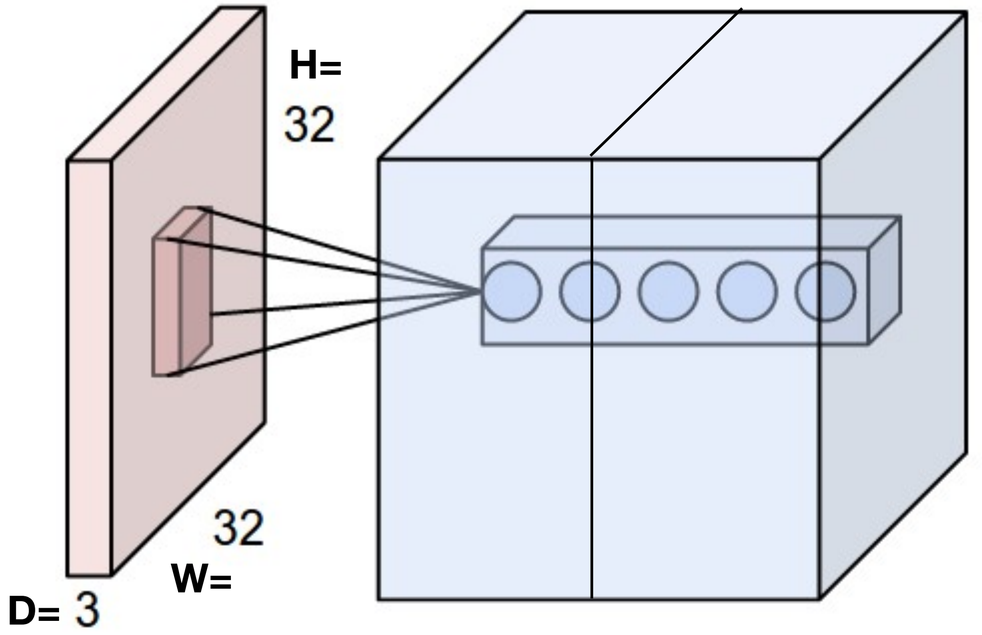
- Neural networks can be powerful **feature extractors** and classifiers
- Feature extraction is important for **images classification.**
- We observe that image classification have some “**properties**”
  - **Spatial structure** - the pixels position in relation to other pixels matters
  - **Translation invariant** - the position of a cat does not change the fact that it is a cat
  - **Large** - images are generally quite large
  - **Rich** - there are a lot of information in images
- Some of these properties makes FC layers **inadequate**
  - Expensive: Too many parameters
  - Do not consider spatial structures
- These properties guides the **design** of CNNs



Motivation: image properties

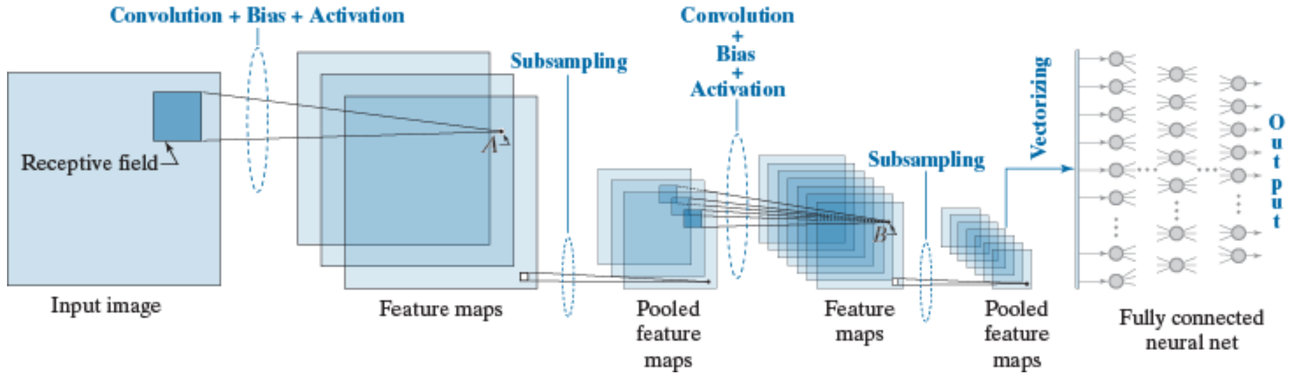
- **Spatial structure**: Work on image data directly and produce maps of where features are in a image
  - *CNNs*: Work on volumes
- **Translation invariant**: Look for the same features everywhere
  - *CNNs*: Use the same filter that looks for the same feature at all spatial positions
- **Large:** Exploit the translation invariance to reduce the number of parameters
  - *CNNs*: The filters looking for the same features naturally share weights, which reduces the parameters needed
- **Rich:**
  - Build up **higher** level features from low level features
    - *CNNs*: Multiple layers
  - Look for **more** than one type of feature in each layers
    - *CNNs*: Multiple filters per layer

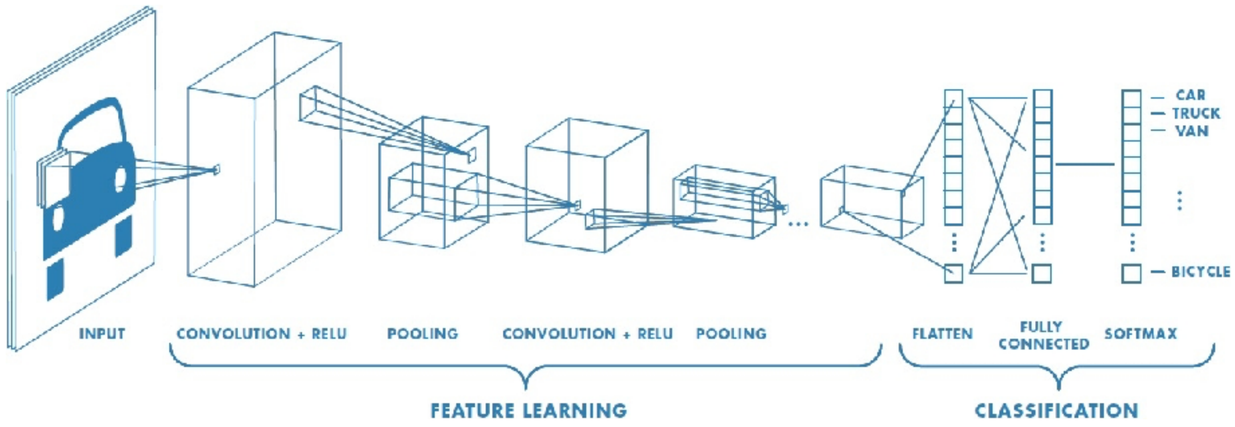
Input tensor vs. output tensor



CNNs for Image Classification

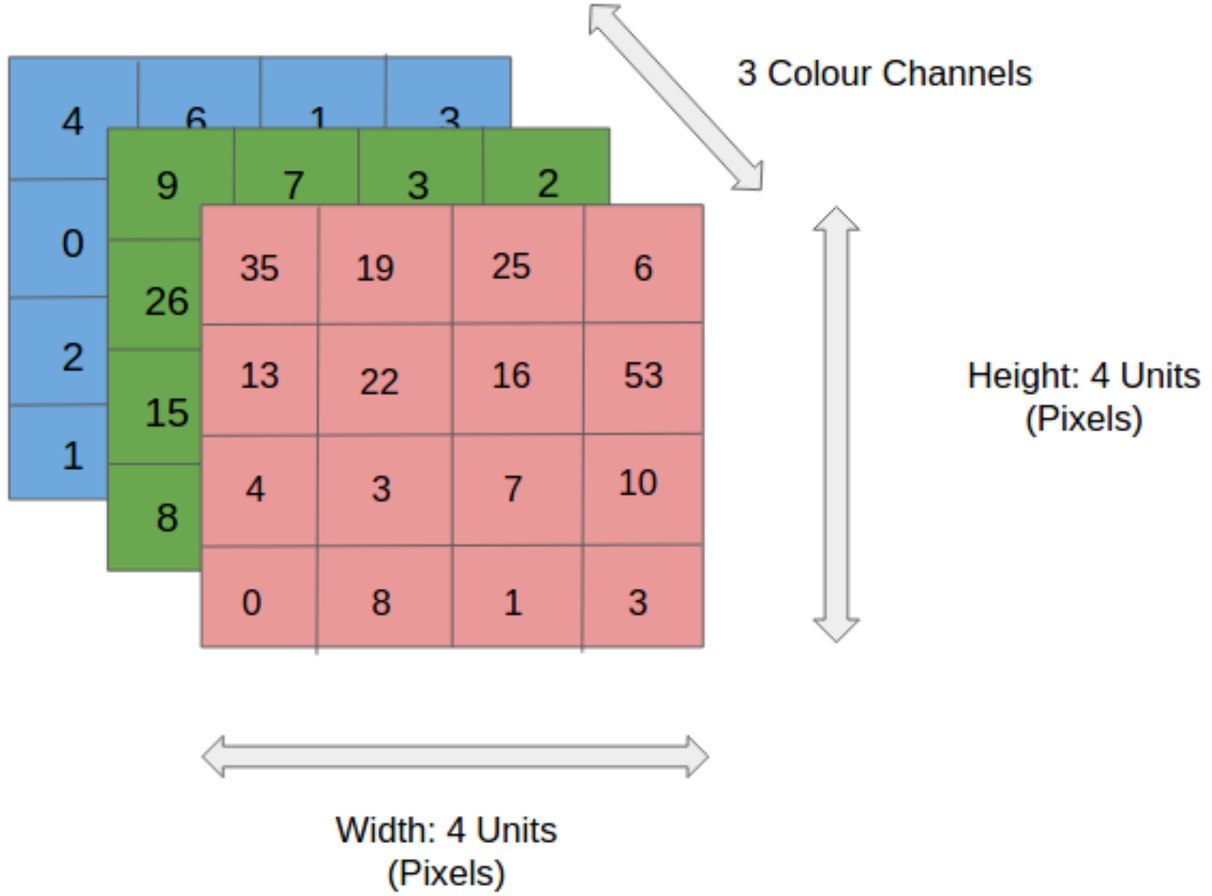
- Feature Learning: Convolution and Pooling
  - Convolution:
    - Receptive field (weighted sum)
    - + bias & activation/ReLU
  - Pooling
    - Max-pooling (average pooling)
- Classification: FCNN





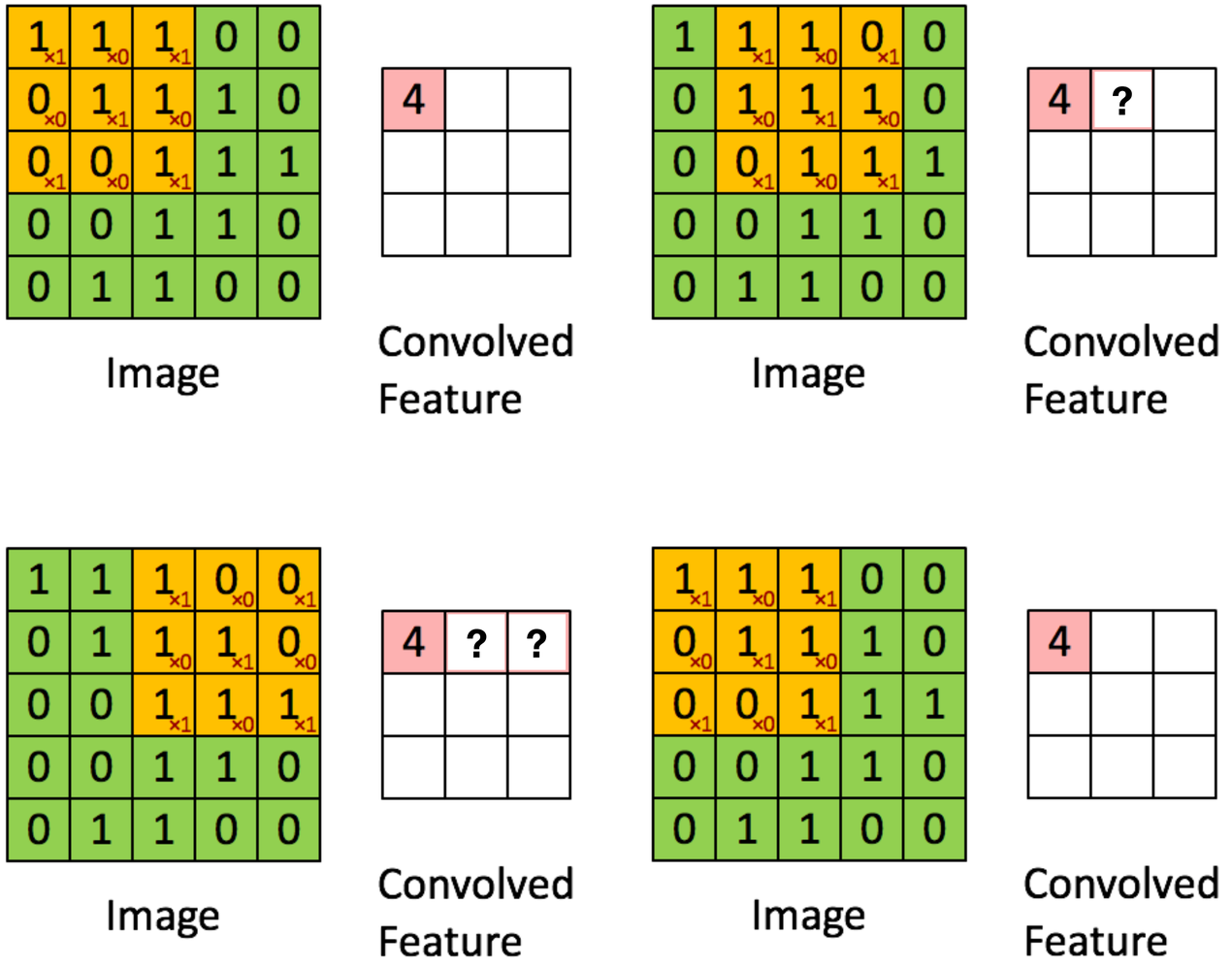
Input (images)

- Image: matrix of pixels (8 bit, 256 values)
- Color images (RGB): 3 channels = dept, input is 3D, i.e. 3 matrices
- Input tensor (3D)



Convolution

- Kernel / filter: smaller-sized matrix
- Kernels are convolved with input volume to obtain feature maps (or activated' regions, i.e. regions where features specific to the kernel have been detected in the input).
- Kernel change with each learning iteration, i.e. network is learning to identify which regions are of significance for extracting features from the data
- Kernel is slided by a certain amount called the '**stride**' over the image, dot produced computed at each point
- Receptive field (e.g. size 5×5 units) extend to the entire depth of the input (5x5x3 for RGB)

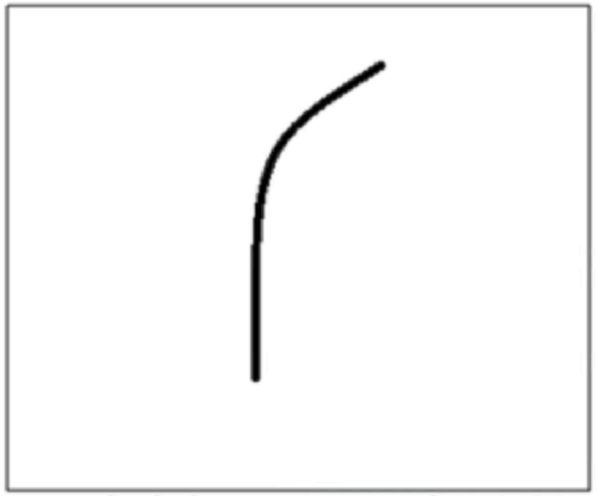


Convolution (2)

- Right: Kernel/filter: Curve detection filter
- Left: pixel representation of filter

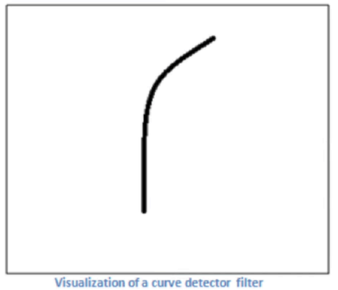
0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

Pixel representation of filter

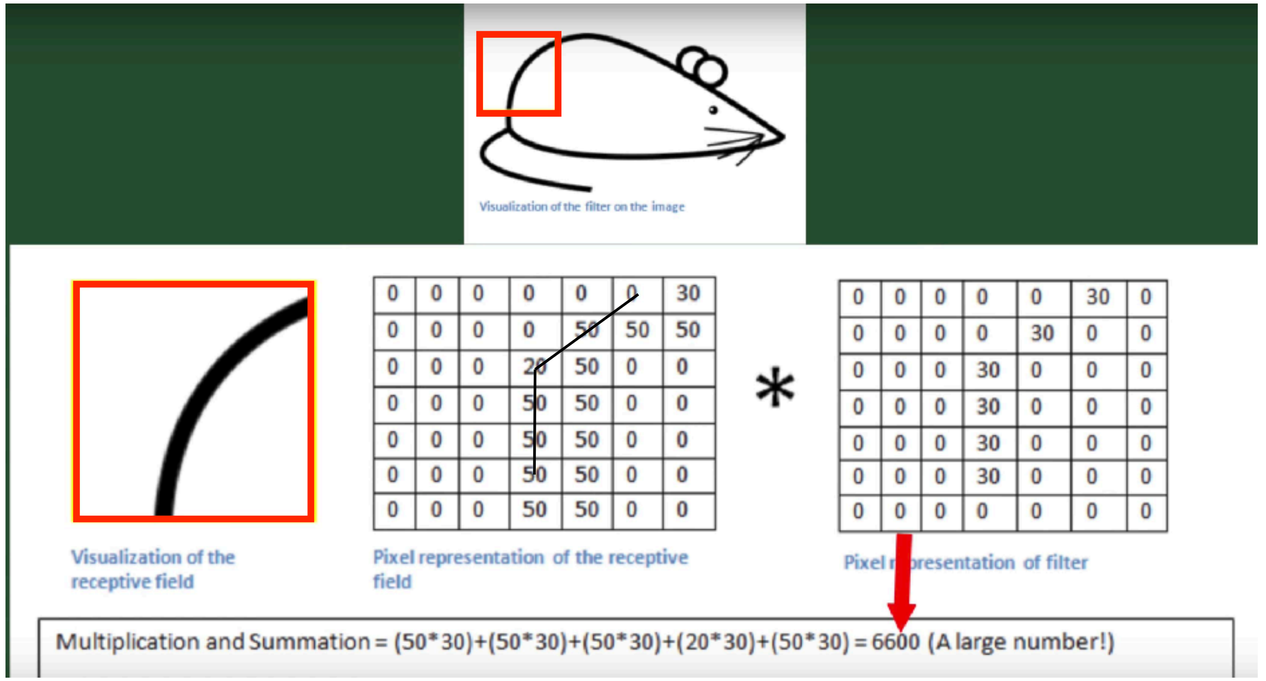


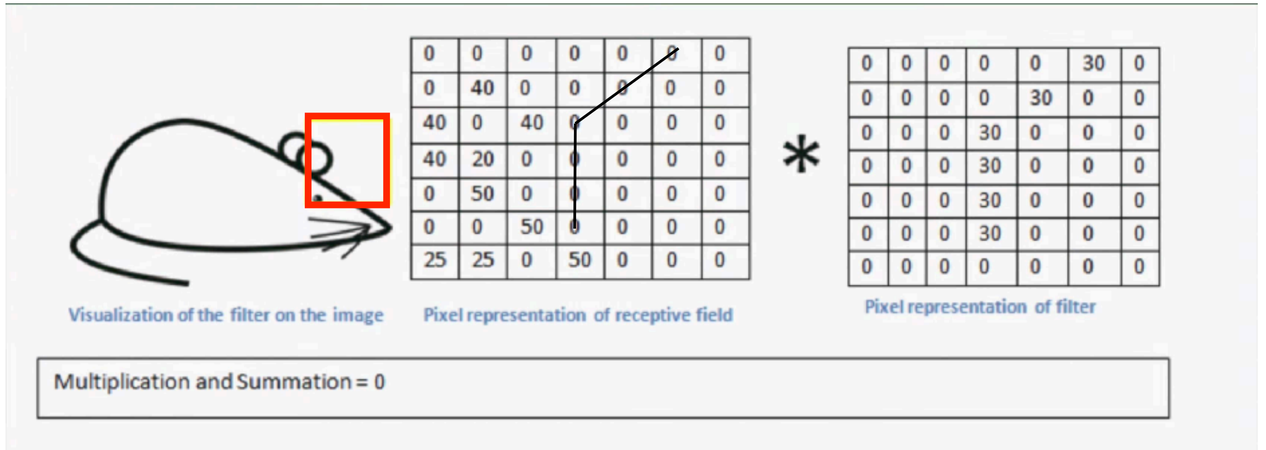
Visualization of a curve detector filter

Convolution (3)


Visualization of a curve detector filter

- Filter / kernel to detect a curve and its pixel representation (previous slide)
- Receptive field over mouse-picture in tow places and their pixel representations
- If image patch under receptive field **is** similar to kernel we get a **high** response (multiplication and summation)
- If image patch under receptive field **is not** similar to kernel we get a **low** response (multiplication and summation)





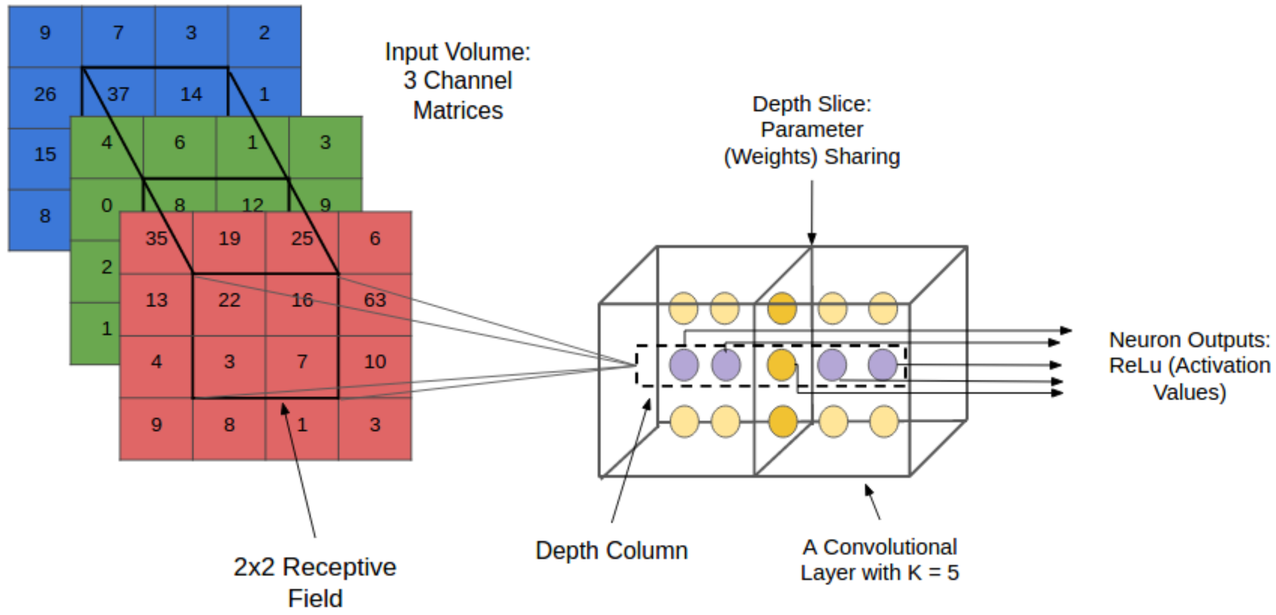
Convolution (4)

- Convolution extend to the entire depth of the input. Depth of RGB image is?
- High/Low respons?
- Dept slice?

  Respons of same filter at different places

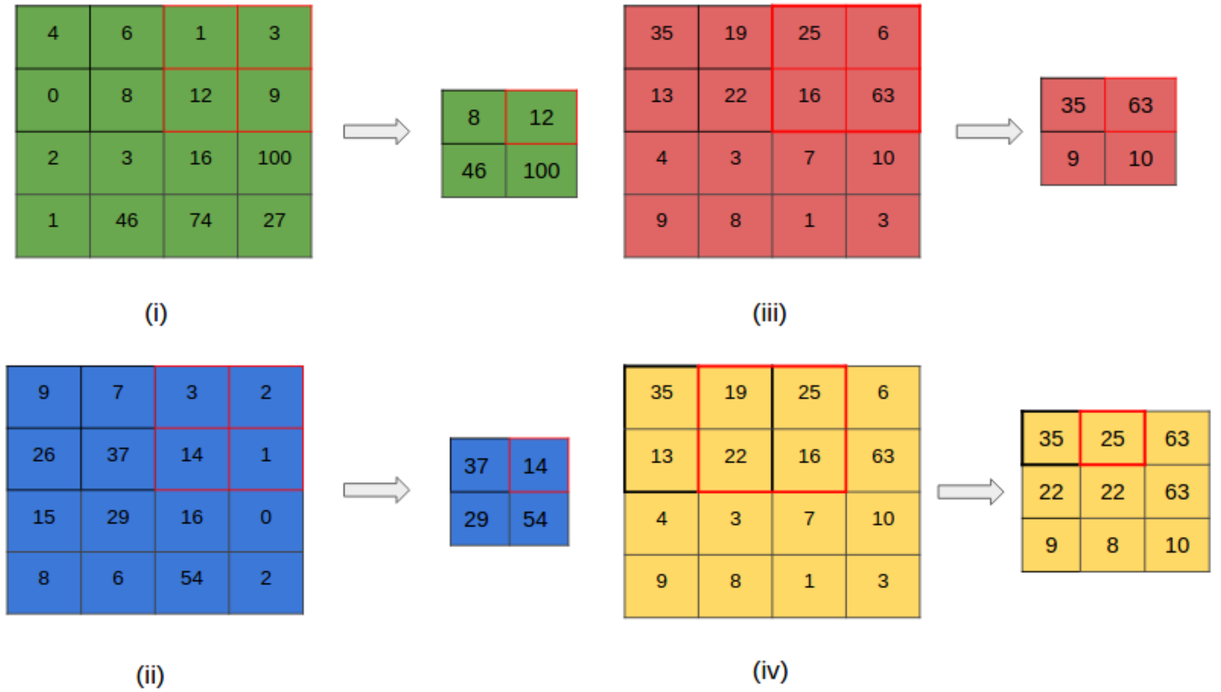
- Dept column?

