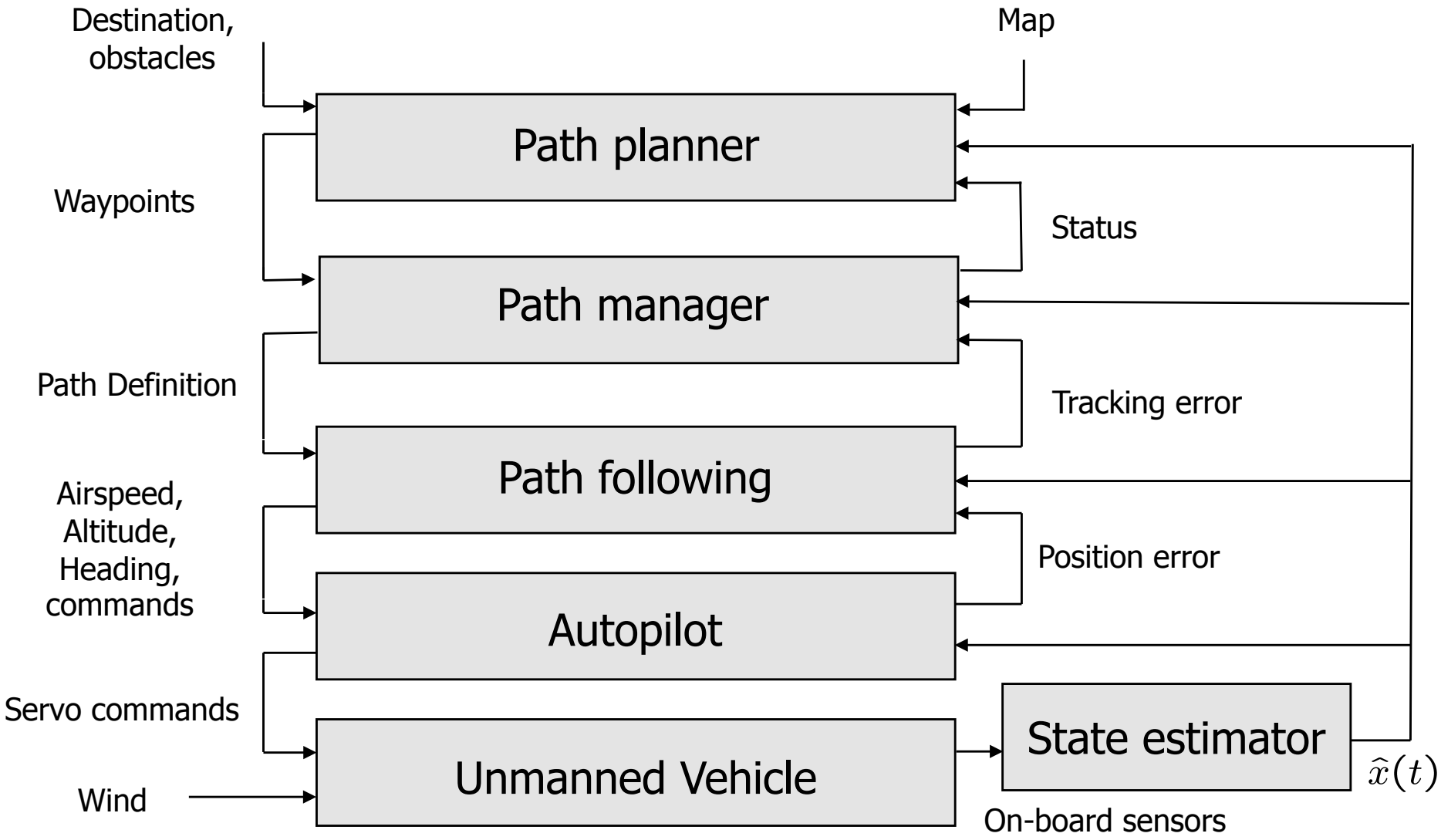
Chapter 6

Autopilot Design

Architecture

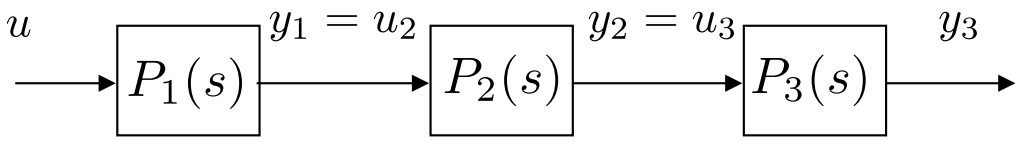


Outline

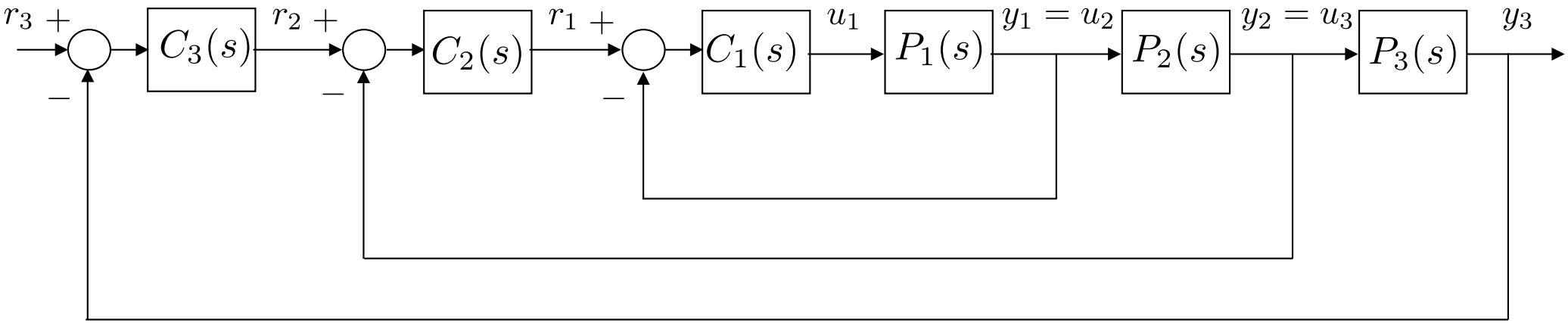
- Different Options for Autopilot Design
  - Successive Loop Closure
  - Total Energy Control
  - LQR Control

