TTK4190 Guidance and Control

Exam Suggested Solution

Spring 2012

Problem 1

A) The weight and buoyancy of the vehicle can be found as follows:

$$W = mg = 46 \times 9.81 = 451.26 \text{ N} \tag{1}$$

$$B = 1025 \times 1.6\pi \left(\frac{0.19}{2}\right)^2 \times 9.81 = 456.1524 \text{ N} \tag{2}$$

The result confirms the statement about the vehicle having slightly positively buoyant behavior.

B)

C) Since $B > W$, we conclude that, when submerged, the vehicle will rise.

D) The vehicle will start rising with constant acceleration:

$$\alpha = \frac{B - W}{m} = 0.1064 \text{ m/s}^2 \tag{3}$$

Therefore, the time needed to reach the surface is:

$$t = \sqrt{\frac{2d}{\alpha}} = 43.36 \text{ s.} \tag{4}$$

E) The minimum and maximum added mass is calculated as follows:

$$M_{q,\min} = 0.5m \tag{5}$$

$$M_{q,\max} = m \tag{6}$$

The minimum and maximum natural frequency and period in pitch can be computed from equations (7.266) – (7.267) in the book:

$$\omega_{\min} = \sqrt{\frac{(z_g - z_b)W}{I_y + m}} = \sqrt{\frac{0.04 * 451.26}{0.18 + 46}} = 0.6252 \quad \text{rad/s} \tag{7}$$

$$T_{\max} = \frac{2\pi}{\omega_{\min}} = \frac{2\pi}{0.6252} = 10.05 \text{ s} \tag{8}$$

$$\omega_{\max} = \sqrt{\frac{(z_g - z_b)W}{I_y + 0.5 * m}} = \sqrt{\frac{0.04 * 451.26}{0.18 + 0.5 * 46}} = 0.8824 \quad \text{rad/s} \tag{9}$$

$$T_{\min} = \frac{2\pi}{\omega_{\max}} = \frac{2\pi}{0.8824} = 7.12 \text{ s} \tag{10}$$

F)Similarly to example 12.5 in the book, we compute the linear damping coefficient in pitch:

$$M_q = -2\sqrt{1 - r^2}\sqrt{(z_g - z_b)WI_y} \tag{11}$$

$$= -2\sqrt{1 - 0.995^2}\sqrt{0.04 \times 451.26 \times (0.18 + 46)} \tag{12}$$

$$= -5.7671 \text{ kg m/s} \tag{13}$$

G)For small angles α and β we have $w = \alpha u$ (eq. 2.116) and $u \approx U$ (eq. 2.43). Next, we assume that the roll and pitch angles ϕ, θ are small and we get:

$$\begin{aligned} \dot{d} &= -u\theta + w, && (14) \\ &= -U(\theta - \alpha). && (15) \end{aligned}$$

H) First, we rewrite the system as:

$$\begin{bmatrix} \dot{d} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & U & 0 \\ 0 & 0 & 1 \\ 0 & \dfrac{W\overline{BG}_z}{(I_y - M_{\dot{q}})} & \dfrac{M_q}{(I_y - M_{\dot{q}})} \end{bmatrix} \begin{bmatrix} d \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} -U\alpha \\ 0 \\ \tau/(I_y - M_{\dot{q}}) \end{bmatrix} \quad (16)$$

The rest of the solution follows the method described in section 13.4.2 of the textbook.

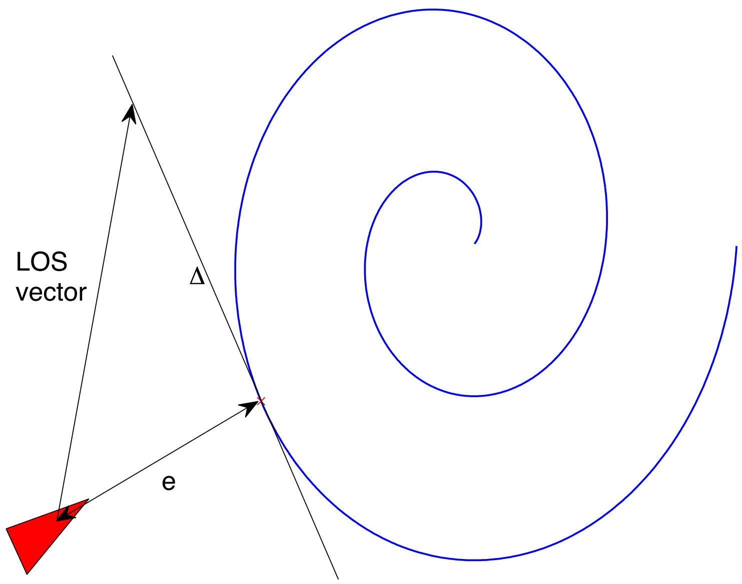

Figure 1: Draft sketch for problem 1I.