  Respons of different filters at same place



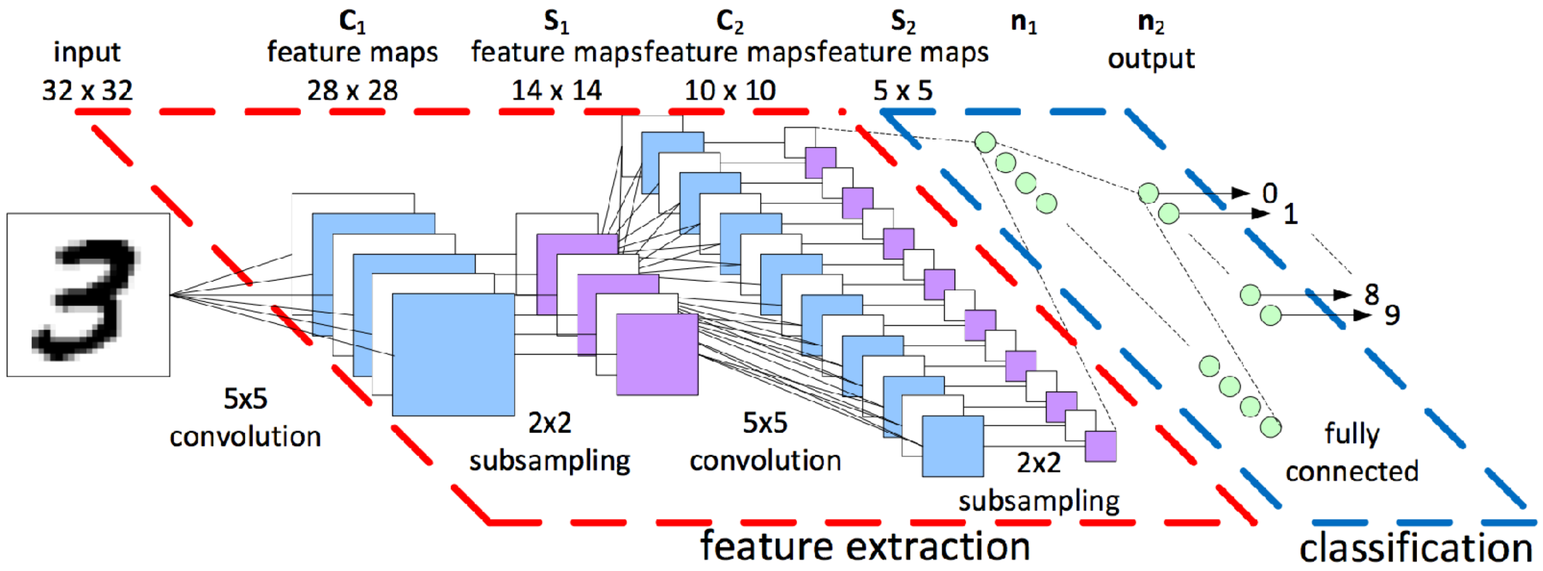
Pooling

- Pooling reduces the spatial dimension (WxH) of input volume but does not affect the depth (D).
- Max-pooling: maximum of values in window / receptive field (can also take the average)
- Usually: Receptive field (F): 2x2 and Stride (S): 2
- Also known as downsampling



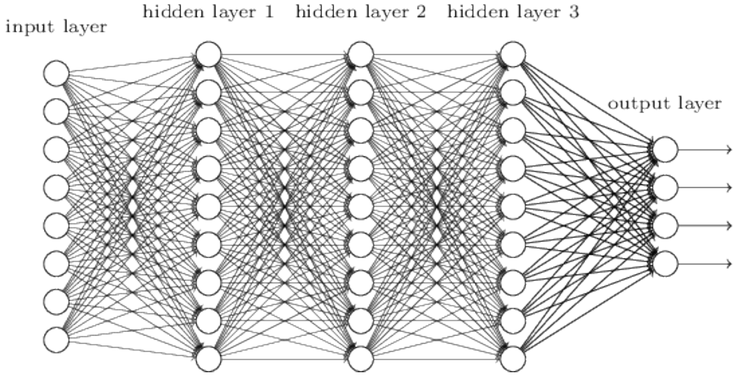
CNNs & ImClass

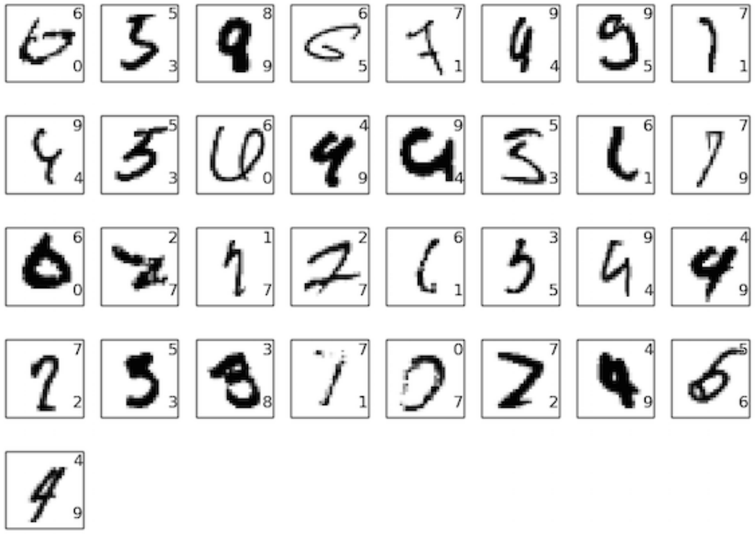
- **CNNs** (Spatial structure):
  - convolutional layer
    - local receptive fields
    - shared weights
  - pooling layer
- **Image Classification:**
  - Feature extraction / learning
  - Classification

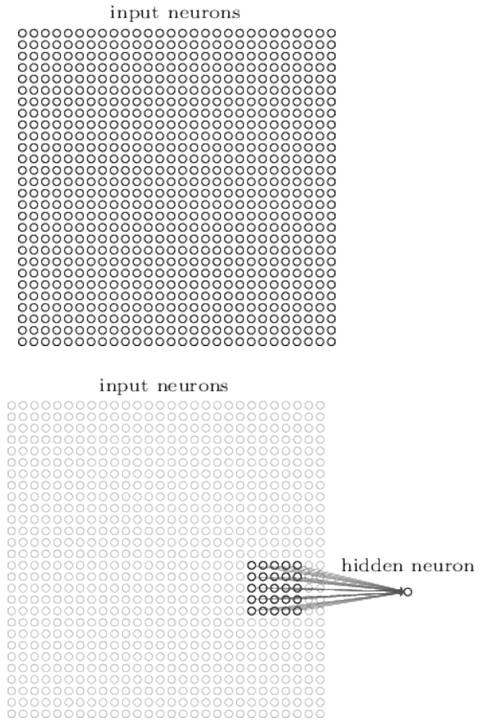


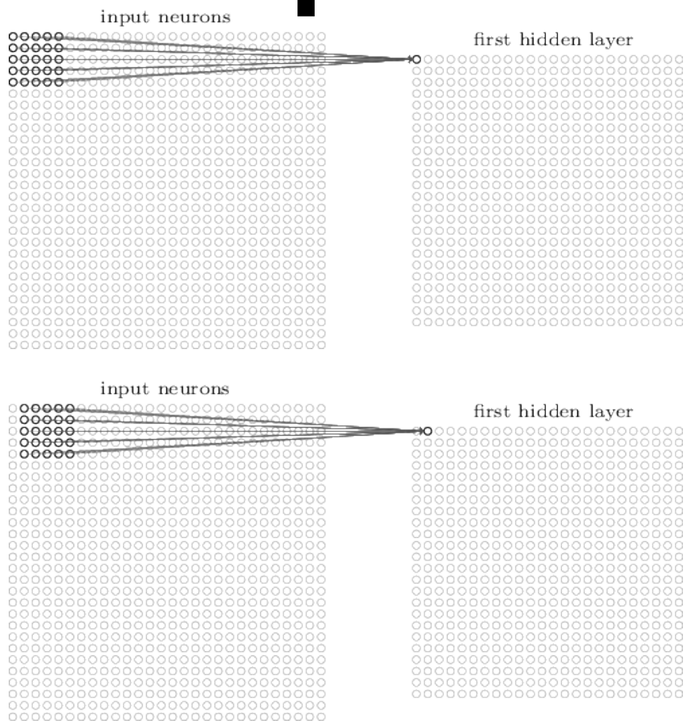
#filters ?

CNNs: Basic concepts







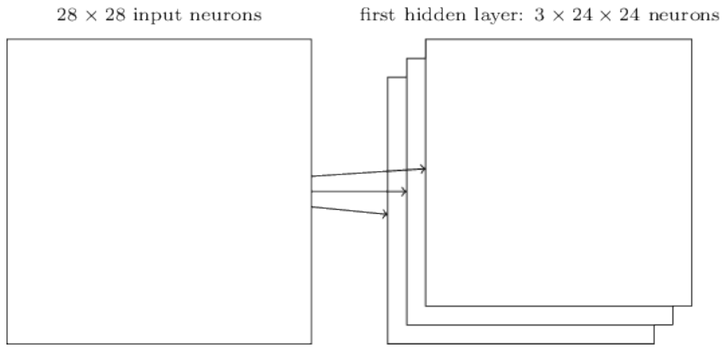


$$f\left(b + \sum_{l=0}^{4}\sum_{m=0}^{4} w_{l,m} a_{j+l,k+m}\right)$$

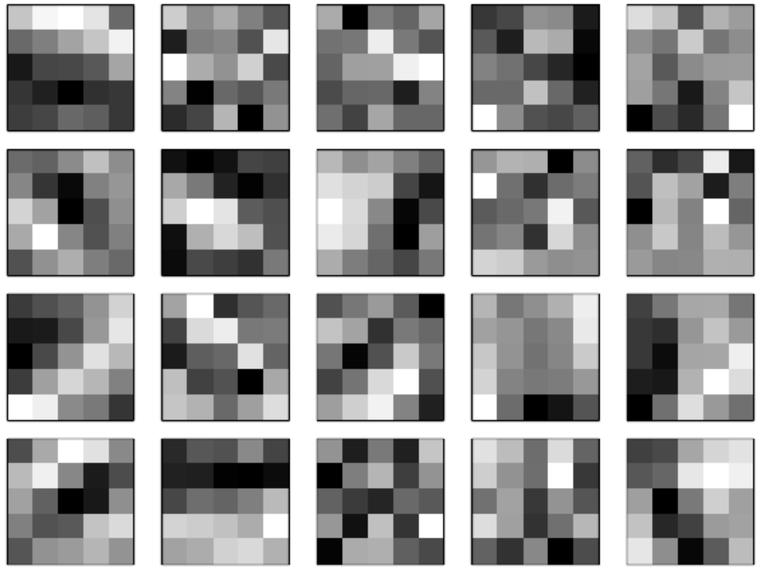
$$a^1 = f(b + w * a^0)$$

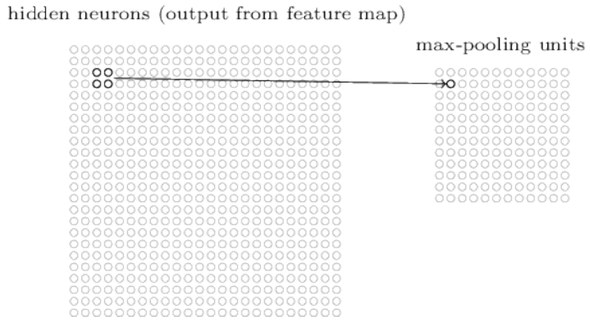
- **CNNs** (Spatial structure)
  - local receptive fields
  - shared weights
  - pooling

CNNs: Basic concepts (2)

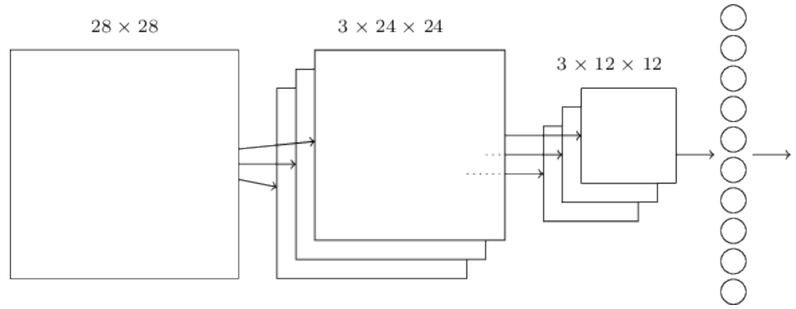


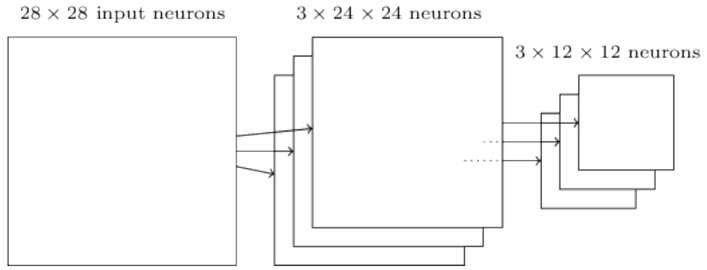
Filter size?

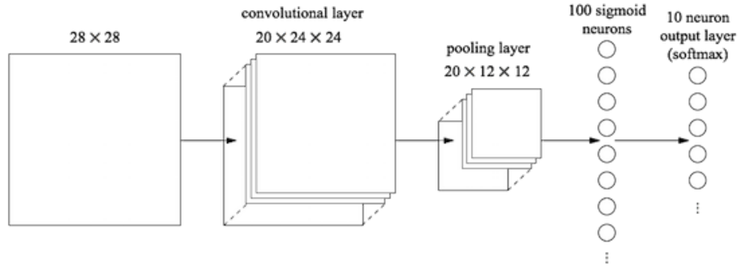




Filter size?

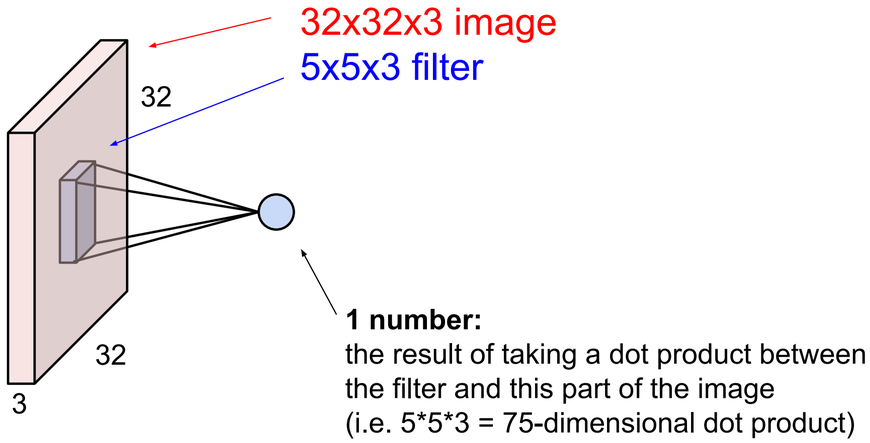


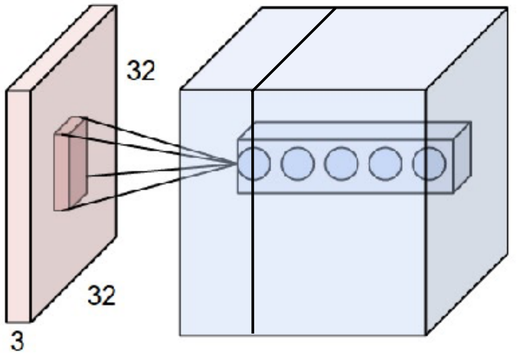


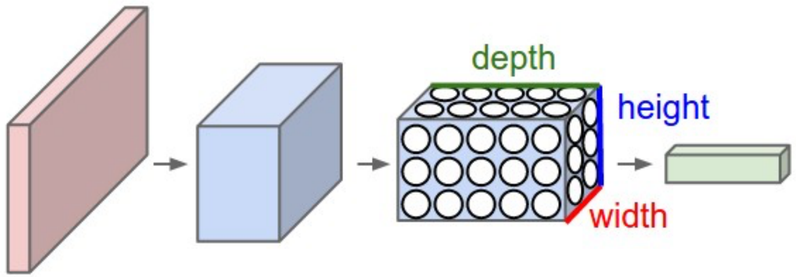


Convolution

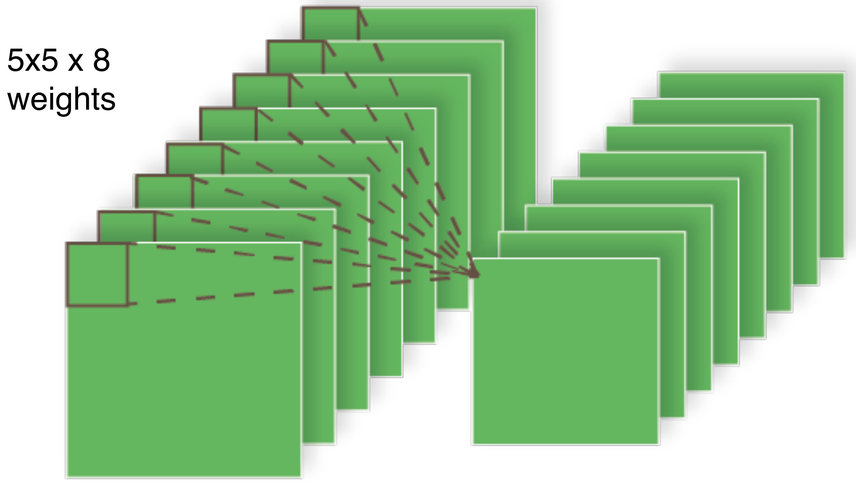
- Filter kernels
  - Small spatial size
  - Same depth
- The filter convolves over each location
- One set of parameters per filter
  - Looks for the same patterns over all spatial locations
- A form of dot product
- Result called **feature maps** (result maps)







Input channels (RGB, features, …) are combined



All filters have #w = F x F x D_in weights (+ 1 bias)
Total number of weights in one layer is #w x K

Convolution: Filters

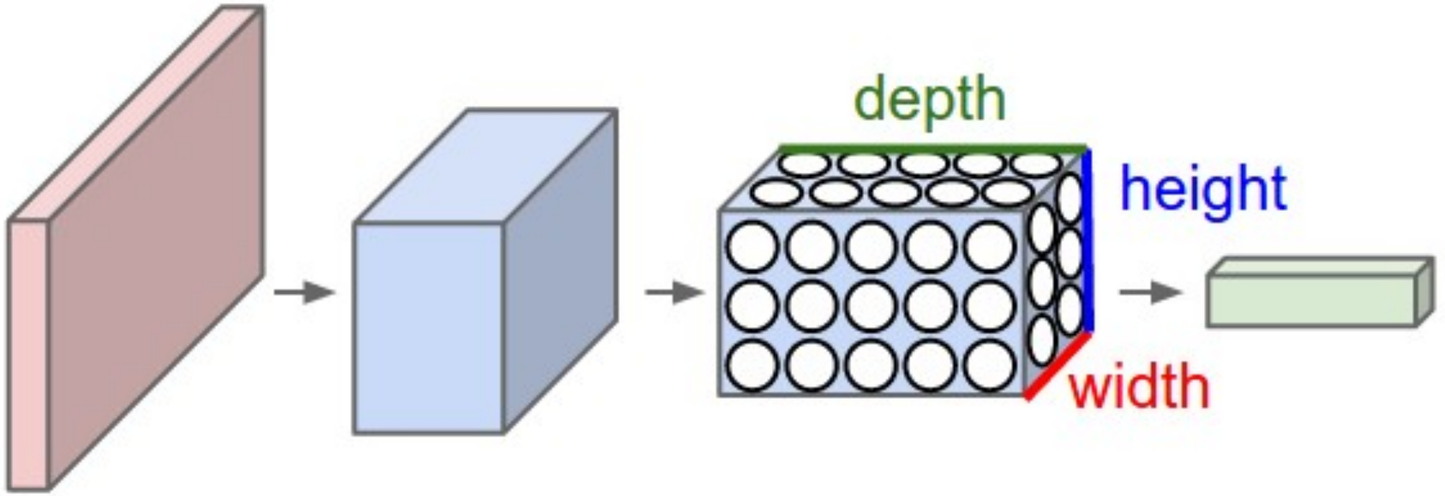
All filters have $F \times F \times D_{in} + 1$ weights
So the total number of parameters in one layer is $F \times F \times D_{in} \times K$

$$W_{out} = \frac{W_{in} - F + 2P}{S} + 1$$

$$H_{out} = \frac{H_{in} - F + 2P}{S} + 1$$

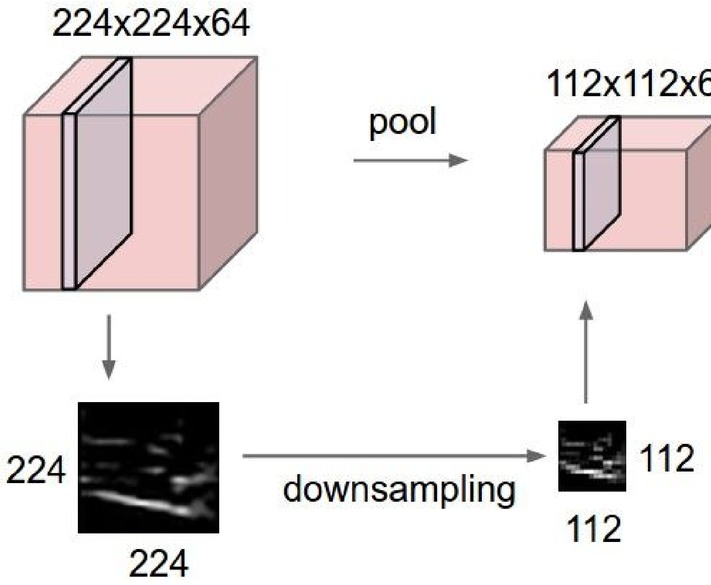
$$D_{out} = K$$

K	number of filters
F	spatial size of filter
S	stride
P	amount of padding
W	width of volume
H	height of volume
D	depth of volume



Pooling

- Downsampling
- Reduces spatial size
  - Aids faster computation
  - Regularization effect



1	1	2	4
5	**6**	7	**8**
3	2	1	0
1	2	3	**4**

2x2 max pooling

6	8
3	4

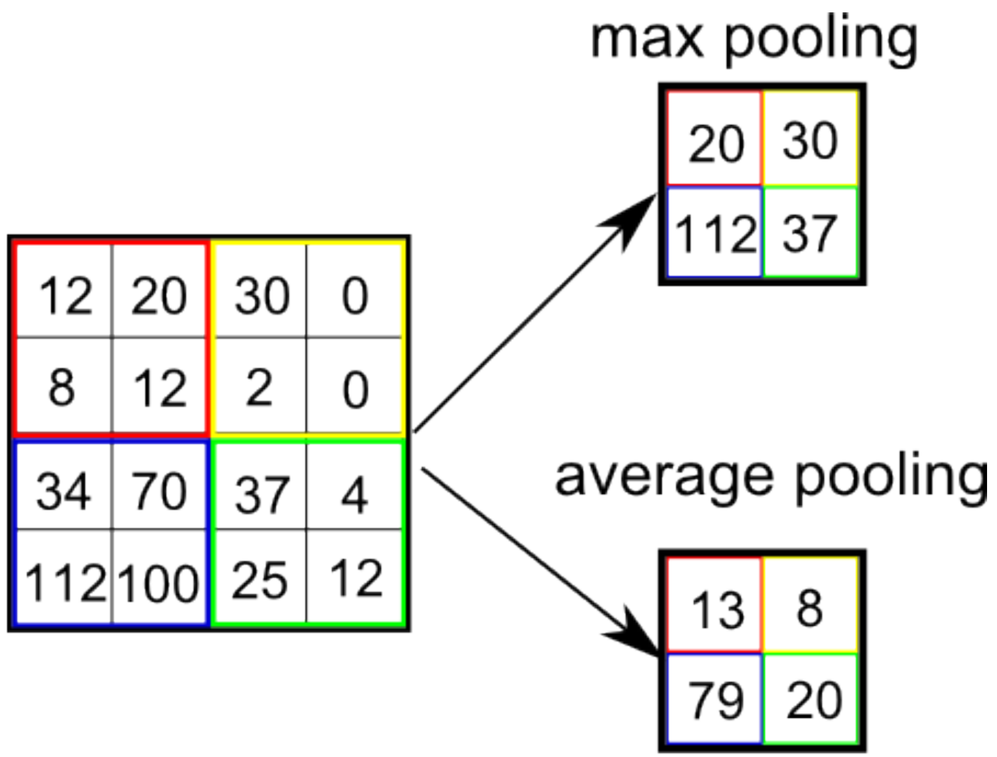
Pooling

The pooling layers is a parameter less function
There are different kinds of pooling max, avg, L2-norm.
Max-pooling have been show to work good in practice.

$$W_{out} = \frac{W_{in} - F}{S} + 1$$

$$H_{out} = \frac{H_{in} - F}{S} + 1$$

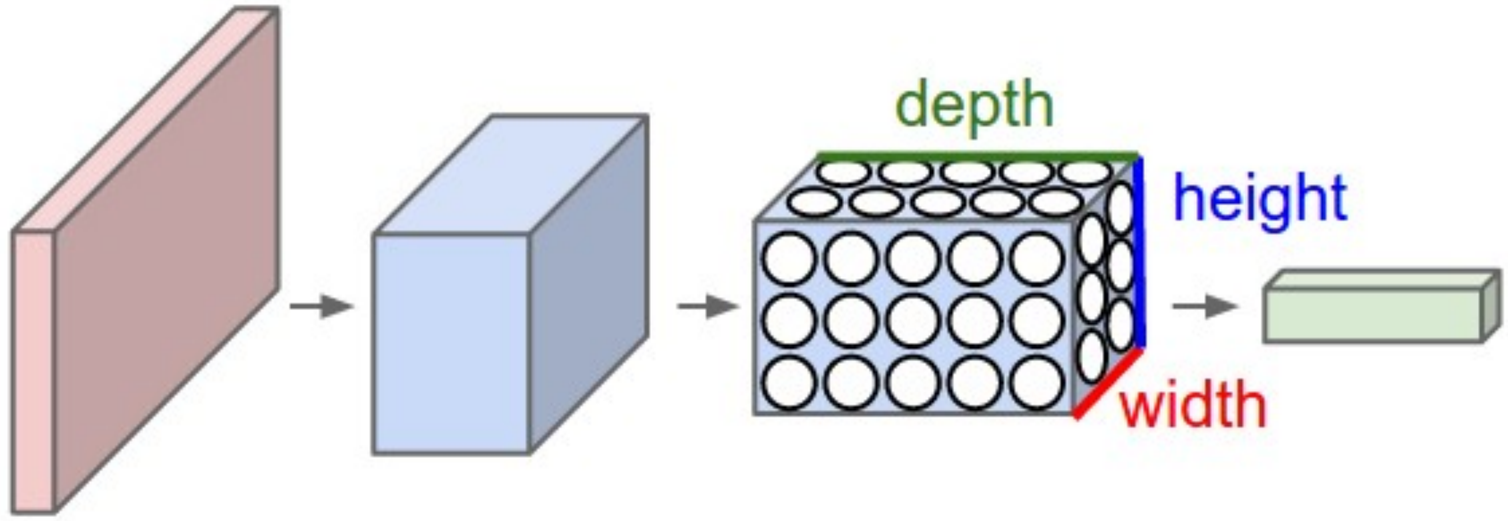
$$D_{out} = D_{in}$$



F spatial size
S stride
W width of volume
H height of volume
D depth of volume

Intuition: cone shape of CNNs

- More filters allow for more features
- Lower spatial dimensions reflect spatial invariance
- Gradually so the network does not need to infer classes directly from edge positions
- This hierarchical extraction of features is what our brain does so well



Numerical Example 1:

CNNs and Image Classification

Input Volume (+pad 1) (7x7x3)

x[:,:,0]

0	0	0	0	0	0	0
0	0	2	1	1	0	0
0	0	2	2	0	1	0
0	0	1	1	1	2	0
0	2	0	0	1	2	0
0	1	0	1	2	1	0
0	0	0	0	0	0	0

i1 (R)

x[:,:,1]

0	0	0	0	0	0	0
0	1	2	2	0	0	0
0	0	2	0	2	1	0
0	1	0	0	2	1	0
0	0	2	0	1	2	0
0	1	0	0	1	2	0
0	0	0	0	0	0	0

i2 (G)

x[:,:,2]

