NTNU

Department of Engineering Cybernetics

Examination paper for TTK4190 Guidance and Control of Vehicles

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- All printed and hand-written support material is allowed.

Other information: All calculators are approved.

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Informasjon om trykking av eksamensoppgave

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Date Signature

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Problem 1: Inertial Navigation System Aided by GNSS (35%)

Consider the kinematic equations:

$$\dot{\boldsymbol{p}}^n = \boldsymbol{R}(\psi)\boldsymbol{v}^b \tag{1}$$
$$\dot{\psi} = r \tag{2}$$

where $\boldsymbol{p}^n = [x, y, z]^\top$ and $\boldsymbol{v}^b = [u, v, w]^\top$. Assume that you measure:

$$\boldsymbol{p}^n_{gnss} = \boldsymbol{p}^n + \boldsymbol{v}_1 \tag{3}$$
$$\psi_{compass} = \psi + v_2 \tag{4}$$
$$\boldsymbol{a}^b_{imu} = \boldsymbol{R}^\top(\psi)\left(\dot{\boldsymbol{v}}^n - \boldsymbol{g}^n\right) + \boldsymbol{b}^b_{\text{acc}} \tag{5}$$

where $\boldsymbol{v}_1 \in \mathbb{R}^3$ and $v_2 \in \mathbb{R}$ are white measurement noise. The gravity vector in NED is $\boldsymbol{g}^n = [0, 0, g]^\top$. The accelerometer bias is modeled as:

$$\dot{\boldsymbol{b}}^b_{\text{acc}} = -\frac{1}{T_b}\boldsymbol{b}^b_{\text{acc}} + \boldsymbol{w} \tag{6}$$

where $\boldsymbol{w} \in \mathbb{R}^3$ is white noise.

1a (2%) Explain what we mean with aided inertial navigation and dead reckoning.

1b (3%) Which assumptions are (1)–(6) based on?

1c (6%) Find a linear time-varying (LTV) state-space model:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} + \boldsymbol{E}\boldsymbol{w} \tag{7}$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{v} \tag{8}$$

where $\boldsymbol{u} = \boldsymbol{a}^b_{imu} + \boldsymbol{R}^\top(\psi)\boldsymbol{g}^n$ for estimation of

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p}^n \\ \boldsymbol{v}^n \\ \boldsymbol{b}^b_{\text{acc}} \end{bmatrix} \tag{9}$$

You are supposed to write down the expressions for all the matrices.

1d (8%) Write down the pseudocode for a discrete-time Kalman filter (for-loop) using the LTV model under 1c.

1e (8%) Assume that you have a single axis yaw gyro measuring:

$$r_{gyro} = r + b_{gyro} \tag{10}$$

Furthermore, assume that b_{gyro} is constant and show by using a Lyapunov function that the fixed-gain estimator:

$$\dot{\hat{\psi}} = r_{gyro} - \hat{b}_{gyro} + K_1(\psi - \hat{\psi}) \tag{11}$$
$$\dot{\hat{b}}_{gyro} = -K_2(\psi - \hat{\psi}) \tag{12}$$

renders the equilibrium point $(\psi - \hat{\psi}, b_{gyro} - \hat{b}_{gyro}) = \mathbf{0}$ exponentially stable for proper choices of the observer gains K_1 and K_2. State the conditions for choosing the gains.

1f (8%) Show that:

$$\dot{\boldsymbol{v}}^n = \boldsymbol{R}(\psi)\left(\dot{\boldsymbol{v}}^b + \boldsymbol{S}(r)\boldsymbol{v}^b\right) \tag{13}$$

where

$$\boldsymbol{S}(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

and show how you can find expressions for $\hat{\boldsymbol{v}}^b$ and $\dot{\hat{\boldsymbol{v}}}^b$.

Figure 1: Semi-submersible drilling rig. The position and heading of the semi-submersible are controlled by using 4 azimuth thrusters, which have the following coordinates (-40,-20, 10), (-40, 20, 10), (40, -20, 10) and (40, 20, 10). The body-fixed coordinate system CO is located on the centerline midships.

