TTK4190 Guidance and Control

Exam Draft Solution

Spring 2013

Problem 1

A) Derivating the position vector w.r.t. gives:

$$\mathbf{v}_t^n = \dot{\mathbf{p}}_t^n = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \tag{1}$$

B) Since the heading angle ψ is held constant, we have $\dot{\psi} = r = 0$. The 3 states of the system can be written in vector form as:

$$\boldsymbol{\nu}_t^n = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix} \tag{2}$$

which can be expressed in the body-frame:

$$\boldsymbol{\nu}_t^b = \begin{bmatrix} \cos(30°) & \sin(30°) & 0 \\ -\sin(30°) & \cos(30°) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 11.16 \\ -0.6699 \\ 0 \end{bmatrix} \tag{3}$$

C) It is possible to compute the course angle of the vehicle from the velocity vector given in the NED frame:

$$\chi = \text{arctan2}\left(\frac{5}{10}\right) = 26.56°. \tag{4}$$

Consequently, the sideslip angle is:

$$\beta = \chi - \psi = 26.56° - 30° = -3.44°. \tag{5}$$

D) The total speed can be computed as:

$$U = \sqrt{u^2 + v^2} \approx 11.18 \text{ m/s}. \tag{6}$$

E) Substituting T yields:

$$(m - x_{\dot{u}})\dot{\tilde{u}} - x_{|u|u}|u|\tilde{u} + K_d\tilde{u} = 0, \tag{7}$$

where $\tilde{u} = u - u_d$. The Lyapunov Function Candidate (LFC):

$$V = \frac{1}{2}(m - x_{\dot{u}})\tilde{u}^2 > 0, \quad \forall\, \tilde{u} \neq 0. \tag{8}$$

Derivating w.r.t. gives:

$$\dot{V} = -(K_d - x_{|u|u}|u|)\tilde{u}^2 < 0, \quad \forall\, \tilde{u} \neq 0, \tag{9}$$

which implies that the system (7) is Globally Asymptotically Stable (GAS) since $K_d - x_{|u|u} > 0$.

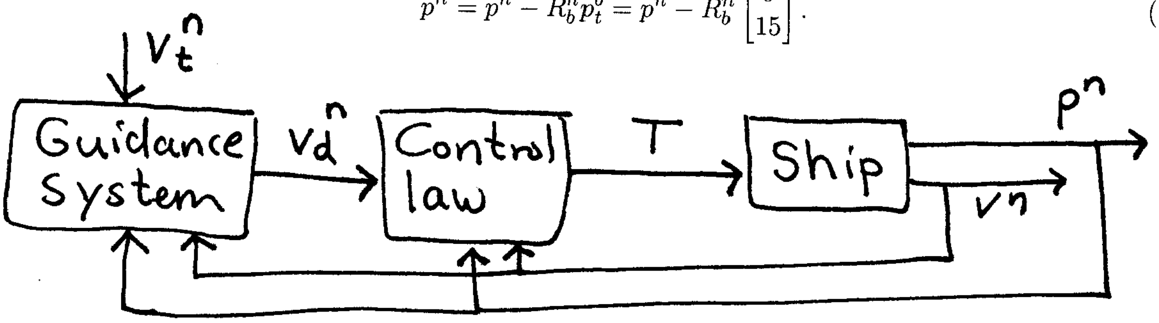
F) The desired velocity in the NED frame can be written as follows:

$$v_d^n = v_t^n + v_a^n, \tag{10}$$

$$v_a^n = -\kappa\frac{\tilde{p}^n}{||\tilde{p}^n||}, \tag{11}$$

where $\kappa > 0$ and

$$\tilde{p}^n = p^n - R_b^n p_t^b = p^n - R_b^n \begin{bmatrix} 0 \\ 15 \end{bmatrix}. \tag{12}$$



Problem 2

A) First, we use the path parametrization equations to eliminate the parameter t:

$$x_d = x_0 + 2(y_d - y_0) \Leftrightarrow \tag{13}$$

$$y_d - y_0 = \frac{1}{2}(x_d - x_0) \tag{14}$$

therefore

$$\tan(\alpha_k) = \frac{1}{2} \Leftrightarrow \tag{15}$$

$$\alpha_k = 26.56^\circ. \tag{16}$$

B) Sideslipping refers to the case where the heading angle of the ship is different compared to its course angle. This occurs due to a nonzero sway velocity which, in this particular case, is a result of the external disturbances that are acting on the ship. Therefore, when a ship is assigned to follow a path (a straight line, for instance) under the influence of external forces, it will have to adjust its heading angle in a way such that the course angle (orientation of the velocity vector) will coincide with the tangent of the desired path.