0	0	0	0	0	0	0
0	1	2	2	1	0	0
0	0	2	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	2	1	0
0	1	2	2	2	1	0
0	0	0	0	0	0	0

i3 (B)

Filter W0 (3x3x3)

w0[:,:,0]

1	-1	1
-1	0	1
0	-1	1

w11

w0[:,:,1]

-1	0	1
1	0	1
1	-1	-1

w12

w0[:,:,2]

0	-1	0
-1	1	-1
1	-1	1

w13

Bias b0 (1x1x1)

b0[:,:,0]

1

Filter W1 (3x3x3)

w1[:,:,0]

0	1	-1
-1	0	1
1	-1	-1

w21

w1[:,:,1]

1	-1	0
-1	0	0
-1	0	0

w22

w1[:,:,2]

0	1	-1
1	-1	1
1	0	-1

w23

Bias b1 (1x1x1)

b1[:,:,0]

0

Output Volume (3x3x2)

o[:,:,0]

6	0	-2
4	7	-3
0	1	-4

o1

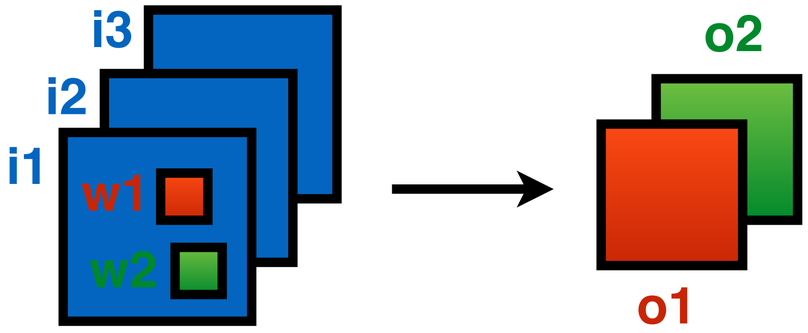
o[:,:,1]

-1	-2	-3
-7	0	-1
2	4	0

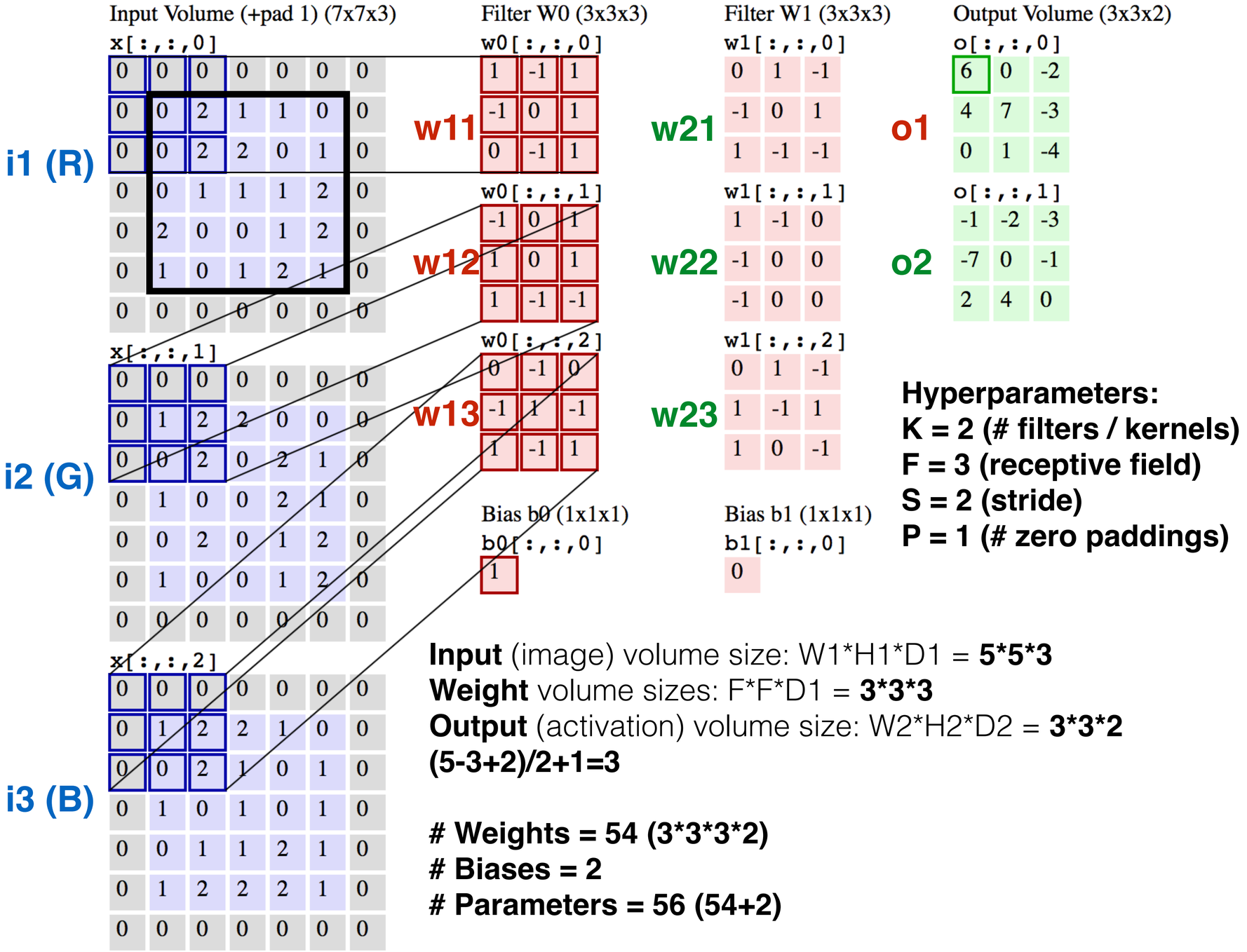
o2

Hyperparameters:
K = ? (?)
F = ? (?)
S = 2 (?)
P = ? (?)

 # Weights, # Biases, # Parameters?



Input (image) volume size: W1*H1*D1 = ?
Weight volume sizes: F*F*D1 = ?
Output (activation) volume size: W2*H2*D2 = ?



Hyperparameters:
K = 2 (# filters / kernels)
F = 3 (receptive field)
S = 2 (stride)
P = 1 (# zero paddings)

Input (image) volume size: W1*H1*D1 = **5*5*3**
Weight volume sizes: F*F*D1 = **3*3*3**
Output (activation) volume size: W2*H2*D2 = **3*3*2**
(5-3+2)/2+1=3

 # Weights = 54 (3*3*3*2)
 # Biases = 2
 # Parameters = 56 (54+2)

Parameters

Hyperparameters:
K = 2 (# filters / kernels)
F = 3 (receptive field)
S = 2 (stride)
P = 1 (# zero paddings)

Input (image) volume size: W1*H1*D1 = **5*5*3**
Weight volume sizes: F*F*D1 = **3*3*3**
Output (activation) volume size: W2*H2*D2 = **3*3*2**
(5-3+2)/2+1 = 3 = (W1-F+2P)/S + 1

 # Weights = 54 = (3*3*3)*2 = (F*F*D1)*K
 # Biases = 2 = K
 # Parameters = 56 (54+2)

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

A common setting of the hyperparameters is $F = 3, S = 1, P = 1$. However, there are common conventions and rules of thumb that motivate these hyperparameters.

AlexNet (winner of the 2012 ImageNet com.):
- Input volume: 227*227*3
- K = 96, F = 11, S = 4, P = 0
- Output Volume = **??**

Parameters

Hyperparameters:
K = 2 (# filters / kernels)
F = 3 (receptive field)
S = 2 (stride)
P = 1 (# zero paddings)

Input (image) volume size: W1*H1*D1 = **5*5*3**
Weight volume sizes: F*F*D1 = **3*3*3**
Output (activation) volume size: W2*H2*D2 = **3*3*2**
(5-3+2)/2+1 = 3 = (W1-F+2P)/S + 1

 # Weights = 54 = (3*3*3)*2 = (F*F*D1)*K
 # Biases = 2 = K
 # Parameters = 56 (54+2)

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

A common setting of the hyperparameters is $F = 3, S = 1, P = 1$. However, there are common conventions and rules of thumb that motivate these hyperparameters.

AlexNet (winner of the 2012 ImageNet com.):
- Input volume: 227*227*3
- K = 96, F = 11, S = 4, P = 0
- Output Volume = **55*55*96 = (227-11)/4 + 1 = 55**
- **Depth column = ?, Size = ?, Connected to = ?**
- **Parameter Sharing ?**

Input Volume (+pad 1) (7x7x3)

x[:,:,0]

0	0	0	0	0	0	0
0	0	2	1	1	0	0
0	0	2	2	0	1	0
0	0	1	1	1	2	0
0	2	0	0	1	2	0
0	1	0	1	2	1	0
0	0	0	0	0	0	0

x[:,:,1]

0	0	0	0	0	0	0
0	1	2	2	0	0	0
0	0	2	0	2	1	0
0	1	0	0	2	1	0
0	0	2	0	1	2	0
0	1	0	0	1	2	0
0	0	0	0	0	0	0

x[:,:,2]

0	0	0	0	0	0	0
0	1	2	2	1	0	0
0	0	2	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	2	1	0
0	1	2	2	2	1	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

w0[:,:,0]

1	-1	1
-1	0	1
0	-1	1

w0[:,:,1]

-1	0	1
1	0	1
1	-1	-1

w0[:,:,2]

0	-1	0
-1	1	-1
1	-1	1

Filter W1 (3x3x3)

w1[:,:,0]

0	1	-1
-1	0	1
1	-1	-1

w1[:,:,1]

1	-1	0
-1	0	0
-1	0	0

w1[:,:,2]

0	1	-1
1	-1	1
1	0	-1

Output Volume (3x3x2)

o[:,:,0]

6	0	-2
4	7	-3
0	1	-4

o[:,:,1]

-1	-2	-3
-7	0	-1
2	4	0

4

0

1

Bias b0 (1x1x1)

b0[:,:,0]

1

Bias b1 (1x1x1)

b1[:,:,0]

0

W*i + b

w(-1,-1) * i(-1,-1)		
	w(0,0) * i(0,0)	
		w(1,1) * i(1,1)

1*0	-1*0	1*0
-1*0	0*0	**1*2**
0*0	-1*0	**1*2**

-1*0	0*0	1*0
1*0	0*1	**1*2**
1*0	-1*0	**-1*2**

0*0	-1*0	0*0
-1*0	**1*1**	**-1*2**
1*0	-1*0	**1*2**

1

4 (2+2) + 0 (2-2) + 1 (1-2+2) + 1 = 6

Input Volume (+pad 1) (7x7x3)

x[:,:,0]

0	0	0	0	0	0	0
0	0	2	1	1	0	0
0	0	2	2	0	1	0
0	0	1	1	1	2	0
0	2	0	0	1	2	0
0	1	0	1	2	1	0
0	0	0	0	0	0	0

x[:,:,1]

0	0	0	0	0	0	0
0	1	2	2	0	0	0
0	0	2	0	2	1	0
0	1	0	0	2	1	0
0	0	2	0	1	2	0
0	1	0	0	1	2	0
0	0	0	0	0	0	0

x[:,:,2]

0	0	0	0	0	0	0
0	1	2	2	1	0	0
0	0	2	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	2	1	0
0	1	2	2	2	1	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

w0[:,:,0]

1	-1	1
-1	0	1
0	-1	1

w0[:,:,1]

-1	0	1
1	0	1
1	-1	-1

w0[:,:,2]

0	-1	0
-1	1	-1
1	-1	1

Filter W1 (3x3x3)

w1[:,:,0]

0	1	-1
-1	0	1
1	-1	-1

w1[:,:,1]

1	-1	0
-1	0	0
-1	0	0

w1[:,:,2]

0	1	-1
1	-1	1
1	0	-1

Output Volume (3x3x2)

o[:,:,0]

6	0	-2
4	7	-3
0	1	-4

o[:,:,1]

-1	-2	-3
-7	0	-1
2	4	0

?

?

?

Bias b0 (1x1x1)

b0[:,:,0]

1

Bias b1 (1x1x1)

b1[:,:,0]

0

W*i + b

w(-1,-1) * i(-1,-1)		
	w(0,0) * i(0,0)	
		w(1,1) * i(1,1)

?	?	?
?	?	?
?	?	?

?	?	?
?	?	?
?	?	?

?	?	?
?	?	?
?	?	?

?

? (?) + ? (?) + ? (?) + ? = ?

Input Volume (+pad 1) (7x7x3)

x[:,:,0]

0	0	0	0	0	0	0
0	0	2	1	1	0	0
0	0	2	2	0	1	0
0	0	1	1	1	2	0
0	2	0	0	1	2	0
0	1	0	1	2	1	0
0	0	0	0	0	0	0

x[:,:,1]

0	0	0	0	0	0	0
0	1	2	2	0	0	0
0	0	2	0	2	1	0
0	1	0	0	2	1	0
0	0	2	0	1	2	0
0	1	0	0	1	2	0
0	0	0	0	0	0	0

x[:,:,2]

0	0	0	0	0	0	0
0	1	2	2	1	0	0
0	0	2	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	2	1	0
0	1	2	2	2	1	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

w0[:,:,0]

1	-1	1
-1	0	1
0	-1	1

w0[:,:,1]

-1	0	1
1	0	1
1	-1	-1

w0[:,:,2]

0	-1	0
-1	1	-1
1	-1	1

Filter W1 (3x3x3)

w1[:,:,0]

0	1	-1
-1	0	1
1	-1	-1

w1[:,:,1]

1	-1	0
-1	0	0
-1	0	0

w1[:,:,2]

0	1	-1
1	-1	1
1	0	-1

Output Volume (3x3x2)

o[:,:,0]

6	0	-2
4	7	-3
0	1	-4

o[:,:,1]

-1	-2	-3
-7	0	-1
2	4	0

-3 **2** **0**

Bias b0 (1x1x1)

b0[:,:,0]

1

Bias b1 (1x1x1)

b1[:,:,0]

0

W*i + b

w(-1,-1) * i(-1,-1)		
	w(0,0) * i(0,0)	
		w(1,1) * i(1,1)

1*0	-1*0	1*0
-1*2	0*1	**1*1**
0*2	**-1*2**	1*0

-1*0	0*0	1*0
1*2	0*2	1*0
1*2	-1*0	**-1*2**

0*0	-1*0	0*0
-1*2	**1*2**	**-1*1**
1*2	**-1*1**	1*0

1

-3 (-2+1-2) + 2 (2+2-2) + 0 (-2+2-1+2-1) + 1 = 0

Input Volume (+pad 1) (7x7x3)

x[:,:,0]

0	0	0	0	0	0	0
0	0	2	1	1	0	0
0	0	2	2	0	1	0
0	0	1	1	1	2	0
0	2	0	0	1	2	0
0	1	0	1	2	1	0
0	0	0	0	0	0	0

x[:,:,1]

0	0	0	0	0	0	0
0	1	2	2	0	0	0
0	0	2	0	2	1	0
0	1	0	0	2	1	0
0	0	2	0	1	2	0
0	1	0	0	1	2	0
0	0	0	0	0	0	0

x[:,:,2]

0	0	0	0	0	0	0
0	1	2	2	1	0	0
0	0	2	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	2	1	0
0	1	2	2	2	1	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

w0[:,:,0]

1	-1	1
-1	0	1
0	-1	1

w0[:,:,1]

-1	0	1
1	0	1
1	-1	-1

w0[:,:,2]

0	-1	0
-1	1	-1
1	-1	1

Bias b0 (1x1x1)

b0[:,:,0]

1

Filter W1 (3x3x3)

w1[:,:,0]

0	1	-1
-1	0	1
1	-1	-1

w1[:,:,1]

1	-1	0
-1	0	0
-1	0	0

w1[:,:,2]

0	1	-1
1	-1	1
1	0	-1

Bias b1 (1x1x1)

b1[:,:,0]

0

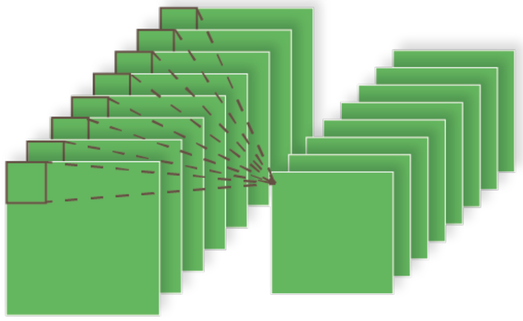
Output Volume (3x3x2)

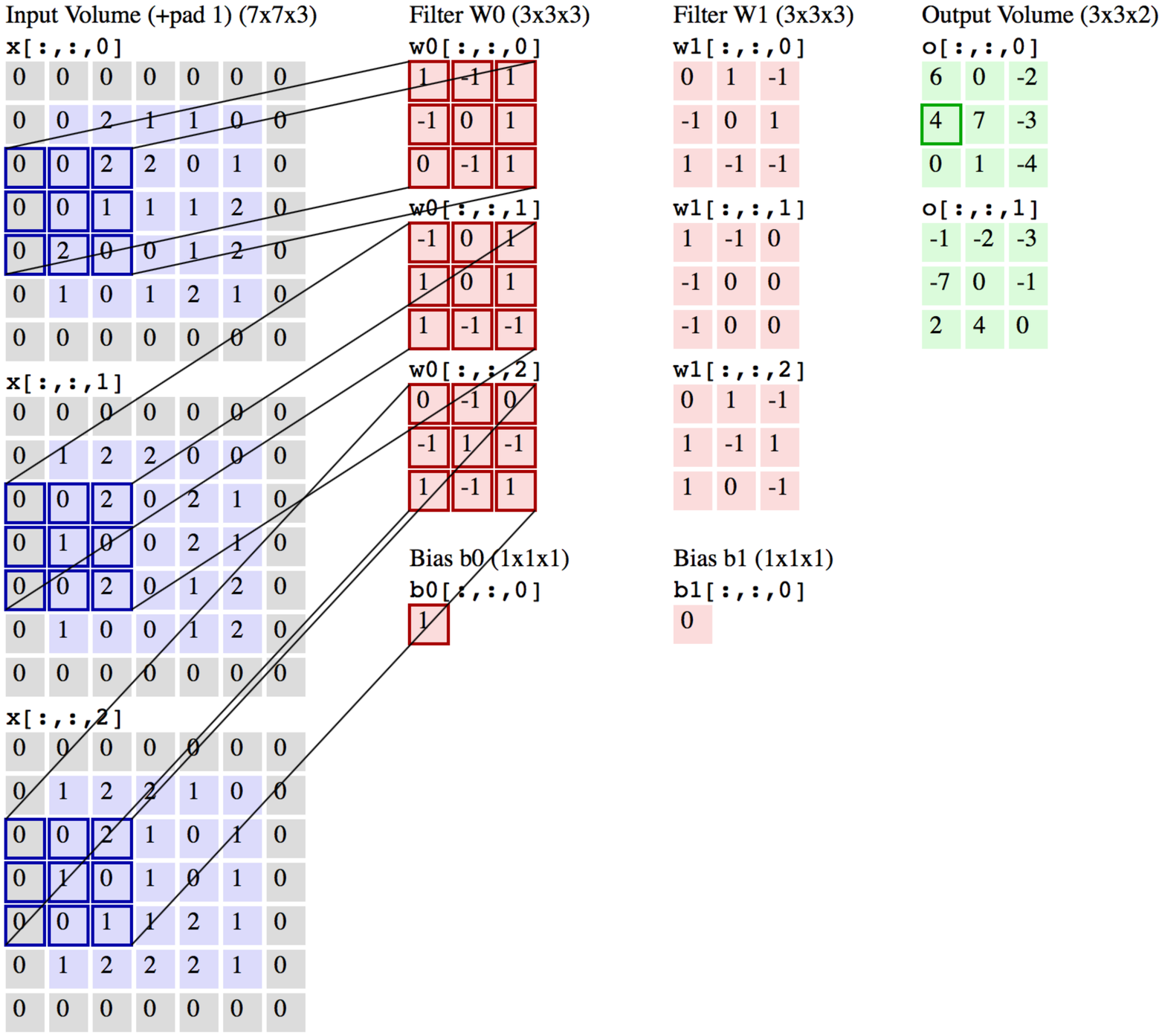
o[:,:,0]

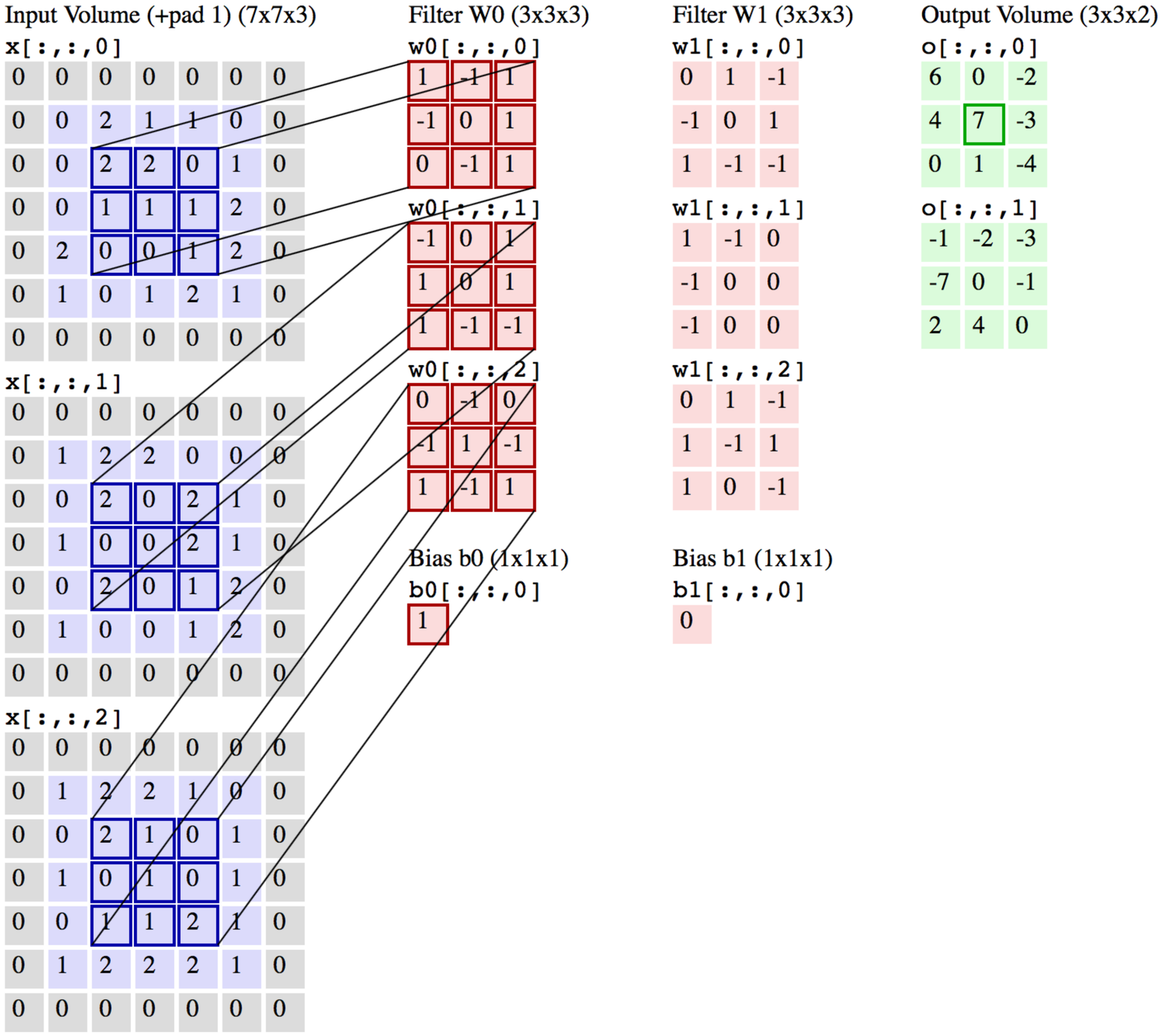
6	0	-2
4	7	-3
0	1	-4

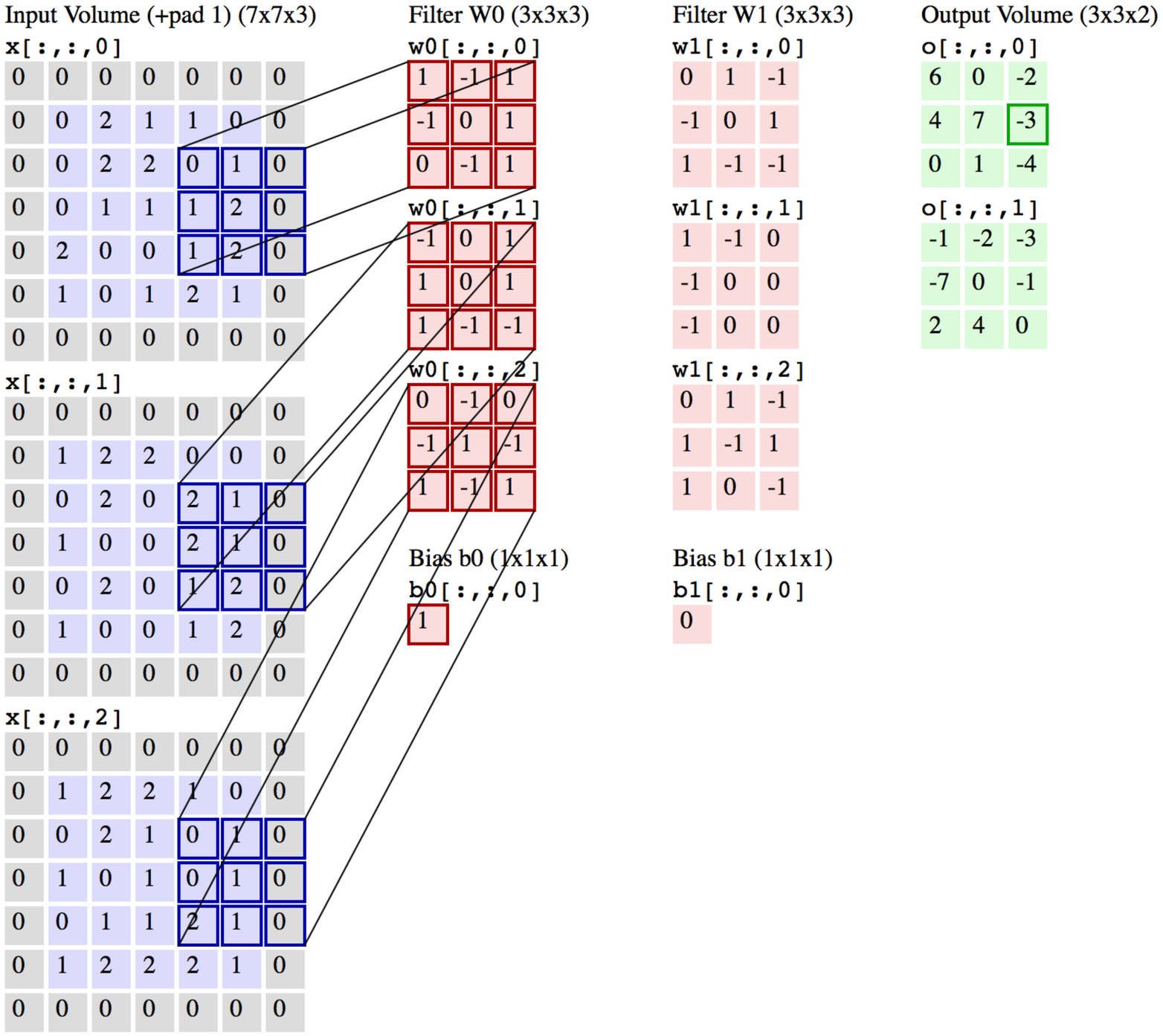
o[:,:,1]

