Exam Fall 2014

Solution

December 8, 2014

1 Problem 1

1A Equation (8.156) gives:

$$V_x = V_c \cos(\beta_c) = 1\cos(40^\circ) \approx 0.77 \text{ m/s} \tag{1}$$
$$V_y = V_c \sin(\beta_c) = 1\sin(40^\circ) \approx 0.64 \text{ m/s} \tag{2}$$

1B Equation (8.159) gives

$$u_c = V_c \cos(\beta_c - \psi) = 1\cos(40^\circ - 20^\circ) \approx 0.94 \text{ m/s} \tag{3}$$
$$v_c = V_c \sin(\beta_c - \psi) = 1\sin(40^\circ - 20^\circ) \approx 0.34 \text{ m/s} \tag{4}$$

1C The ship moves on a straight line and the current is constant. Hence, sideslip is only caused by the current (not induced by turning, which gives a nonzero v as well). Moreover,

$$u = U = 10 \text{ m/s} \tag{5}$$
$$v = 0 \tag{6}$$

1d Equation (2.98) gives

$$\beta = \frac{v_r}{U_r} = \frac{v - v_c}{\sqrt{(u-u_c)^2 + (v-v_c)^2}} = \frac{-0.34}{\sqrt{9.06^2 + 0.34^2}} \approx -2.1^\circ \tag{7}$$

For the zero current case this simplifies to

$$\beta = \frac{v}{U} = \frac{0}{U} = 0^\circ \tag{8}$$

β will only be nonzero if the ship turns.

1e The course angle is:

$$\chi = \psi + \beta = 20.0^\circ - 2.1^\circ = 17.9^\circ \tag{9}$$

The course angle can be measured using GNSS, that is COG = course over ground.

1f The proof is:

$$\begin{aligned}
\dot{\eta} &= \mathbf{R}(\psi)\nu \\
&= \mathbf{R}(\psi)(\nu_r + \nu_c) \\
&= \mathbf{R}(\psi)\nu_r + \mathbf{R}(\psi)\nu_c \\
&= \mathbf{R}(\psi)\nu_r + \dot{\eta}_c \\
&= \mathbf{R}(\psi)\nu_r + \begin{bmatrix} V_x \\ V_y \\ 0 \end{bmatrix}
\end{aligned} \tag{10}$$

q.e.d.

1g For $\tau_{prop} = \mathbf{0}$, we have that

$$\dot{\nu}_r = -\mathbf{M}^{-1}\mathbf{D}\nu_r \tag{11}$$

In steady state $\nu_r = \mathbf{0}$ and consequently $\nu = \nu_c$. This means that the ship will drift with the current velocity.

2 Problem 2

2a

$$\begin{aligned}
\dot{y} &= U\sin(\chi - \gamma_p) && (12)\\
&= U\sin\left(\arctan\left(-\frac{y}{\Delta}\right)\right) && (13)\\
&= U\frac{-\frac{y}{\Delta}}{\sqrt{1+\left(-\frac{y}{\Delta}\right)^2}} && (14)\\
&= -\frac{U}{\sqrt{\Delta^2+y^2}}y && (15)
\end{aligned}$$

q.e.d.

2b Linearization about $y = 0$ gives

$$\dot{y} = -\frac{U}{\Delta}y \tag{16}$$

Assume that $U/\Delta > 0$, then the equilibrium point $y = 0$ is (locally) exponentially stable (LES). This is only a local result since the system has been linearized. Notice that both U and Δ must be positive. Large Δ gives a slow convergence rate and vice versa. Large U gives a fast convergence rate and vice versa.

2c

$$V = \frac{1}{2}y^2 \tag{17}$$

gives

$$\begin{aligned}
\dot{V} &= y\dot{y} && (18)\\
&= -\frac{U}{\sqrt{\Delta^2+y^2}}y^2 && (19)\\
&\leq 0 && (20)
\end{aligned}$$

The system is semiglobally exponentially stable for constant $U > 0$ and $\Delta > 0$. The equilibrium point cannot be proven to be GES since the trigonometric function in (12) is a saturating element. In addition it follows that the system is GAS/LES or so-called globally κ-exponentially stable. To obtain a full score the student only needs to argue that the system is GAS using Lyapunov's direct method. LES has already been proven under 2b.

2d This implies that the guidance law is chosen as a heading command:

$$\psi = \arctan\left(-\frac{y}{\Delta}\right) \tag{21}$$

Hence,

$$\chi = \psi + \beta = \arctan\left(-\frac{y}{\Delta}\right) + \beta \tag{22}$$

where β is unknown. We can remove the sideslip angle β by using integral LOS. Moreover,

$$\psi = \arctan\left(-K_p y - K_i z\right), \quad K_p = \frac{1}{\Delta}, \quad K_i > 0 \tag{23}$$

$$\dot{z} = y \tag{24}$$

2e LOS guidance laws can be used to follow a curve by replacing γ_p with a time-varying value representing the tangent to the path. This will, however, introduce time-varying sideslip β, which must be compensated by direct measurement of β or integral action (integral LOS).

3 Problem 3

3a The state-space model is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{E}\mathbf{w} \tag{25}$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{26}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{27}$$

The KF continuous-time equations are:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{K}(\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}) \tag{28}$$

$$\dot{\mathbf{P}} = \mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^\top + \mathbf{E}\mathbf{Q}\mathbf{E}^\top - \frac{1}{r}\mathbf{P}\mathbf{H}^\top\mathbf{H}\mathbf{P} \tag{29}$$

$$\mathbf{K} = \frac{1}{r}\mathbf{P}\mathbf{H}^\top \tag{30}$$

where the process covariance $q \geq 0$ and measurement covariance $r > 0$ are tunable parameters.