Successive Loop Closure

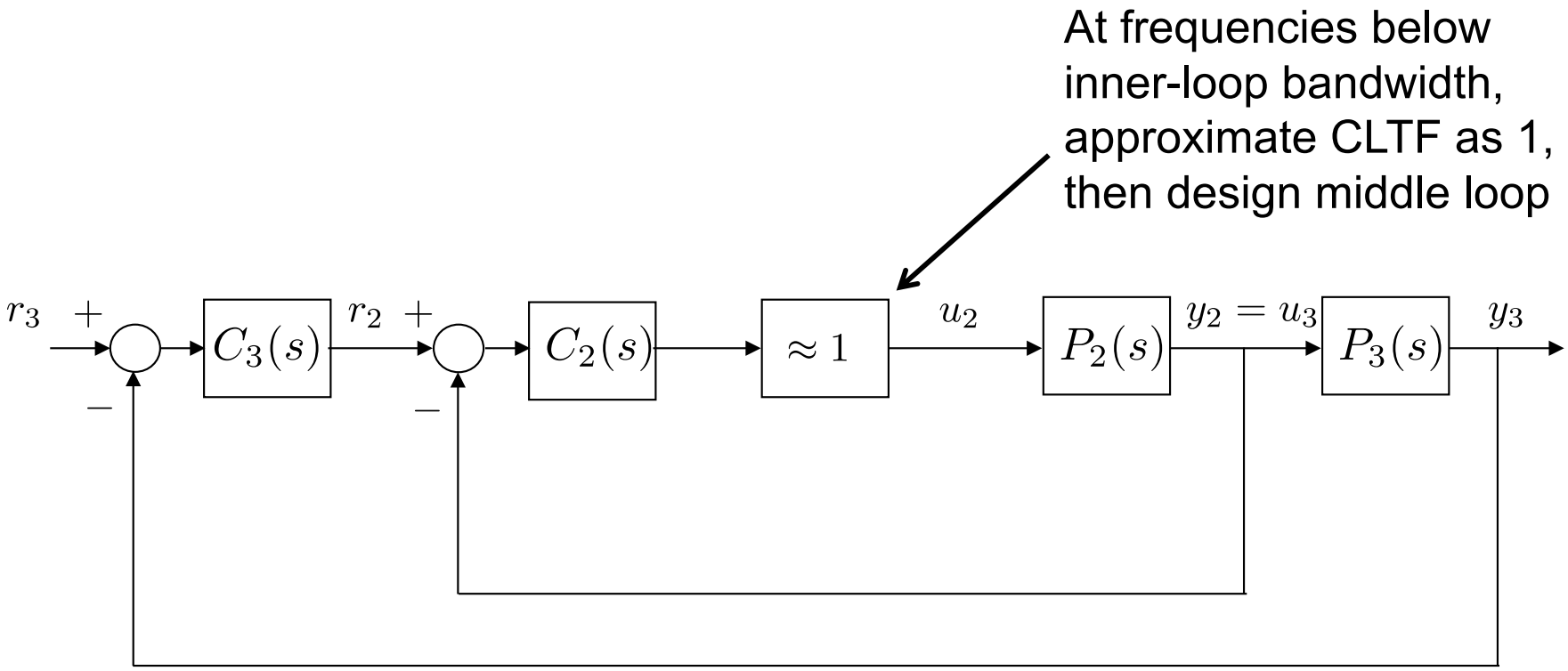
Open-loop system



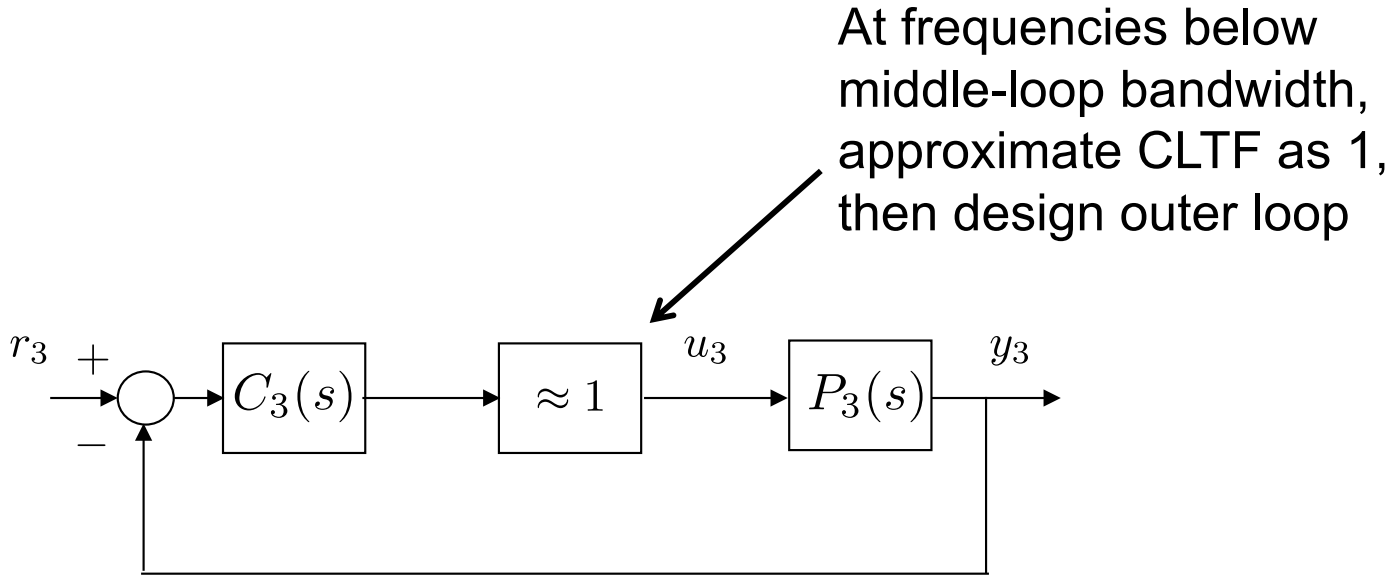
Closed-loop system



SLC: Inner Loop Closed

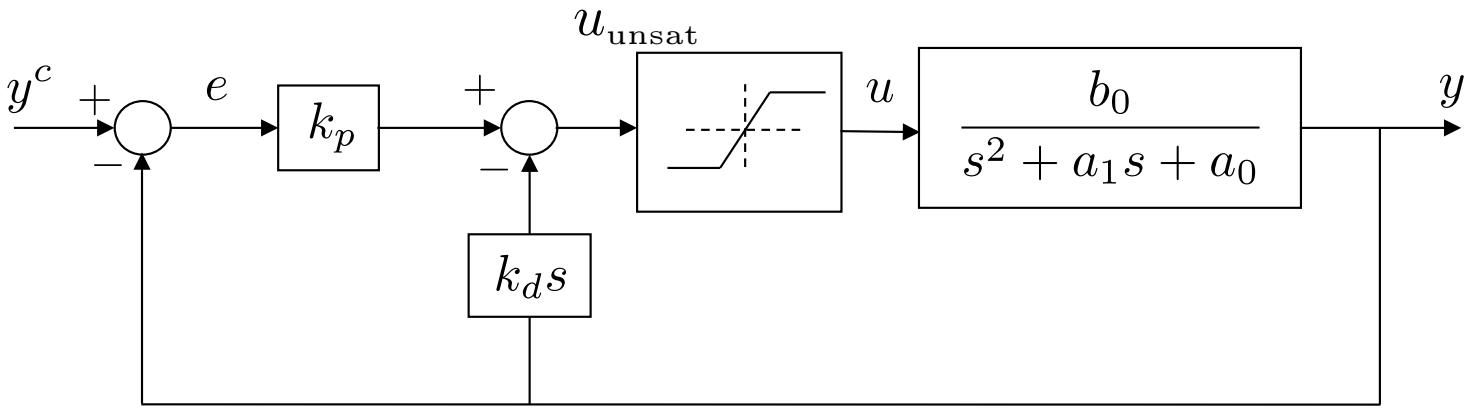


SLC: Two Loops Closed



Key idea: Each successive loop must be lower in bandwidth
--- typically by a factor of 5 to 10

Saturation Limits



The control signal u is largest immediately after a step on y_c, at which point the output of the differentiator is essentially zeros. Therefore $u \approx k_p e$. Let $u^{\max}$ be the input saturation limit, and $e^{\max}$, the largest expected step, then set

$$\boxed{k_p = \frac{u^{\max}}{e^{\max}}.}$$

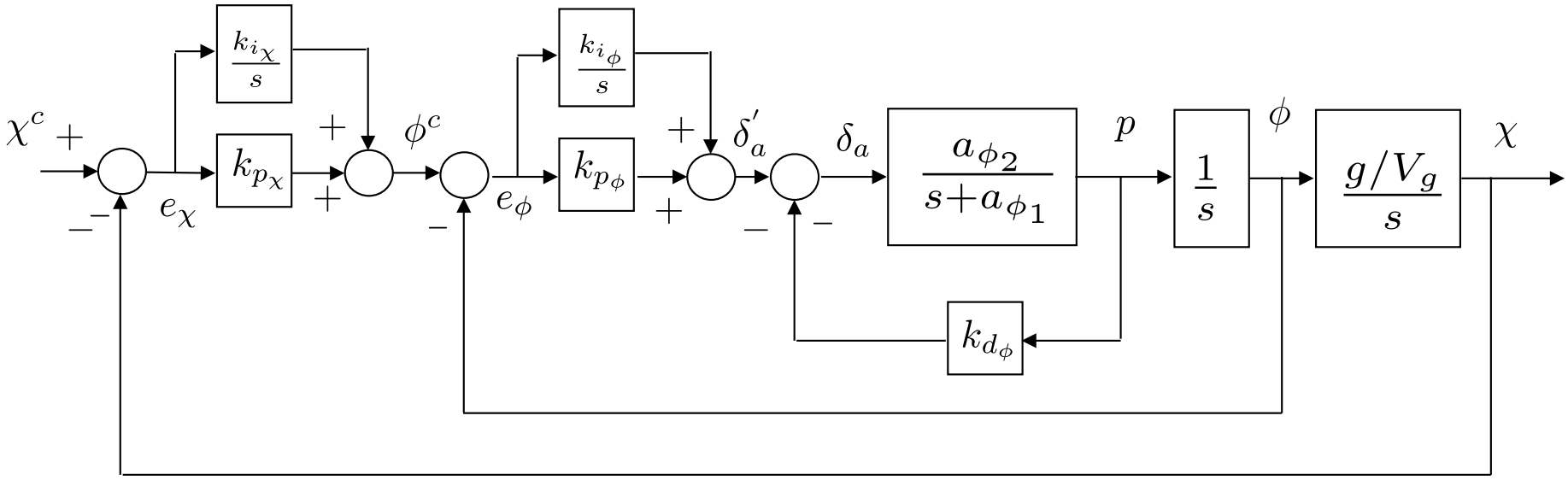
The closed loop transfer function is

$$Y(s) = \frac{b_0 k_p}{s^2 + (a_1 + b_0 k_d)s + (a_0 + b_0 k_p)} Y^c(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} Y^c(s)$$

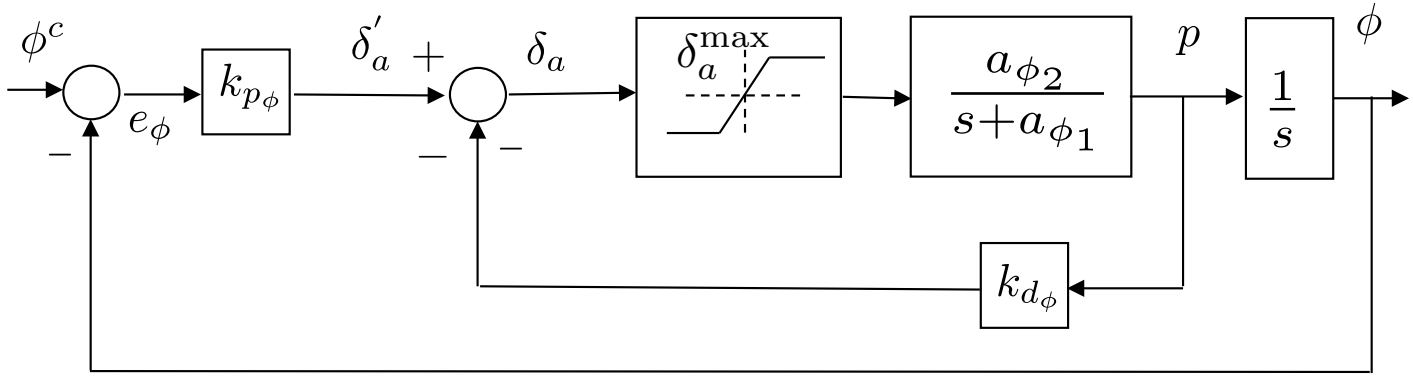
Equating terms gives

$$\omega_n = \sqrt{a_0 + b_0 k_p} \qquad \boxed{k_d = \frac{2\zeta\omega_n - a_1}{b_0}.}$$

Lateral-directional Autopilot



Roll Autopilot



$$H_{\phi/\phi^c}(s) = \underbrace{\frac{k_{p_\phi} a_{\phi_2}}{s^2 + (a_{\phi_1} + a_{\phi_2} k_{d_\phi})s + k_{p_\phi} a_{\phi_2}}}_{\text{Closed Loop TF}} = \underbrace{\frac{\omega_{n_\phi}^2}{s^2 + 2\zeta_\phi \omega_{n_\phi} s + \omega_{n_\phi}^2}}_{\text{Canonical } 2^{nd}\text{-order TF}}$$

Design parameters are $e_\phi^{\max}$ and ζ_ϕ

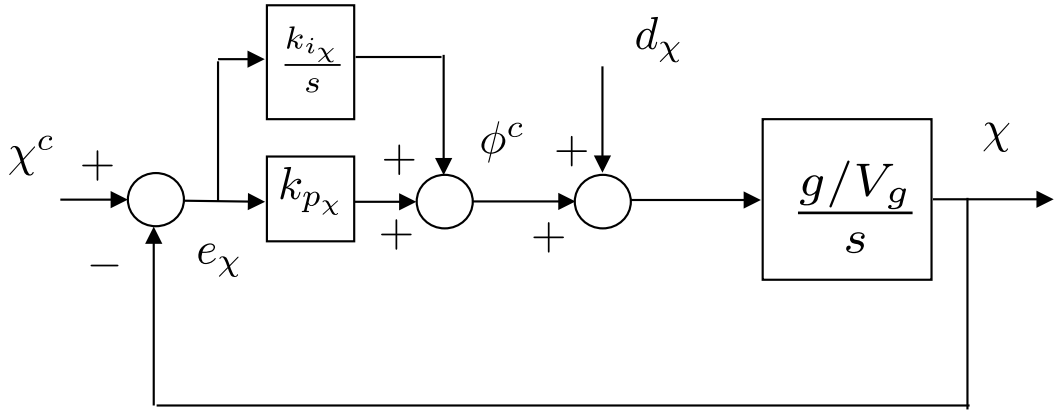
Gains are given by

$$\boxed{k_{p_\phi} = \frac{\delta_a^{\max}}{e_\phi^{\max}}} \qquad \boxed{k_{d_\phi} = \frac{2\zeta_\phi \omega_{n_\phi} - a_{\phi_1}}{a_{\phi_2}}}$$

Roll Autopilot

- The book suggests using an integrator on roll in the roll loop to correct for steady state error.
- Our current suggestion is to not have an integrator on inner loops including the roll loop.
    - Integrators add delay and instability -> not a good idea for the inner-most loops.
    - An integrator will be used on the course loop to correct for steady state values.

Course Hold Loop



For the course loop, note the presence of the input disturbance.

