NTNU

Department of Engineering Cybernetics

Examination paper for TTK4190 Guidance and Control of Vehicles

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Examination date: Thursday 14 December 2017

Examination time (from-to): 09:00-13:00

Permitted examination support material: Code C

- Textbooks (or printed versions) of Fossen (2011) and Beard & McLain (2012)
- Printed lecture notes/slides.
- Printed assignments, problems and examination sheets.
- All handwritten materials and digital notes are allowed.

Other information: All type of calculators is approved

Language: English

Number of pages (front page excluded): 5

Number of pages enclosed:

Informasjon om trykking av eksamensoppgave	
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5/12-2017, Svein P. Berge

Date Signature

Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

Problem 1: Ship Path-Following Control System (35%)

Figure 1: NTNU's research vessel, R/V Gunnerus,

Consider the kinematic equations:

$$
\begin{aligned}
\dot{N} &= u\cos(\psi)\cos(\theta) + v[\cos(\psi)\sin(\theta)\sin(\phi) - \sin(\psi)\cos(\phi)] \\
&\quad + w\left[\sin(\psi)\sin(\phi) + \cos(\psi)\cos(\phi)\sin(\theta)\right] \\
\dot{E} &= u\sin(\psi)\cos(\theta) + v[\cos(\psi)\cos(\phi) + \sin(\phi)\sin(\theta)\sin(\psi)] \\
&\quad + w[\sin(\theta)\sin(\psi)\cos(\phi) - \cos(\psi)\sin(\phi)] \\
\dot{D} &= -u\sin(\theta) + v\cos(\theta)\sin(\phi) + w\cos(\theta)\cos(\phi) \\
\dot{\phi} &= p + q\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta) \\
\dot{\theta} &= q\cos(\phi) - r\sin(\phi) \\
\dot{\psi} &= q\frac{\sin(\phi)}{\cos(\theta)} + r\frac{\cos(\phi)}{\cos(\theta)}, \quad \theta \neq \pm 90^o
\end{aligned}
$$

and Nomoto model:

$$T\dot{r} + r = K\delta \tag{1}$$

with $T = 22.0$ s and $K = 0.1$ s^{-1}.

1a (2%) The ship is moving at $U = 10$ m/s. What is the steady-state turning radius of the ship for $\delta = 10$ deg.?

1b (2%) The control objective is to track a straight line at constant forward speed $U =$ constant. A 2-D Cartesian system is oriented such that the $x-$axis points Northwards, while the $y-$axis points Eastwards. Explain under which conditions:

$$\dot{\psi} = r \tag{2}$$
$$\dot{y}_e = U\psi \tag{3}$$

is a good approximation for the ship cross-track error y_e.

1c (5%) Find the expressions for $\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$ and the state vector $\mathbf{x}$ in

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\,\delta \tag{4}$$

$$y_e = \mathbf{c}^\top \mathbf{x} \tag{5}$$

where the control objective is $y_e = 0$. Show how the linear optimal regulator δ can be computed and explain how you will choose the weighting matrices.

1d (5%) Modify the state-space model (expressions $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}$) such that the resulting optimal control law includes integral action.

1e (8%) Assume that

$$\dot{y}_e = U \sin(\psi) \tag{6}$$

and choose the line-of-sight (LOS) guidance law according to

$$\psi_d = \tan^{-1}(-K_p y_e) \tag{7}$$

where $K_p > 0$. Assume that $\psi = \psi_d$ and find an expression for the function $f(y_e, U)$ such that:

$$\dot{y}_e = f(y_e, U) \tag{8}$$

What is the equilibrium point of (8)? Linearize the cross-track error dynamics (8) about the equilibrium point. Under what conditions is the equilibrium point of the linearized system exponentially stable?

Hint: $\sin\left(\tan^{-1}(x)\right) = \frac{x}{\sqrt{1+x^2}}$.

1f (5%) Use pole-placement to design a PD-controller for the system (1) such that

$$\frac{\psi}{\psi_d}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{9}$$

Compute the numerical values for the PD gains K_p and K_d for $\zeta = 1.0$ such that the bandwidth of the closed-loop system is 0.3 rad/s.

1g (3%) Draw a block diagram showing how the LOS guidance algorithm (Problem 1e) can be combined with the pole-placement algorithm (Problem 1f) to solve the path-following control problem of a ship moving on a straight line.