I) The polar coordinates are converted to the Cartesian coordinates as follows:

$$\begin{aligned} x &= r\cos(\theta), && (17) \\ y &= r\sin(\theta). && (18) \end{aligned}$$

At the point $\theta_0 = 10$ rad we have:

$$\begin{aligned} x_0 &= r_0\cos(\theta_0) = -9.2298, && (19) \\ y_0 &= r_0\sin(\theta_0) = -5.9842. && (20) \end{aligned}$$

We also compute the slope of the tangent as:

$$\frac{dy}{dx} = -1.8996. \tag{21}$$

Therefore, the equation of the tangent at $\theta_0 = 10$ rad is:

$$y - y_0 = \frac{dy}{dx}(x - x_0) \Rightarrow \tag{22}$$

$$y = -1.8996x - 23.5172. \tag{23}$$

Problem 2

A) For Σ_1 we can propose the following fixed gain observer:

$$\dot{\hat{\mathbf{p}}}^n = \mathbf{R}_b^n(\mathbf{\Theta})\hat{\mathbf{v}}^b + \mathbf{K}_1\tilde{\mathbf{p}}^n \tag{24}$$

$$\dot{\hat{\mathbf{v}}}^b = (-d/m)\hat{\mathbf{v}}^b + (\boldsymbol{\tau}/m) + \mathbf{K}_2\tilde{\mathbf{p}}^n \tag{25}$$

which gives the following error dynamics, in matrix form:

$$\begin{bmatrix} \dot{\tilde{\mathbf{p}}}^n \\ \dot{\tilde{\mathbf{v}}}^b \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_1 & \mathbf{R}_b^n(\mathbf{\Theta}) \\ -\mathbf{K}_2 & -(d/m) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{p}}^n \\ \tilde{\mathbf{v}}^n \end{bmatrix} \tag{26}$$

B) We transform the second system so as to make it more similar with the one given by eq. (11.232) - (11.235) in the textbook. More specifically, we multiply both sides of the second equation by $\mathbf{R}_b^n(\mathbf{\Theta})$ and we get:

$$\dot{\mathbf{p}}^n = \mathbf{v}^n, \tag{27}$$

$$\dot{\mathbf{v}}^n = \mathbf{R}_b^n(\mathbf{\Theta})[\boldsymbol{\alpha}_{\text{imu}}^b - \mathbf{b}^b] + \mathbf{g}^n, \tag{28}$$

$$\dot{\mathbf{b}}^b = \mathbf{0}. \tag{29}$$

Next, we propose the following fixed-gain observer:

$$\dot{\hat{\mathbf{p}}}^n = \hat{\mathbf{v}}^n + \mathbf{K}_1\tilde{\mathbf{p}}^n, \tag{30}$$

$$\dot{\hat{\mathbf{v}}}^n = \mathbf{R}_b^n(\mathbf{\Theta})[\boldsymbol{\alpha}_{\text{imu}}^b - \hat{\mathbf{b}}^b] + \mathbf{g}^n + \mathbf{K}_2\tilde{\mathbf{p}}^n, \tag{31}$$

$$\dot{\hat{\mathbf{b}}}^b = \mathbf{K}_3\mathbf{R}_b^n(\mathbf{\Theta})^{\mathrm{T}}\tilde{\mathbf{p}}^n. \tag{32}$$

which results in the error dynamics:

$$\begin{bmatrix} \dot{\tilde{\mathbf{p}}}^n \\ \dot{\tilde{\mathbf{v}}}^n \\ \dot{\tilde{\mathbf{b}}}^b \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_1 & \mathbf{I} & \mathbf{0} \\ -\mathbf{K}_2 & \mathbf{0} & -\mathbf{R}_b^n(\mathbf{\Theta}) \\ -\mathbf{K}_3\mathbf{R}_b^n(\mathbf{\Theta})^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{p}}^n \\ \tilde{\mathbf{v}}^n \\ \tilde{\mathbf{b}}^b \end{bmatrix} \tag{33}$$

C) The second model would be preferred from a company that sells motion sensors. The main reason is that Σ_2 requires accelerometer measurements. The first model circumvents this necessity by using a mass-damper model of the vehicle in coordination with the control forces, which are known. As a consequence his implies that Σ_1 requires that system identification techniques have been applied before the vehicle's operation. This method might be more susceptible to give large errors in the case where the operational conditions are not constant and the model is not representative of the system anymore.