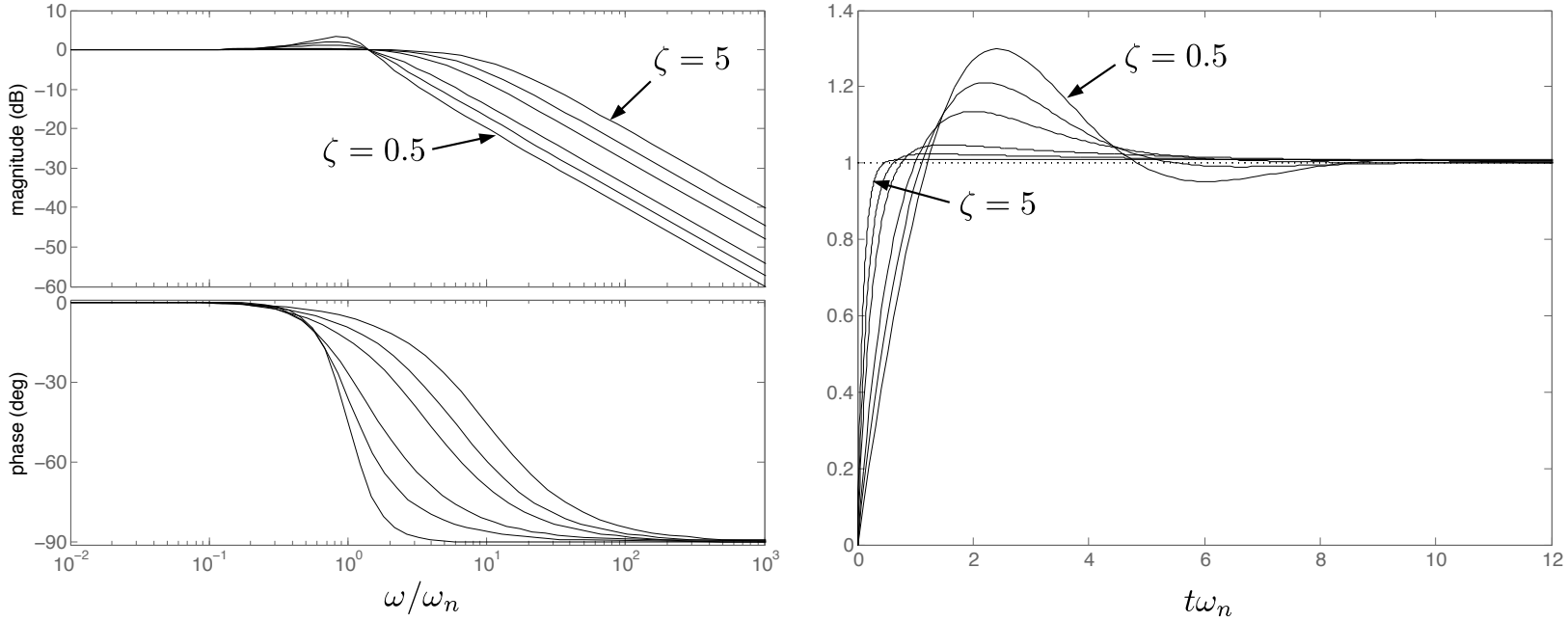
Using a PI controller for course, the response to the course command and disturbance is given by

$$\chi = \frac{k_{p_\chi} g/V_a s + k_{i_\chi} g/V_a}{s^2 + k_{p_\chi} g/V_a s + k_{i_\chi} g/V_a} \chi^c + \frac{g/V_a s}{s^2 + k_{p_\chi} g/V_a s + k_{i_\chi} g/V_a} d_\chi$$

Note:

- There is a zero in the response to the course command χ^c.
- The presence of the zero at the origin ensures rejection of low frequency disturbances.

TF Zero Affects Response

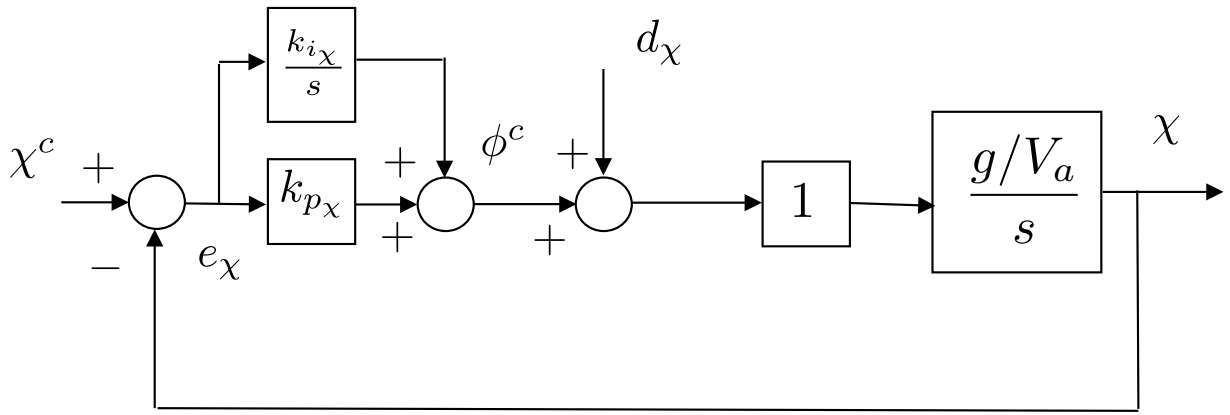


With a zero, the canonical 2^{nd}-order TF is given by

$$H = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Note that ζ has a different effect when the zero is present.

Course Hold Loop



$$\chi = \underbrace{\frac{(k_{p_\chi} g/V_g)s + (k_{i_\chi} g/V_g)}{s^2 + (k_{p_\chi} g/V_g)s + (k_{i_\chi} g/V_g)}\chi^c}_{\text{Response to course command}} + \underbrace{\frac{(g/V_g)s}{s^2 + (k_{p_\chi} g/V_g)s + (k_{i_\chi} g/V_g)}d_\chi}_{\text{Response to disturbance}}$$

Equating coefficients to canonical TF gives:

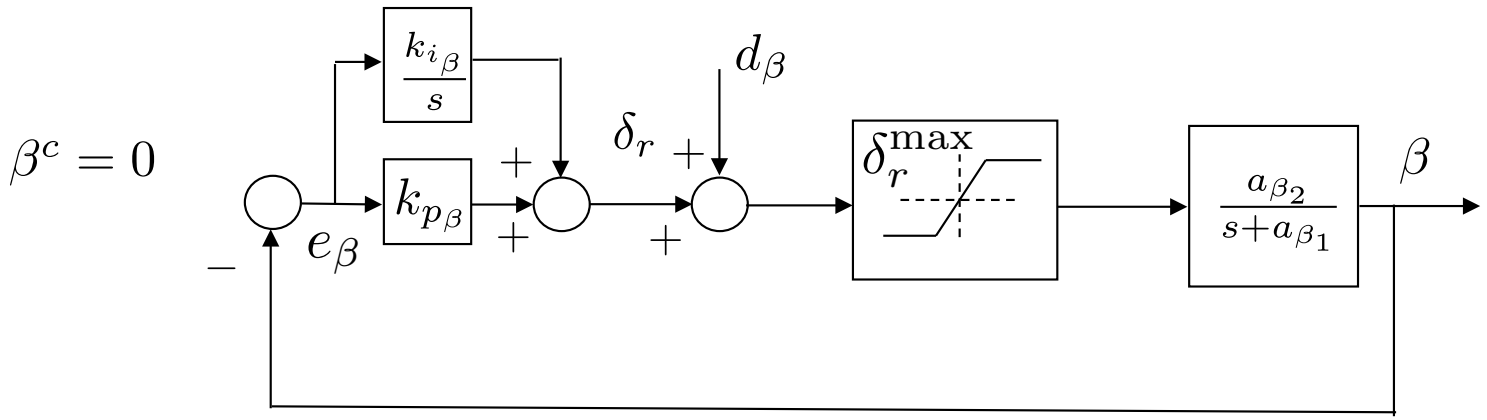
$$\omega_{n_\chi}^2 = k_{i_\chi} g/V_g \qquad \text{and} \qquad 2\zeta_\chi \omega_{n_\chi} = k_{p_\chi} g/V_g$$

or

$$\omega_{n_\chi} = \frac{1}{W_\chi}\omega_{n_\phi} \qquad \boxed{k_{p_\chi} = 2\zeta_\chi \omega_{n_\chi} V_g/g} \qquad \boxed{k_{i_\chi} = \omega_{n_\chi}^2 V_g/g}$$

Design parameters are bandwidth separation W_χ and damping ratio ζ_χ

Sideslip Hold



The transfer function for sideslip hold is

$$H_{\beta/\beta^c}(s) = \frac{a_{\beta_2} k_{p_\beta} s + a_{\beta_2} k_{i_\beta}}{s^2 + (a_{\beta_1} + a_{\beta_2} k_{p_\beta}) s + a_{\beta_2} k_{i_\beta}}$$

Equating coefficients to canonical TF with zero gives

$$\boxed{k_{p_\beta} = \frac{\delta_r^{\max}}{e_\beta^{\max}}} \qquad \omega_{n_\beta} = \frac{a_{\beta_1} + a_{\beta_2} k_{p_\beta}}{2\zeta_\beta} \qquad \boxed{k_{i_\beta} = \frac{\omega_{n_\beta}^2}{a_{\beta_2}}}$$

Design parameters are maximum error $e_\beta^{\max}$ and damping ratio ζ_β

Lateral Autopilot - Summary

If model is known, the the design parameters are

Inner Loop (roll attitude hold)

- $e_\phi^{\max}$ - Error in roll when aileron just saturates.
- ζ_ϕ - Damping ratio for roll attitude loop.

Outer Loop (course hold)

- $W_\chi > 1$ - Bandwidth separation between roll and course loops.
- ζ_χ - Damping ratio for course hold loop.

Sideslip hold (if rudder is available)

- $e_\beta^{\max}$ - Error in sideslip when rudder just saturates.
- ζ_β - Damping ratio for sideslip loop.

Lateral Autopilot – In Flight Tuning

If model is not known, and autopilot must be tuned in flight, then the following gains are tuned one at a time, in this specific order:

Inner Loop (roll attitude hold)

- k_{d_ϕ} - Increase k_{d_ϕ} until onset of instability, and then back off by 20%.
- k_{p_ϕ} - Tune k_{p_ϕ} to get acceptable step response.