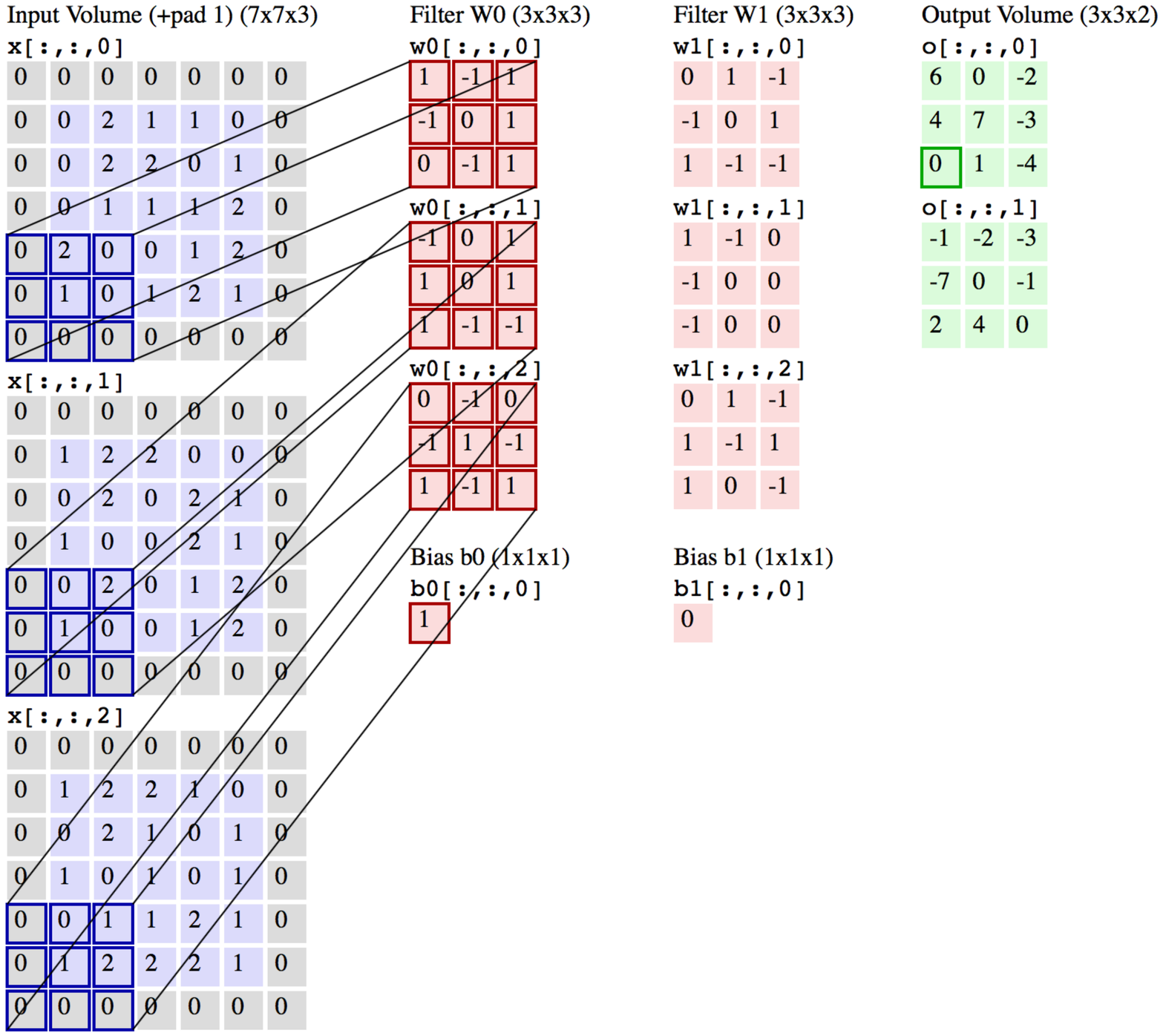
-1	-2	-3
-7	0	-1
2	4	0

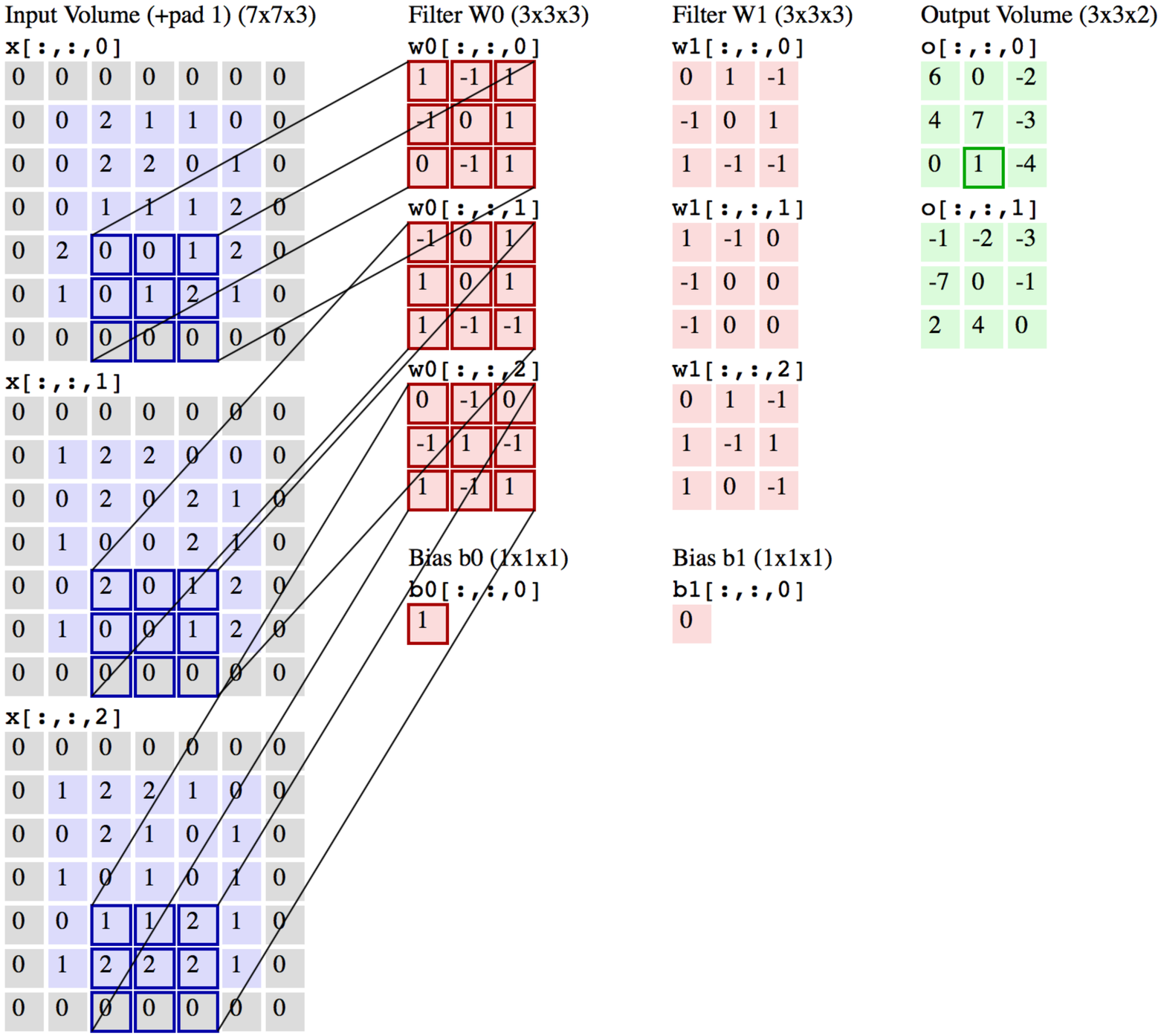


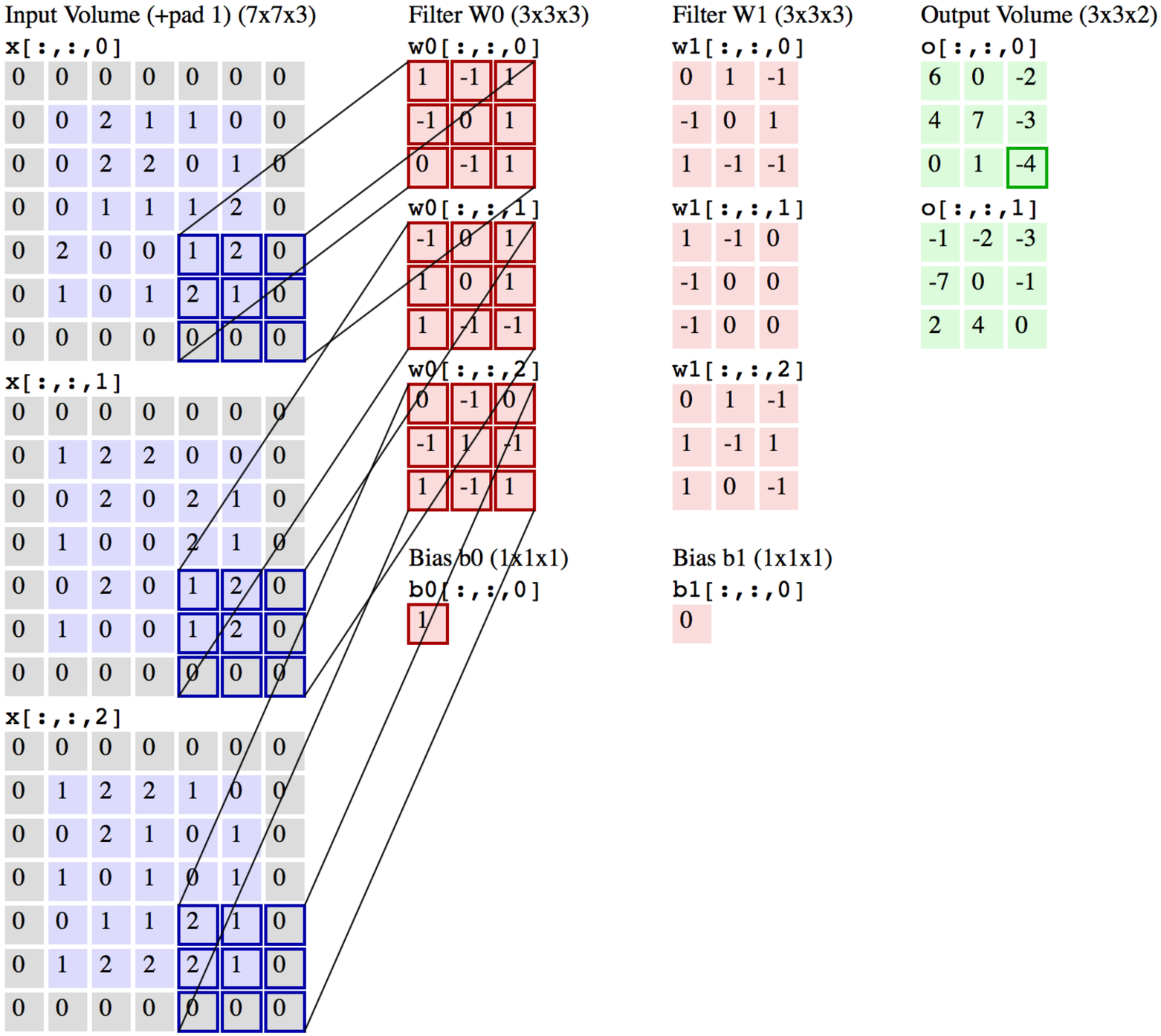

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

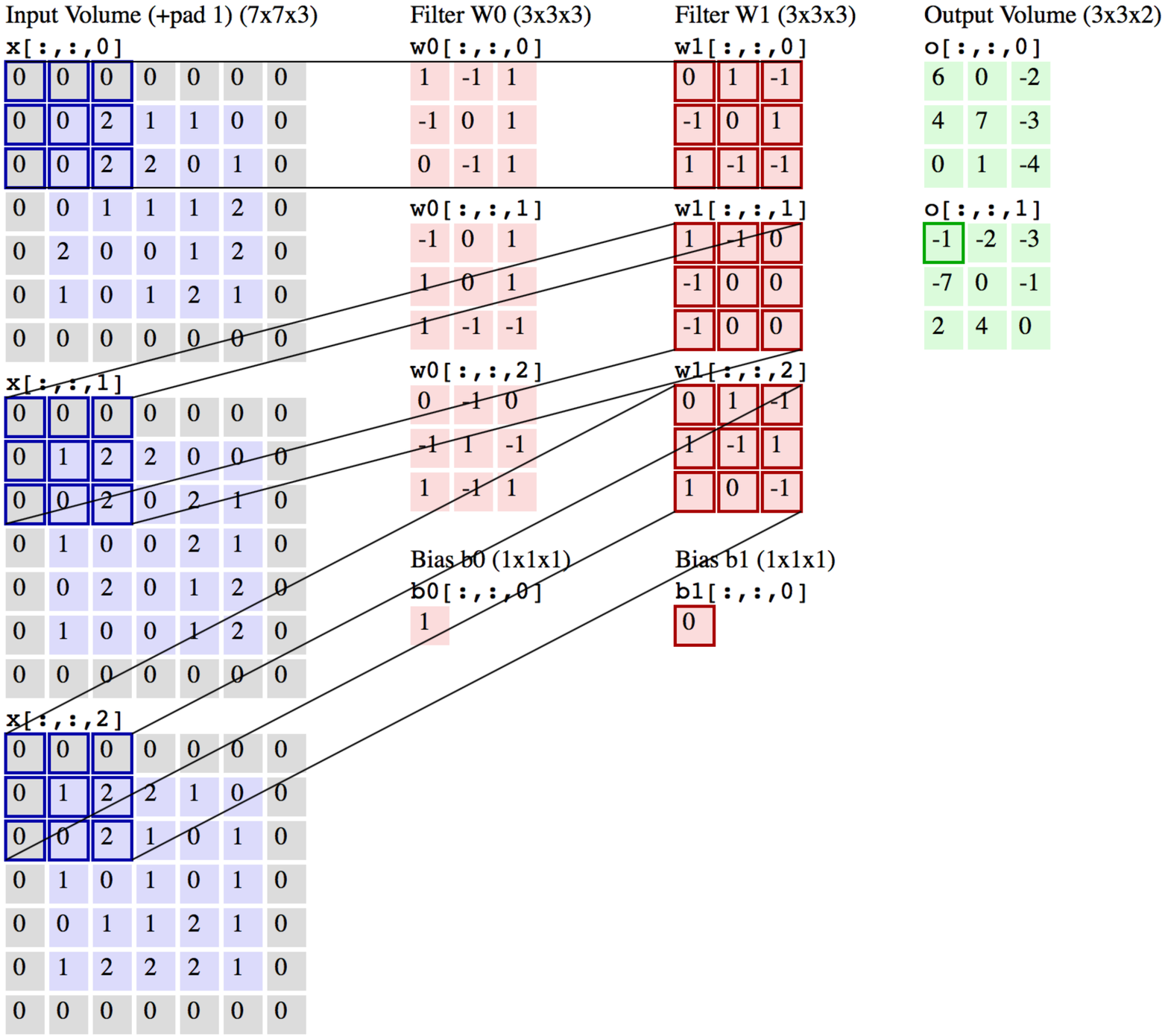

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

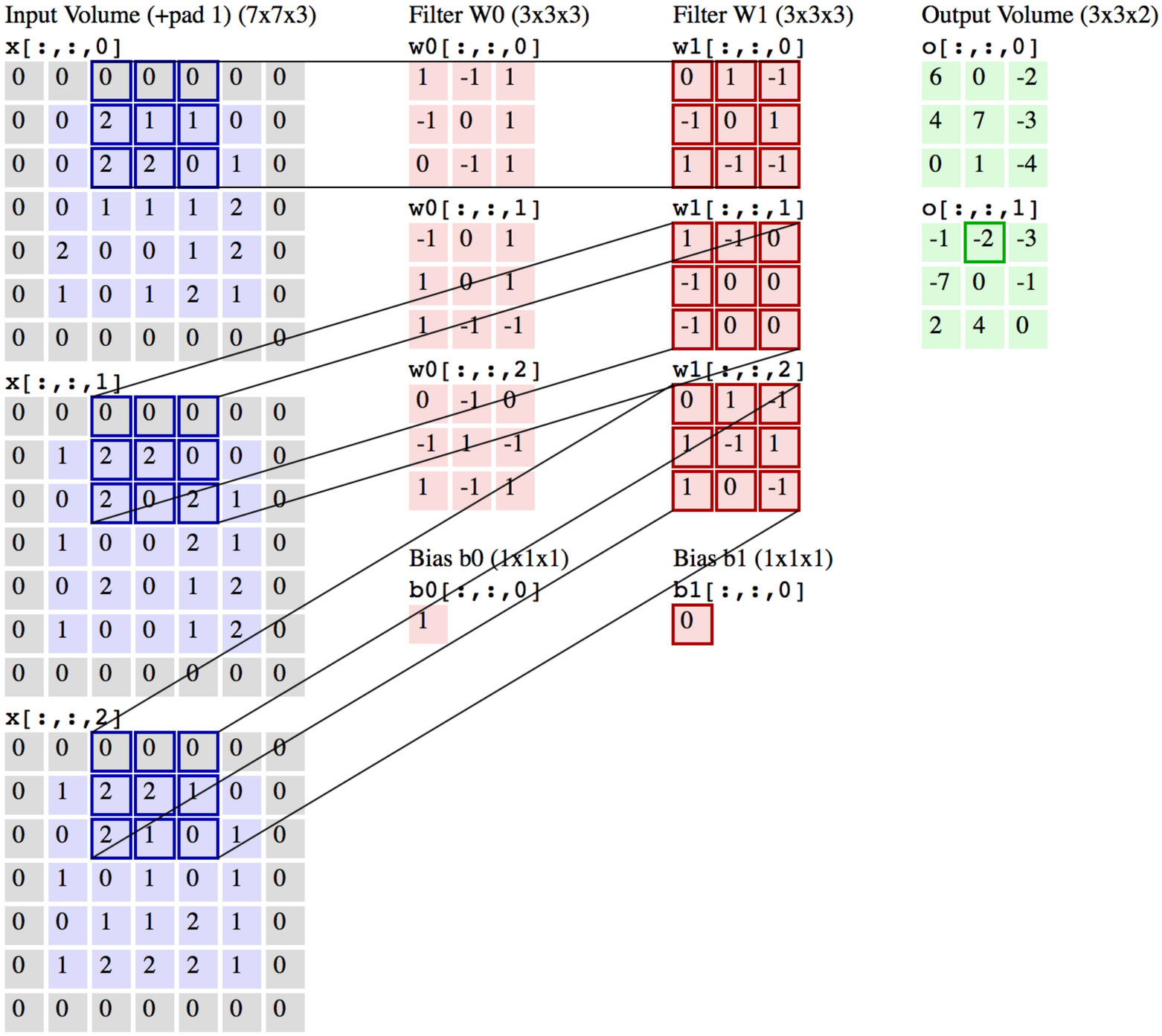

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

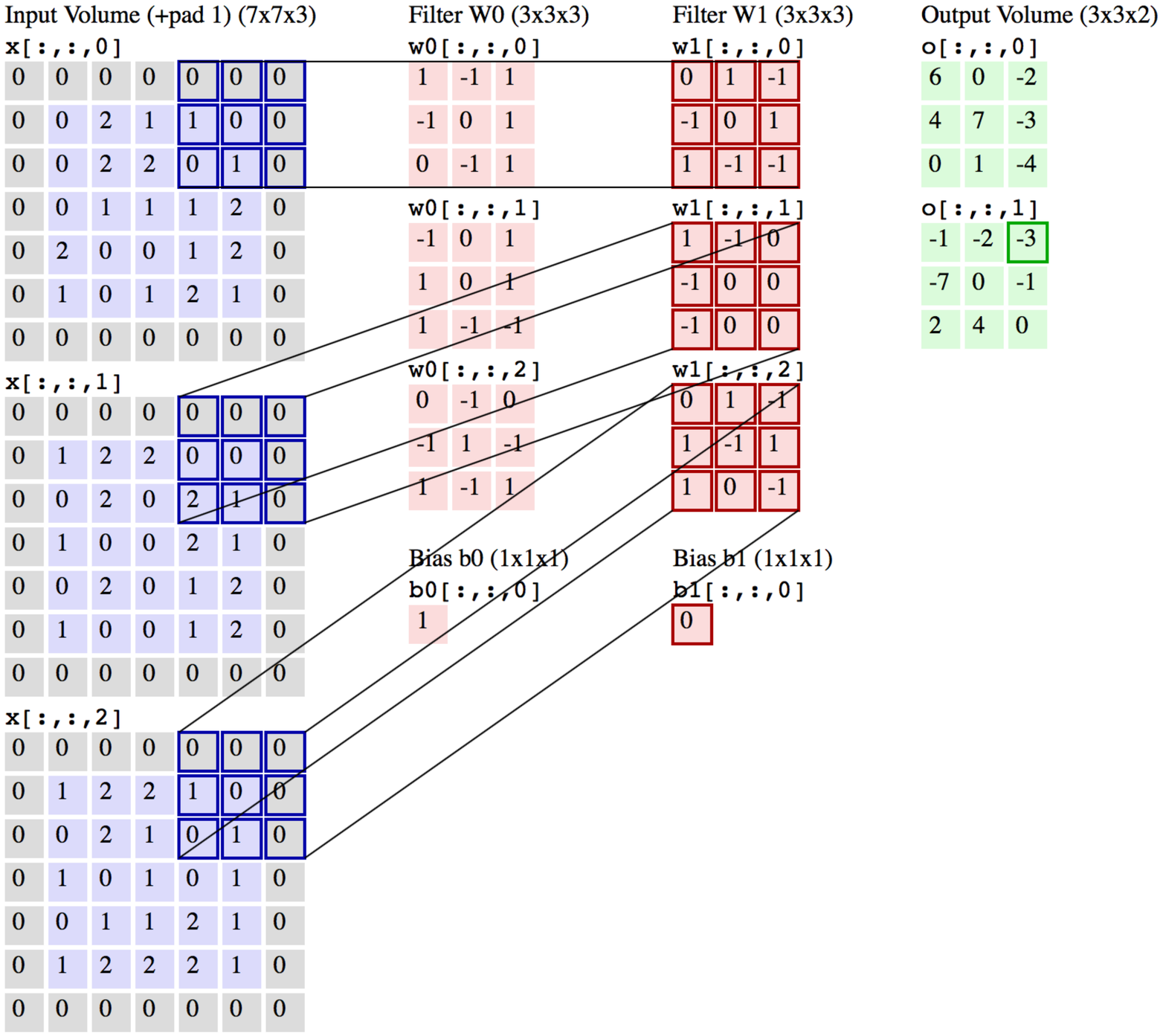

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

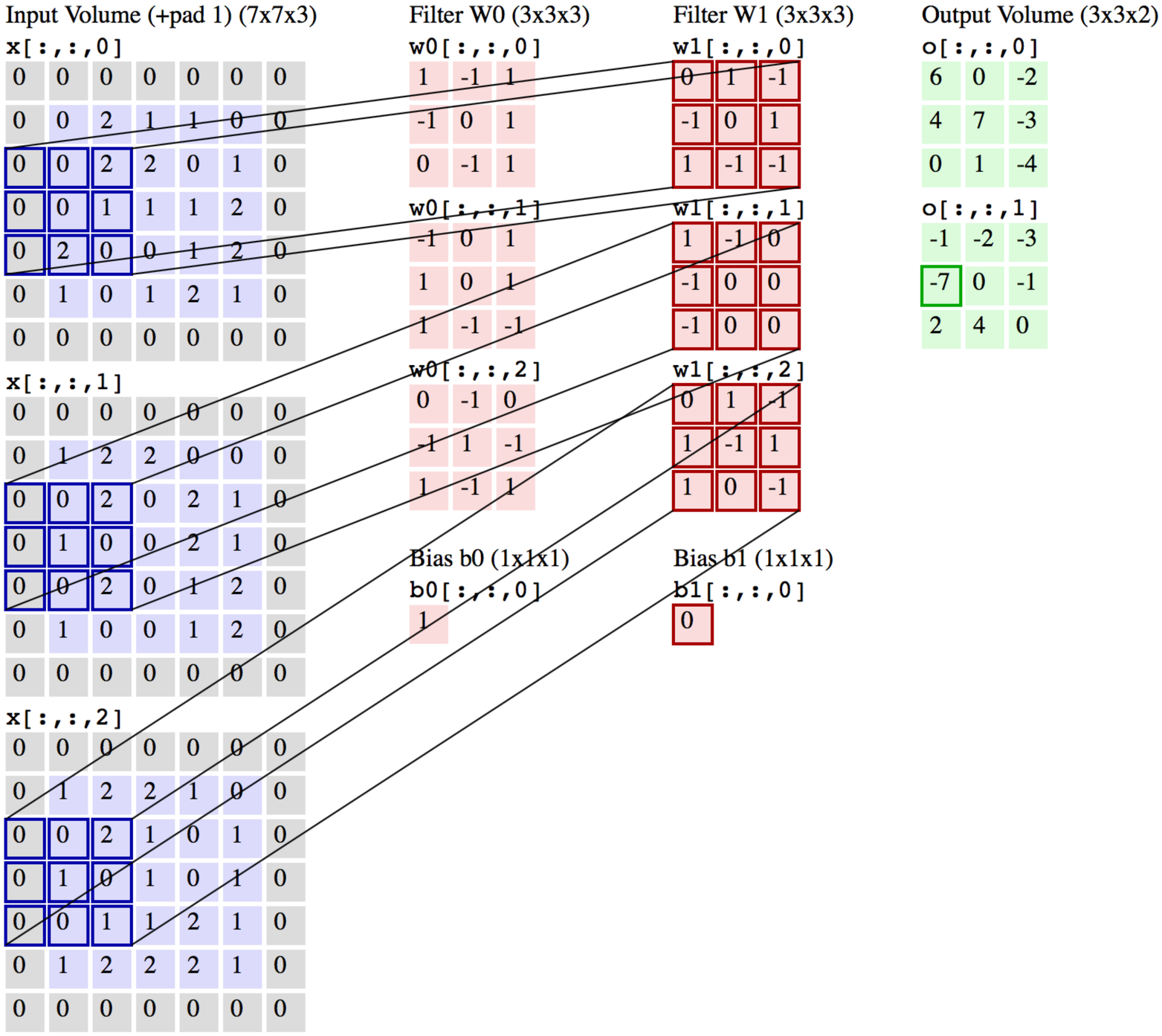

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

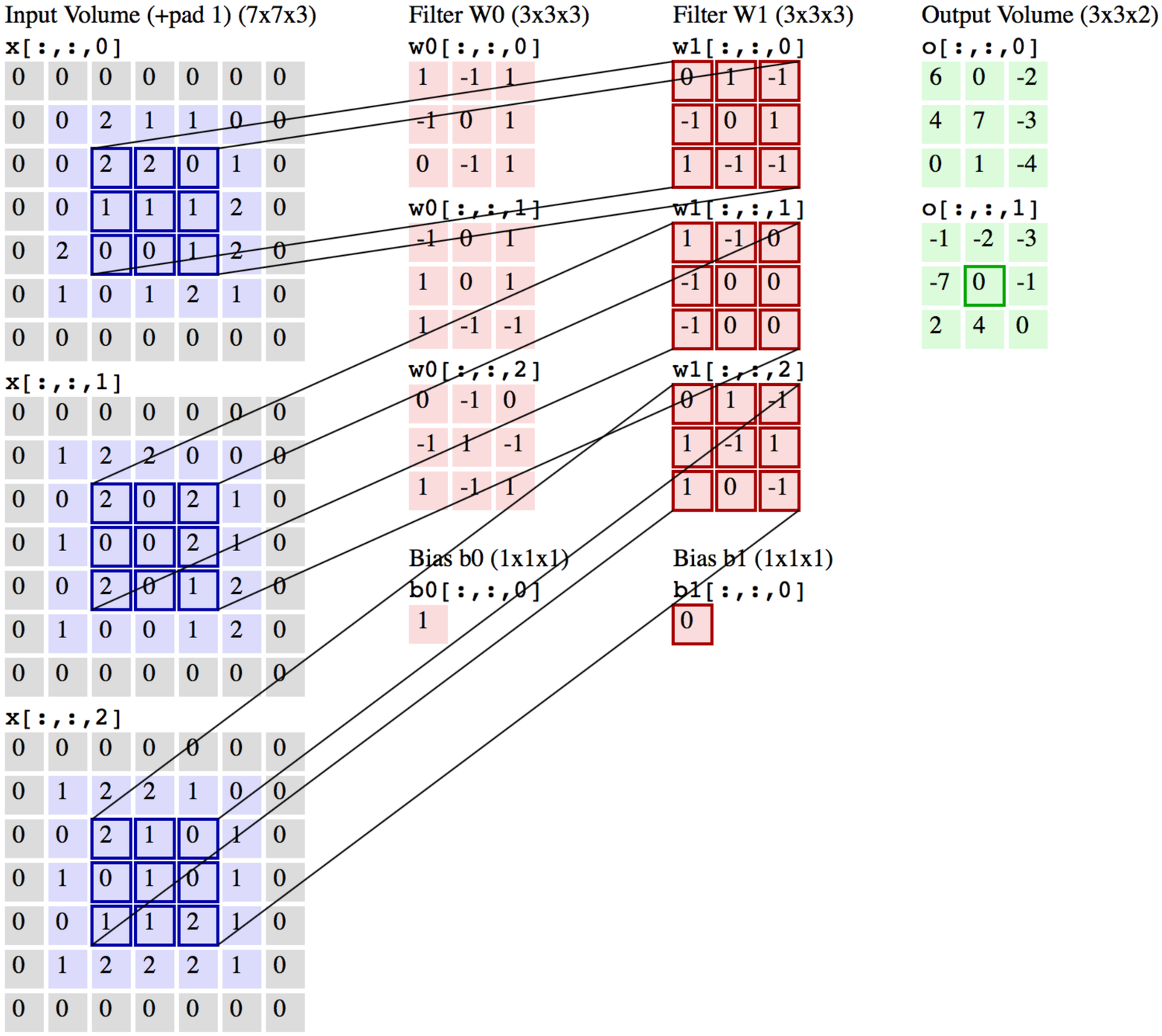

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

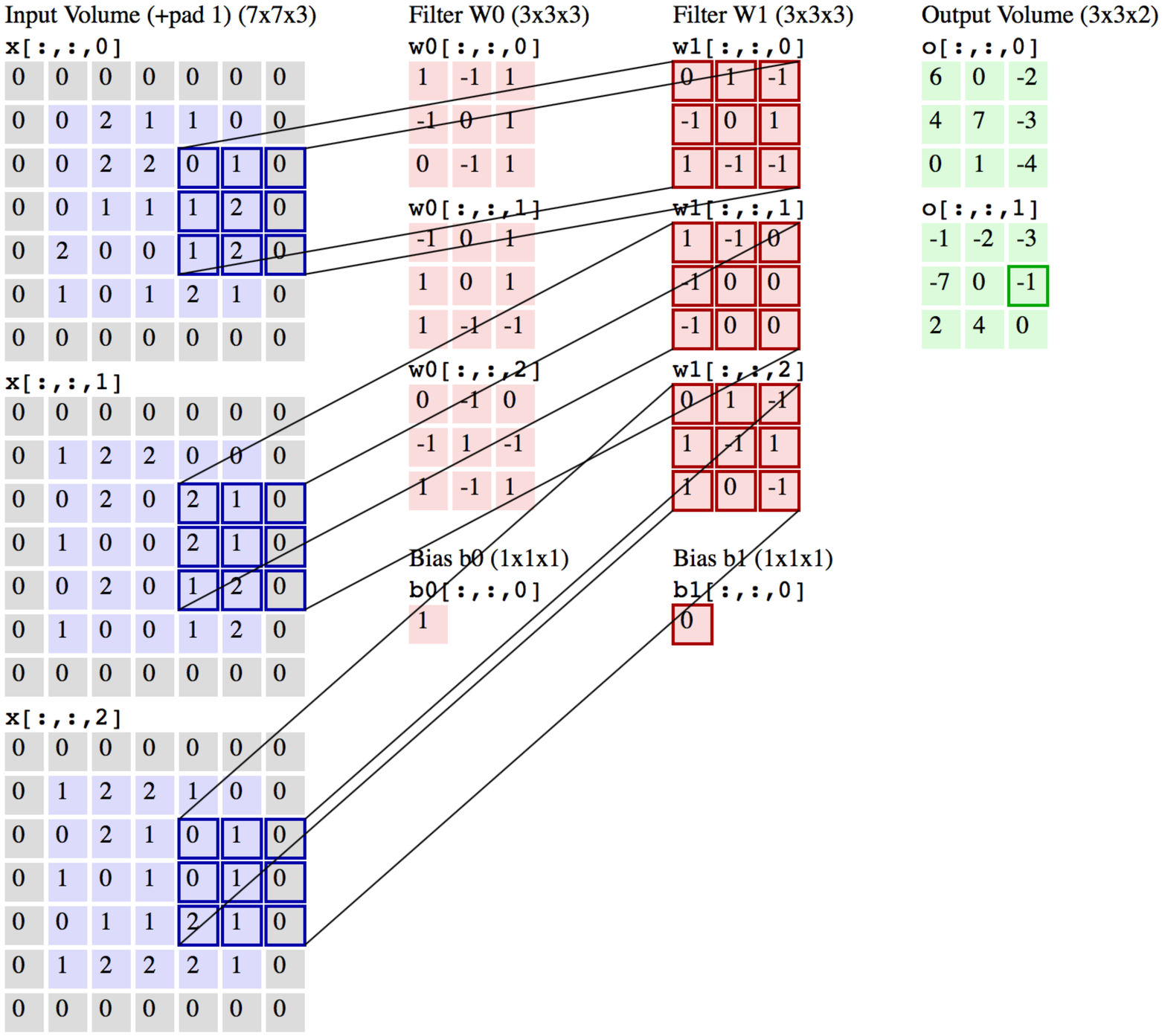

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

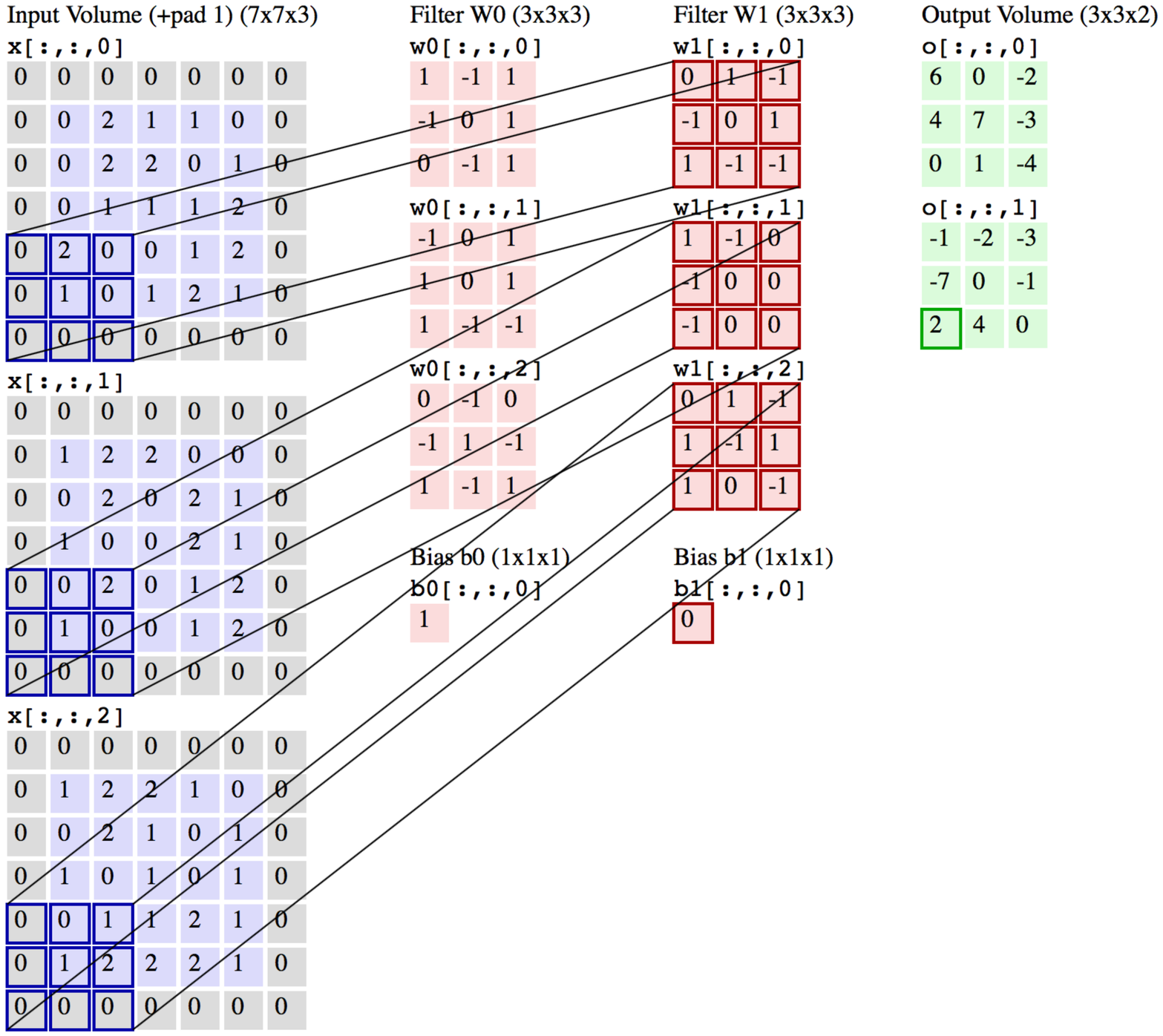

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b0 (1x1x1)
b0[:,:,0]
1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

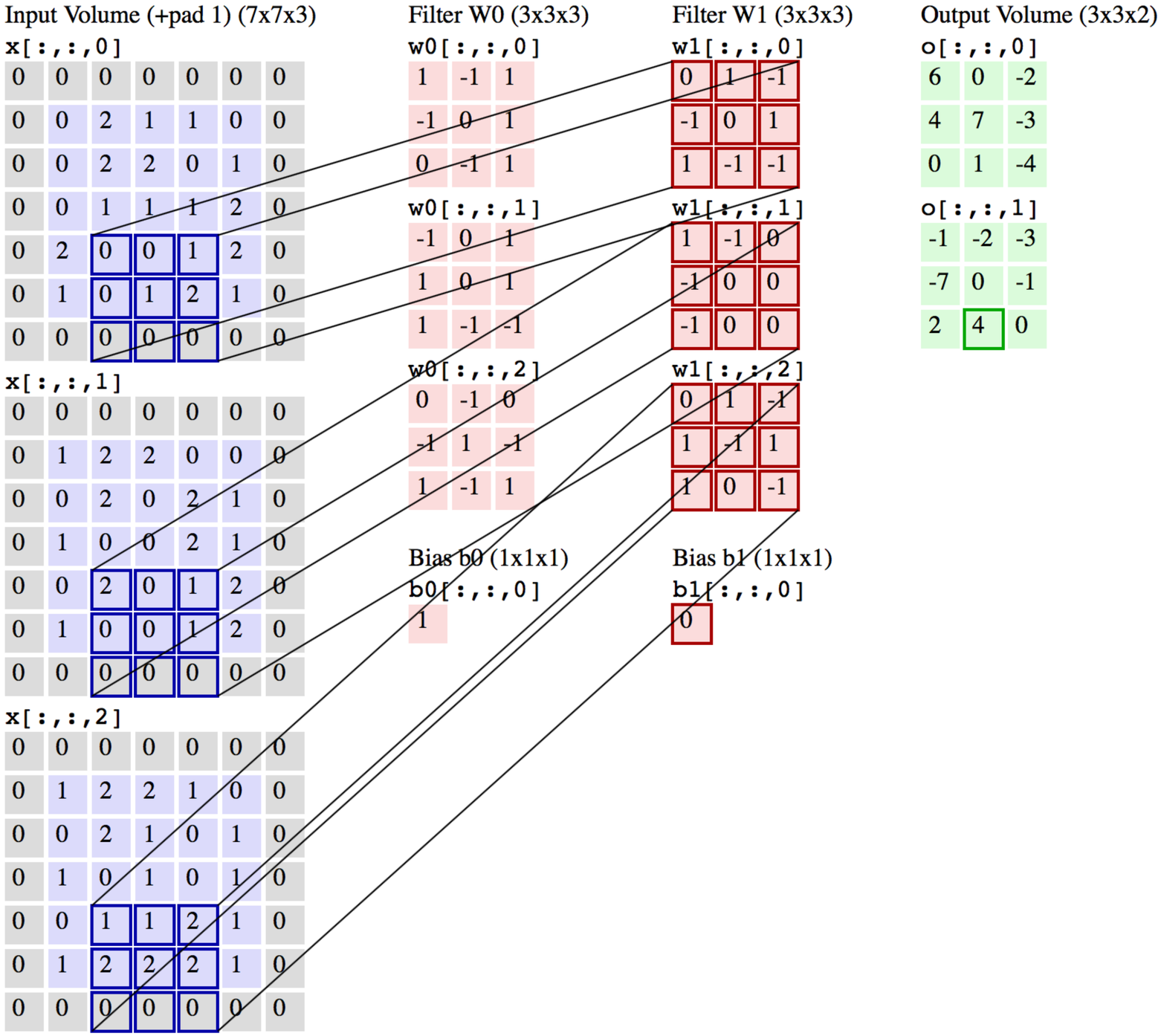

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b0 (1x1x1)
b0[:,:,0]
1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

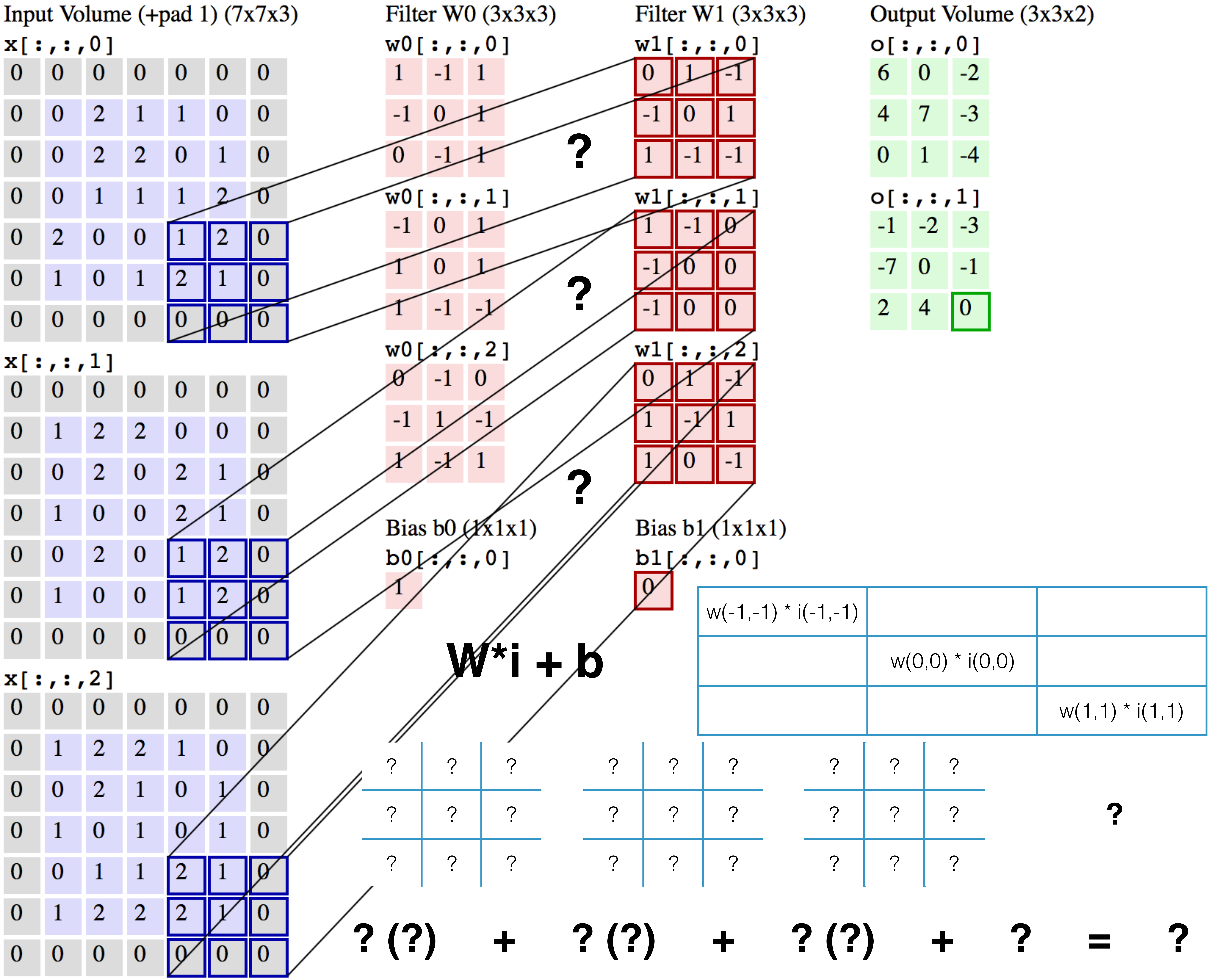

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0

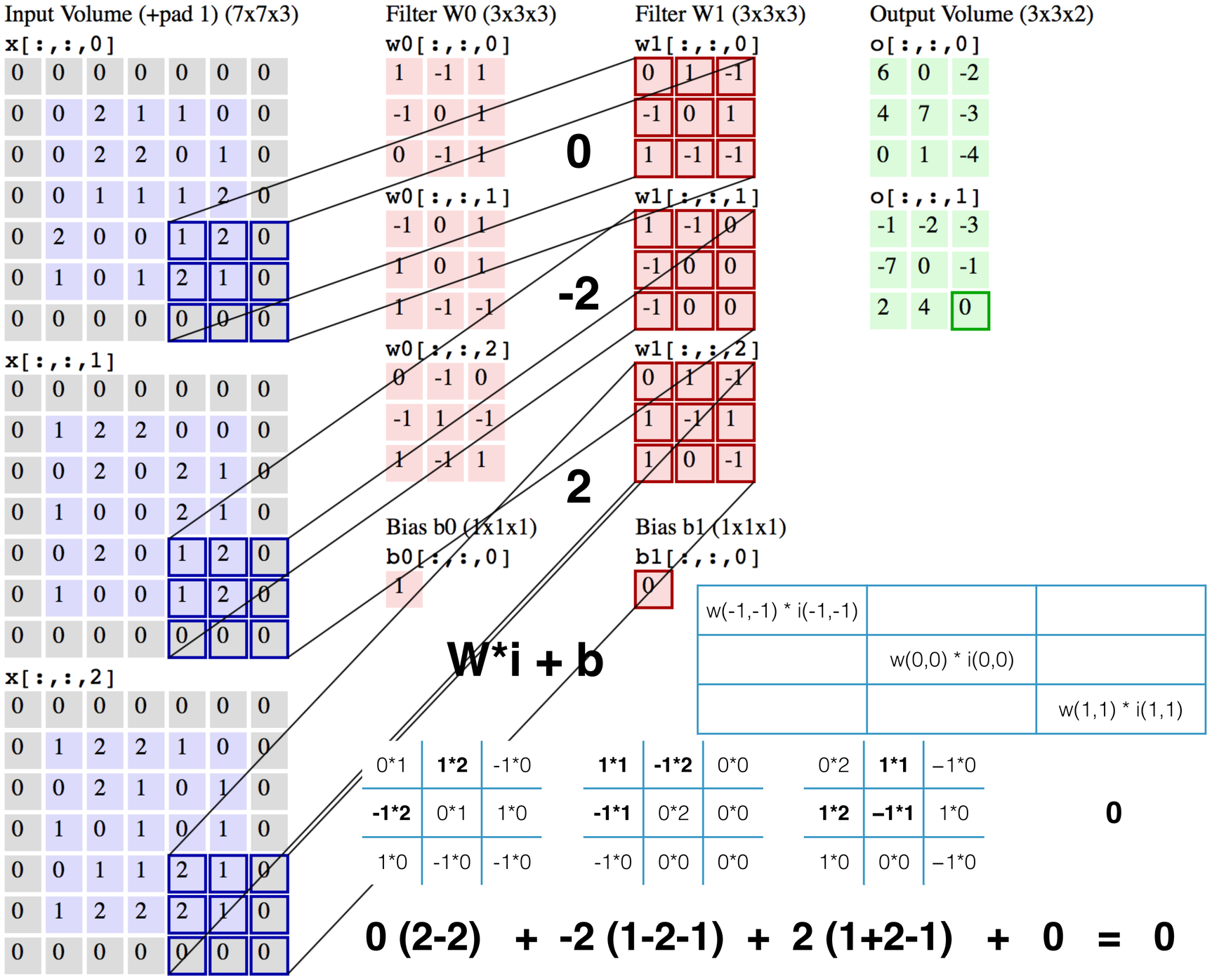

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0


Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b0 (1x1x1)
b0[:,:,0]
1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0


Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b0 (1x1x1)
b0[:,:,0]