Problem 2: Semi-Submersible Drilling Rig (40%)

Consider the semi-submersible drilling rig shown in Figure 1. The 6-DOF model is chosen as:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{15}$$

$$(\boldsymbol{M}_{RB} + \boldsymbol{M}_A)\dot{\boldsymbol{\nu}} + \boldsymbol{D}\boldsymbol{\nu} + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau} \tag{16}$$

where the numerical values are given in Appendix A and $\boldsymbol{J}(\boldsymbol{\eta})$ is the 6-DOF kinematic transformation matrix.

2a (6%) Compute the decoupled time constants and natural periods in 6 DOF.

2b (5%) Assume that the drilling rig is exposed to an irrotational ocean current, which is constant in NED. Show how you will modify (16) to include the effect of ocean currents.

2c (5%) Reduce the 6-DOF model to a 3-DOF horizontal-plane model for surge, sway and yaw motions. State the necessary assumptions to write the model as:

$$\dot{\boldsymbol{\eta}}_3 = \boldsymbol{R}(\psi)\boldsymbol{\nu}_3 \tag{17}$$

$$\boldsymbol{M}_3\dot{\boldsymbol{\nu}}_3 + \boldsymbol{D}_3\boldsymbol{\nu}_3 = \boldsymbol{\tau}_3 \tag{18}$$

where $\boldsymbol{\eta}_3 = [x, y, \psi]^\top$ and $\boldsymbol{\nu}_3 = [u, v, r]^\top$. Write down the expressions for $\boldsymbol{M}_3$ and $\boldsymbol{D}_3$. Numerical values are required.

2d (6%) Design a nonlinear PD controller for regulation of generalized position and velocity to zero using pole placement. Write down the formula for the control law and find the expressions for the controller gains $\boldsymbol{K}_p$ and $\boldsymbol{K}_d$ as a function of the model parameters. Specify, which numerical value you would choose for the natural frequency in surge, sway and yaw.

2e (6%) The semi-submersible is controlled by 4 azimuth thrusters as shown in Figure 1. The azimuth thrust is modeled as $(i = 1, 2, 3, 4)$:

$$F_i = 200\,|n_i|n_i \quad [\text{kN}] \tag{19}$$

and all thrusters can be rotated an angle α_i. Find a formula for the roll moment produced by the 4 thrusters.

2f (6%) The control forces and moment acting on the semi-submersible can be written as:

$$\boldsymbol{\tau}_3 = k\boldsymbol{T}_e\boldsymbol{u}_e \tag{20}$$

with

$$\boldsymbol{T}_e = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ -20 & 40 & -20 & -40 & 20 & -40 & 20 & 40 \end{bmatrix} \tag{21}$$

where $\boldsymbol{\tau}_3 = [\tau_x, \tau_y, \tau_\psi]^\top \in \mathbb{R}^3$, $k \in \mathbb{R}$ and $\boldsymbol{u}_e \in \mathbb{R}^8$. Find the numerical value for k. Assume that the PD controller generates the signal $\boldsymbol{\tau}_3 \in \mathbb{R}^3$ and show how you can compute the control inputs n_i for $(i = 1, 2, 3, 4)$ and azimuth angles α_i for $(i = 1, 2, 3, 4)$ using an unconstrained control allocation method. It is not necessary to compute the numerical values. Only the formulas are required.

2g (3%) What are the main limitations/problems with the solution in 2f?

2h (3%) Will the PD control law $\boldsymbol{\tau}_3$ generate roll and pitch motions during stationkeeping when applied to (16)? Justify your answer.

Problem 3: Guidance and Target Tracking Systems (25%)

Consider the simplified ship model:

$$\dot{\psi} = r \tag{22}$$

$$T\dot{r} + r = K\delta \tag{23}$$

$$m\dot{u} + d|u|u = \tau \tag{24}$$

where the rudder angle δ and thrust τ are the control inputs. It is assumed that $u \gg 0$ and $v = 0$ such that speed $U = u$. The model parameters m, d, K and T are known.