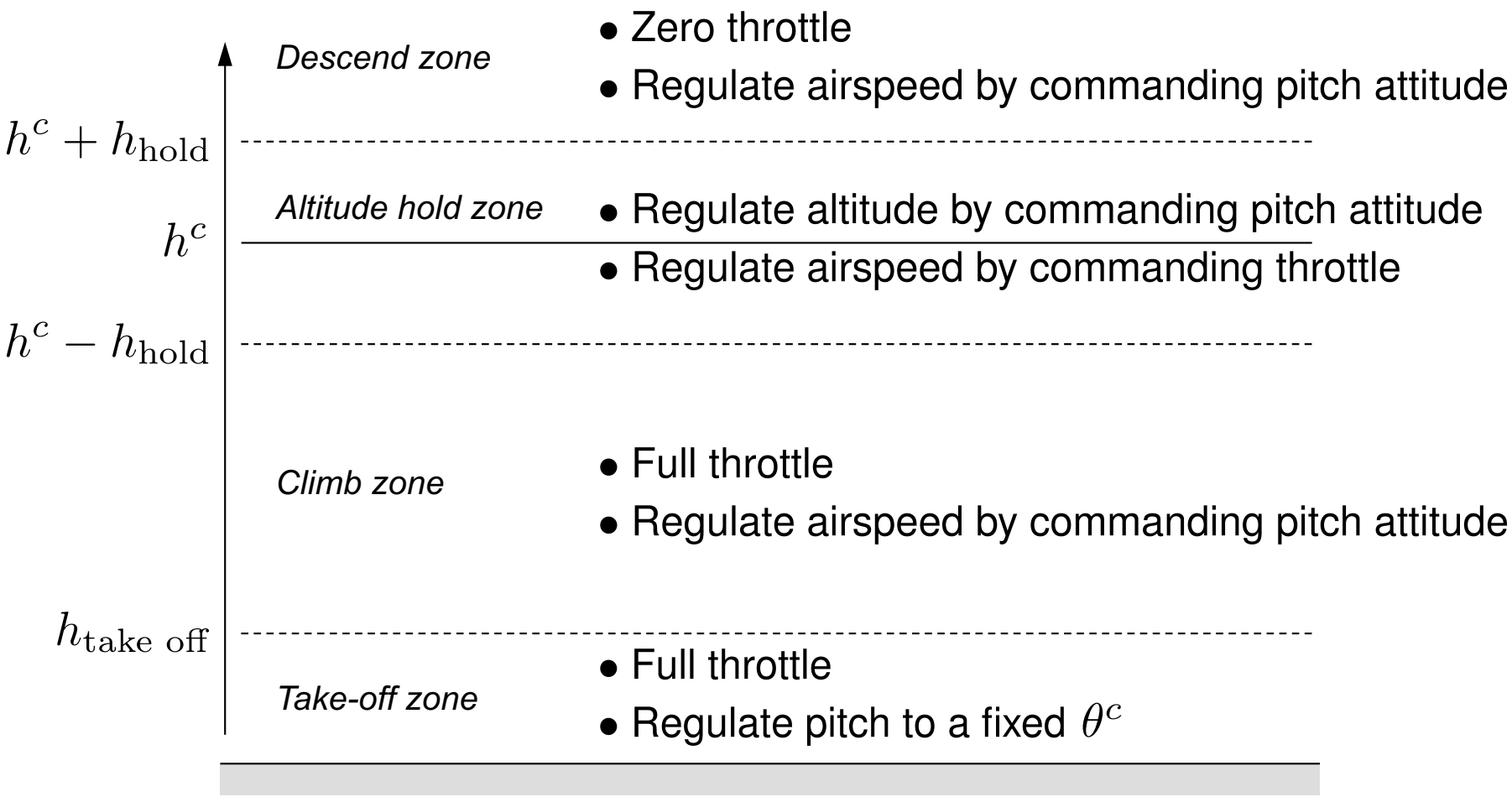
Outer Loop (course hold)

- k_{p_χ} - Tune k_{p_χ} to get acceptable step response.
- k_{i_χ} - Tune k_{i_χ} to remove steady state error.

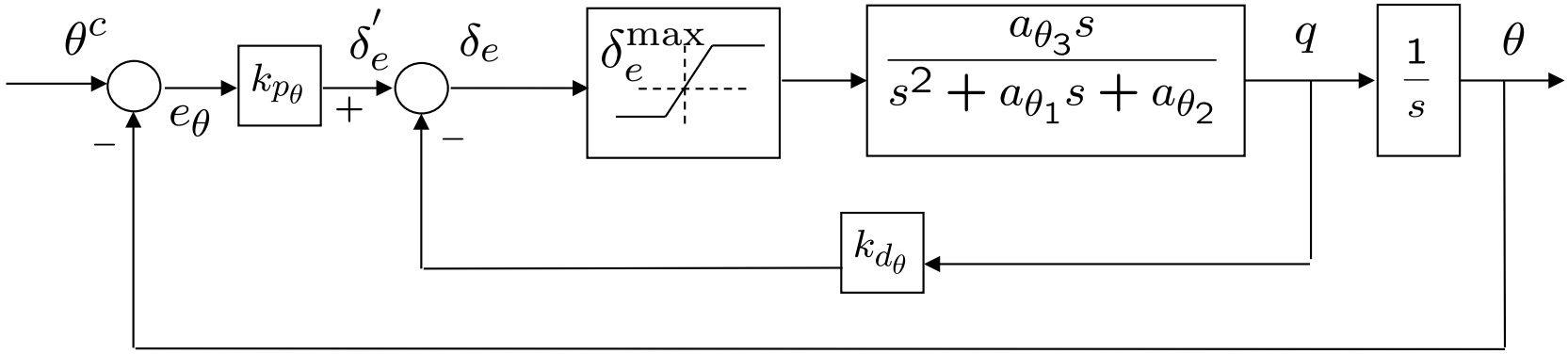
Sideslip hold (if rudder is available)

- k_{p_β} - Tune k_{p_β} to get acceptable step response.
- k_{i_β} - Tune k_{i_β} to remove steady state error.

Longitudinal Flight Regimes



Pitch Attitude Hold



$$H_{\theta/\theta^c}(s) = \underbrace{\frac{k_{p_\theta}a_{\theta_3}}{s^2+(a_{\theta_1}+k_{d_\theta}a_{\theta_3})s+(a_{\theta_2}+k_{p_\theta}a_{\theta_3})}}_{\text{Closed Loop TF}} = \underbrace{\frac{K_{\theta_{\rm DC}}\omega_{n_\theta}^2}{s^2+2\zeta_\theta\omega_{n_\theta}s+\omega_{n_\theta}^2}}_{\text{Note: Non-unity DC Gain}}$$

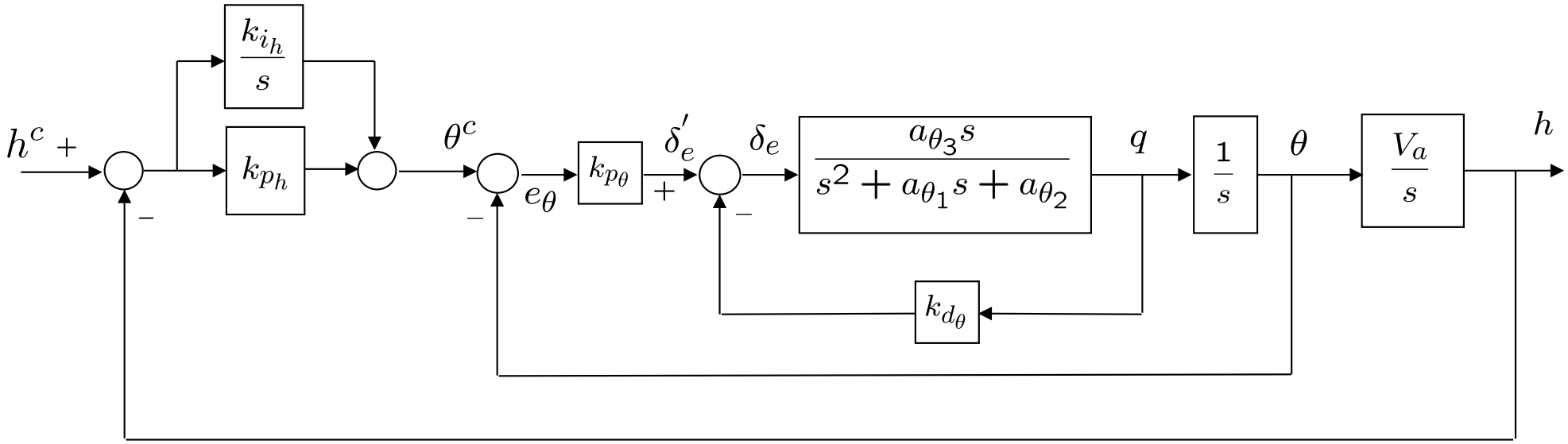
Equating coefficients, the gains are given by

$$\boxed{k_{p_\theta} = \frac{\delta_e^{\max}}{e_\theta^{\max}}\text{sign}(a_{\theta_3})} \qquad \omega_{n_\theta} = \sqrt{a_{\theta_2}+k_{p_\theta}a_{\theta_3}} \qquad \boxed{k_{d_\phi} = \frac{2\zeta_\theta\omega_{n_\theta}-a_{\theta_1}}{a_{\theta_3}}}$$

Design parameters are $e_\theta^{\max}$ and ζ_θ
The DC gain is

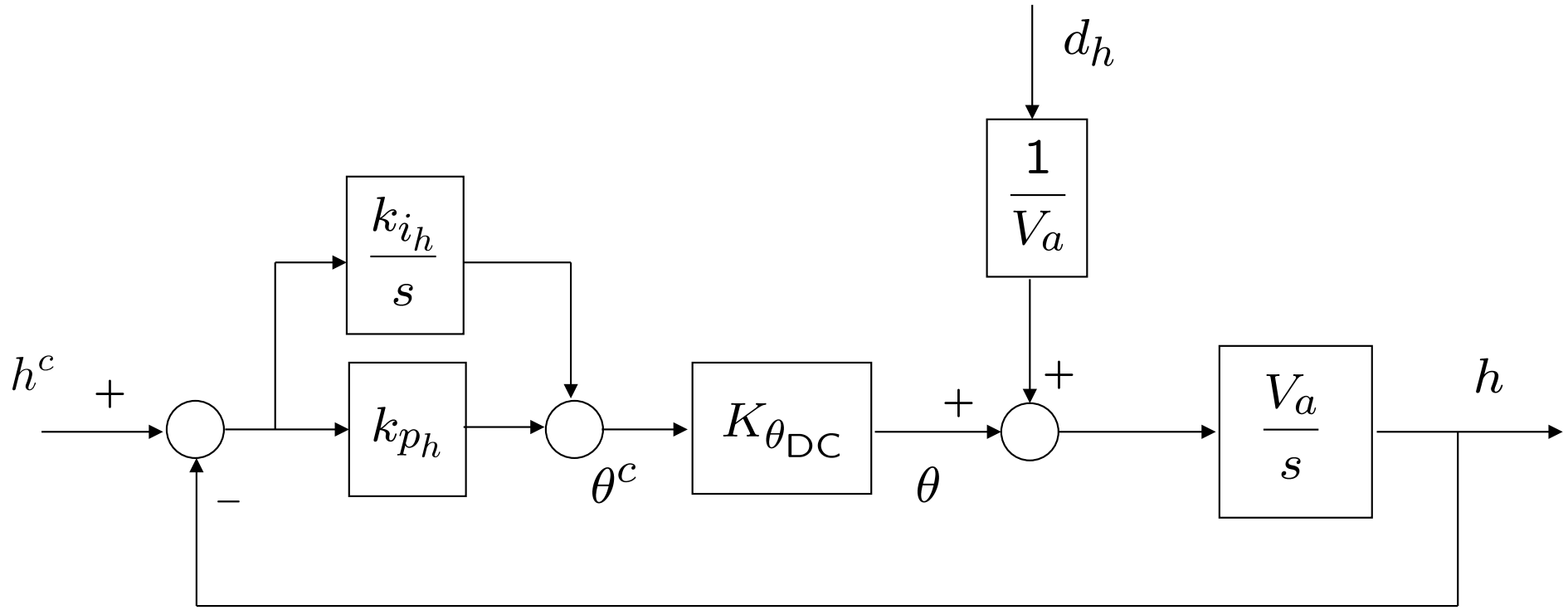
$$\boxed{K_{\theta_{DC}} = \frac{k_{p_\theta}a_{\theta_3}}{a_{\theta_2}+k_{p_\theta}a_{\theta_3}}}$$

Altitude Hold Using Commanded Pitch



Provided pitch loop functions as intended, we can simplify the inner-loop dynamics to $\theta^c/\theta \approx K_{\theta_{\rm DC}}$.