C) Yes, a ship will also sideslip when there are no external disturbances present. This happens during a turn because of the nonzero sway velocity that is developed due to the lateral acceleration. Note that if the ship was to move just on a straight line, then there would be no sideslip angle, but this is not a realistic assumption for any vessel. If the ship is highly maneuverable then the lateral acceleration will be higher and more noticeable. In those cases, if high performance is required, the sideslip angle should be taken into account and compensated for. In order to demonstrate the difference between the sideslip angle due to external disturbances and the sideslip angle due to lateral acceleration, the following two equations can be mentioned, respectively:

$$\beta = \text{arctan2}\left(\frac{v}{u}\right) \tag{17}$$

$$\beta_r = \text{arctan2}\left(\frac{v_r}{u_r}\right) \tag{18}$$

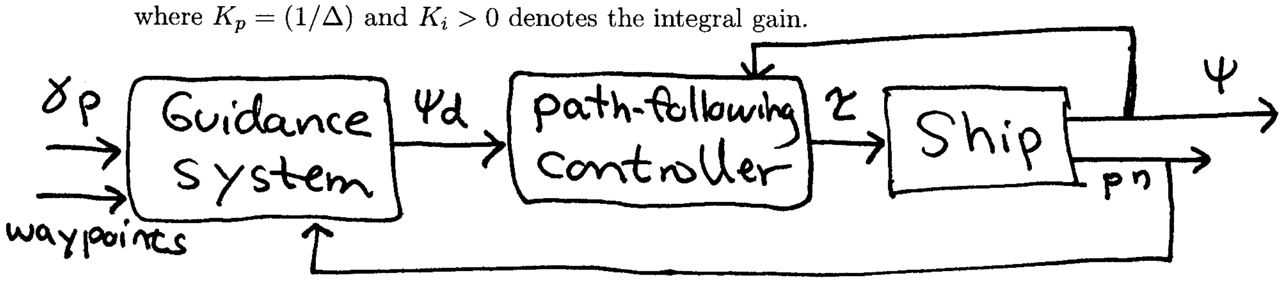
D) This problem can be solved in different ways, depending on the measurement data available and whether it is possible to control the course angle, or just the heading angle of the ship. First, a guidance strategy has to be designed that will generate the necessary reference trajectories for the controller. Since the problem refers to a path following scenario Assuming a surface vessel with GNNS position and velocity measurements available, it is possible to remove the uncertainty due to the sideslip angle by controlling the course angle directly. A line-of-sight (LOS) guidance law to demonstrate this would be:

$$\chi_d = \alpha_k + \arctan\left(\frac{-y_e}{\Delta}\right). \tag{19}$$

Equation (19) presupposes that the course angle can be available for feedback (via GNNS velocity measurements). However this is not always possible or easy to implement in practice (path following of AUV's, for instance). In that case it is possible to control the heading angle and consider the sideslip angle to be a disturbance. Then, the guidance system can be augmented with integral action in order to adjust the heading angle in a way such that the cross-track error will converge to zero:

$$\psi_d = \gamma_p + \arctan\left(-K_p y_e - K_i \int_{t_0}^{t} y_e d\tau\right). \tag{20}$$

where $K_p = (1/\Delta)$ and $K_i > 0$ denotes the integral gain.



Problem 3

A) Since it is possible to know the direction of the magnetic field at any location on Earth, you can align the xyz-axes in the NED-directions on your desk. Then the magnetometer measures m^n. In addition to this, $\mathbf{m}^n = \mathbf{R}_b^n(\phi, \theta, \psi)\mathbf{m}^b$ and for $\mathbf{R}_b^n(0,0,0) = \mathbf{I} \Rightarrow \quad \mathbf{m}^n = \mathbf{m}^b$.

B) An IMU acceleration measurement including bias can be written as:

$$a_{\text{imu}}^b = a^b + b_{\text{acc}}. \tag{21}$$

Integrating (21) will result in drift in the velocity and position computation. More specifically, the drift is proportional with time t for velocity and $(1/2)t^2$ for position. This will occur in any case, but the bias might vary in different operational conditions.

C) Assuming that the accelerometer biases are compensated for and that the 3-axis accelerometer measurements vector is:

$$a_{\text{imu}}^b = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \tag{22}$$

then the roll and pitch angle can be computed as:

$$\phi = \operatorname{atan}\left(\frac{a_y}{a_z}\right), \tag{23}$$

$$\theta = -\operatorname{atan}\left(\frac{a_z}{\sqrt{a_x^2 + a_z^2}}\right). \tag{24}$$

However, the IMU acceleration measurements can be written in the following form:

$$a^b_{\text{imu}} = R^n_b(\dot{\mathbf{v}}^n + g^n) + b^b_{\text{acc}}. \tag{25}$$

During a hard turn $\dot{\mathbf{v}}^n \neq 0$ and therefore the formulas for θ, ϕ will be inaccurate.

D) The gyroscopes measure $\boldsymbol{\omega}^b_{ib}$. Since $\dot{\boldsymbol{\Theta}} = \mathbf{T}_\Theta(\boldsymbol{\Theta})\boldsymbol{\omega}^b$ it is not possible to calculate $\boldsymbol{\Theta}$ from $\boldsymbol{\omega}^b_{ib}$. The integral of $\boldsymbol{\omega}^b_{ib}$ is not angular velocity, both statements are wrong.

