Graphics & Visualization

Chapter 12

ILLUMINATION MODELS & ALGORITHMS

Introduction

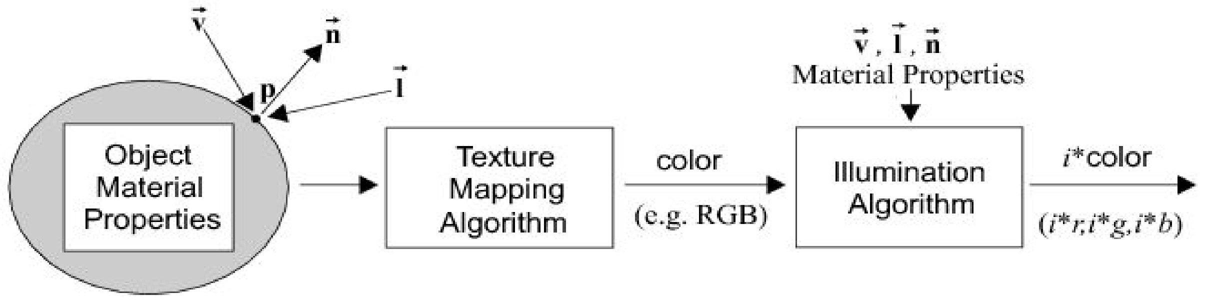
- Representation of illumination phenomena in CG :
  - Based on the laws of optics
- Laws that make the most difference in practice are implemented
- Computational cost must be considered

Role of an *Illumination Model*:

- Light illuminates a point **p** of an object (directly or via reflections) $\rightarrow$ changes object's color at **p** according to:
  - Direction of the incident light
  - Direction of observation
  - Surface normal at **p**
  - Reflectivity of the material
  - …

Introduction (2)

- Effects of illumination and texturing should not be confused:



- Two trends in Computer Graphics:
  - 1st uses practical illumination models:
    - Produces acceptable illumination effects
    - Low computational cost
    - Suitable for real-time applications
  - 2nd implements a large part of available illumination theory:
    - Produces the most convincing illumination effects
    - High computational cost
    - Suitable for demanding, non real-time applications

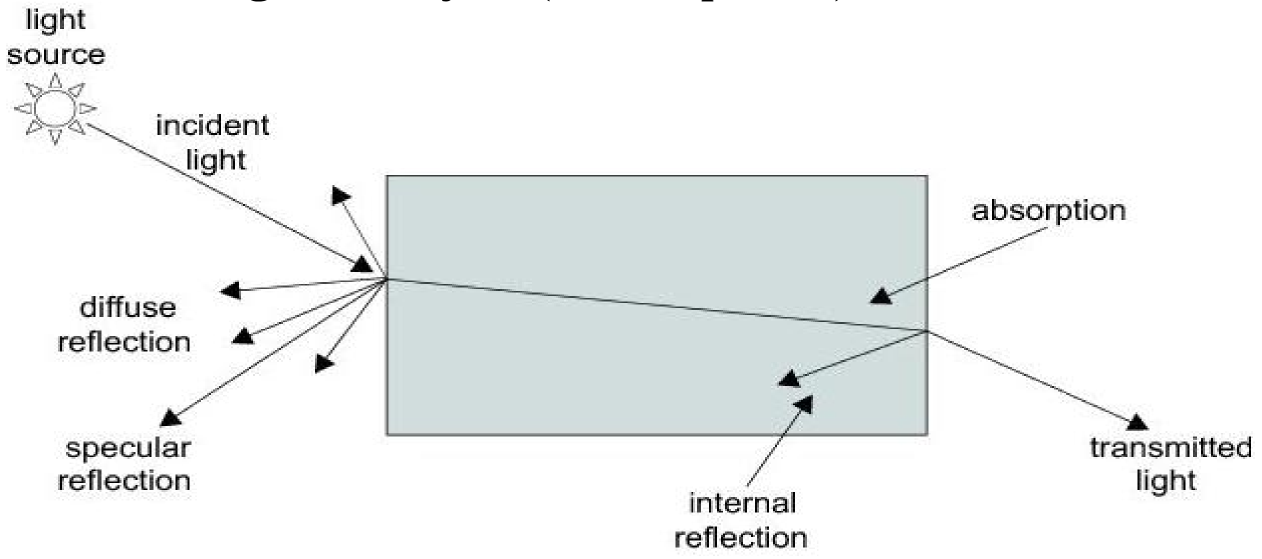
Introduction (3)