Altitude from Pitch – Simplified



$$h(s) = \left(\frac{K_{\theta_{\rm DC}} V_a k_{p_h} \left(s + \frac{k_{i_h}}{k_{p_h}} \right)}{s^2 + K_{\theta_{\rm DC}} V_a k_{p_h} s + K_{\theta_{\rm DC}} V_a k_{i_h}} \right) h^c(s) + \left(\frac{s}{s^2 + K_{\theta_{\rm DC}} V_a k_{p_h} s + K_{\theta_{\rm DC}} V_a k_{i_h}} \right) d_h(s)$$

A PI control on altitude ensures that h tracks constant h^c with zero steady state error, and rejects low frequency disturbances.

Altitude from Pitch Gain Calculations

Equating the transfer functions

$$H_{h/h^c} = \left(\frac{K_{\theta_{\rm DC}} V_a k_{p_h} \left(s + \frac{k_{i_h}}{k_{p_h}} \right)}{s^2 + K_{\theta_{\rm DC}} V_a k_{p_h} s + K_{\theta_{\rm DC}} V_a k_{i_h}} \right) h^c(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

gives the coefficients

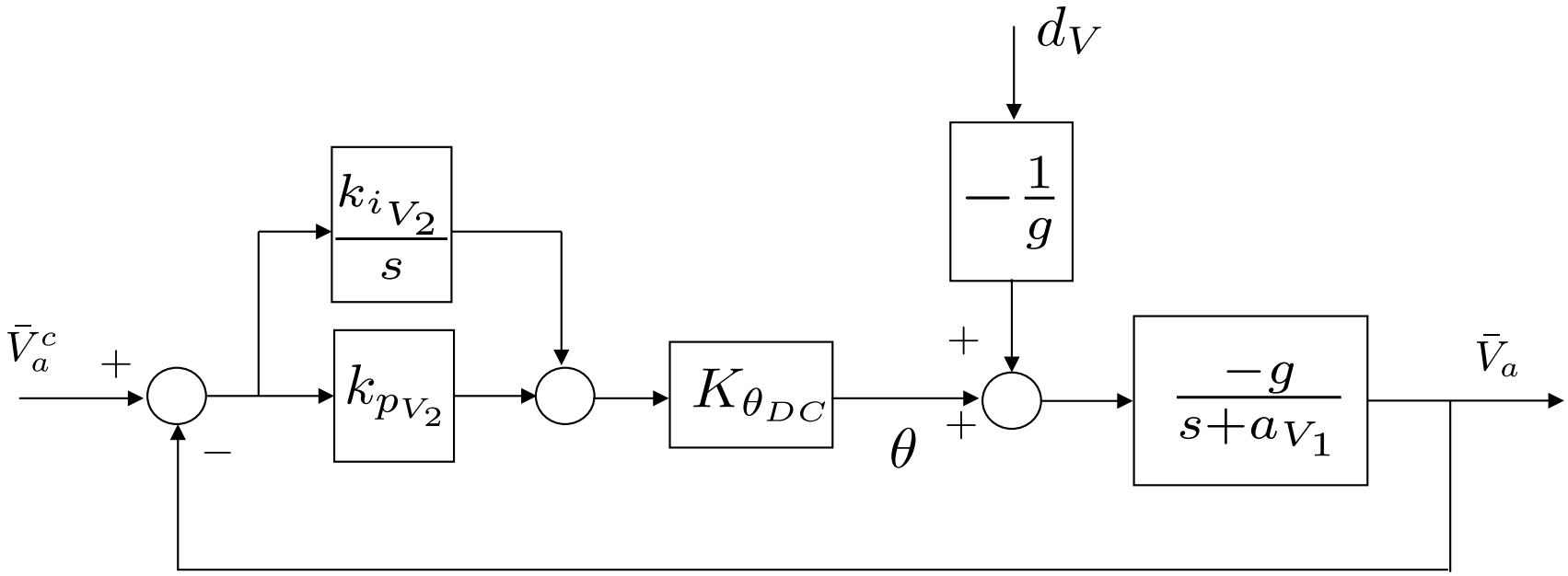
$$\boxed{k_{i_h} = \frac{\omega_{n_h}^2}{K_{\theta_{\rm DC}} V_a}} \qquad \boxed{k_{p_h} = \frac{2\zeta_h \omega_{n_h}}{K_{\theta_{\rm DC}} V_a}}$$

where bandwidth separation is achieved by selecting

$$\omega_{n_h} = \frac{1}{W_h} \omega_{n_\theta}.$$

Design parameters are bandwidth separation W_h and damping ratio ζ_θ

Airspeed Hold Using Commanded Pitch



$$V_a(s) = \left(\frac{-K_{\theta_{\rm DC}} g k_{p_{V_2}} \left(s + \frac{k_{i_{V_2}}}{k_{p_{V_2}}} \right)}{s^2 + (a_{V_1} - K_{\theta_{\rm DC}} g k_{p_{V_2}}) s - K_{\theta_{\rm DC}} g k_{i_{V_2}}} \right) V_a^c(s) + \left(\frac{s}{s^2 + (a_{V_1} - K_{\theta_{\rm DC}} g k_{p_{V_2}}) s - K_{\theta_{\rm DC}} g k_{i_{V_2}}} \right) d_V(s)$$

A PI control on the pitch to airspeed loop ensures that V_a tracks a constant V_a^c with zero steady state error, and rejects low frequency disturbances.

Airspeed from Pitch Gain Calculations

Equating the transfer functions

$$H_{V_a/V_a^c}(s) = \left(\frac{-K_{\theta_{\rm DC}} g k_{p_{V_2}} \left(s + \frac{k_{i_{V_2}}}{k_{p_{V_2}}} \right)}{s^2 + (a_{V_1} - K_{\theta_{\rm DC}} g k_{p_{V_2}}) s - K_{\theta_{\rm DC}} g k_{i_{V_2}}} \right) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

gives the coefficients

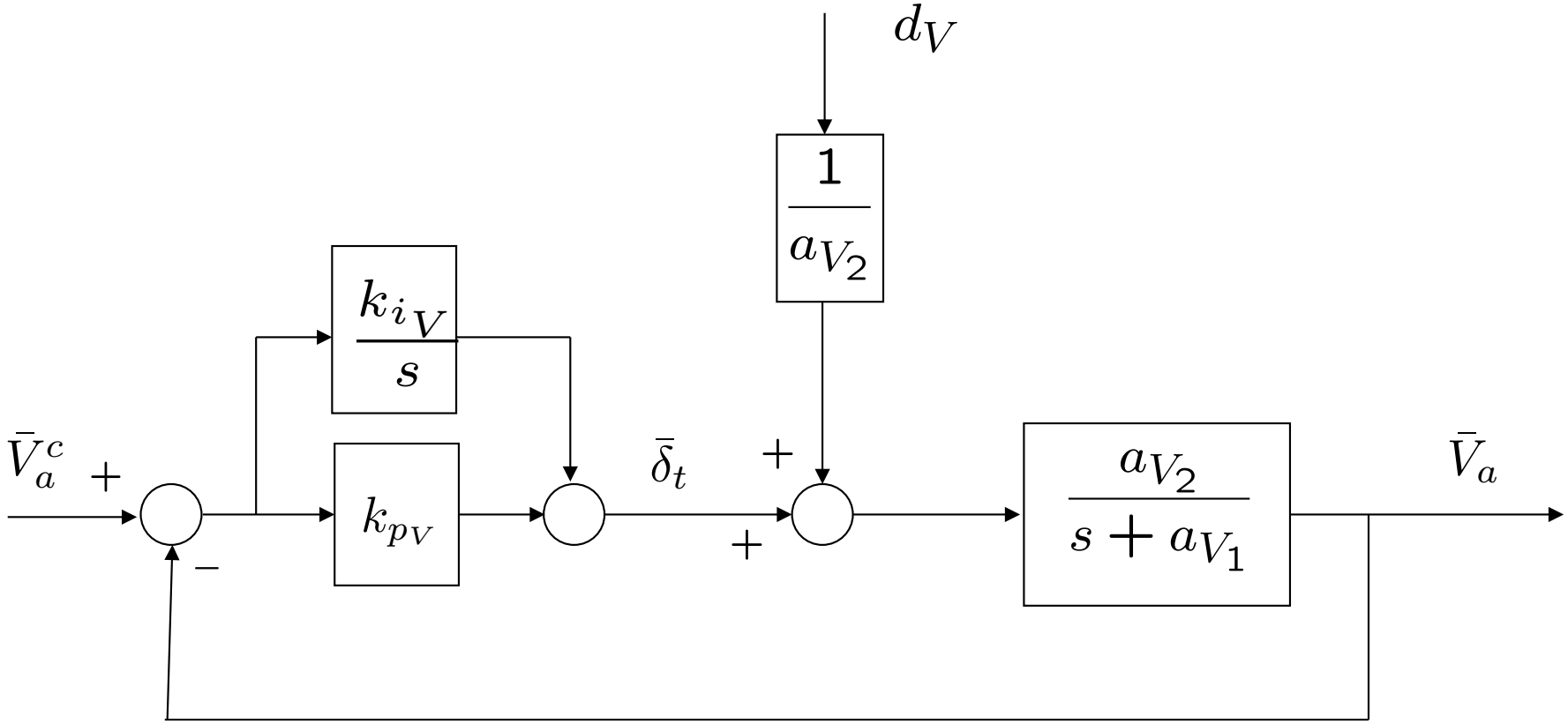
$$\boxed{k_{i_{V_2}} = \frac{\omega_{n_{V_2}}^2}{K_{\theta_{\rm DC}} g}} \qquad \boxed{k_{p_{V_2}} = \frac{a_{V_1} - 2\zeta_{V_2}\omega_{n_{V_2}}}{K_{\theta_{\rm DC}} g}}$$

where bandwidth separation is achieved by selecting

$$\omega_{n_{V_2}} = \frac{1}{W_{V_2}} \omega_{n_\theta}$$

Design parameters are bandwidth separation W_{V_2} and damping ratio ζ_{V_2}

Airspeed Hold Using Throttle



$$V_a = \left(\frac{a_{V_2}(k_{p_V} s + k_{i_V})}{s^2 + (a_{V_1} + a_{V_2} k_{p_V})s + a_{V_2} k_{i_V}} \right) V_a^c + \left(\frac{s}{s^2 + (a_{V_1} + a_{V_2} k_{p_V})s + a_{V_2} k_{i_V}} \right) d_V$$

A PI control on the throttle to airspeed loop ensures that V_a tracks a constant V_a^c with zero steady state error, and rejects low frequency disturbances.