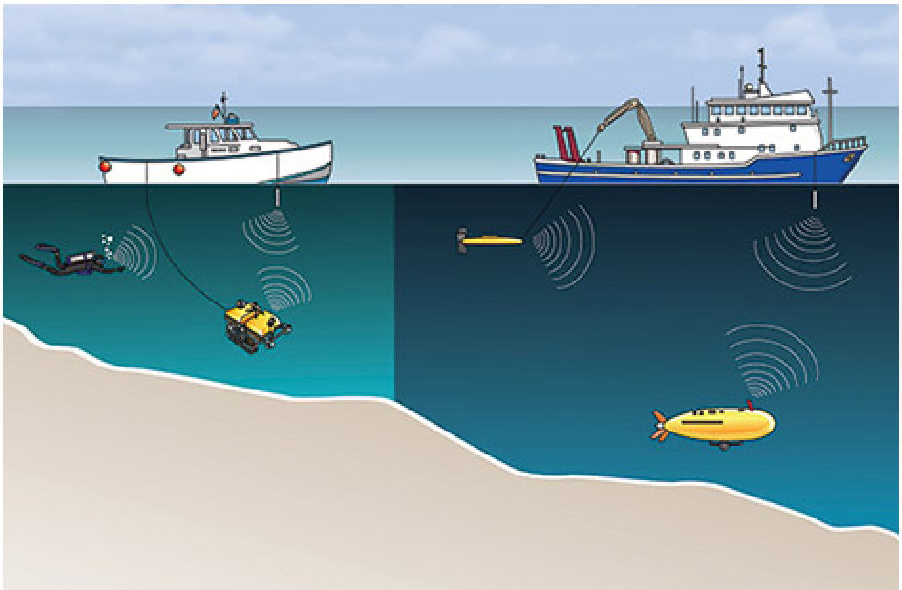

Figure 2: ROV and AUV tracking systems.

3a (4%) Design a feedback linearizing controller for τ such that u converges to u_d exponentially.

3b (4%) Design a PD controller for δ such that ψ converges to ψ_d exponentially.

3c (7%) Assume that you measure the NED position $\boldsymbol{p}_t^n$ and velocity $\boldsymbol{v}_t^n$ of a moving surface vehicle (target), which you want to follow with your ship. It is also assumed that you measure your own position $\boldsymbol{p}^n$ and heading angle ψ. Show how you can use the constant bearing guidance law:

$$\boldsymbol{v}_d^n = \boldsymbol{v}_t^n - k\frac{\tilde{\boldsymbol{p}}^n}{\|\tilde{\boldsymbol{p}}^n\|} \tag{25}$$

where $\tilde{\boldsymbol{p}}^n = \boldsymbol{p}^n - \boldsymbol{p}_t^n$ together with the control laws from 3a and 3b to track a moving surface vessel. The control objective is $u = u_t$ and $v = 0$.

3d (5%) Explain what will happen if you replace $\tilde{\boldsymbol{p}^n}$ in (25) with:

$$\tilde{\boldsymbol{p}^n} = \boldsymbol{p}^n - \left(\boldsymbol{p}_t^n + \boldsymbol{R}(\psi) \begin{bmatrix} 100 \\ 0 \end{bmatrix} \right) \tag{26}$$

3e (5%) Assume that you measure range and bearing from the ship to the target instead of $\boldsymbol{p}^n$ and $\boldsymbol{p}_t^n$. Show how the constant bearing guidance law (25) can be modified to use range-bearing measurements instead of positions.

A Numerical Values for Semi-Submersible Drilling Rig

```
Mass of the vessel = 27 162 500 kg

MRB =
   1.0e+10 *
    0.0027         0         0         0   -0.0530         0
         0    0.0027         0    0.0530         0   -0.0014
         0         0    0.0027         0    0.0014         0
         0    0.0530         0    3.4775         0   -0.0265
   -0.0530         0    0.0014         0    3.8150         0
         0   -0.0014         0   -0.0265         0    3.7192

MA =
   1.0e+10 *
    0.0017         0         0         0   -0.0255         0
         0    0.0042         0    0.0365         0         0
         0         0    0.0021         0         0         0
         0    0.0365         0    1.3416         0         0
   -0.0255         0         0         0    2.2267         0
         0         0         0         0         0    3.2049

D =
   1.0e+09 *
    0.0004         0         0         0   -0.0085         0
         0    0.0003         0    0.0067         0   -0.0002
         0         0    0.0034         0    0.0017         0
         0    0.0067         0    4.8841         0   -0.0034
   -0.0085         0    0.0017         0    7.1383         0
         0   -0.0002         0   -0.0034         0    0.8656

G =
   1.0e+10 *
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0    0.0006         0         0         0
         0         0         0    1.4296         0         0
         0         0         0         0    2.6212         0
         0         0         0         0         0         0
```