E) If the GNNS position measurements become unavailable the position measurements will start to drift. This will not affect the velocity estimation since the velocity measurements are available for feedback.

Problem 4

A) We can use the following simple model structure:

$$C_L \approx C_{L0} + C_{L\alpha} = 0.2 + 0.0134\alpha, \tag{26}$$

where

$$C_{L\alpha} = \left.\frac{\partial C_L(\alpha)}{\partial \alpha}\right|_{\alpha=0}. \tag{27}$$

Linearizing the drag equation for $\alpha = 0$ we get $\dot{C}_D = 0$. Therefore we can linearize at $\alpha = (15-0)/2 = 7.5$ deg (Other values can also be accepted). Consequently:

$$C_{D_\alpha} \approx 2 * 0.0035\alpha\big|_{\alpha=7.5} = 0.0525. \tag{28}$$

and finally

$$C_D(\alpha) \approx C_{D_\alpha}\alpha. \tag{29}$$

The comparison between the linearized equations and their nonlinear counterparts can be seen in Fig. 1.

B) Stall is the maximum $C_L(\alpha)$ value, or

$$\frac{\partial C_L(\alpha)}{\partial \alpha} = 0. \tag{30}$$

This occurs for $\alpha = 15$ deg, therefore $C_L(15) = 1.5$.

C) According to Eq. (2.86) from the aircraft lecture notes:

$$\begin{bmatrix} C_D \\ 0 \\ C_L \end{bmatrix} = R^{\mathrm{T}}(\alpha) \begin{bmatrix} -C_X \\ 0 \\ -C_Z \end{bmatrix}, \Leftrightarrow \tag{31}$$

$$\begin{bmatrix} C_X \\ 0 \\ C_X \end{bmatrix} = -R^{\mathrm{T}}(\alpha) \begin{bmatrix} C_D \\ 0 \\ -C_L \end{bmatrix}, \tag{32}$$

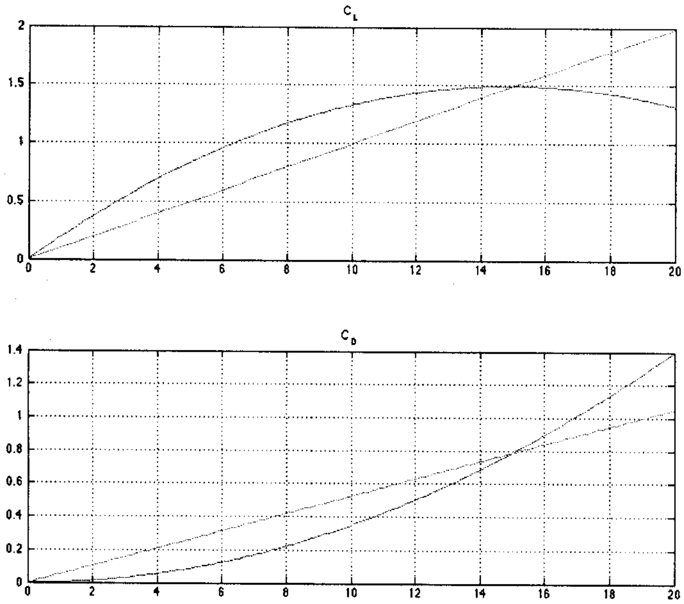


Figure 1:

where X and Z are aerodynamic forces in BODY frame due to lift and drag, and finally:

$$C_X = -\cos(\alpha)C_D + \sin(\alpha)C_L, \tag{33}$$
$$C_Z = -\sin(\alpha)C_D - \cos(\alpha)C_L. \tag{34}$$

D) Since the forward speed time derivative and the applied thrust force are given by the equations:

$$\dot{U} = VR - WQ - g\sin(\theta) + \frac{T}{m} + \frac{\bar{q}S}{m}C_X(\alpha), \tag{35}$$
$$T = K_t\rho_\alpha n^2 D^4. \tag{36}$$

Choosing the following control law:

$$T = m(-VR + WQ + g\sin(\theta) - \frac{\bar{q}S}{m}C_X(\alpha)) + \dot{U}_d - K_p(U - U_d). \tag{37}$$

yields

$$\dot{\tilde{U}} + K_p\tilde{U} = 0. \tag{38}$$

Therefore the propeller RPM input, n can be computed:

$$n = \left|\sqrt{\frac{T}{K_t\rho_\alpha n^2 D^4}}\right| > 0. \tag{39}$$

E) See Eq. (2.130) in the lecture notes:

$$\gamma := U(\theta - \alpha). \tag{40}$$

F) The solution should be based on the model:

$$\dot{\Theta} = Q \tag{41}$$

$$\dot{Q} = \frac{I_x - I_z}{I_y} PR + \frac{\bar{q}S\bar{c}}{I_y} C_m(\alpha) + b_2\delta_E. \tag{42}$$

with the control objective:

$$\gamma := U(\theta - \alpha). \tag{43}$$

There are several ways this can be solved, one of them being backstepping, look for instance Section 13.3 in the textbook.

G) The procedure is exactly the same as that of a conventional horizontal LOS guidance law.