Airspeed Hold Using Throttle

Equating the transfer functions

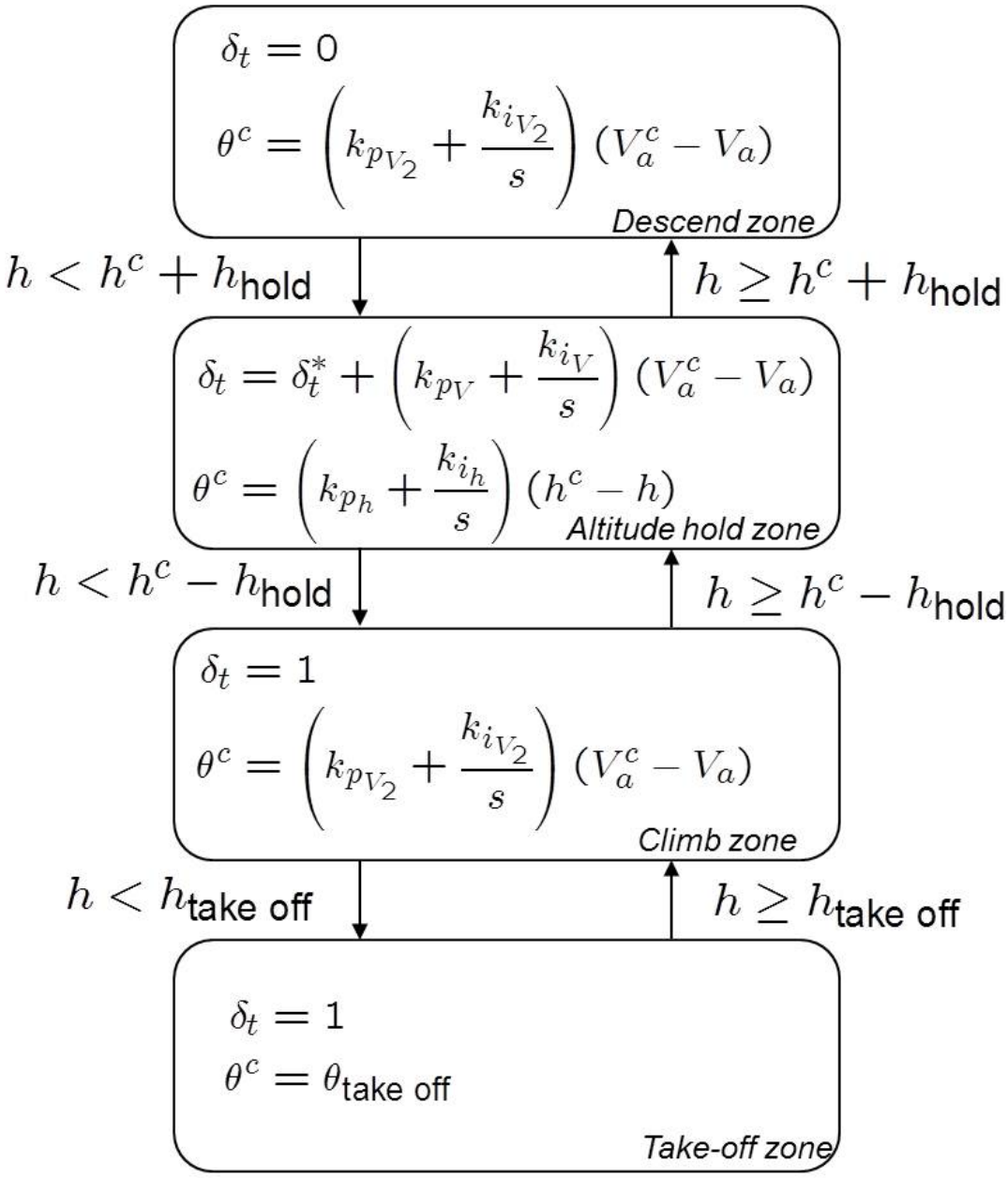
$$H_{V_a/V_a^c}(s) = \left(\frac{a_{V_2}(k_{p_V} s + k_{i_V})}{s^2 + (a_{V_1} + a_{V_2} k_{p_V})s + a_{V_2} k_{i_V}} \right) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

gives the coefficients

$$\boxed{k_{i_V} = \frac{{\omega_{n_V}}^2}{a_{V_2}}} \qquad \boxed{k_{p_V} = \frac{2\zeta_V \omega_{n_V} - a_{V_1}}{a_{V_2}}}$$

Design parameters are natural frequency ω_{n_V} and damping ratio ζ_V.

Altitude Control State Machine



Alternative to State Machine

- An alternative is to eliminate the climb and descend modes, and to saturate the altitude command as

```
h_c_filtered = sat(h_c, h+P.altitude_hold_zone, h-P.altitude_hold_zone);
theta_c = altitude_hold(h_c_filtered, h, 0, P);
delta_t = airspeed_with_throttle_hold(Va_c, Va, 0, P);
```

- This scheme seems to eliminate much of the adverse coupling between altitude an airspeed.

Longitudinal Autopilot - Summary

If model is known, the the design parameters are

Inner Loop (pitch attitude hold)

- $e_\theta^{\max}$ - Error in pitch when elevator just saturates.
- ζ_θ - Damping ratio for pitch attitude loop.

Altitude Hold Outer Loop

- $W_h > 1$ - Bandwidth separation between pitch and altitude loops.
- ζ_h - Damping ratio for altitude hold loop.

Airspeed Hold Outer Loop

- $W_{V_2} > 1$ - Bandwidth separation between pitch and airspeed loops.
- ζ_{V_2} - Damping ratio for airspeed hold loop.

Throttle hold (inner loop)

- ω_{nV} - Natural frequency for throttle loop.
- ζ_V - Damping ratio for throttle loop.

Longitudinal Autopilot – In Flight Tuning

If model is not known, and autopilot must be tuned in flight, then the following gains are tuned one at a time, in this specific order:

Inner Loop (pitch attitude hold)

- k_{d_θ} - Increase k_{d_θ} until onset of instability, and then back off by 20%.
- k_{p_θ} - Tune k_{p_θ} to get acceptable step response.

Altitude Hold Outer Loop

- k_{p_h} - Tune k_{p_h} to get acceptable step response.
- k_{i_h} - Tune k_{i_h} to remove steady state error.

Airspeed Hold Outer Loop

- $k_{p_{V_2}}$ - Tune $k_{p_{V_2}}$ to get acceptable step response.
- $k_{i_{V_2}}$ - Tune $k_{i_{V_2}}$ to remove steady state error.

Throttle hold (inner loop)

- k_{p_V} - Tune k_{p_V} to get acceptable step response.
- k_{i_V} - Tune k_{i_V} to remove steady state error.

PID Loop Implementation

PID control in continuous time is given by

$$u(t) = k_p e(t) + k_i \int_{-\infty}^{t} e(\tau) d\tau + k_d \frac{de}{dt}(t)$$

where

$$e(t) = y^c(t) - y(t).$$

Taking the Laplace transform gives

$$U(s) = k_p E(s) + k_i \frac{E(s)}{s} + k_d s E(s).$$

Use a dirty derivative for causality and to reduce noise:

$$U(s) = k_p E(s) + k_i \frac{E(s)}{s} + k_d \frac{s}{\tau s + 1} E(s),$$

where $1/\tau$ is the bandwidth of the differentiator.

PID Loop Implementation

To convert to discrete time implementation, use the Tustin (or trapezoidal) rule

$$s \mapsto \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right).$$

The integrator $I(s) = \frac{1}{s}E(s)$ becomes

$$I(z) = \frac{T_s}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)E(z).$$

Taking the inverse z-transform gives

$$I[n] = I[n-1] + \frac{T_s}{2}\left(E[n] + E[n-1]\right).$$

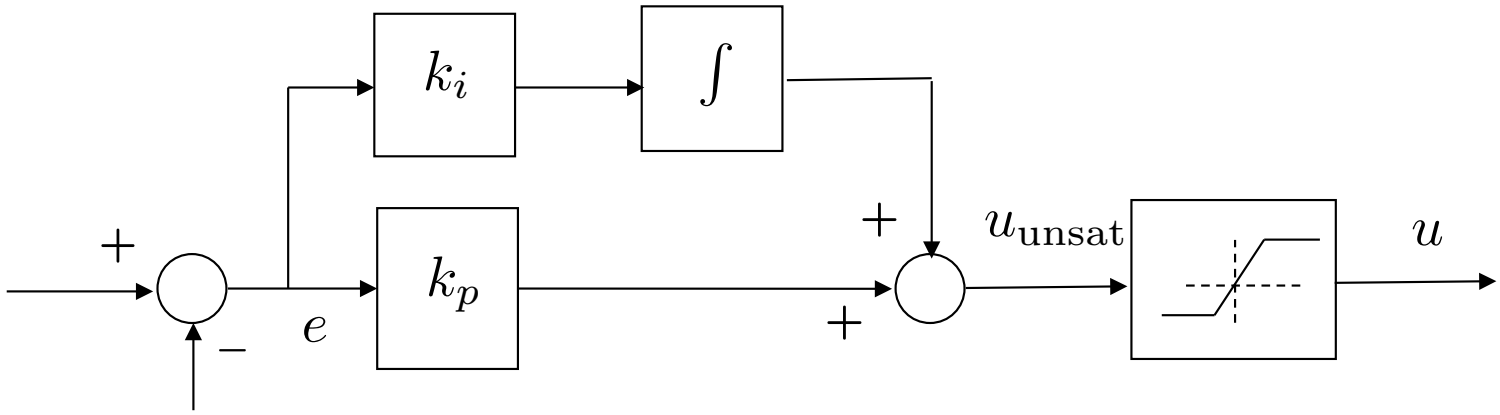
The differentiator $D(s) = \frac{s}{\tau s+1}E(s)$ becomes

$$D(z) = \frac{\frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2\tau}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1}E(z) = \frac{\left(\frac{2}{2\tau+T_s}\right)(1-z^{-1})}{1-\left(\frac{2\tau-T_s}{2\tau+T_s}\right)z^{-1}}E(z).$$

Taking the inverse z-transform gives

$$D[n] = \left(\frac{2\tau-T_s}{2\tau+T_s}\right)D[n-1] + \left(\frac{2}{2\tau+T_s}\right)\left(E[n]-E[n-1]\right).$$

Integrator Anti-wind-up



Integrator wind-up happens when the error $e(t)$ persists, causing the integrator to add area, so that u_{unsat} is beyond saturation. When the error changes sign, so that u should also change sign, the positive area under the integrator hold u at the wrong sign until the integrator un-winds.

Anti-wind-up schemes are intended to limit the integrator from winding-up after u is in saturation.

Integrator Anti-wind-up

Let the control before the anti-wind-up update be given by

$$u^-_{\text{unsat}} = k_p e + k_d D + k_i I^-$$

and the control after the anti-wind-up update be given by

$$u^+_{\text{unsat}} = k_p e + k_d D + k_i I^+.$$

Subtracting the two gives

$$u^+_{\text{unsat}} - u^-_{\text{unsat}} = k_i (I^+ - I^-).$$

Therefore

$$I^+ = I^- + \frac{1}{k_i}(u^+_{\text{unsat}} - u^-_{\text{unsat}}),$$

where u^+_{unsat} is selected to be the saturation limit $\bar{u}\text{sign}(u^-_{\text{unsat}})$.

Anti-wind-up is applied when $|u^-_{\text{unsat}}| \geq \bar{u}$.