1h (5%) Explain how the path-following guidance system can be modified to track several straight-line segments. Include equations describing your approach and make a drawing (block diagram) showing how the equations can be used to implement the guidance and control systems.

Problem 2: UAV Altitude Control System (25%)

Figure 2: NTNU's Penguin UAV system.

Consider the following UAV model:

$$
\begin{aligned}
\dot{p}_n &= (\cos\theta\cos\psi)u + (\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi)v + (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)w \\
\dot{p}_e &= (\cos\theta\sin\psi)u + (\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi)v + (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)w \\
\dot{h} &= u\sin\theta - v\sin\phi\cos\theta - w\cos\phi\cos\theta \\
\dot{u} &= rv - qw - g\sin\theta + \frac{\rho V_a^2 S}{2\mathsf{m}}\left[C_X(\alpha) + C_{X_q}(\alpha)\frac{cq}{2V_a} + C_{X_{\delta_e}}(\alpha)\delta_e\right] + \frac{\rho S_{\text{prop}} C_{\text{prop}}}{2\mathsf{m}}\left[(k_{\text{motor}}\delta_t)^2 - V_a^2\right] \\
\dot{v} &= pw - ru + g\cos\theta\sin\phi + \frac{\rho V_a^2 S}{2\mathsf{m}}\left[C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{bp}{2V_a} + C_{Y_r}\frac{br}{2V_a} + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r\right] \\
\dot{w} &= qu - pv + g\cos\theta\cos\phi + \frac{\rho V_a^2 S}{2\mathsf{m}}\left[C_Z(\alpha) + C_{Z_q}(\alpha)\frac{cq}{2V_a} + C_{Z_{\delta_e}}(\alpha)\delta_e\right] \\
\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\
\dot{\theta} &= q\cos\phi - r\sin\phi \\
\dot{\psi} &= q\sin\phi\sec\theta + r\cos\phi\sec\theta \\
\dot{p} &= \Gamma_1 pq - \Gamma_2 qr + \frac{1}{2}\rho V_a^2 Sb\left[C_{p_0} + C_{p_\beta}\beta + C_{p_p}\frac{bp}{2V_a} + C_{p_r}\frac{br}{2V_a} + C_{p_{\delta_a}}\delta_a + C_{p_{\delta_r}}\delta_r\right] \\
\dot{q} &= \Gamma_5 pr - \Gamma_6(p^2 - r^2) + \frac{\rho V_a^2 Sc}{2J_y}\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{cq}{2V_a} + C_{m_{\delta_e}}\delta_e\right] \\
\dot{r} &= \Gamma_7 pq - \Gamma_1 qr + \frac{1}{2}\rho V_a^2 Sb\left[C_{r_0} + C_{r_\beta}\beta + C_{r_p}\frac{bp}{2V_a} + C_{r_r}\frac{br}{2V_a} + C_{r_{\delta_a}}\delta_a + C_{r_{\delta_r}}\delta_r\right]
\end{aligned}
$$

2a (4%) Explain under which conditions

$$\dot{h} = u\theta - w \tag{10}$$

$$\dot{\theta} = q \tag{11}$$

is a good approximation for aircraft altitude.

2b (8%) Assume that δ_t is chosen such that V_a = constant. Furthermore, assume that $\dot{u} = 0$, and that the lateral motions and wind can be neglected. Design a backstepping controller for altitude control using elevator δ_e as control input. Use the kinematic equation (10) and explain why you choose u, θ or w as virtual controller. The desired altitude h_d = constant. It is not necessary to include integral action when you design the control law.

2c (2%) Find an expression for the time derivative of stabilizing function, $\dot{\alpha}_1$, which is only function of the states and not the time derivative of the states.

2d (6%) What is the equilibrium point of the closed-loop system? Write the error dynamics in matrix-vector form and discuss if the equilibrium point is locally/globally asymptotically/exponentially stable by using Lyapunov stability theory.

2e (3%) What kind of navigation and sensor system do you need to implement the backstepping controller.

2f (2%) Is the backstepping controller robust? Explain why/why not.