3b The KF is based on the assumption that $\dot{\mathbf{v}}_t^n = 0$, which implies that $\mathbf{v}_n^t =$ constant. However, the KF will be able to estimate a slowly varying $\mathbf{v}_n^t$ if properly tuned.

3c

$$\begin{aligned} \mathbf{v}_d^n &= \mathbf{v}_t^n + \mathbf{v}_a^n \\ &= \mathbf{v}_t^n - \kappa \frac{\tilde{\mathbf{p}}^n}{\|\tilde{\mathbf{p}}^n\|} \end{aligned} \tag{31}$$

where $\tilde{\mathbf{p}}^n = \mathbf{p}^n - \mathbf{p}_d^n$ and

$$\kappa = U_{a,\max} \frac{\|\tilde{\mathbf{p}}^n\|}{\sqrt{(\tilde{\mathbf{p}}^n)^\top \tilde{\mathbf{p}}^n + \Delta_p^2}} \tag{32}$$

The variables are defined in Section 10.1.3.

3d System

$$(m - X_{\dot{u}})\dot{u} - X_{|u|u}\,|u|\,u = T \tag{33}$$

The feedback linearizing controller

$$T = (m - X_{\dot{u}})a - X_{|u|u}\,|u|\,u \tag{34}$$

gives

$$\dot{u} = a \tag{35}$$

Hence, we choose

$$a = \dot{u}_d - K_p(u - u_d) - K_i \int_0^t (u - u_d) d\tau \tag{36}$$

The feedback velocity controller is expressed in BODY and the control objective is $u = u_d$ and $v = 0$. Transforming this to NED gives the following desired velocities

$$\mathbf{v}_d^n = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} u_d \\ 0 \end{bmatrix} \tag{37}$$

3e Block diagram

4 Problem 4

4a Stall is the maximum $C_L(\alpha)$-value. Hence,

$$\frac{\partial C_L(\alpha)}{\partial \alpha} = 0.18 - 2 \cdot 0.007\alpha = 0 \tag{38}$$

$$\alpha \approx 12.9^\circ \tag{39}$$

4b See pages 72 and 107 of Beard & McLain (2012).

See slide 16 Ch. 5 of Beard & McLain (2012). Consider that $\alpha = \theta - \gamma$:

$$\dot{q} = \frac{I_x - I_z}{I_y} pr + \frac{\bar{q}S\bar{c}}{m}(C_{m_0} + C_{m_\alpha}(\theta - \gamma) + C_{m_q}\frac{cq}{2V_a}) + b_2\delta_e \tag{40}$$

and going to the Laplace domain and doing some math:

$$s^2\theta = d_{\theta_2} - a_{\theta_1} s\theta - a_{\theta_2}\theta + a_{\theta_3}\delta_e \tag{41}$$

$$\theta = \frac{a_{\theta_3}}{s^2 + a_{\theta_1}s + a_{\theta_2}} \left(\frac{1}{a_{\theta_3}} d_{\theta_2} + \delta_e \right) \tag{42}$$

where

$$d_{\theta_2} = \frac{I_x - I_z}{I_y} pr + \frac{\bar{q}S\bar{c}}{m}(C_{m_0} - C_{m_\alpha}\gamma) \tag{43}$$

$$a_{\theta_1} = -\frac{\bar{q}S\bar{c}C_{m_q}c}{2mV_a} \tag{44}$$

$$a_{\theta_2} = -\frac{\bar{q}S\bar{c}}{m}C_{m_\alpha} \tag{45}$$

$$a_{\theta_3} = b_2 \tag{46}$$

4c The controller equation and the various k gains are then found in slide 16 Ch. 6:

$$k_{p_\theta} = \frac{\delta_e^{\max}}{e_\theta^{\max}} \operatorname{sign}(a_{\theta_3}) \tag{47}$$

$$\omega_{n_\theta} = \sqrt{a_{\theta_2} + k_{p_\theta} a_{\theta_3}} \tag{48}$$

$$k_{d_\theta} = \frac{2\zeta_\theta \omega_{n_\theta} - a_{\theta_1}}{a_{\theta_3}} \tag{49}$$

$$K_{\theta_{DC}} = \frac{k_{p_\theta} a_{\theta_3}}{a_{\theta_2} + k_{p_\theta} a_{\theta_3}} \tag{50}$$

Integral action can be used to compensate for the unmodeled dynamics d_{θ_3}.

4d The block diagram is on slide 17 Ch. 6. When designing a controller using successive loop closure, the innermost loop has to be the one with the highest bandwidth, and each surrounding loop has to be designed so that it has a bandwidth lower than the ones it includes (typically 5-10 times smaller in frequency). This guarantees that, when designing a specific loop, the transfer function of the inner one can be approximated with a unitary gain, thus simplifying the design. Therefore, following the notation of slide 19,

$$\omega_{n_h} = \frac{1}{W_h}\omega_{n_\theta} \tag{51}$$

and W_h is the bandwidth separation, a number between 5 and 10.

4e The following sensors can be used:

- A sensor for measuring V_a, like a Pitot tube.
- IMU (accelerometer, gyros) for estimation of q and θ.
- Altimeter and/or GNSS for h.