- Essential difference between the two approaches:
  - The 2nd considers the interaction of light between objects or how objects are indirectly illuminated by light reflected from other objects
  - 1st approach constitutes the *local illumination models*
  - 2nd approach constitutes the *global illumination models*

- Distinction between illumination models and algorithms:
  - An illumination model encapsulates a set of physical illumination laws
  - An illumination algorithm implements an illumination model efficiently

Physics of Light-Object Interaction I

- Incident Light= reflected light + scattered light + absorbed light + transmitted light
- Depending on the structure (roughness) of the object’s surface portions of the incident light will be:
  - Reflected in the “mirror” of the incident direction (*specular reflection)*
  - Scattered in all directions (*diffuse reflection)*
  - Absorbed, increasing the object’s temperature
  - Transmitted through the object (if transparent)



Physics of Light-Object Interaction I (2)

- *Radiant energy (Q):* emitted from a light source or reflected from a surface and transferred through space as photons
  - Radiant energy is the total energy emitted as radiation of all wavelengths in a defined period of time and is measured in *joules*
- *Radiant power* (or *flux* Φ): the rate at which radiant energy passes a spatial reference and is measured in *watts*:

$$\Phi = dQ / dt \qquad (12.1)$$

- Energy emitted or reflected from a point:
  - may be restricted to certain directions or
  - may be spreading equally in all directions
- *Radiant intensity* (I_r): radiant power per unit of solid angle ω_r in a certain direction:

$$I_r = d\Phi_r / d\omega_r \qquad (12.2)$$

  - *Intensity* is measured in *watts/steradian* (overloaded term)

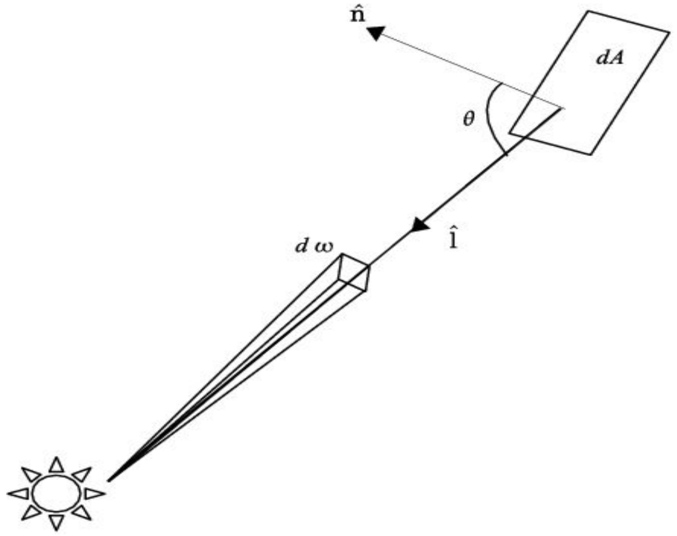
Physics of Light-Object Interaction I (3)

- *Radiance (L):* Assume an infinitesimal surface *dA* with normal vector $\hat{\mathbf{n}}$ forming an angle θ with the direction of incident or outgoing illumination $\hat{\mathbf{l}}$. Radiance is defined as the radiant power per unit solid angle leaving or entering the infinitesimal area *dA* from a certain direction per unit projected surface area in that direction:

$$L = d\Phi / (d\omega dA \cos\theta) = d\Phi / (d\omega dA(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})) \qquad (12.3)$$