1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0


Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b0 (1x1x1)
b0[:,:,0]
1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0


Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Bias b0 (1x1x1)
b0[:,:,0]
1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Bias b1 (1x1x1)
b1[:,:,0]
0
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0
?
?
?
W*i + b
w(-1,-1) * i(-1,-1)
w(0,0) * i(0,0)
w(1,1) * i(1,1)
? ? ?
? ? ?
? ? ?
? ? ?
? ? ?
? ? ?
? ? ?
? ? ?
? ? ?
?
? (?) + ? (?) + ? (?) + ? = ?

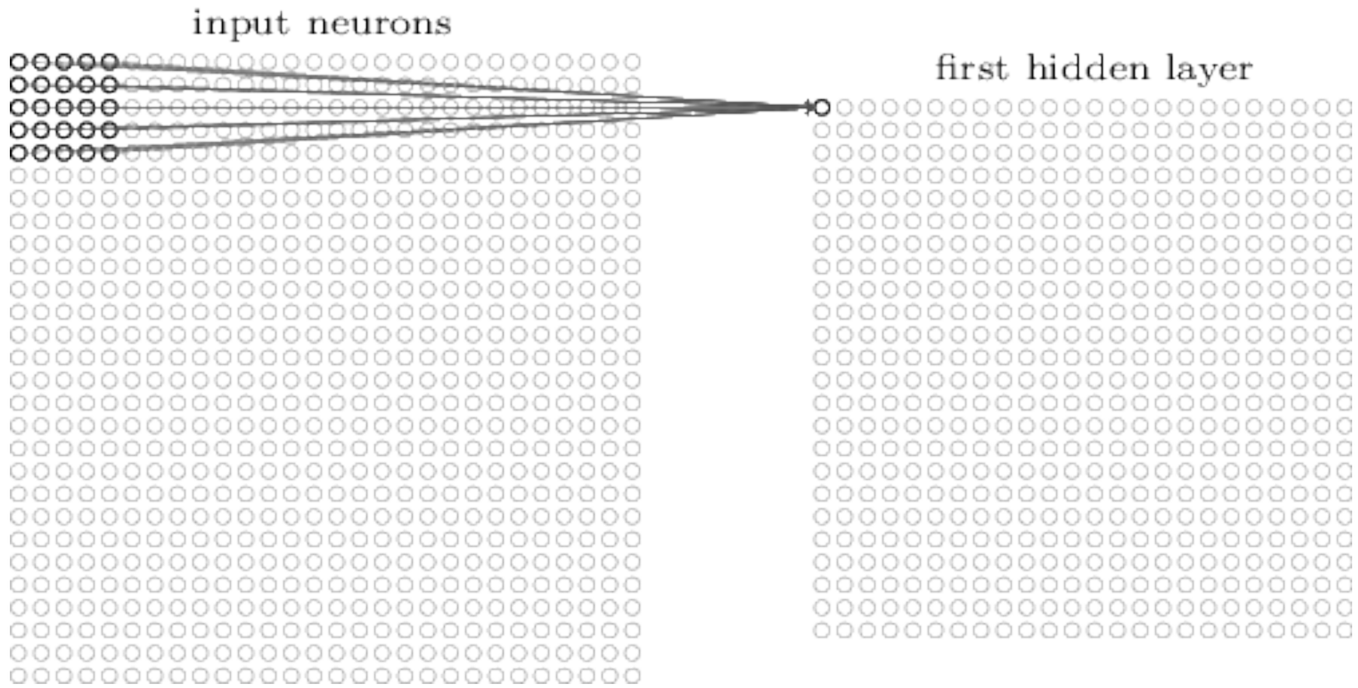

Input Volume (+pad 1) (7x7x3)
x[:,:,0]
0 0 0 0 0 0 0
0 0 2 1 1 0 0
0 0 2 2 0 1 0
0 0 1 1 1 2 0
0 2 0 0 1 2 0
0 1 0 1 2 1 0
0 0 0 0 0 0 0
x[:,:,1]
0 0 0 0 0 0 0
0 1 2 2 0 0 0
0 0 2 0 2 1 0
0 1 0 0 2 1 0
0 0 2 0 1 2 0
0 1 0 0 1 2 0
0 0 0 0 0 0 0
x[:,:,2]
0 0 0 0 0 0 0
0 1 2 2 1 0 0
0 0 2 1 0 1 0
0 1 0 1 0 1 0
0 0 1 1 2 1 0
0 1 2 2 2 1 0
0 0 0 0 0 0 0
Filter W0 (3x3x3)
w0[:,:,0]
1 -1 1
-1 0 1
0 -1 1
w0[:,:,1]
-1 0 1
1 0 1
1 -1 -1
w0[:,:,2]
0 -1 0
-1 1 -1
1 -1 1
Filter W1 (3x3x3)
w1[:,:,0]
0 1 -1
-1 0 1
1 -1 -1
w1[:,:,1]
1 -1 0
-1 0 0
-1 0 0
w1[:,:,2]
0 1 -1
1 -1 1
1 0 -1
Output Volume (3x3x2)
o[:,:,0]
6 0 -2
4 7 -3
0 1 -4
o[:,:,1]
-1 -2 -3
-7 0 -1
2 4 0
0
-2
2
Bias b0 (1x1x1)
b0[:,:,0]
1
Bias b1 (1x1x1)
b1[:,:,0]
0
W*i + b
w(-1,-1) * i(-1,-1)
w(0,0) * i(0,0)
w(1,1) * i(1,1)
0*1 1*2 -1*0
-1*2 0*1 1*0
1*0 -1*0 -1*0
1*1 -1*2 0*0
-1*1 0*2 0*0
-1*0 0*0 0*0
0*2 1*1 −1*0
1*2 −1*1 1*0
1*0 0*0 −1*0
0
0 (2-2) + -2 (1-2-1) + 2 (1+2-1) + 0 = 0