PID Implementation

```
function u = pidloop(y_c, y, flag, kp, ki, kd, limit, Ts, tau)
 persistent integrator;
 persistent differentiator;
 persistent error_d1;
 if flag==1, % reset (initialize) persistent variables
             % when flag==1
   integrator = 0;
   differentiator = 0;
   error_d1  = 0; % _d1 means delayed by one time step
 end
 error = y_c - y; % compute the current error
 integrator = integrator + (Ts/2)*(error + error_d1);
 % update integrator
 differentiator = (2*tau-Ts)/(2*tau+Ts)*differentiator...
   + 2/(2*tau+Ts)*(error - error_d1);
 % update differentiator
 error_d1 = error; % update the error for next time through
                   % the loop
 u = sat(...                 % implement PID control
   kp * error +...           % proportional term
   ki * integrator +...      % integral term
   kd * differentiator,...   % derivative term
   limit...                  % ensure abs(u)<=limit
   );
 % implement integrator anti-windup
 if ki~=0
  u_unsat = kp*error + ki*integrator + kd*differentiator;
  integrator = integrator + Ts/ki * (u - u_unsat);
 end

function out = sat(in, limit)
 if in > limit,    out = limit;
 elseif in < -limit;  out = -limit;
 else         out = in;
 end
```

Simulation Project

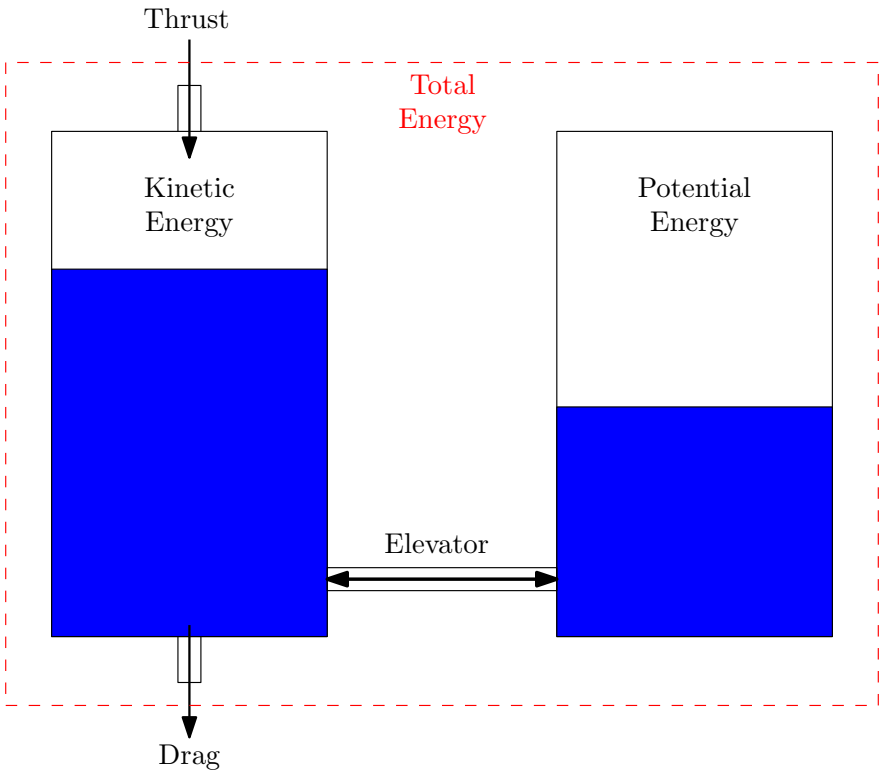
- Roll attitude loop. For Aerosonde model use $V_a = 17\ m/s$. Note that $\phi^{\max}$ is a design parameter. Put aircraft in trim, and command steps on roll.
- Course attitude loop. Command steps in χ. Can by-pass simplified simulink files.
- For sideslip, assume no rudder, i.e., set $\delta_r = 0$.
- Pitch attitude loop. Note that $e_\theta^{\max}$ is a design parameter. Don't use simplified simulink file. Command steps in pitch angle.
- Altitude using pitch, airspeed using pitch, and airspeed using throttle: implement directly on full Simulink model.
- Implement full autopilot using state machine for longitudinal control. Simulation should be from take-off to altitude hold.

Outline

- Successive Loop Closure
- Total Energy Control
- LQR Control

Total Energy Control

- Developed in the 1980's by Antonius Lambregts
- Based on energy manipulation techniques from the 1950's
- Control the energy of the system instead of the altitude and airspeed



Total Energy Control

- Kinetic Energy: $E_K \triangleq \frac{1}{2} m V_a^2$
- Potential Energy: $E_P \triangleq mgh$
- Total Energy: $E_T \triangleq E_P + E_K$
- Energy Difference: $E_D \triangleq E_P - E_K$

Total Energy Control

Original TECS proposed by Lambregts is based on energy rates:

- $T^c = T_D + k_{p,t}\dot{E}_t + k_{i,t}\int_{t_0}^{t} \dot{\tilde{E}}_t \delta_\tau$
  - T_D is thrust needed to counteract drag
  - PI controller based on total energy rate

- $\theta^c = k_{p,\theta}\dot{E}_d + k_{i,\theta}\int_{t_0}^{t} \dot{\tilde{E}}_d \delta_\tau$
  - PI controller based on energy distribution rate

- Stability shown for linear systems

We will show that the performance of this scheme is less than desirable.

Total Energy Control

If the trust needed to counteract drag is unknown, then one possibility is to use an integrator to find T_D:

- $T^c = k_{p,t}\tilde{E}_t + k_{i,t}\int \tilde{E}_t d\tau + k_{d,t}\dot{E}_t$
  - PID controller based on total energy (not energy rate)

- $\theta^c = k_{p,\theta}\tilde{E}_d + k_{i,\theta}\int \tilde{E}_d d\tau + k_{d,\theta}\dot{E}_d$
  - PID controller based on energy distribution (not rate)

Total Energy Control

Nonlinear re-derivation:

- Error Definitions

$$\tilde{E}_K = \frac{1}{2} m \left(\left(V_a^d \right)^2 - V_a^2 \right)$$
$$\tilde{E}_P = mg \left(h^d - h \right)$$

- Lyapunov Function

$$V = \frac{1}{2} \tilde{E}_T^2 + \frac{1}{2} \tilde{E}_D^2$$

- Controller

$$T^c = D + \frac{\tilde{E}_T^d}{V_a} + k_T \frac{\tilde{E}_T}{V_a}$$
$$\gamma^c = \sin^{-1} \left(\frac{\dot{h}^d}{V_a} + \frac{1}{2mgV_a} \left(-k_1 \tilde{E}_K + k_2 \tilde{E}_P \right) \right)$$

Total Energy Control

- Original: $$T^c = D + k_{p,t}\frac{\dot{E}_T}{mgV_a} + k_{i,t}\frac{\tilde{E}_T}{mgV_a}$$

- Nonlinear: $$T^c = D + \frac{\dot{E}_T^d}{V_a} + k_T\frac{\tilde{E}_T}{V_a}$$

Similar if $k_{p,T} = mg$ and $k_{i,T} = mgk_T$.

The nonlinear controller uses the desired energy rate.

Total Energy Control

- Modified Original (Ardupilot):

$$\theta^c = \frac{k_{p,\theta}}{V_a mg}\left((2-k)\,\dot{E}_P - k\dot{E}_K\right) + \frac{k_{i,\theta}}{V_a mg}\tilde{E}_D$$
$$k \in [0,2]$$

- Nonlinear:

$$\gamma^c = \sin^{-1}\left(\frac{\dot{h}^d}{V_a} + \frac{1}{2mgV_a}\left(-k_1\tilde{E}_K + k_2\tilde{E}_P\right)\right)$$
$$k_1 \triangleq |k_T - k_D|$$
$$k_2 \triangleq k_T + k_D$$
$$0 < k_T \leq k_D$$

Lyapunov derivation suggests potential energy error should be weighted more than kinetic energy

Total Energy Control

If the drag is unknown, then we can add an adaptive estimate:

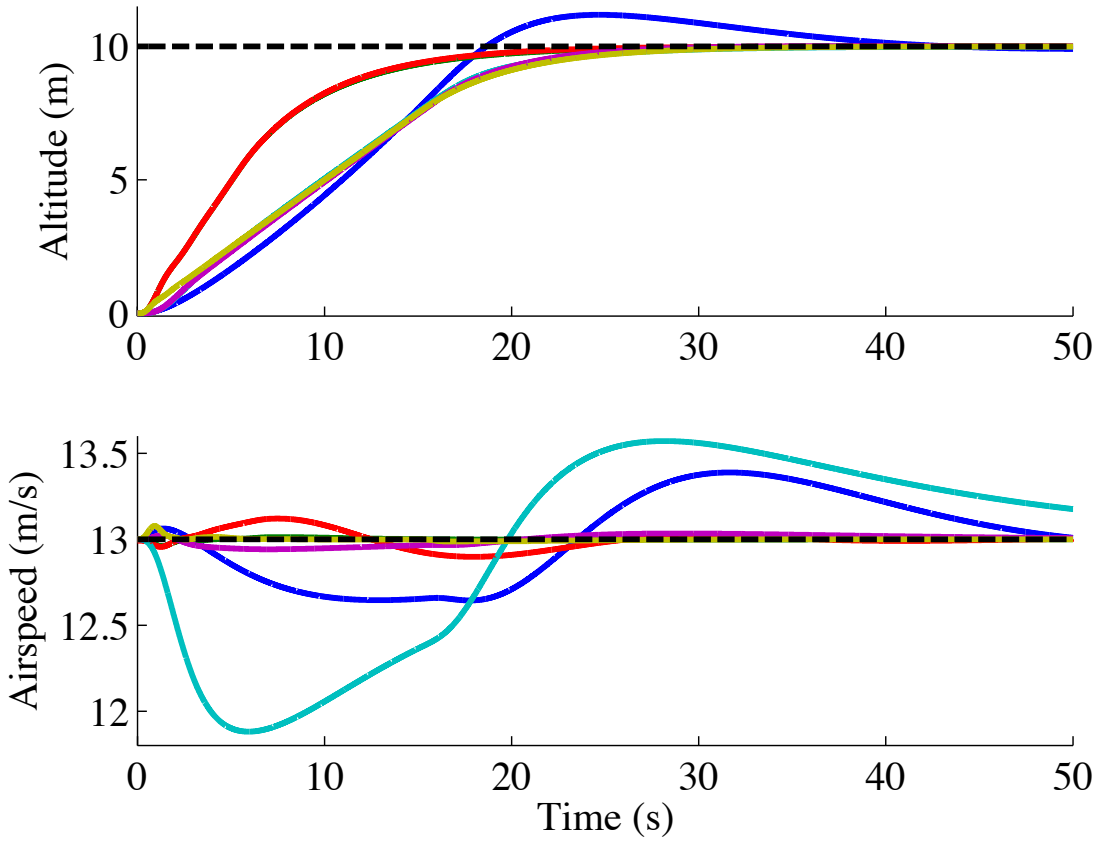
$$T^c = \hat{D} + \mathbf{\Phi}^\top \hat{\mathbf{\Psi}} + \frac{\dot{E}_T^d}{V_a} + k_T \frac{\tilde{E}_T}{V_a}$$

$$\gamma^c = \sin^{-1}\left(\frac{\dot{h}^d}{V_a} + \frac{1}{2mgV_a}\left(-k_1\tilde{E}_K + k_2\tilde{E}_P\right)\right)$$

$$\dot{\hat{\mathbf{\Psi}}} = \left(\Gamma_T\tilde{E}_T - \Gamma_D\tilde{E}_D\right)\mathbf{\Phi}V_a$$