Physics of Light-Object Interaction I (4)

- Radiance:



- Radiance is inversely proportional to the square of the distance from the light source and is measured in *watts/(steradians · m*^{2}*)*
- *Albedo (ρ):* Ratio of scattered to incident electromagnetic radiation across the spectrum
  - Albedo defines the color of a material without the effect of illumination

Physics of Light-Object Interaction I (5)

- *Irradiance (E_i):* Incident flux per unit area in the vicinity of a point
  - Irradiance can be visualized as the power per unit area incident from all directions within a hemisphere onto an elementary surface located at the center of the base of that hemisphere:

$$E_i = d\Phi_i / dA \qquad (12.4)$$

  - Measured in *watts/m*^{2}
- *Radiosity (B):* flux per unit area exiting a surface:

$$E_r = B = d\Phi_r / dA \qquad (12.5)$$

  - Also measured in *watts/m*^{2}
- *Incident intensity (I_i):* incident flux on a point per unit solid angle:

$$I_i = d\Phi_i / d\omega_i \qquad (12.6)$$

- Relation between incident intensity & irradiance by combining (12.4) & (12.6): $E_i = I_i d\omega_i / dA.$ (12.7)

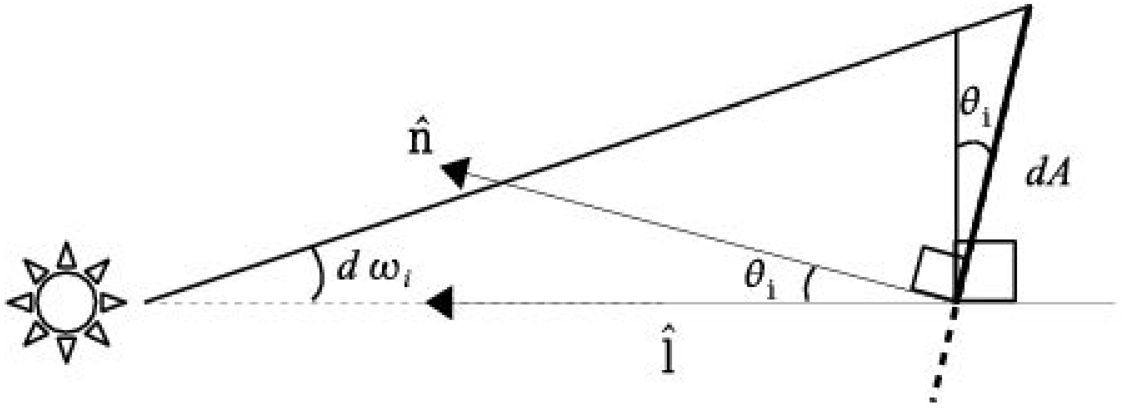
Physics of Light-Object Interaction I (6)

- From the definition of solid angle:

$$d\omega_i = \frac{dA\cos\theta_i}{d^2}$$

where $dA \cdot \cos\theta_i$: projection of the elementary surface dA onto a plane normal to the direction of illumination

d: distance from the light source to the elementary surface



Physics of Light-Object Interaction I (7)

- Photometry law (from (12.7) and solid angle definition):

$$E_i = I_i \frac{\cos\theta_i}{d^2} = I_i \frac{(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{d^2} \qquad (12.8)$$

- In Computer Graphics we are interested in the relationship between the incident light from a certain direction onto a surface and:
    - The reflected light in another direction
    - The transmitted light through the object
- Relationship is captured by the *bidirectional reflectance distribution function (BRDF)*