Problem 3: Estimation and Navigation (30%)

Consider the vehicle model:

$$\dot{\mathbf{p}}^n = \mathbf{R}_b^n(\mathbf{\Theta})\mathbf{v}^b \tag{12}$$

$$\mathbf{M}\dot{\mathbf{v}}^b + \mathbf{D}\mathbf{v}^b = \boldsymbol{\tau}^b \tag{13}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix and $\boldsymbol{\tau}^b$ is the control input. Furthermore, $\mathbf{p}^n = [x, y, z]^\top$ and $\mathbf{v}^b = [u, v, w]^\top$. Assume that you measure the attitude vector $\mathbf{\Theta} = [\phi, \theta, \psi]^\top$ perfectly such that:

$$\mathbf{R}_b^n(\mathbf{\Theta}(t)) := \mathbf{R}(t) \tag{14}$$

3a (2%) The measurement equations for linear acceleration and position are:

$$\mathbf{z}_1 = \mathbf{p}^n + \mathbf{w}_1 \tag{15}$$

$$\mathbf{z}_2 = \mathbf{R}_n^b(\mathbf{\Theta})(\dot{\mathbf{v}}^n - \mathbf{g}^n) + \mathbf{w}_2 \tag{16}$$

where $\mathbf{w}_1$ and $\mathbf{w}_2$ are Gaussian white noise and $\mathbf{g}^n = [0, 0, 9.81]^\top$. What kind of sensors/navigation systems can provide measurements $\mathbf{z}_1$ and $\mathbf{z}_2$ for:

- underwater vehicles
- surface ships

3b (4%) Show that $\mathbf{z}_2$ can be rewritten as

$$\mathbf{z}_2 = \dot{\mathbf{v}}^b + \mathbf{S}(t)\mathbf{v}^b - \mathbf{R}(t)^\top\mathbf{g}^n + \mathbf{w}_2 \tag{17}$$

if $\boldsymbol{\omega}_{b/n}^b$ is known and

$$\boldsymbol{\omega}_{b/n}^b \times \mathbf{v}^b := \mathbf{S}(t)\mathbf{v}^b \tag{18}$$

3c (8%) Find the expressions for $\mathbf{A}(t)$, $\mathbf{B}$, $\mathbf{C}(t)$ and $\mathbf{D}(t)$, the state vector $\mathbf{x}$ and $\mathbf{u}$ in

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{v} \tag{19}$$

$$\mathbf{z} = \mathbf{C}(t)\mathbf{x} + \mathbf{D}(t)\mathbf{u} + \mathbf{w} \tag{20}$$

where the objective is to estimate $\mathbf{x}$ from the measurements $\mathbf{z}$ and $\mathbf{u}$. Explain how you will model the terms $\mathbf{E}$ and $\mathbf{v}$ if the goal is to estimate $\mathbf{x}$ using a linear time-varying (LTV) Kalman filter.

3d (6%) Explain how you will modify the measurement equation $\mathbf{z}_2$ and the state-space model under 3b) to estimate acceleration bias.

3e (6%) Assume that the model (12)–(13) is unknown. Show that:

$$\dot{\mathbf{p}}^n = \mathbf{v}^n \tag{21}$$

$$\dot{\mathbf{v}}^n = \mathbf{u}_a^n \tag{22}$$

$$\mathbf{z}_1 = \mathbf{p}^n + \mathbf{w}_1 \tag{23}$$

where

$$\mathbf{u}_a{}^n := \mathbf{R}(t)\mathbf{z}_2 + \mathbf{g}^n - \mathbf{R}(t)\mathbf{w}_2 \tag{24}$$

Propose a state-estimator for $\mathbf{p}^n$ and $\mathbf{v}^n$, which is globally exponentially stable.

3f (4%) What are the conceptual differences of the estimators in 3c and 3e. Also set up a list of advantages and disadvantages of these two approaches.

Problem 4: Multiple-Choice Problems (10%)

The YES and NO questions below give you 2 points for correct answer, -1 point for wrong answer and 0 points for no answer. Please answer only YES or NO, alternately no answer at all.

4a (2%) The roll and pitch periods of a ship depends on the sea state and load condition.

4b (2%) It is possible to use rudders in dynamic positioning systems even though lift and drag are zero for zero water speed.

4c (2%) The Nomoto model

$$h(s) = \frac{K}{s(Ts+1)} + d(s) \tag{25}$$

of a ship where $d(s)$ is a constant wind disturbance has an open-loop integrator so it is not necessary to include integral action in the control law in order to avoid steady-state errors.

4d (2%) The added mass matrix is independent of the location of the coordinate system.

4e (2%) Coriolis forces can destabilize ships and underwater vehicles.