Numerical Example 2:

CNNs and Image Classification

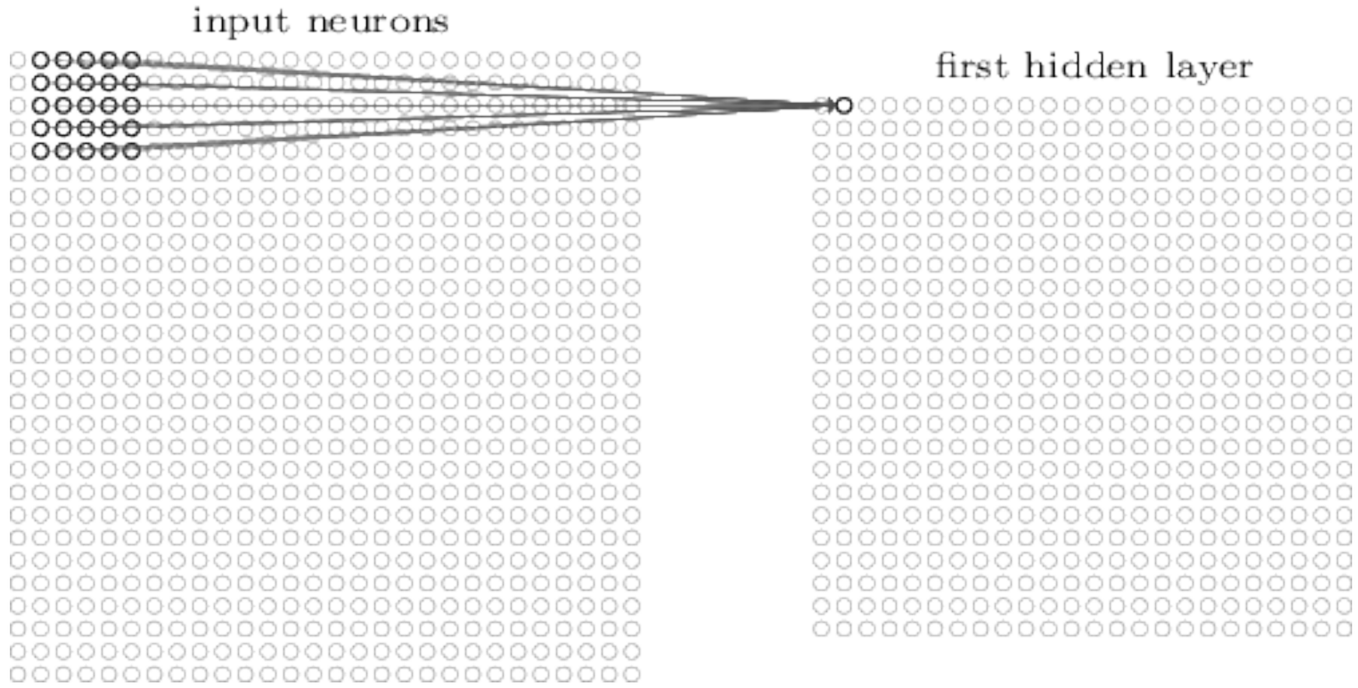
Convolution: Local Receptive Fields

- Each neuron is connected to a region in the previous layer
- 5x5 local receptive field



Convolution: Local Receptive Fields

- Each neuron is connected to a region in the previous layer
- 5x5 local receptive field

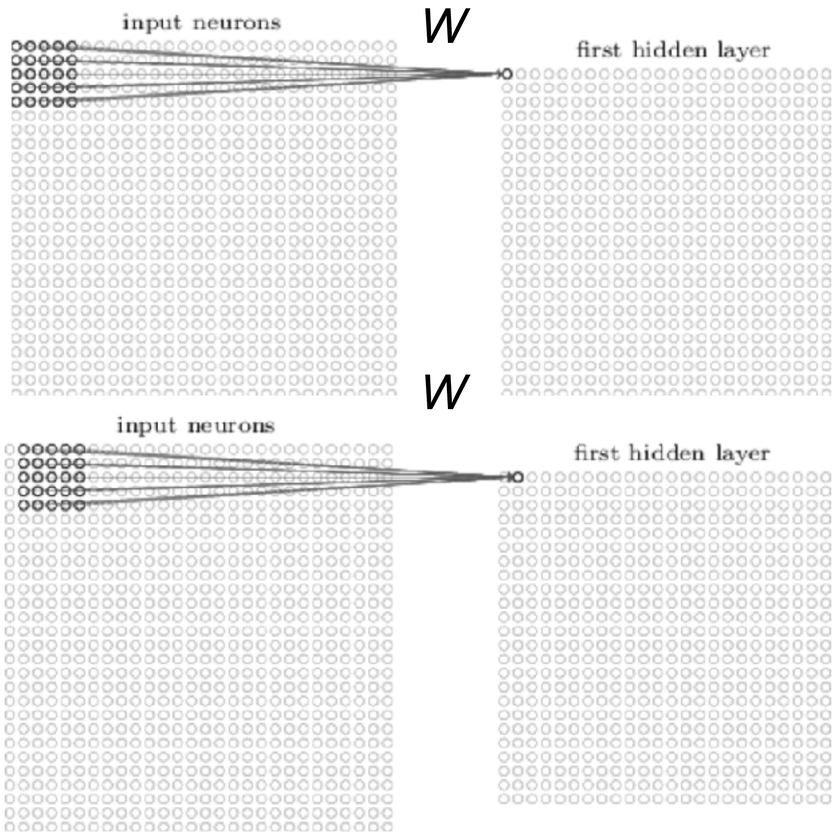


Input image *I*

Feature map *S*

Convolution: Shared Weights

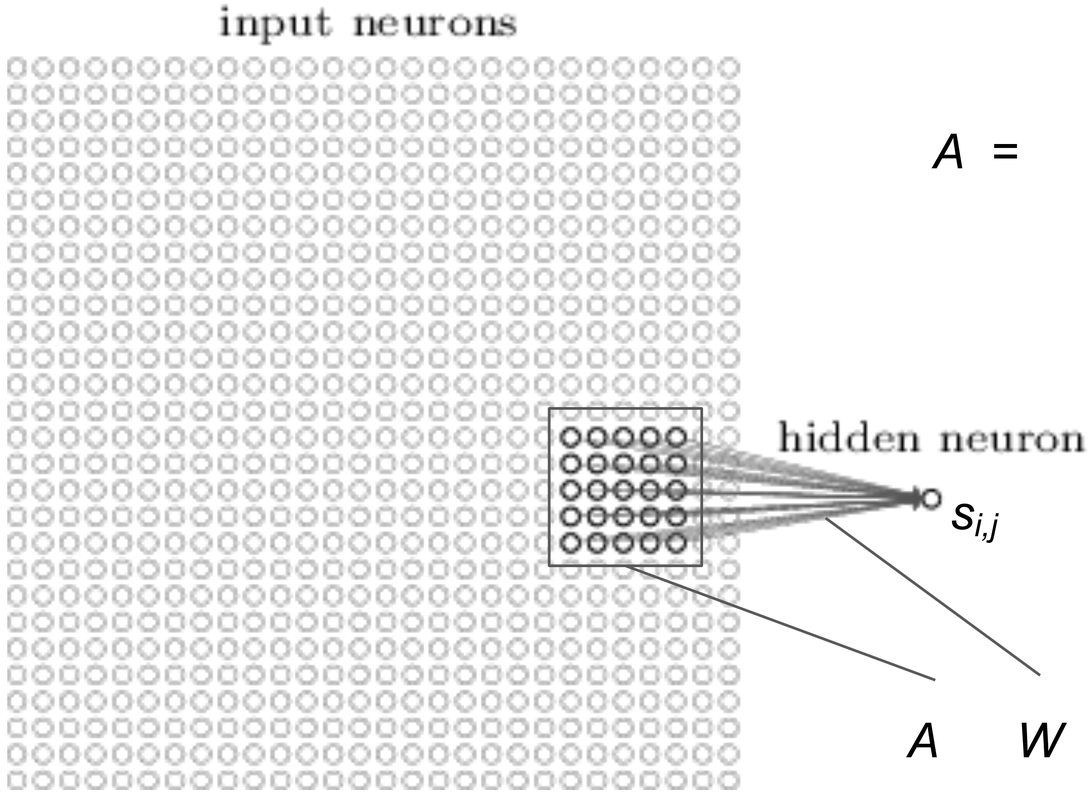
- The weights *W* connecting the neurons in the local receptive field and the hidden neuron are the same across the feature map



$$W = \begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,5} \\ w_{2,1} & w_{2,2} & w_{2,3} & w_{2,4} & w_{2,5} \\ w_{3,1} & w_{3,2} & w_{3,3} & w_{3,4} & w_{3,5} \\ w_{4,1} & w_{4,2} & w_{4,3} & w_{4,4} & w_{4,5} \\ w_{5,1} & w_{5,2} & w_{5,3} & w_{5,4} & w_{5,5} \end{bmatrix}$$

Convolution: Convolution Operation

- The values of the neurons ($s_{i,j}$) in feature map *S* are calculated with convolution. Given input image *I*, local receptive field *A*, shared weights *W and* shared bias *b,* we have*:*



$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} \\ a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} \end{bmatrix} \quad W = \begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,5} \\ w_{2,1} & w_{2,2} & w_{2,3} & w_{2,4} & w_{2,5} \\ w_{3,1} & w_{3,2} & w_{3,3} & w_{3,4} & w_{3,5} \\ w_{4,1} & w_{4,2} & w_{4,3} & w_{4,4} & w_{4,5} \\ w_{5,1} & w_{5,2} & w_{5,3} & w_{5,4} & w_{5,5} \end{bmatrix}$$

$$s_{i,j} = (I * W)_{i,j} + b = \sum_m \sum_n i_{i+m,j+n} w_{m,n} + b$$

$$s_{i,j} = \langle A, W \rangle + b = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

Convolution: Example

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{1:3,1:3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_{1,1} = (1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 2 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 3) - 2 =$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} & \\ & \end{bmatrix}$$

Convolution: Example

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{1:3,1:3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_{1,1} = (1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 2 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 3) - 2 = 4$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 & & \\ & & \\ & & \end{bmatrix}$$

Convolution: Example (2)

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{1:3,2:4} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$s_{1,2} = (0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 2 + 1 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 3) - 2 =$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} & 4 & \\ & & \\ & & \end{bmatrix}$$

Convolution: Example (2)

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{1:3,2:4} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$s_{1,2} = (0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 2 + 1 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 3) - 2 = 1$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} -4 & 1 \\ & \end{bmatrix}$$

Convolution: Example (3)

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{1:3,3:5} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$s_{1,3} = (1 \cdot 2 + 1 \cdot 3) - 2 =$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} -4 & 1 & \square \\ & & \\ & & \end{bmatrix}$$

Convolution: Example (3)

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{1:3,3:5} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$s_{1,3} = (1 \cdot 2 + 1 \cdot 3) - 2 = 3$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} -4 & 1 & 3 \\ & & \\ & & \end{bmatrix}$$

Convolution: Example (4)

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{6:8,6:8} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_{6,6} = (1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3) - 2 =$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 & 1 & 3 & 1 & -1 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ -1 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & -1 & \square \end{bmatrix}$$

Convolution: Example (4)

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, b = -2$$

$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b$$

kernel size = 3x3, stride = 1

$$A = I_{6:8,6:8} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

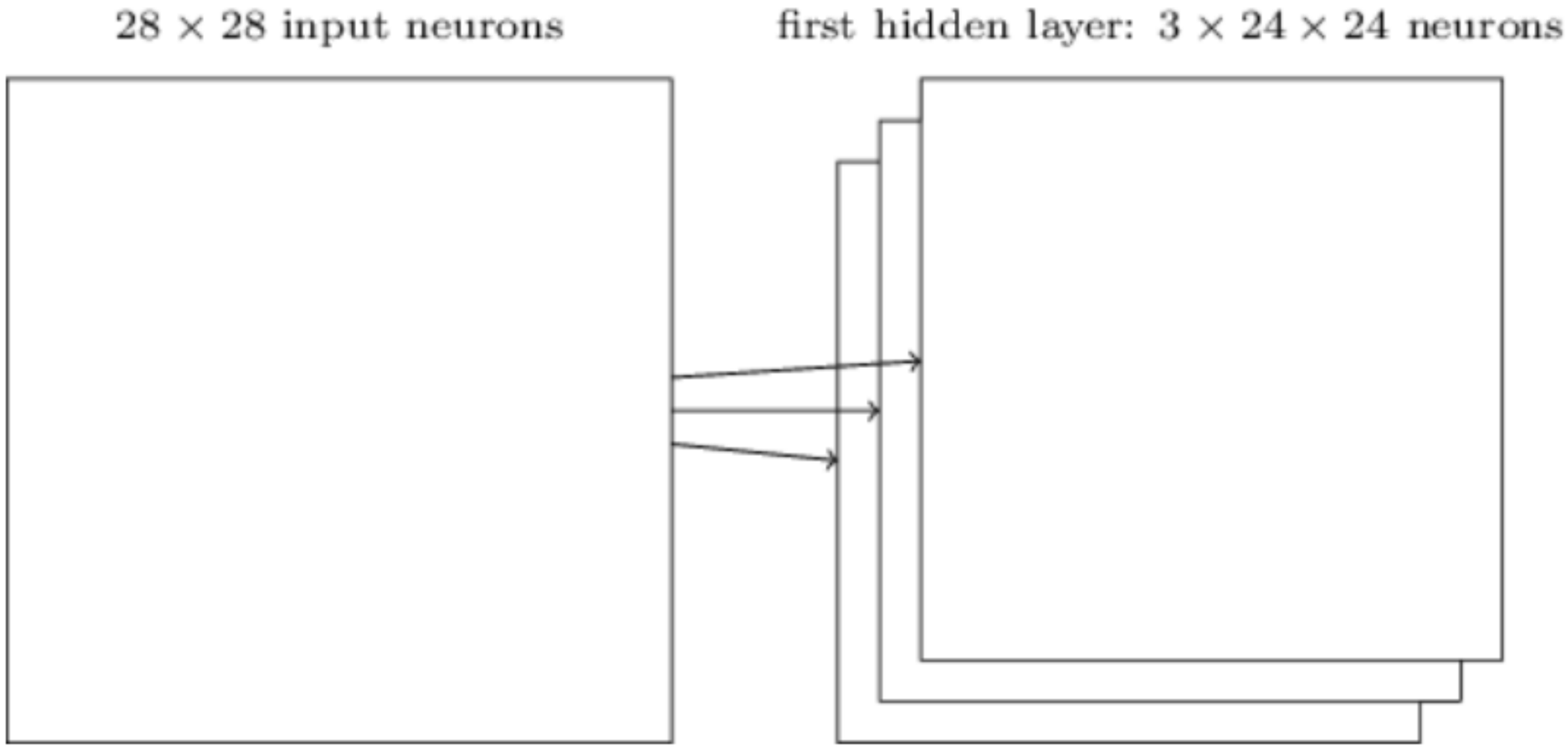
$$s_{6,6} = (1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3) - 2 = 4$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 & 1 & 3 & 1 & -1 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ -1 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & -1 & 4 \end{bmatrix}$$

Convolution: Feature Maps

- 3 5x5 kernels, each with its own set of weights, results in 3 feature maps



Convolution: Comparison

- Compared with fully-connected layer
  - **20x20** input neurons
    - Fully-connected layer with 10 neurons: (20 * 20) * 10 = **4 000 weights**
    - Convolutional layer with 10 kernels of size 3x3: (3 * 3) * 10 = **90 weights**
  - **200x200** input neurons
    - Fully-connected layer with 10 neurons: (200 * 200) * 10 = **400 000 weights**
    - Convolutional layer with 10 kernels of size 3x3: (3 * 3) * 10 = **90 weights**

What about biases?

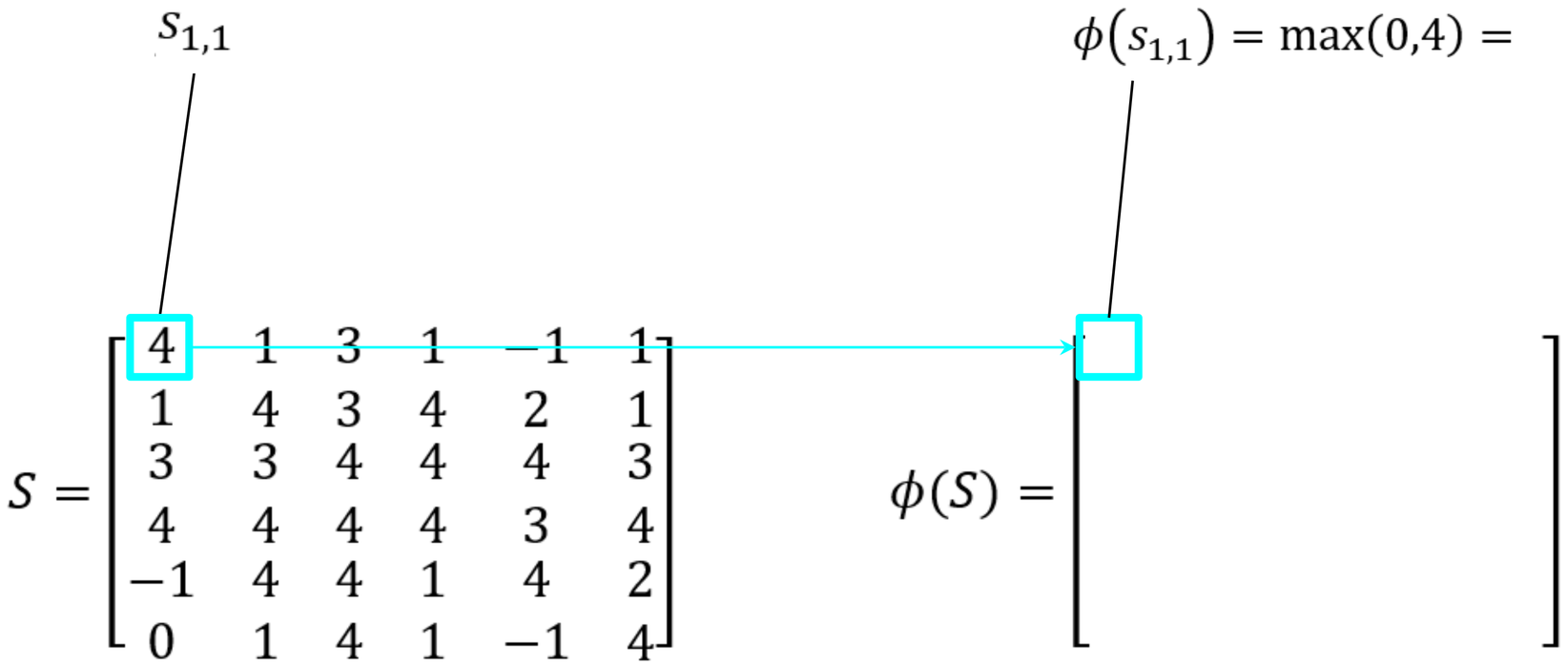
Rectified Linear Unit (ReLU)

- Following convolution a non-linearity operation is performed
  - Non-linearity to learn non-linear patterns
- ReLU
  - Replace all negative values with 0

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$

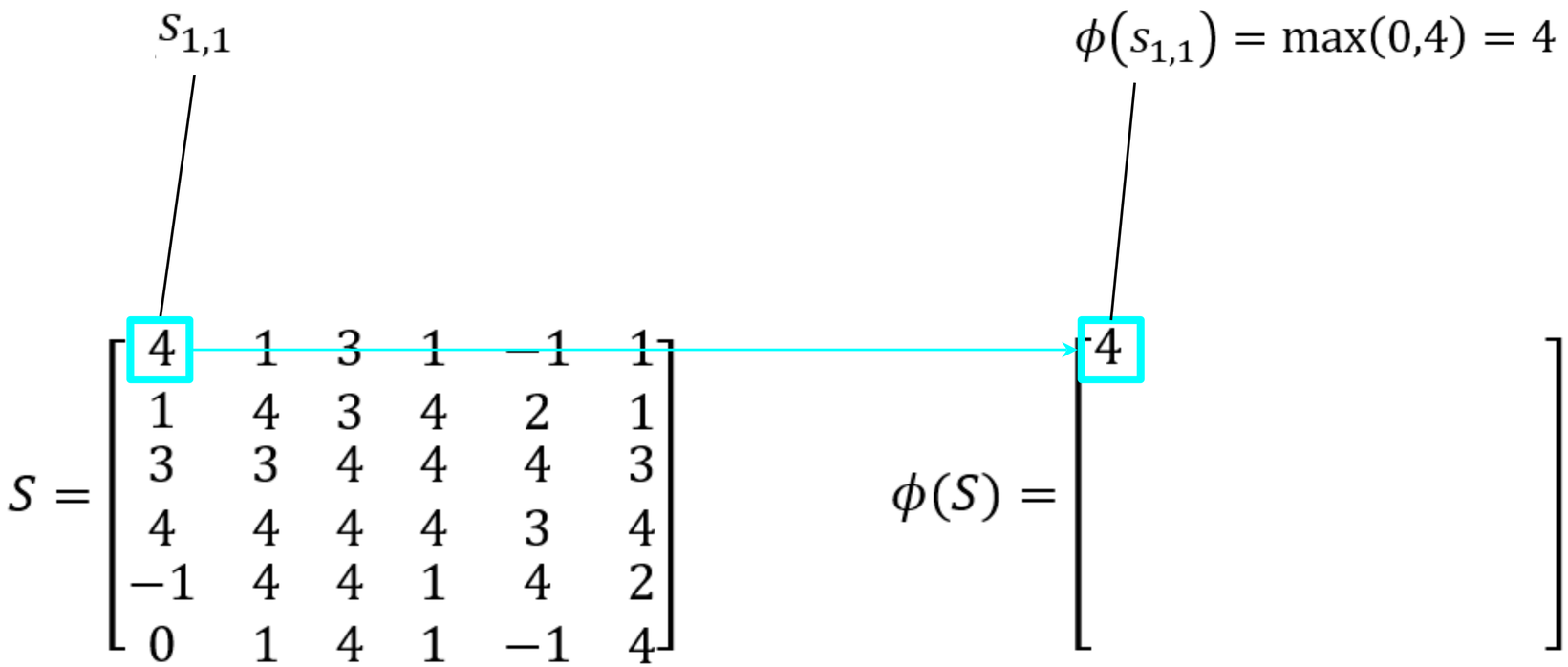
ReLU: Example

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$



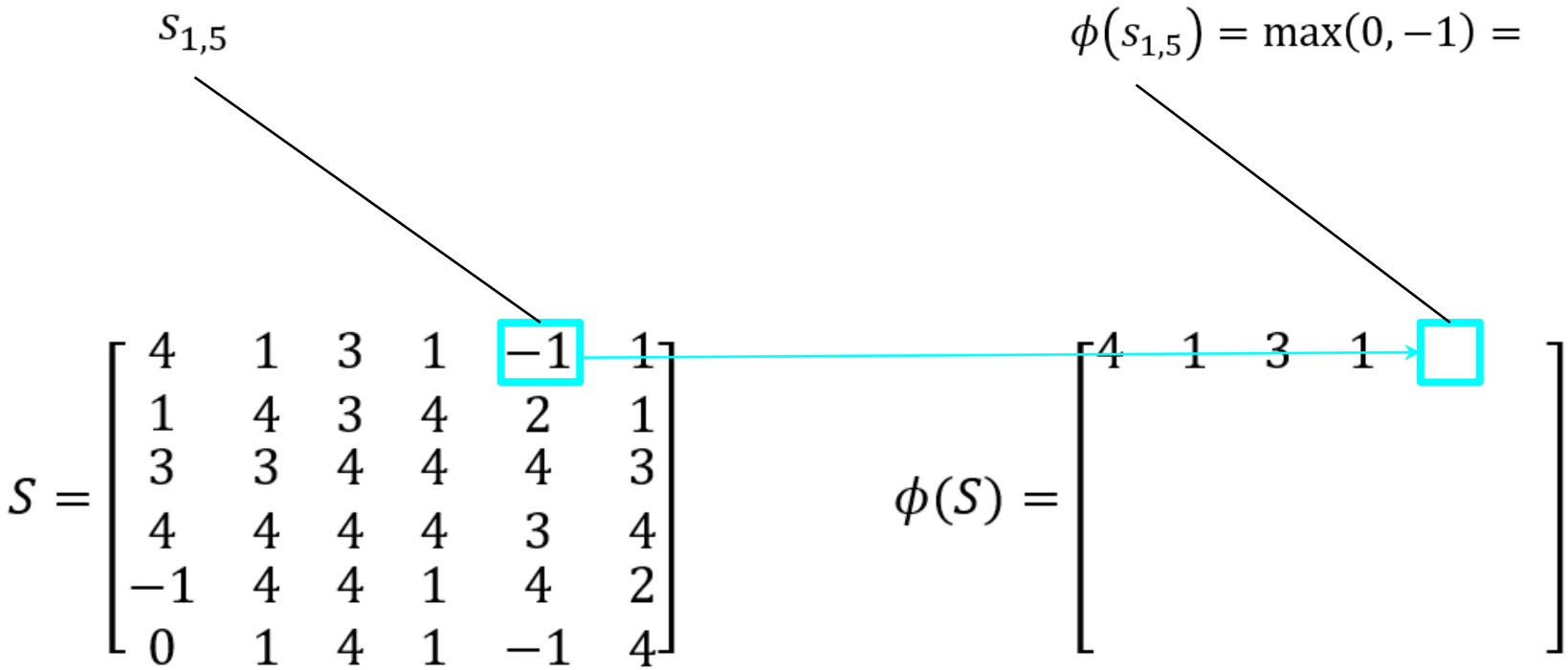
ReLU: Example

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$



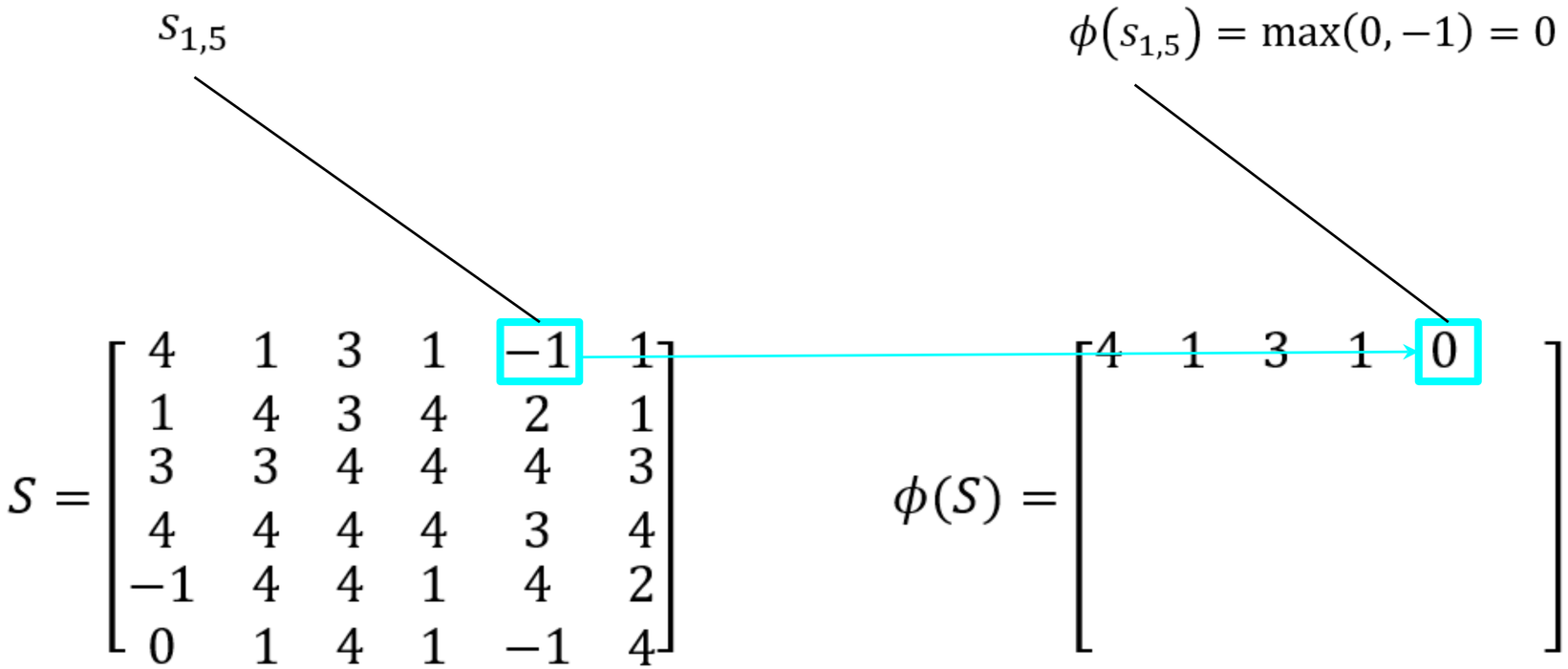
ReLU: Example (2)