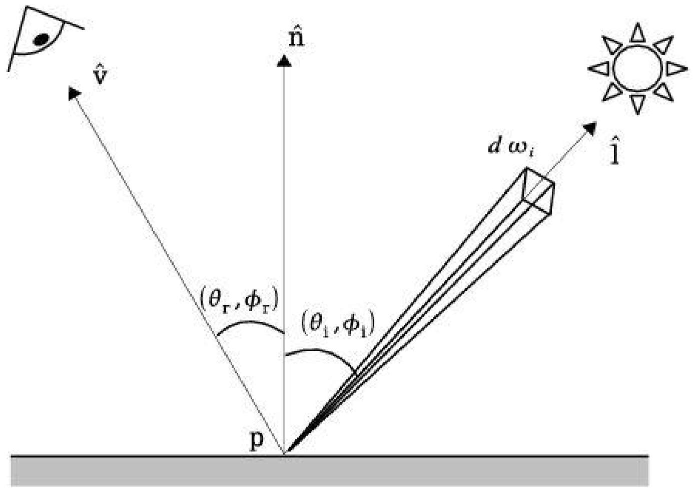
Physics of Light-Object Interaction I (8)

- BRDF depends on:
  - Lighting and observation directions
  - Wavelength
  - Shadow casting
  - The optical properties of the object
  - Reflectivity
  - Absorption
  - Emission
- BRDF can only be approximated
- BRDF associates the outgoing radiance dL_r in direction (θ_r,φ_r) to the irradiance dE_i from the incident direction (θ_i,φ_i):

$$BRDF = \frac{dL_r}{dE_i} \qquad (12.9)$$

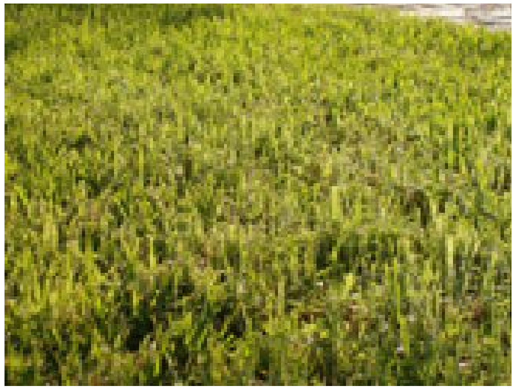
Physics of Light-Object Interaction I (9)

- BRDF

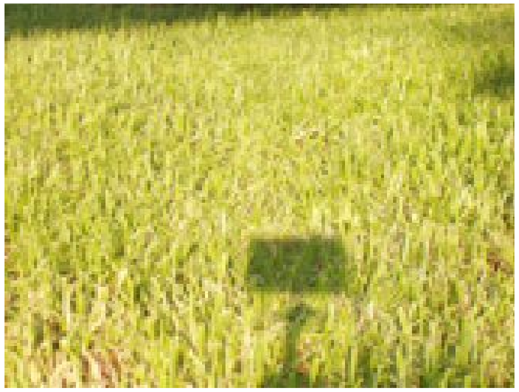


- BRDF expresses how objects look differently when seen from different angles or when illuminated from different directions, e.g.:

Light source opposite observer



Light source behind observer

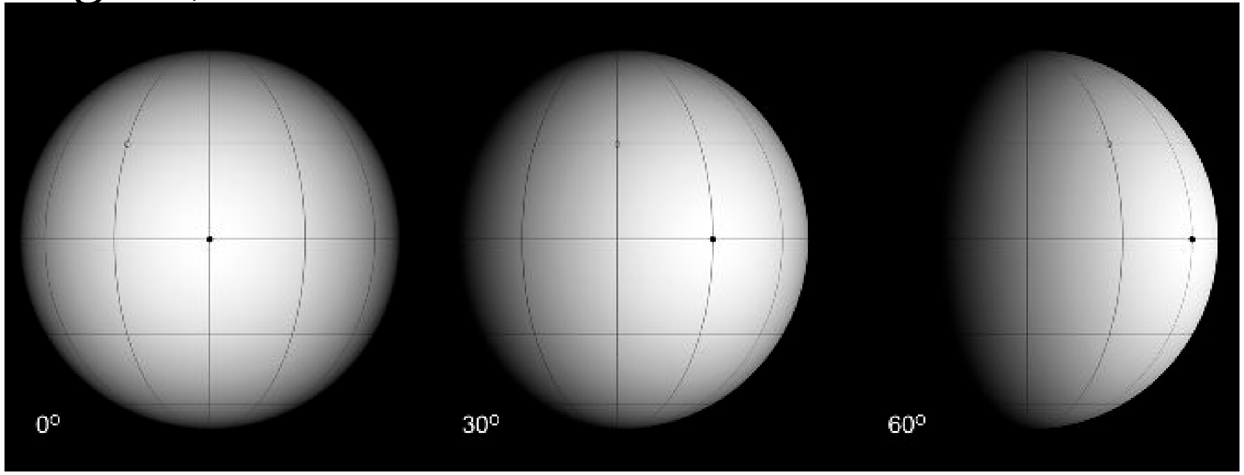


The Lambert Illumination Model*

- Simplest illumination model for body reflection
- Assumes that incident light at vicinity of a point **p** on a surface is equally diffused in all directions on the incident hemisphere
- BRDF of the body surface is constant in all directions and invariant with respect to wavelength and polarization
- A perfectly diffuse surface is called *Lambertian*
- *Diffuse illumination* mostly accounts for reflected light due to body reflectance
  - (In contrast, *specular illumination* corresponds to light reflected off the surface)
- *Lambert's cosine law:* The total radiant power observed from a Lambertian surface is directly proportional to the cosine of the angle θ_r between the observer's line of sight and the surface normal.

The Lambert Illumination Model (2)

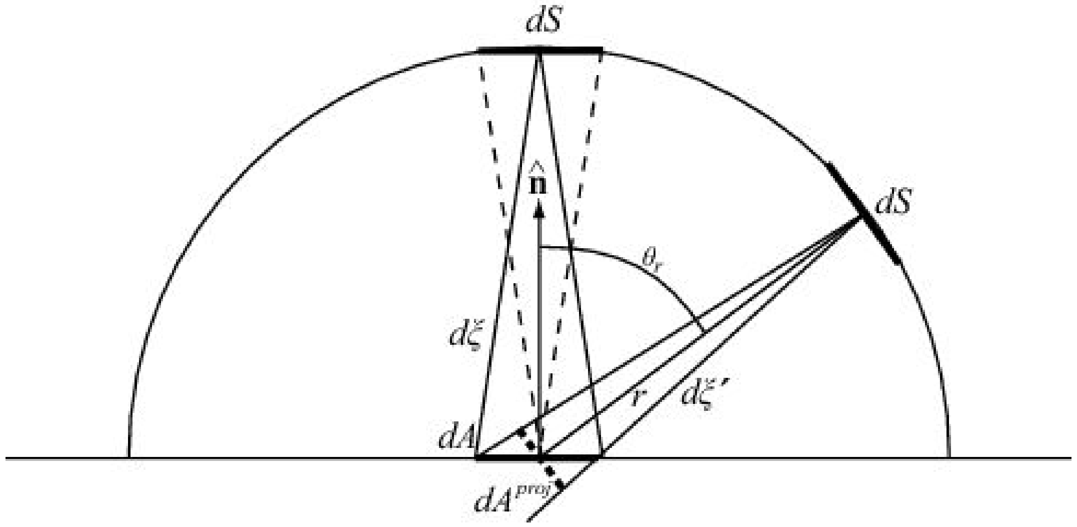
- Consequence of Lambert's law: when an elementary surface *dA* is viewed from an arbitrary direction within the hemisphere Ω surrounding *dA*, it exhibits the same radiance