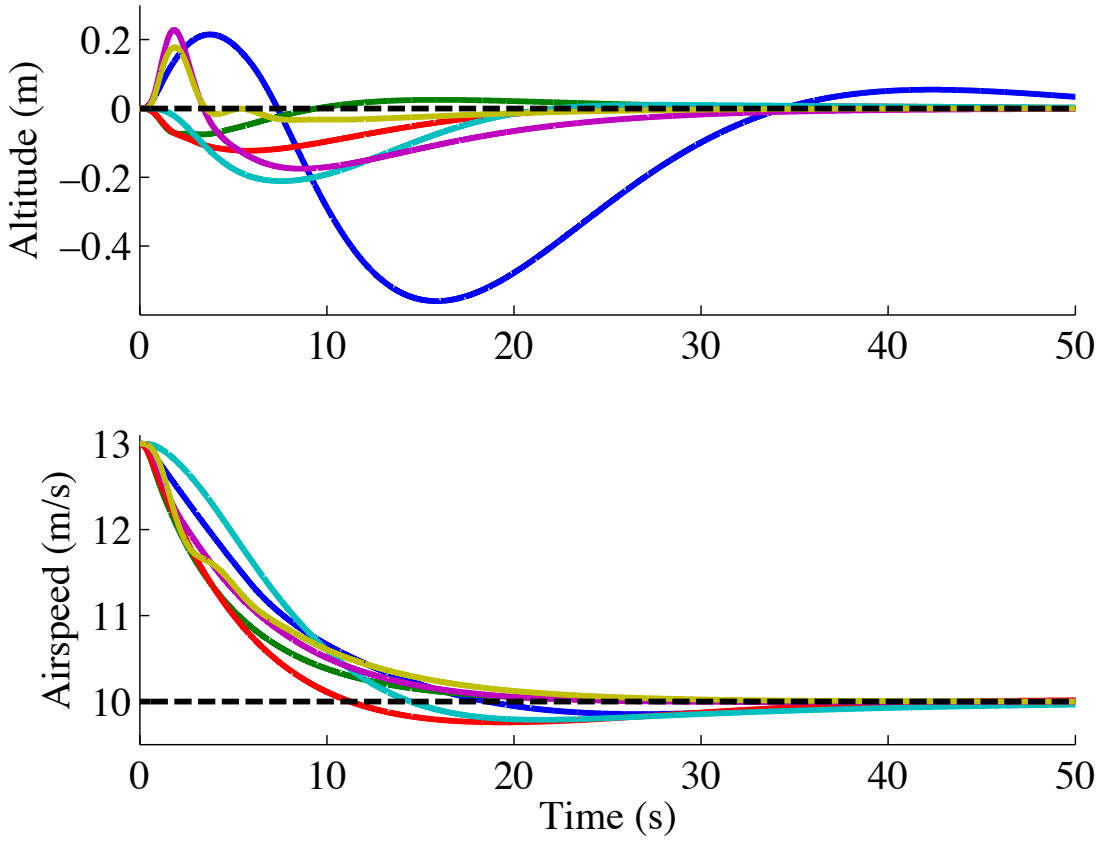
Total Energy Control

Step in Altitude, Constant Airspeed



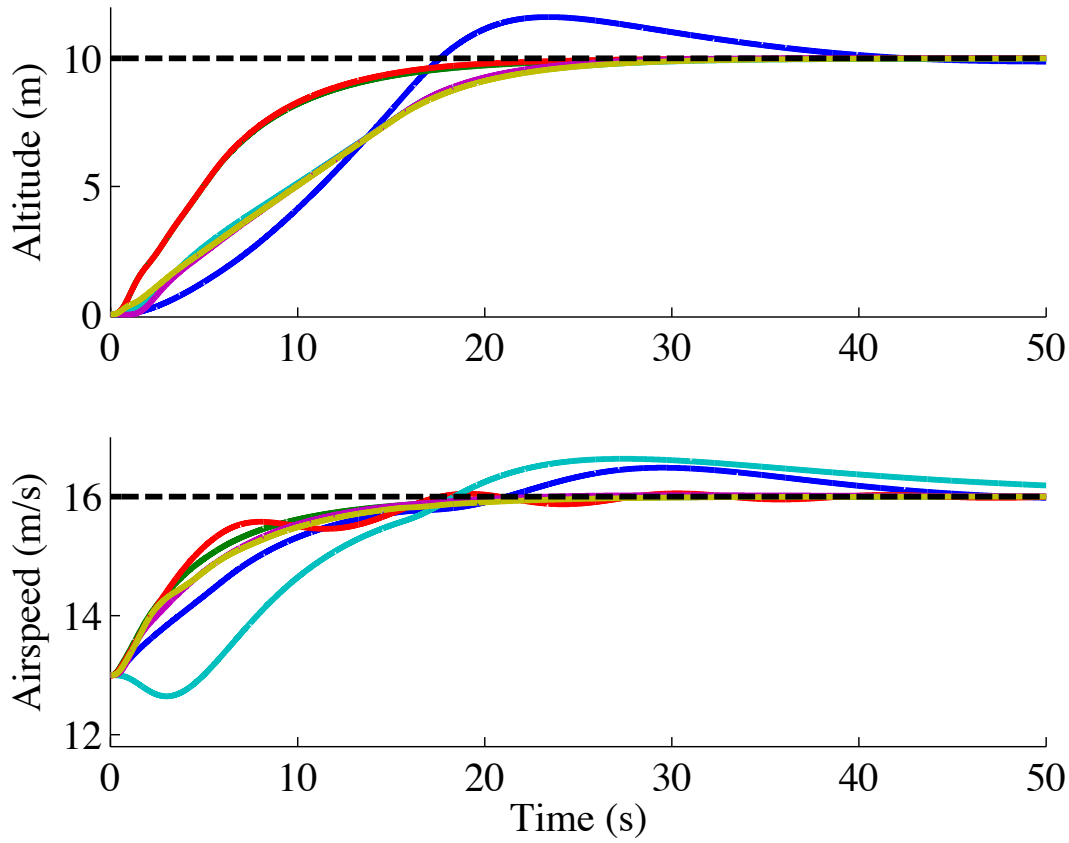
Total Energy Control

Step in Airspeed, Constant Altitude



Total Energy Control

Step in Altitude and Airspeed



Total Energy Control

- Observations
  - TECS seems to work better than successive loop closure.
  - Removes needs for different flight modes.
  - Nonlinear TECS seems to better, but the Ardupilot controller works very well.

Outline

- Successive Loop Closure
- Total Energy Control
- LQR Control

LQR Control

Augment the States with an Integrator.

Given the state space system

$$\dot{x} = Ax + Bu$$
$$z = Hx$$

where z represents the controlled output. Suppose that the objective is to drive z to a reference signal z_r and further suppose that z_r is a step, i.e., $\dot{z}_r = 0$. The first step is to augment the state with the integrator

$$x_I = \int_{-\infty}^{t} (z(\tau) - z_r)\, d\tau.$$

LQR Control

Augment the States with an Integrator. (cont)

Defining the augmented state as $\xi = (x^\top, x_I^\top)^\top$, results in the augmented state space equations

$$\dot{\xi} = \bar{A}\xi + \bar{B}u,$$

where

$$\bar{A} = \begin{pmatrix} A & 0 \\ H & 0 \end{pmatrix} \qquad \bar{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}.$$

LQR Control

Linear Quadratic Regulator Theory

Given the state space equation

$$\dot{x} = Ax + Bu$$

and the symmetric positive semi-definite matrix Q, and the symmetric positive definite matrix R, the LQR problem is to minis the cost index

$$J(x_0) = \min_{u(t), t \geq 0} \int_0^\infty x^\top(\tau) Q x(\tau) + u^\top(\tau) R u(\tau) d\tau.$$

If (A, B) is controllable, and $(A, Q^{1/2})$ is observable, then a unique optimal control exists and is given in linear feedback form as

$$u^*(t) = -K_{lqr} x(t).$$

LQR Control

Linear Quadratic Regulator Theory (cont)

The LQR gain is given by

$$K_{lqr} = R^{-1}B^\top P,$$

where P is the symmetric positive definite solution of the Algebraic Riccati Equation

$$PA + A^\top P + Q - PBR^{-1}B^\top P = 0.$$

LQR Control

Linear Quadratic Regulator Theory (cont)

It should be noted that K_{lqr} is the optimal feedback gains given Q and R. The controller is tuned by changing Q and R.

Typically we choose Q and R to be diagonal matrices

$$Q = \begin{pmatrix} q_1 & 0 & \dots & 0 \\ 0 & q_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & q_n \end{pmatrix} \qquad R = \begin{pmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & r_m \end{pmatrix},$$

where n is the number of states, m is the number of inputs, and $q_i \geq 0$ ensures Q is positive semi-definite, and $r_i > 0$ ensure R is positive definite.

LQR Control

Lateral Autopilot

As derived in Chapter 5, the state space equations for the lateral equation of motion are given by

$$\dot{x}_{lat} = A_{lat} x_{lat} + B_{lat} u_{lat},$$

where $x_{lat} = (v, p, r, \phi, \psi)^\top$ and $u_{lat} = (\delta_a, \delta_r)^\top$.

The objective of the lateral autopilot is to drive course χ to commanded course χ_c. Therefore, we augment the state with

$$x_I = \int (\chi - \chi_c) dt.$$

Since $\chi \approx \psi$, we approximate x_I as

$$x_I = \int (H_{lat} x_{lat} - \chi_c) dt,$$

where $H_{lat} = (0, 0, 0, 0, 1)$.

LQR Control

Lateral Autopilot (cont)

The augmented lateral state equations are therefore

$$\dot{\xi}_{lat} = \bar{A}_{lat}\xi_{lat} + \bar{B}_{lat}u_{lat},$$

where

$$\bar{A}_{lat} = \begin{pmatrix} A_{lat} & 0 \\ H_{lat} & 0 \end{pmatrix} \qquad \bar{B}_{lat} = \begin{pmatrix} B_{lat} \\ 0 \end{pmatrix}$$

The LQR controller designed using

$$Q = \text{diag}\left([q_v, q_p, q_r, q_\phi, q_\chi, q_I]\right)$$
$$R = \text{diag}\left([r_{\delta_a}, r_{\delta_r}]\right).$$

LQR Control

Longitudinal Autopilot

As derived in Chapter 5, the state space equations for the longitudinal equations of motion are given by

$$\dot{x}_{lon} = A_{lon} x_{lon} + B_{lon} u_{lon},$$

where $x_{lon} = (u, w, q, \theta, h)^\top$ and $u_{lat} = (\delta_e, \delta_t)^\top$.

The objective of the longitudinal autopilot is to drive altitude h to commanded altitude h_c, and airspeed V_a to commanded airspeed V_{ac}. Therefore, we augment the state with

$$\begin{aligned} x_I &= \begin{pmatrix} \int (h - h_c) dt \\ \int (V_a - V_{ac}) dt \end{pmatrix} \\ &= \int \left(H_{lon} x_{lon} - \begin{pmatrix} h_c \\ V_{ac} \end{pmatrix} \right) dt, \end{aligned}$$

where

$$H_{lon} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ \frac{1}{V_a} & \frac{1}{V_a} & 0 & 0 & 0 \end{pmatrix}.$$

LQR Control

Longitudinal Autopilot (cont)

The augmented longitudinal state equations are therefore

$$\dot{\xi}_{lon} = \bar{A}_{lon}\xi_{lon} + \bar{B}_{lon}u_{lon},$$

where

$$\bar{A}_{lon} = \begin{pmatrix} A_{lon} & 0 \\ H_{lon} & 0 \end{pmatrix} \qquad \bar{B}_{lon} = \begin{pmatrix} B_{lon} \\ 0 \end{pmatrix}$$

The LQR controller designed using

$$Q = \text{diag}\left([q_u, q_w, q_q, q_\theta, q_h, q_I]\right)$$
$$R = \text{diag}\left([r_{\delta_e}, r_{\delta_t}]\right).$$