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$



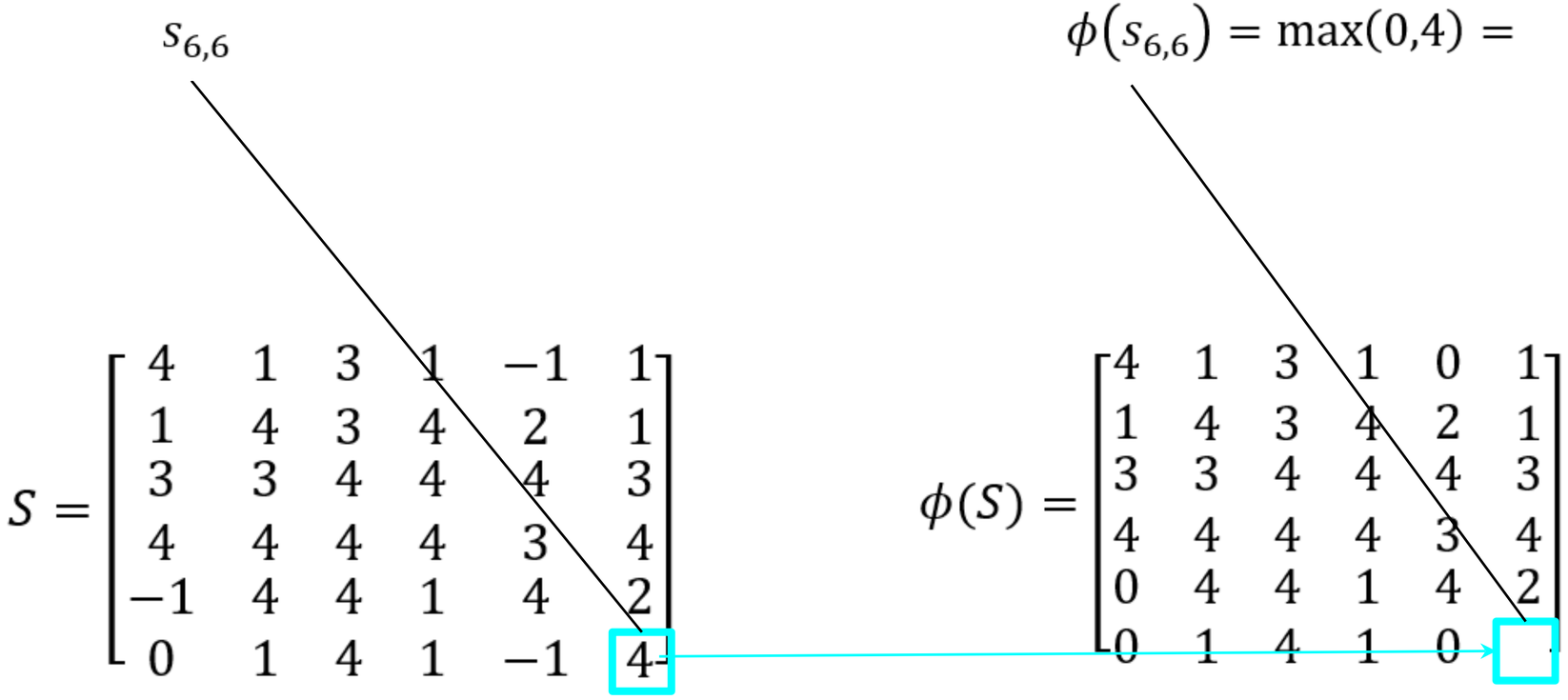
ReLU: Example (2)

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$



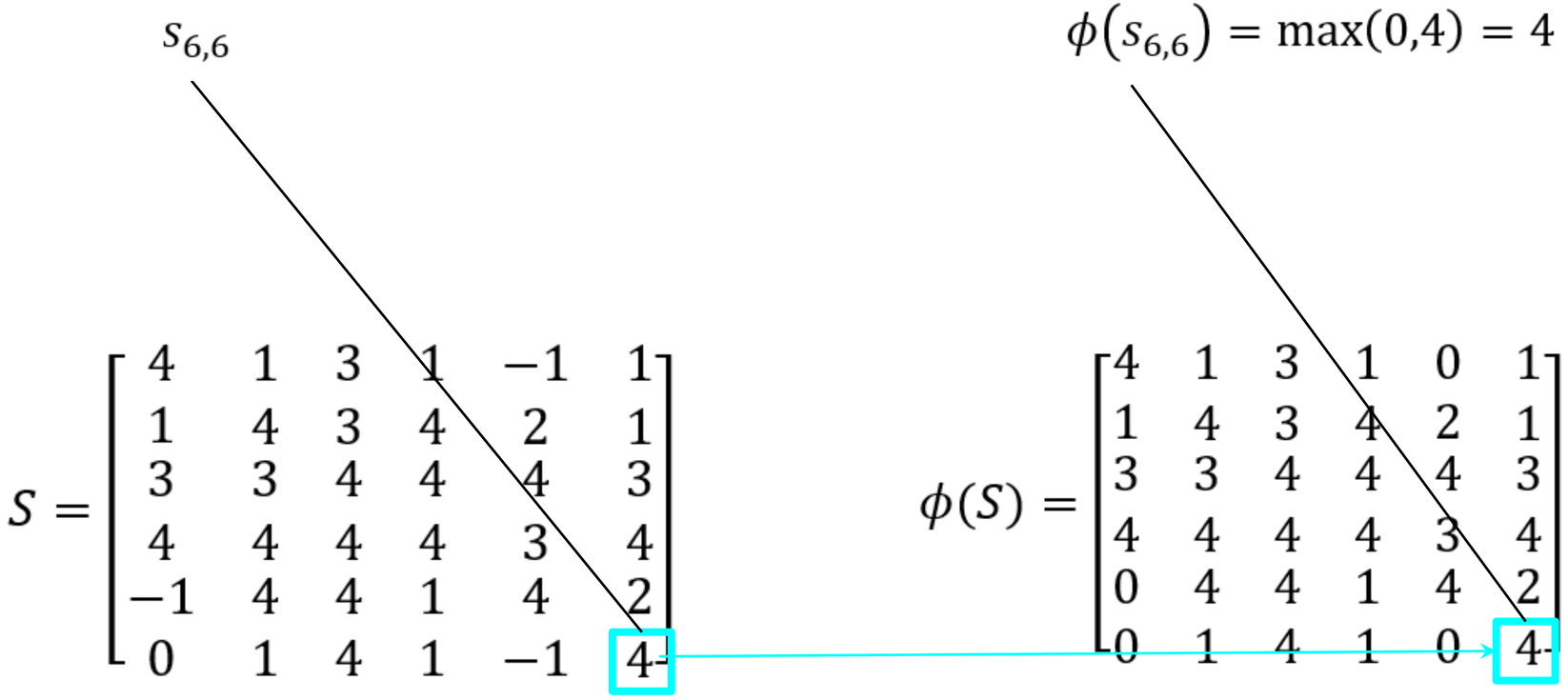
ReLU: Example (3)

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$



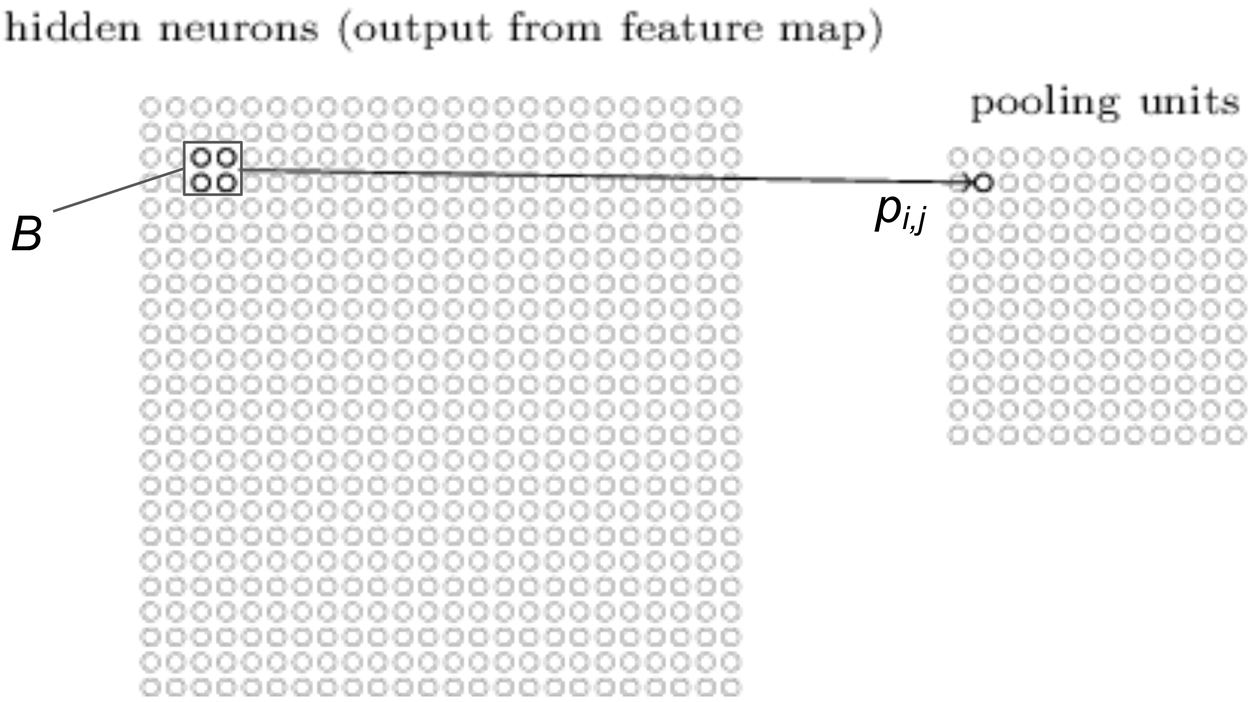
ReLU: Example (3)

$$\phi(s_{i,j}) = \max(0, s_{i,j})$$



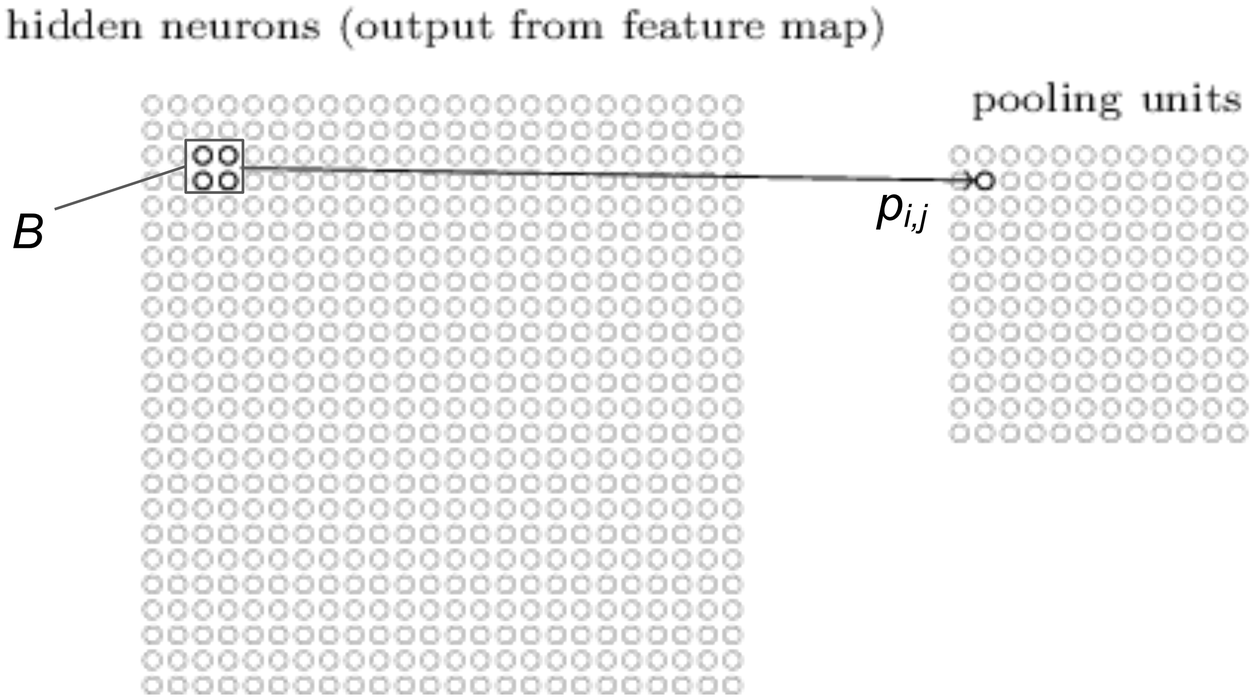
Pooling

- Reduce dimension of feature map while preserving the most important information



Pooling

- The values of the neurons ($p_{i,j}$) in the pooled feature map P are calculated as a function of the region B:



$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Max pooling: $p_{i,j} = \max(B) = \max(b_{1,1}, b_{1,2}, b_{2,1}, b_{2,2})$

Avg pooling: $p_{i,j} = \text{avg}(B) = \frac{b_{1,1} + b_{1,2} + b_{2,1} + b_{2,2}}{4}$

L2 pooling: $p_{i,j} = l2(B) = \sqrt{{b_{1,1}}^2 + {b_{1,2}}^2 + {b_{2,1}}^2 + {b_{2,2}}^2}$

Max Pooling: Example

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{1:2,1:2} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

$$p_{1,1} = \max(B) = \max\left(\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\right) =$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} \square & \end{bmatrix}$$

Max Pooling: Example

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{1:2,1:2} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

$$p_{1,1} = \max(B) = \max\left(\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\right) = 4$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & \\ & \end{bmatrix}$$

Max Pooling: Example (2)

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{1:2,3:4} = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}$$

$$p_{1,2} = \max\left(\begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}\right) =$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & \square \\ & \end{bmatrix}$$

Max Pooling: Example (2)

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{1:2,3:4} = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}$$

$$p_{1,2} = \max\left(\begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}\right) = 4$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 4 \\ & \end{bmatrix}$$

Max Pooling: Example (3)

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{1:2,5:6} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$p_{1,3} = \max\left(\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}\right) =$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 4 & \\ & & \end{bmatrix}$$

Max Pooling: Example (3)

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{1:2,5:6} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$p_{1,3} = \max\left(\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}\right) = 2$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 4 & 2 \\ & & \end{bmatrix}$$

Max Pooling: Example (4)

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{5:6,5:6} = \begin{bmatrix} 4 & 2 \\ 0 & 4 \end{bmatrix}$$

$$p_{3,3} = \max\left(\begin{bmatrix} 4 & 2 \\ 0 & 4 \end{bmatrix}\right) =$$

$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 4 \\ 4 & 4 & \square \end{bmatrix}$$

Max Pooling: Example (4)

$$p_{i,j} = \max(B)$$

region size = 2x2, stride = 1

$$B = \phi(S)_{5:6,5:6} = \begin{bmatrix} 4 & 2 \\ 0 & 4 \end{bmatrix}$$

$$p_{3,3} = \max\left(\begin{bmatrix} 4 & 2 \\ 0 & 4 \end{bmatrix}\right) = 4$$

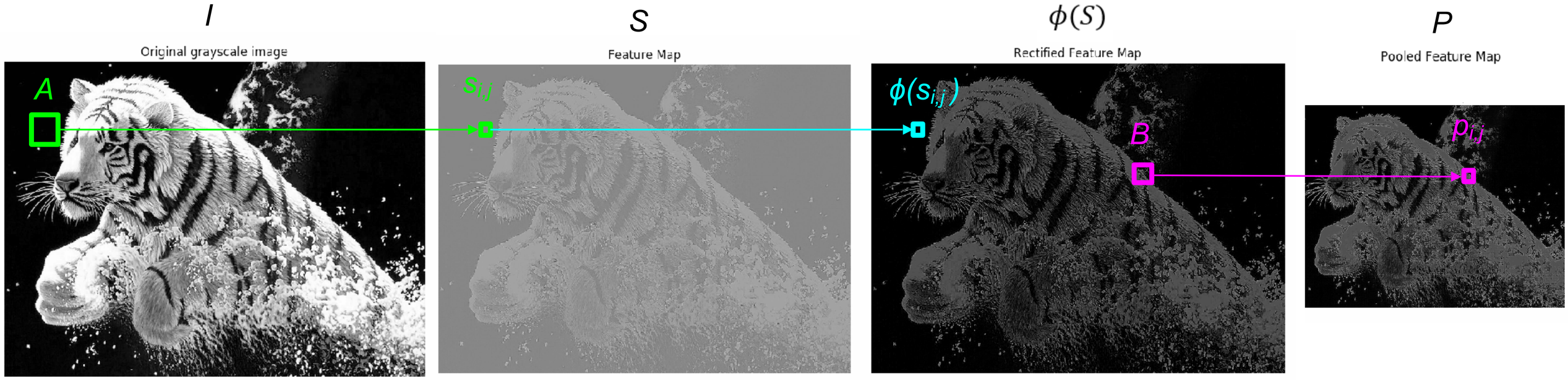
$$\phi(S) = \begin{bmatrix} 4 & 1 & 3 & 1 & 0 & 1 \\ 1 & 4 & 3 & 4 & 2 & 1 \\ 3 & 3 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 3 & 4 \\ 0 & 4 & 4 & 1 & 4 & 2 \\ 0 & 1 & 4 & 1 & 0 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{bmatrix}$$

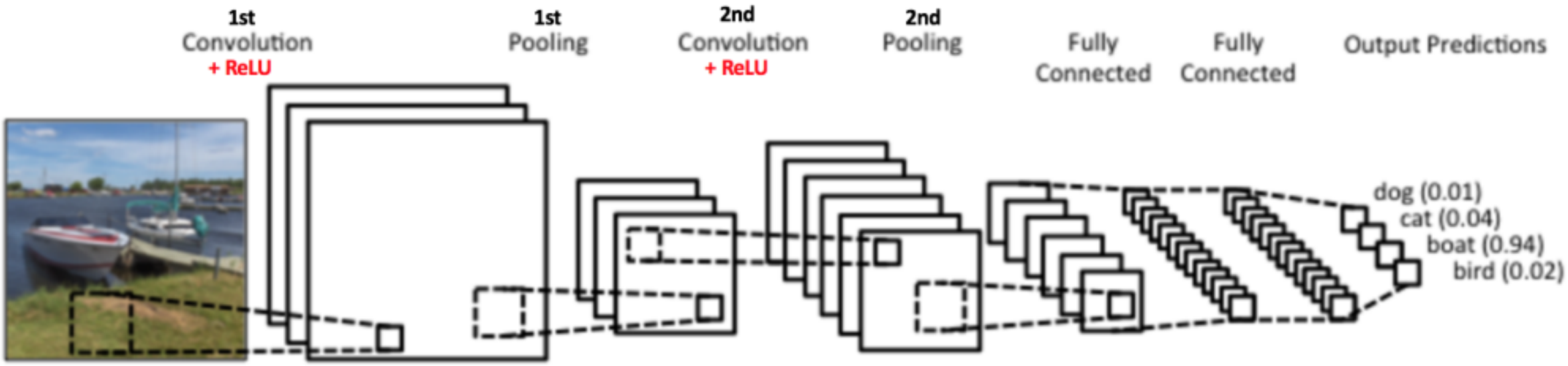
Putting the Pieces Together

$$W = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}, b = 0$$

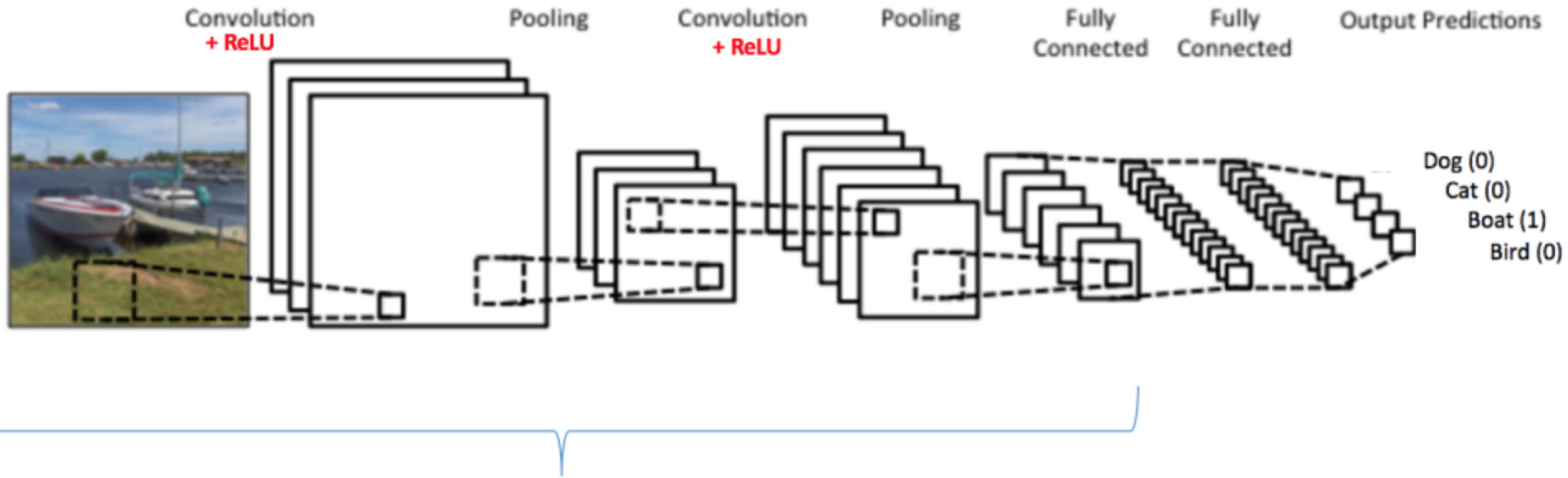
$$s_{i,j} = \sum_m \sum_n a_{m,n} w_{m,n} + b \qquad \phi(s_{i,j}) = \max(0, s_{i,j}) \qquad p_{i,j} = \max(B)$$



Putting the Pieces Together



Putting the Pieces Together



Putting the Pieces Together: **Feature Extractor**

- Combining several convolutional layers
  - The deeper in the network, the more complex structures of the image can be learnt

1st Convolution **2nd Convolution** **3rd Convolution**

Putting the Pieces Together: **Classification**

- Fully-connected layers combined with softmax in the output layer

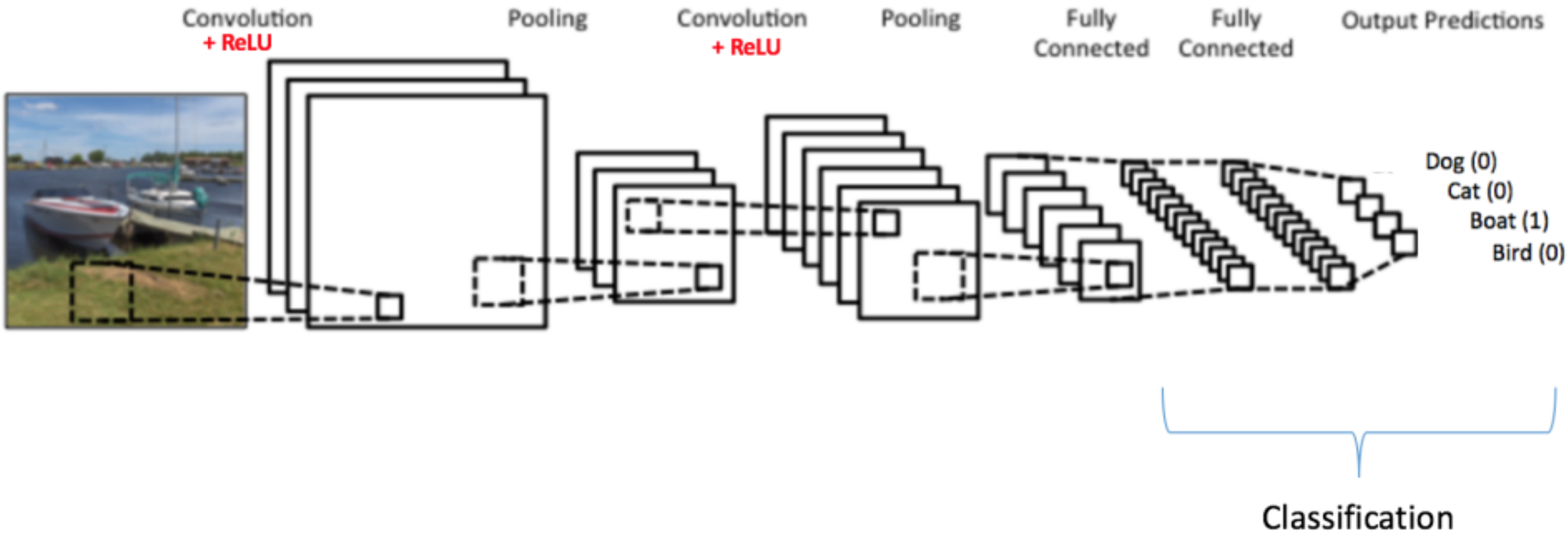


Image Classification

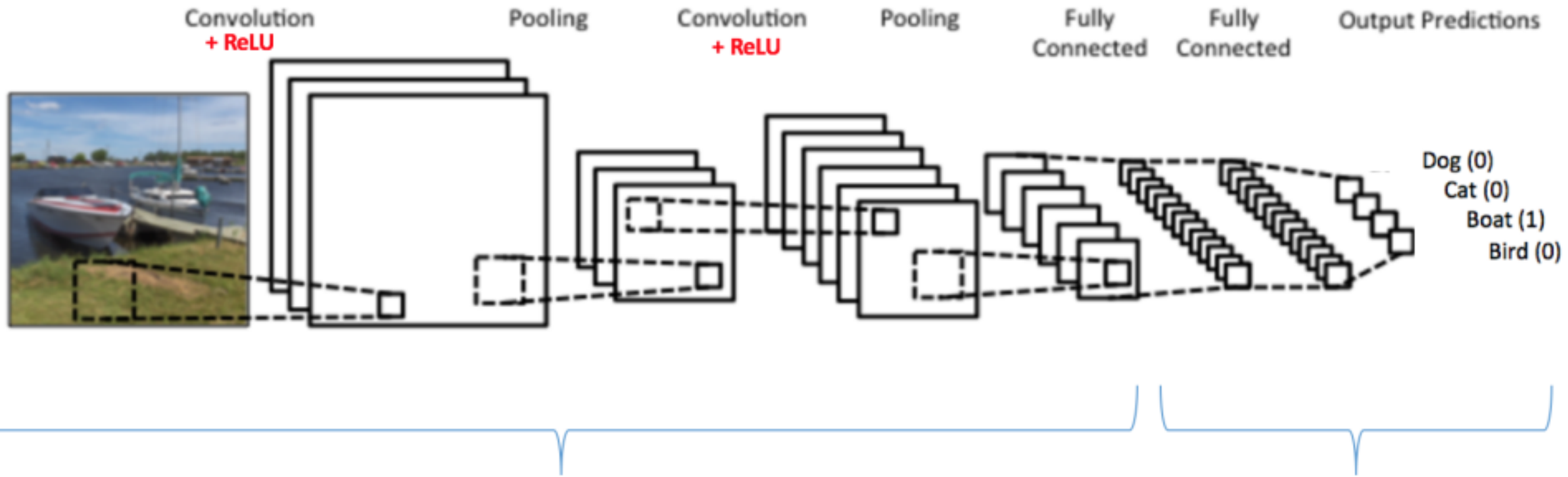


Image Classification: Example

- MNIST
  - Predict what number (0-9) an image contains

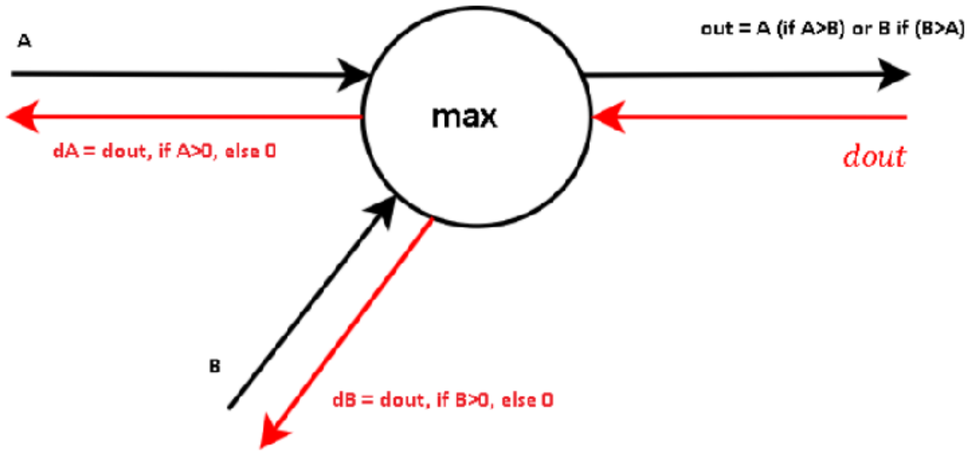
NB: Not part of the syllabus (just for orientation)