- Explanation: As the radiant power $d\Phi_r$ observed at a direction (θ_r,ϕ_r) diminishes according to Lambert's cosine law, so does the solid angle $d\xi$ subtended by the surface patch *dA* and viewed from a distant patch *dS* around the observer location → equal decrease of both terms, which eventually cancel out

The Lambert Illumination Model (3)

- Imagine that receiving patch *dS* were positioned directly above *dA*, perpendicular to the normal vector of *dA*



- Since $\theta_r = 0$, from the definition of radiance (12.3) → the observed radiance is:

$$L_0 = \frac{d\Phi_0}{dS\,d\xi}$$

The Lambert Illumination Model (4)

- Let us position *dS* at a different viewing angle, away from the normal direction of *dA*, always perpendicular to the corresponding viewing direction vector (see previous figure)
- According to Lambert's cosine law, the new radiance at this arbitrary outbound direction is:

$$L = \frac{d(\Phi_0 \cos\theta_r)}{dSd\xi'} = \frac{\cos\theta_r d\Phi_0}{dSd\xi'} \qquad (12.11)$$

- As *dA* is very small, the new solid angle $d\xi'$ is proportional to the projection of *dA* on the light transfer direction ($d\xi' = dA\cos\theta_r/r^2$), and therefore:

$$d\xi' = \cos\theta_r d\xi \qquad (12.12)$$

The Lambert Illumination Model (5)

- Replacing the new solid angle in (12.11) yields:

$$L = \frac{\cos\theta_r d\Phi_0}{dSd\xi'} = \frac{\cos\theta_r d\Phi_0}{\cos\theta_r dSd\xi} = \frac{d\Phi_0}{dSd\xi} = L_0 \qquad (12.13)$$

- We next derive constant BRDF (f_d) for the Lambertian surface
- Radiant flux is evenly distributed over the hemisphere subtended by the surface patch at vicinity of **p**, <u>BUT</u> f_d is not equal to 1/2π
- Outgoing radiance is constant → does not depend on the reflected light direction on the hemisphere: $L_r(\theta_r,\phi_r) = L_r$
- Irradiance is not attenuated by the material & is equally spread to every outgoing differential solid angle → reflectance factor $\rho(\vec{\omega}_i \to \Omega)$ (ratio of total reflected light to incident light from $d\vec{\omega}_i$) equals 1

The Lambert Illumination Model (6)

- From definition of irradiance, radiosity, and radiance ((12.4), (12.5), (12.3)) we get:

$$d\Phi_i = E_i dA \qquad (12.14)$$

$$L_r(\theta_r,\varphi_r) = \frac{dE_r(\theta_r,\varphi_r)}{d\vec{\omega}_r \cos\theta_r} = \frac{dE_r(\theta_r,\varphi_r)}{d\vec{\omega}_r^{proj}} \Rightarrow$$

$$dE_r = L_r d\vec{\omega}_r^{proj} \Rightarrow E_r = \int_\Omega L_r d\vec{\omega}_r^{proj} \qquad (12.15)$$

- Using the results from (12.14) & (12.15), the unit reflectance becomes:

$$\rho(\vec{\omega}_i \to \Omega) = 1 = \frac{d\Phi_r}{d\Phi_i} = \frac{dA\int_\Omega L_r d\vec{\omega}_r^{proj}}{E_i dA} = \frac{L_r dA\int_\Omega d\vec{\omega}_r^{proj}}{E_i dA} \qquad (12.16)$$

The Lambert Illumination Model (7)

- From definition of BRDF & taking into account that BRDF for the Lambertian surface is constant, we have:

$$f_d = \frac{dL_r}{L_i \cos\theta_i d\vec{\omega}_i} \Rightarrow dL_r = f_d L_i d\vec{\omega}_i^{proj} \Rightarrow$$

$$L_r = \int_\Omega f_d L_i d\vec{\omega}_i^{proj} = f_d \int_\Omega L_i d\vec{\omega}_i^{proj} = f_d E_i \qquad (12.17)$$

