**NTNU Trondheim**
**Norwegian University of Science and Technology**
*Department of Engineering Cybernetics*

Contact persons: Christian Holden/Thor I. Fossen
Cell: 996 18 881

# Exam TTK4190
# Guidance and Control

Tuesday May 24, 2011

Hours: 09:00-13:00

**On the Exam Day:**
- All printed and handwritten materials are allowed
- Type A calculators approved by NTNU can be used

**Language:** English

**Number of Pages:** 4

**Sensor Deadline:** 3 weeks

You may write your answers in *Norwegian* or *English.*

Perfect scores amounts to 100 %.

# Problem 1: Autonomous Underwater Vehicles (AUV) Modeling (40 %)

Consider the Omni-Directional Intelligent Navigator (ODIN) in Figure 1. ODIN is an autonomous underwater vehicle (AUV) developed at the ASL of the University of Hawaii. It has a spherical shape with diameter of 0.62 m. Its dry weight is 125 kg, resulting in a slightly positively buoyant behavior. Assume that the CO is located in the center of the sphere and that CB = CO and that CG is located 0.05 m below the CO. The density of water is $\rho = 1026$ kg/m$^3$.

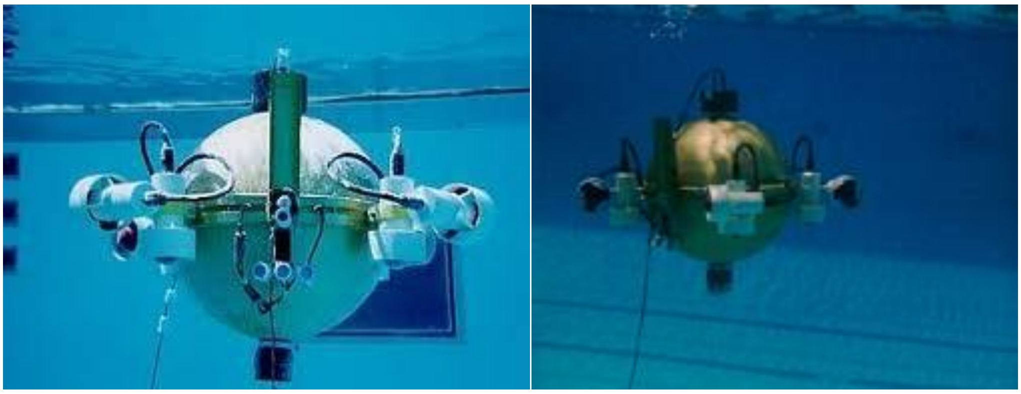

**Figure 1.** The ODIN underwater robotic vehicle. Courtesy to the Autonomous Systems Laboratory (ASL), University of Hawaii.

A. (2 %) What is the weight and buoyancy of the vehicle? (Hint: $V = \frac{4}{3}\pi r^3$)

B. (4 %) For simplicity assume that the mass distribution is homogenous and that the shift in CG downwards can be neglected when computing the moment of inertia. Compute the moment of inertia and products of inertia about CO and find an expression for $\mathbf{M}_{RB}$. Also compute the diagonal added mass matrix $\mathbf{M}_A$.
(Hint: for a sphere the moment of inertia is $I = \frac{2}{5}mr^2$ and the added mass: $\rho\frac{2}{3}\pi\, r^3$).

C. (3 %). Assume that $W = B$ and find an approximation for the linear spring stiffness matrix **G** satisfying:

$$\mathbf{g}(\boldsymbol{\eta}) \approx \mathbf{G}\boldsymbol{\eta}$$

Numerical values for **G** are required.

D. (3 %) The dissipative forces in surge, sway and yaw can be modeled as quadratic damping with drag coefficient $C_d = 0.8$. Compute the hydrodynamic coefficients $X_{|u|u}, Y_{|v|v}$ and $Z_{|w|w}$.

E. (2 %) Compute the natural periods and frequencies in roll and pitch.

F. (4 %) Assume that the linear damping in roll and pitch reduce the frequency of oscillation with 0.05 % such that frequency of oscillation is $\omega = 0.995\omega_n$ where $\omega_n$ is the natural frequency. Use this relationship to compute the linear damping coefficients $K_p$ and $M_q$.