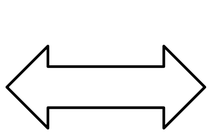
CNNs: Learning

- Updates performed using backpropagation similarly to regular NNs
- Some modifications
  - Account for weight sharing, pooling

1	1	2	4
5	**6**	7	**8**
3	2	1	0
1	2	3	**4**

Backpropagation in case of max pooling





6	8
3	4

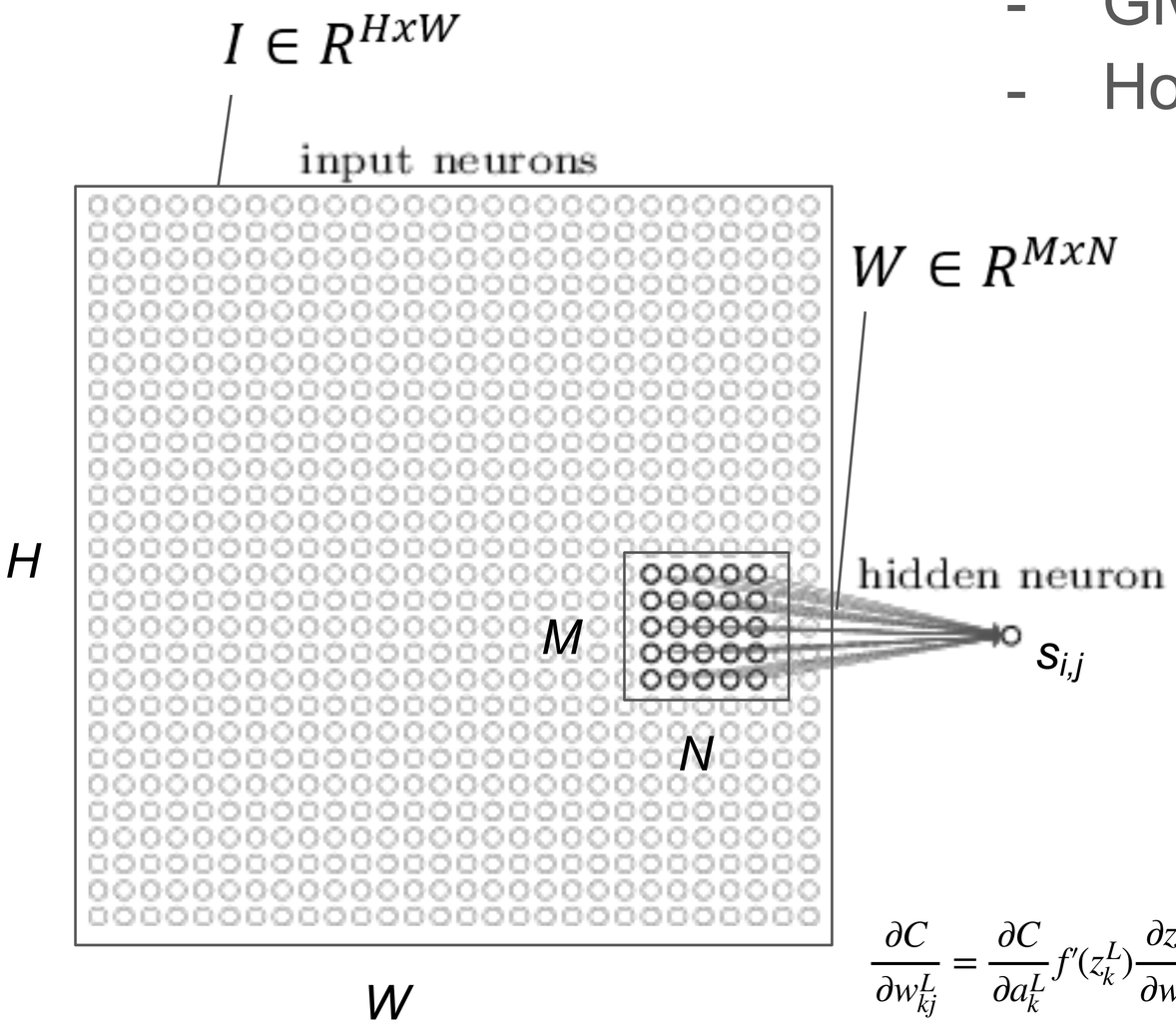
Backpropagation

0	0	0	0
0	dout	0	dout
dout	0	0	0
0	0	0	dout

NB: Not part of the syllabus (just for orientation)

CNNs: Backpropagation

- Given input map *I*, output map *S*, kernel *W*
- How to update a weight $w_{m',n'}$ in *W* ?



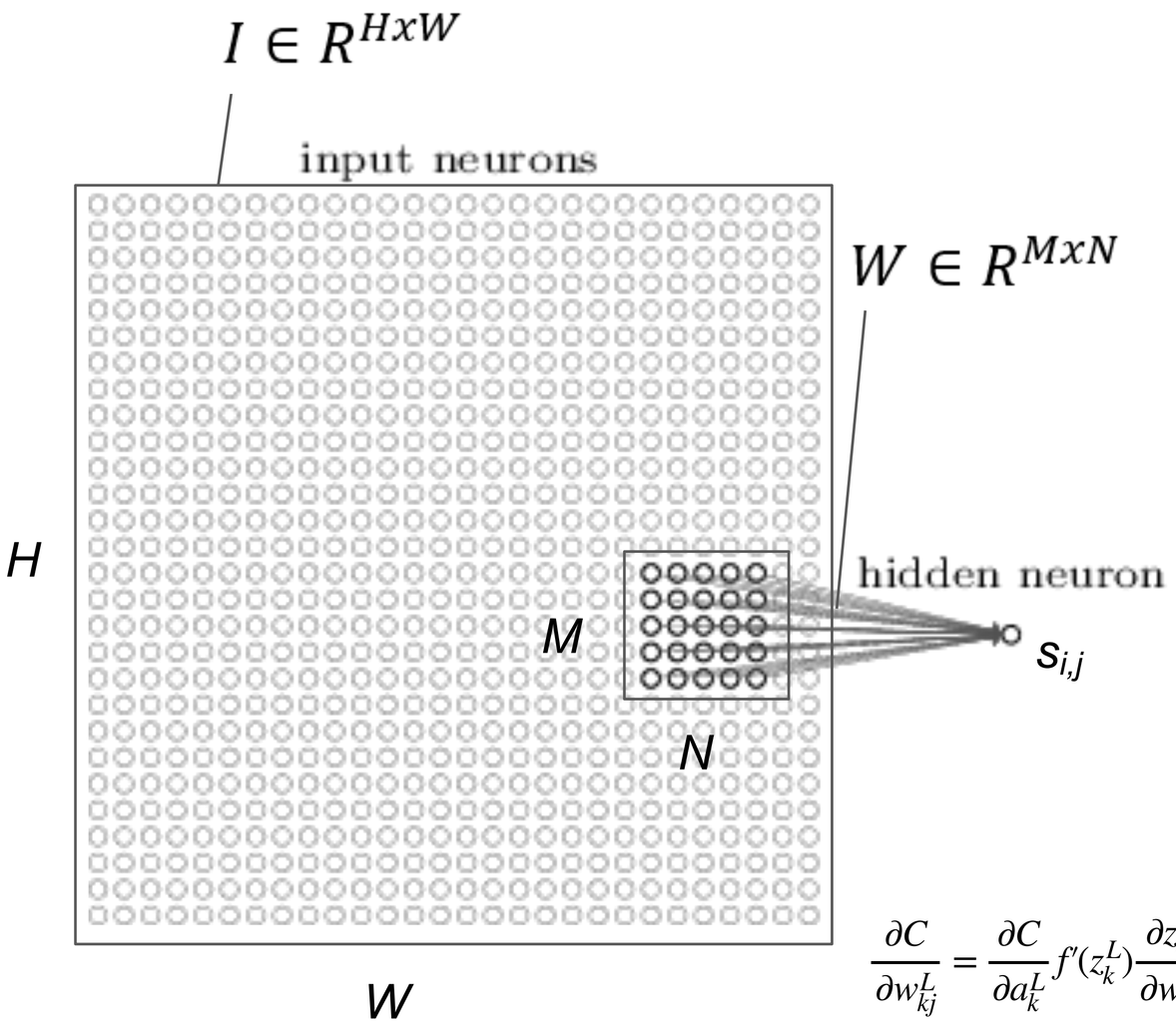
$$\frac{\partial E}{\partial w_{m\prime,n\prime}} = \sum_{i=0}^{H-M+1} \sum_{j=0}^{W-N+1} \frac{\partial E}{\partial s_{i,j}} \frac{\partial s_{i,j}}{\partial w_{m\prime,n\prime}}$$

$\frac{\partial E}{\partial s_{i,j}}$ is known, how to calculate $\frac{\partial s_{i,j}}{\partial w_{m\prime,n\prime}}$

$$\frac{\partial C}{\partial w_{kj}^L} = \frac{\partial C}{\partial a_k^L} f'(z_k^L) \frac{\partial z_k^L}{\partial w_{kj}^L} = \delta_k^L a_j^{L-1}$$

NB: Not part of the syllabus (just for orientation)

CNNs: Backpropagation (2)



- How to calculate $\frac{\partial s_{i,j}}{\partial w_{m\prime,n\prime}}$?

$$s_{i,j} = (I * W)_{i,j} + b = \sum_{m=1}^{M} \sum_{n=1}^{N} i_{i+m,j+n} w_{m,n} + b$$

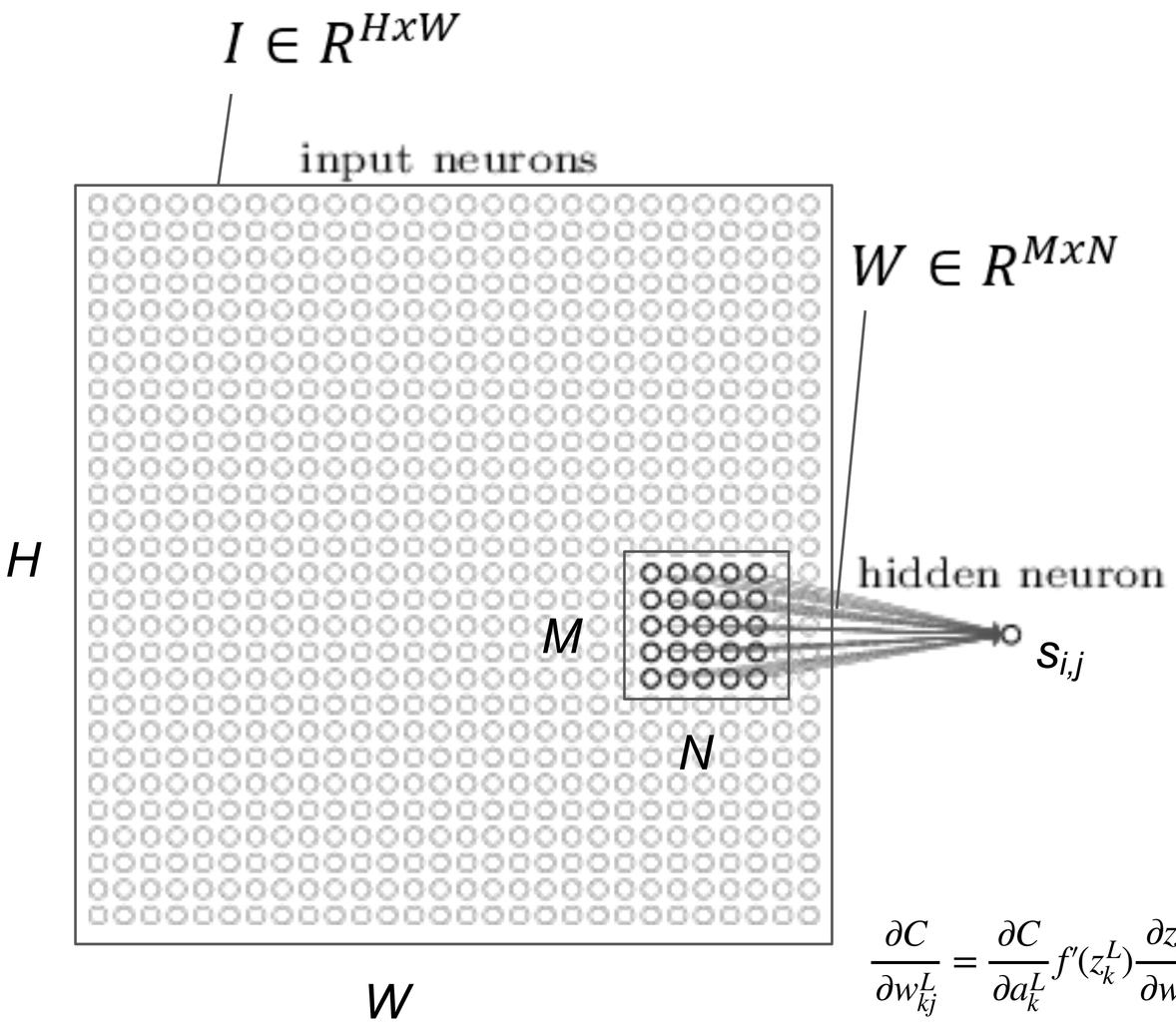
$$\frac{\partial s_{i,j}}{\partial w_{m\prime,n\prime}} = \frac{\partial}{\partial w_{m\prime,n\prime}} \sum_{m} \sum_{n} i_{i+m,j+n} w_{m,n} + b$$

$$= \frac{\partial}{\partial w_{m',n'}} \left(i_{i+m',j+n'} w_{m',n'} \right) = i_{i+m',j+n'}$$

$$\frac{\partial C}{\partial w_{kj}^L} = \frac{\partial C}{\partial a_k^L} f'(z_k^L) \frac{\partial z_k^L}{\partial w_{kj}^L} = \delta_k^L a_j^{L-1}$$

NB: Not part of the syllabus (just for orientation)

CNNs: Backpropagation (3)



- We know that $\frac{\partial s_{i,j}}{\partial w_{m\prime,n\prime}} = i_{i+m',j+n'}$

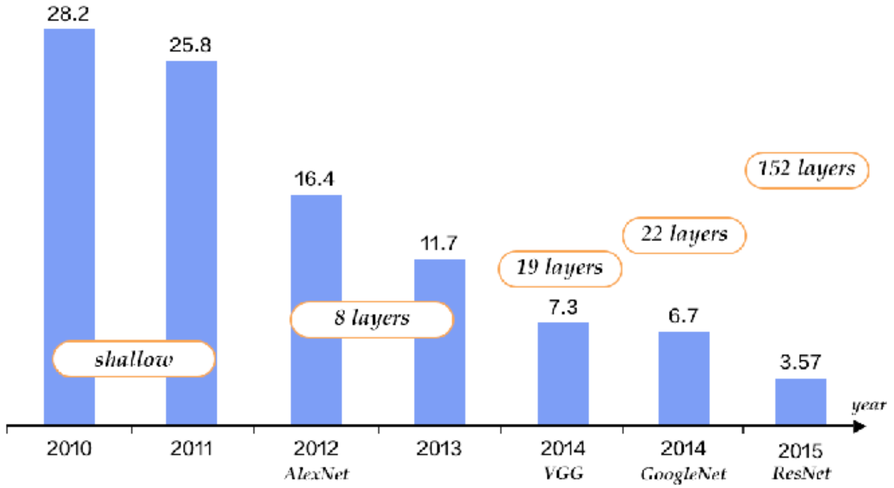
$$\frac{\partial E}{\partial w_{m\prime,n\prime}} = \sum_{i=1}^{H-M+1} \sum_{j=1}^{W-N+1} \frac{\partial E}{\partial s_{i,j}} \frac{\partial s_{i,j}}{\partial w_{m\prime,n\prime}}$$

$$= \sum_{i=1}^{H-M+1} \sum_{j=1}^{W-N+1} \frac{\partial E}{\partial s_{i,j}} i_{i+m',j+n'}$$

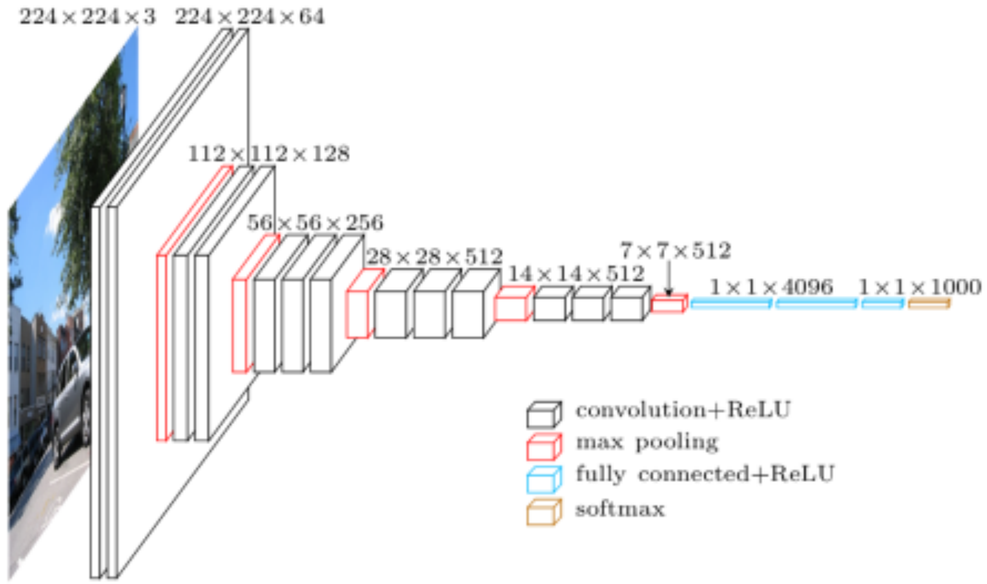
$$\frac{\partial C}{\partial w_{kj}^L} = \frac{\partial C}{\partial a_k^L} f'(z_k^L) \frac{\partial z_k^L}{\partial w_{kj}^L} = \delta_k^L a_j^{L-1}$$

Recent Progress

- GPUs accelerates training of deep neural networks
  - Parallell computations
- Larger datasets
  - ImageNet = Millions of images
- ImageNet competition (ILSVRC)
  - Development of high performing CNNs
  - From AlexNet to ResNet



VGG-16



Recent Progress (2)

- More complex tasks
  - Predict 1000 categories
  - Localization, Object detection and Segmentation

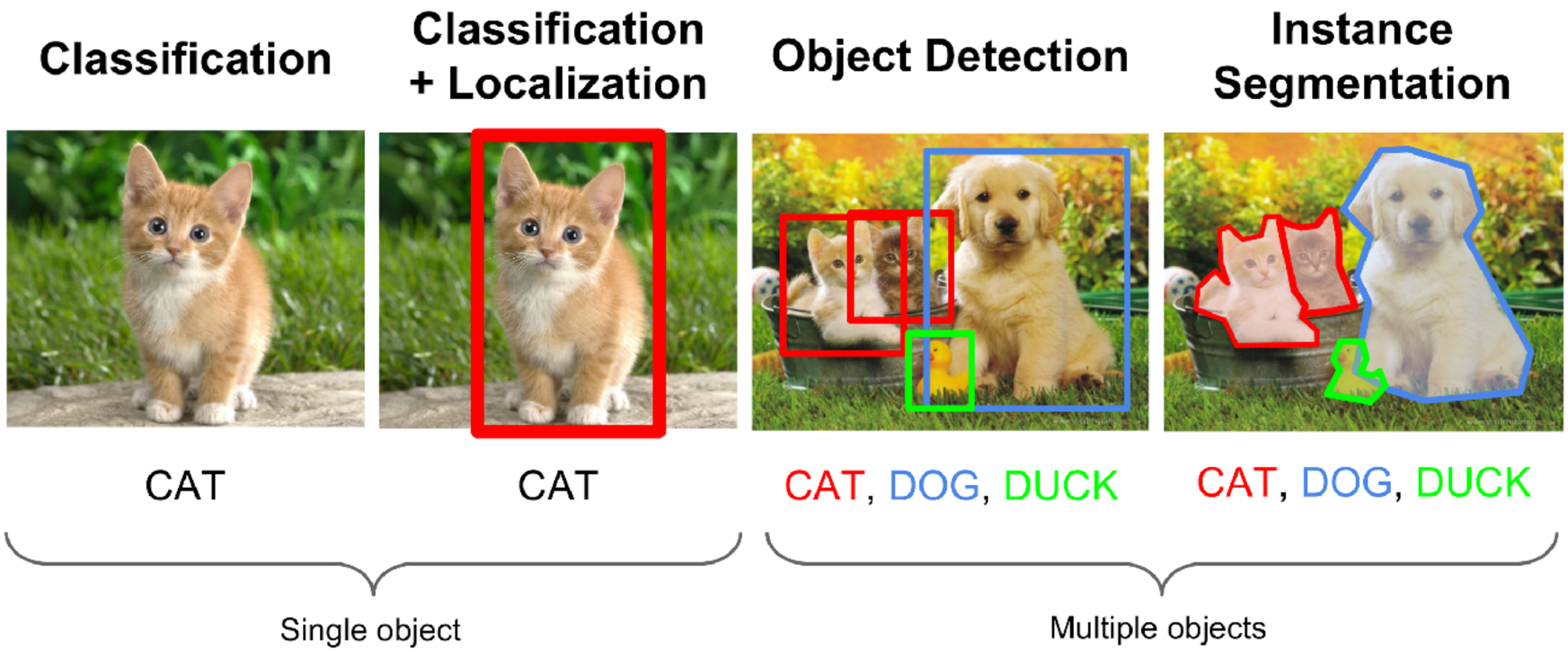
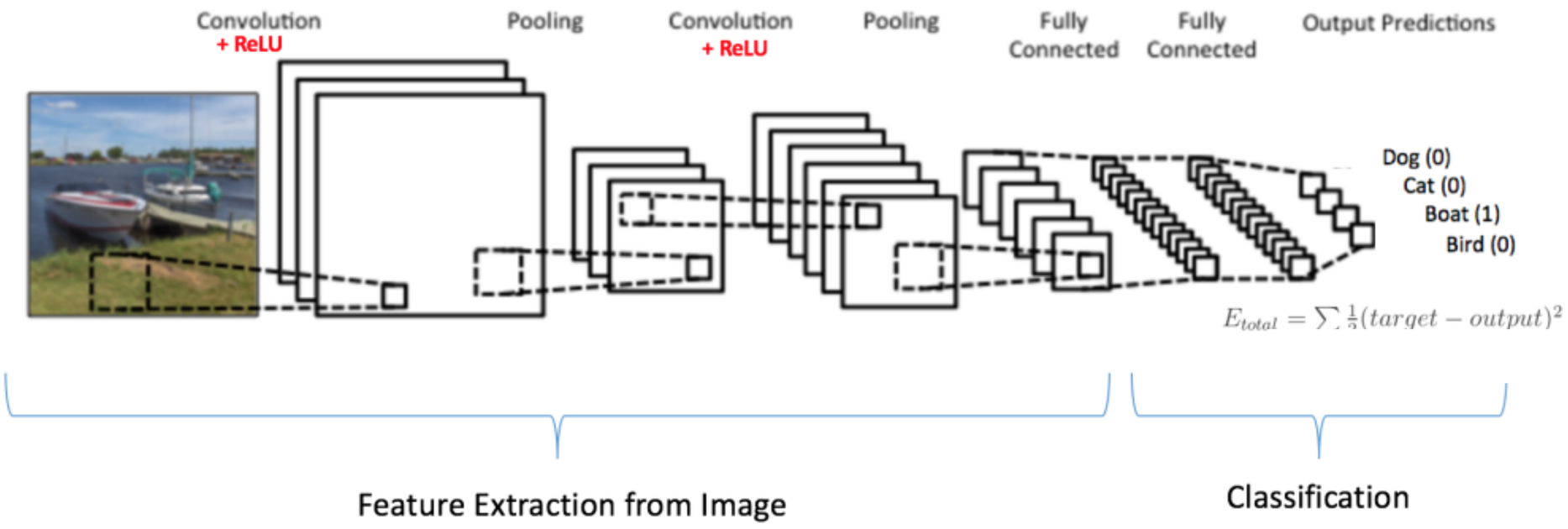
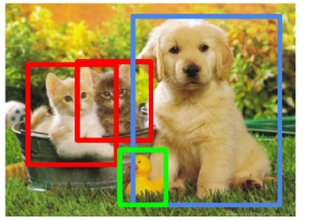


Image Classification (base net)



Classification

Object Detection



Instance Segmentation

Object detection: Faster R-CNN (SSD, YOLO2, etc.)

Image segmentation: Mask R-CNN (FCN, U-net, etc.)

Base model for advanced CV