- Now we can return to (12.16) and substitute L_r from (12.17):

$$1 = \frac{L_r dA \int_\Omega d\vec{\omega}_r^{proj}}{E_i dA} = \frac{f_d E_i dA \int_\Omega d\vec{\omega}_r^{proj}}{E_i dA} =$$

$$= f_d \int_\Omega d\vec{\omega}_r^{proj} = f_d \pi \Leftrightarrow f_d = \frac{1}{\pi} \qquad (12.18)$$

The Lambert Illumination Model (8)

- Summing up:
  - The radiance associated with an infinitesimal surface patch of area *dA* around point ***p*** is proportional to the cosine of the angle θ_i between normal vector at ***p*** and the incident direction
  - The above happens due to the flow of energy that passes through the (projected) area *dA* of the patch with respect to the incident light direction

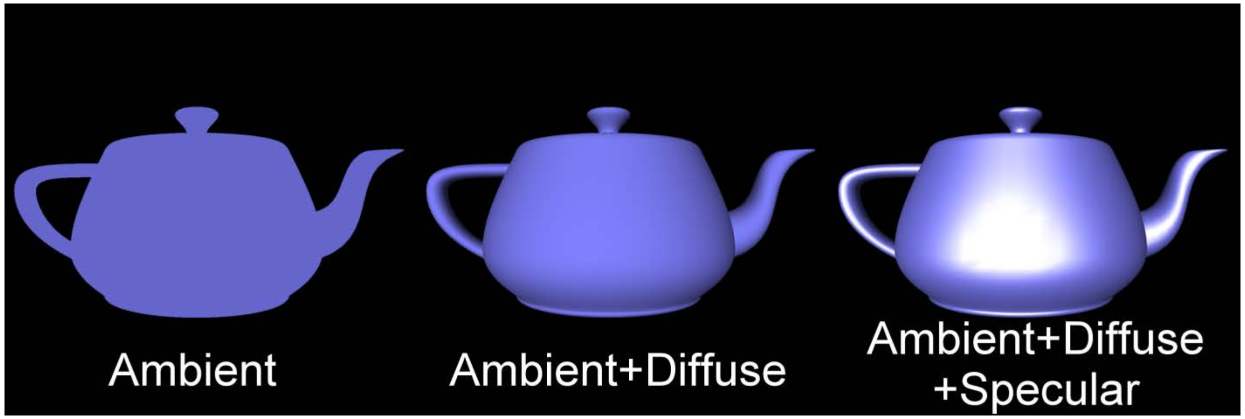
The Phong Illumination Model

- The classic local empirical model
  - Does not take into account the interaction of light between objects
- Some of the terms used do not directly derive from physical laws
- Gives a reasonable approximation of reality
- Modest computational cost → widespread adoption
- Proposes a simplified BRDF which:
  - Relates incoming light intensity from direction (θ_i, φ_i) to reflected light intensity in direction (θ_r, φ_r) for an object point **p**
  - Estimates visible intensity as sum of 4 components:
    - Emission
    - Ambient reflection
    - Diffuse reflection
    - Specular reflection

$$I = I_e + I_g + I_d + I_s \qquad (12.19)$$

The Phong Illumination Model (2)

- Components of the Phong Model:



- **Emission** component I_e: caters for objects with self illumination
- **Ambient** component I_g: compensates for the fact that the Phong is a **local** model (thus a surface not directly illuminated by a light source will not appear completely un-illuminated)
  - ambient light I_a: a constant value is assumed
  - each object reflects ambient light according to its ambient reflectance coefficient k_a: $I_g = I_a k_a \ (0 \le k_a \le 1)$ (12.20)

The Phong Illumination Model (3)