G. (2 %) Assume linear damping in yaw and the time constant is 20 s. Use this information to compute the linear damping coefficient $N_r$.

H. (3 %) The AUV is controlled by using a heading autopilot (PD controller) and the natural period in yaw is 10 s in closed loop. What is the proportional gain $K_p$ of the heading autopilot?

I. (3 %) The relative damping ratio in yaw is $\zeta$ = 0.8. Compute the derivative gain $K_d$ corresponding to this value.

J. (6 %) Write down the 6-DOF equations of motion for the AUV (kinematics and kinetics). Assume diagonal mass and damping matrices and that Coriolis-centripetal forces can be neglected. No numerical values are required. Use zeros in the matrices/vectors to indicate zero elements and SNAME notation for the non-zero elements.

K. (4 %) Show how the equations of motion can be transformed from the CO to the CG.

L. (2 %) Explain why the ODIN AUV is asymptotically stable in roll and pitch. Does this hold locally or globally? Explain why/why not.

M. (2 %) What kind of stability properties does the vehicle have in heave? Explain why.

## Problem 2: Unmanned Aerial Vehicle (UAV) Control System (30 %)

Consider the UAV in Figure 2. The longitudinal motion is given by the following equations:

$$\dot{x} = u\cos(\theta) + w\sin(\theta)$$

$$\dot{z} = -u\sin(\theta) + w\cos(\theta)$$

$$\dot{\theta} = q$$

$$(m - X_{\dot{u}})\dot{u} - Z_{u|u|}u|u| + mg\sin(\theta) = T$$

$$(m - Z_{\dot{w}})\dot{w} - Z_{w|w|}w|w| - mg\cos(\theta) = 0$$

$$\left(I_y - M_{\dot{q}}\right)\dot{q} - M_q q = M_{\delta_s}\delta_S$$

All parameters are assumed to be known and all states are measured.

**Figure 2.** The ODIN Recce 6 Unmanned Aerial Vehicle (UAV).

A. (5 %) Design a feedback linearizing controller for automatic speed control using thrust $\mathrm{T}$ as the control input. Proof that $\mathrm{u}$ converges exponentially to the desired speed $\mathrm{U}_0 =$ constant using a Lyapunov function candidate.

B. (10 %) Assume that $\mathrm{u} = \mathrm{U}_0 > 0$ is constant and design an altitude controller for the aircraft using <u>linear theory</u> such that $\mathrm{h}$ given by

$$\dot{\mathrm{h}} = \mathrm{u}\theta - \mathrm{w}$$

converges to constant reference signal $\mathrm{h}_0$. Is the equilibrium point $\mathrm{h} - \mathrm{h}_0 = 0$ locally or globally exponentially stable (explain why)? Only $\delta_S$ should be used as control input.

C. (12 %) Design a <u>nonlinear backstepping controller</u> that renders the equilibrium point $\mathrm{h} - \mathrm{h}_d = 0$ globally exponentially stable (GES) where $\mathrm{h}_d$ is a time-varying bounded smooth reference signal. Only $\delta_S$ should be used as control input. (Hint: use θ as virtual controller in Step 1 and $q$ as virtual controller in Step 2).

D. (3 %) What is steady-state relationship between $\mathrm{h}$ and angle of attack α?

## Problem 3: Navigation and Estimation (30 %)

A. (3 %) Explain the difference between “flat Earth” and global navigation?

B. (3 %) What are the practical problems when using a magnetic compass for ship navigation?

C. (3 %) What are the main differences of the extended Kalman filter and a deterministic nonlinear state observer?

D. (9 %) Consider an uncontrolled ship exposed to waves, wind and ocean currents. Explain how to estimate the environmental forces (wind, waves and ocean currents) and what kind of instrumentation you need to accomplish this.

E. (10 %) Write down the system model and measurement equations corresponding to Problem 3D and show how these equations can be used in a Kalman filter.

F. (2 %) Is the environmental forces estimation problem observable? Explain why/why not.