D) By taking into account the non-gravitational acceleration we end up with a system of equations that can be very time-consuming to solve:

$$\begin{bmatrix} 1.8018 \\ 2.4958 \\ -9.3151 \end{bmatrix} = \begin{bmatrix} \cos(\psi)\cos(\theta)0.1 - \sin(\theta)g \\ (-\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\theta)\sin(\psi))0.1 + g\sin(\phi)\cos(\phi) \\ (-\sin(\psi)\sin(\phi) + \cos(\psi)\sin(\phi)\sin(\theta))0.1 + \cos(\theta)\cos(\phi)g \end{bmatrix}. \tag{34}$$

It's worth checking whether the non-gravitational acceleration is large enough or we can neglect it. The norm of the total acceleration vector is:

$$\|\alpha^b_{\text{imu}}\| = 9.8105. \tag{35}$$

This value is not much different than that of the gravity vector:

$$\|g^b\| = 9.81. \tag{36}$$

Therefore, we neglect $\dot{\mathbf{v}}^n$ and compute the roll and pitch angles (see eqs. 11.268–11.269 in the textbook):

$$\phi \approx \arctan\left(\frac{\alpha_y}{\alpha_z}\right), \tag{37}$$

$$\theta \approx \arctan\left(\frac{\alpha_x}{\sqrt{\alpha_y^2 + \alpha_z^2}}\right). \tag{38}$$

$$\phi = -0.2618 \text{ rad}, \tag{39}$$

$$\theta = -0.1847 \text{ rad}. \tag{40}$$

E)

Problem 3

A) The main drawback of using straight lines and inscribed circles in waypoint tracking systems is that a jump in the desired yaw rate r_d occurs during the transition from the straight line to the circle arc. As a result, a small offset is produced during cross-tracking. In order to prevent this from happening, smooth trajectories need to be generated. This can be achieved by using interpolating methods, Hermite cubic splines for instance. However it's worth noting that the shortest possible path will always consist of combined straight lines and circle arcs, as it was proven by Dubins in 1957.

B) As mentioned in the textbook, the course (or heading, depending on whether we have drift or not) angle assignment is separated in two parts:

$$\psi_d = \alpha_k + \arctan\left(\frac{-e}{\Delta}\right) \tag{41}$$

When the task is to converge to a straight line, the path-tangential angle α_k is the angle between the straight line and the vertical line and remains constant. However, in the general case where the desired path is curved, then α_k is the derivative (or tangent) of the path at a specific point on the path. The second angle is the one between the line connecting the vehicle's position with the point on the straight line indicated by the lookahead distance Δ and the straight line itself. This angle is time-varying (because the cross-track error is also varying) with the exception of the case where the vehicle has converged to a straight line - in that case it takes the 'permanent' value α_k.

C)There are several ways to tackle this problem. We know that the vehicle is in the position $P_1(4,5)$ and that we want it to converge to the line $y_1 = 3x_1 + 1$. We need two elements in order to solve the problem, namely, the path-tangential angle α_k and the cross-track error e. The first one is obvious, since the slope of a straight line is given by the equation of the line itself. Therefore:

$$\tan(\alpha_k) = 3 \tag{42}$$
$$\alpha_k = 1.249 \text{ rad.} \tag{43}$$

Next, we need to find a line which is perpendicular to y_1 and also passes through the point P_1. We know that the product of the slopes of two perpendicular lines is equal to -1, therefore:

$$y_2 - 5 = -\frac{1}{3}(x_2 - 4) \Rightarrow \tag{44}$$
$$y_2 = -\frac{1}{3}x_2 + \frac{19}{3} \tag{45}$$

The point P_2 where the two lines, y_1 and y_2 meet can be found by solving the system consisting of the two line equations. The result is:

$$P_2(x_2, y_2.z_2) = P_2(1.6, 5.8, 0). \tag{46}$$

Consequently, the cross-track error can be computed as:

$$e = \sqrt{(1.6-4)^2 + (5.8-5)^2} \tag{47}$$
$$= 2.5298 \text{ m.} \tag{48}$$

Finally, the desired course angle is:

$$\chi_d = 1.249 + \arctan\left(\frac{-2.5298}{100}\right) \tag{49}$$
$$= 1.2237 \text{ rad} \tag{50}$$
$$= 70.1128^\circ. \tag{51}$$

D) Since the sideslip angle is known to us, we can simply compute the desired heading angle as follows (see textbook, page 263):

$$\psi_d = \chi_d - \beta \tag{52}$$
$$= 75.1128^\circ. \tag{53}$$

E) The value of the lookahead distance can have a significant impact on the behavior of a vehicle that navigates using a lookahead-based steering law. As it can be concluded from eq. (41), a small Δ implies aggressive steering because the vehicle is commanded to reach a straight line as fast as possible. However, this implies that the vehicle will reach the line faster but, due to its orientation when reaching it, it will also start oscillating around the target line. If a large Δ is chosen instead, the vehicle will follow a smoother trajectory which will converge to the line without oscillating, but in a considerably longer time.

These two behaviors indicate that a combination of both can lead to a better overall performance. Consequently, when the vehicle is very far from its target, a small Δ can be chosen in order to make the vehicle more aggressive and approach the target faster. As the vehicle gets closer, the value for Δ should decrease so as to obtain a smooth behavior without oscillations. The minimum and maximum values for Δ should be determined by taking into account the physical constraints of the vehicle, amongst other things.