- Light that hits an object directly from a light source is split into 2 reflected components:
  - Diffusely reflected light, which is uniformly scattered in all directions
  - Specularly reflected light, which has max value in the “mirror” of the lighting direction
- **Diffuse & Specular** reflection coefficients k_d and k_s depend mainly on the object’s surface properties
  - The rougher the surface the more light is diffusely reflected
  - The shinier the surface the more light is specularly reflected
  - We have:

$$0 \leq kd, ks \leq 1, kd + ks \leq 1$$

  - Sum of k_d and k_s may be slightly smaller than 1 to account for light that is transmitted or absorbed by the object
  - Diffuse component assumes a Lambertian surface & distributes incident light evenly in all directions

The Phong Illumination Model (4)

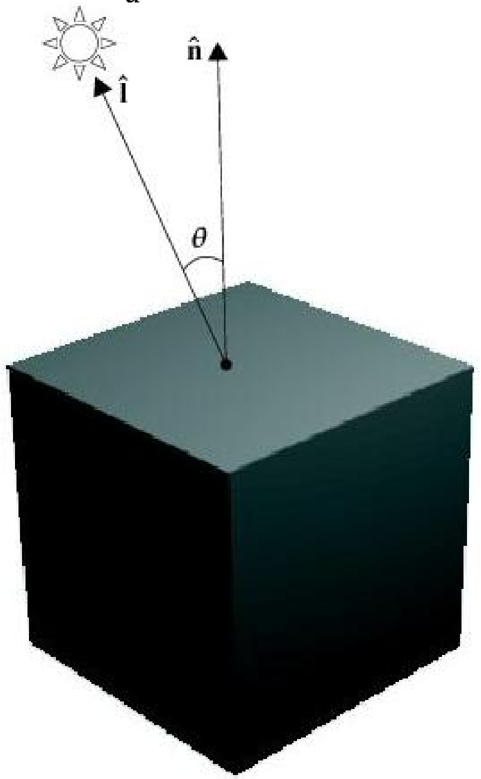
- Diffuse component does not depend on the viewing direction
  - Its value is proportional to the irradiance E_i which is replaced by intensity I_i according to the photometry law (12.8)
- The distance d is ignored by assuming the light source at infinity:

$$I_d = I_i k_d \cos\theta = I_i k_d (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}), \quad (0 \le \theta \le \pi/2, \;\; 0 \le k_d \le 1)$$

where I_i the intensity of the point light source,

θ the angle between $\hat{\mathbf{l}}$ and $\hat{\mathbf{n}}$

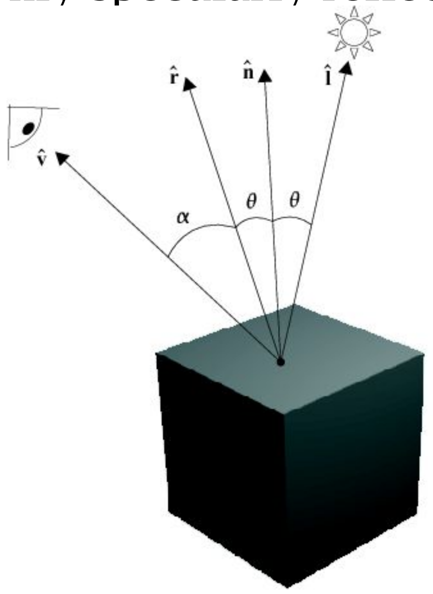
  - (k_d also depends on the wavelength of the incident light, not modeled here)



The Phong Illumination Model (5)

- Vectors $\hat{\mathbf{l}}, \hat{\mathbf{n}}$ should be unit vectors
- I_d is constant over a planar surface since the $\hat{\mathbf{l}}, \hat{\mathbf{n}}$ vectors are constant
- *cosθ* should be non-negative: $I_d = I_i k_d \max(0, \hat{\mathbf{n}} \cdot \hat{\mathbf{l}})$
- Diffuse component alone gives objects a totally matte appearance

- Specular component follows the rule of the mirror
  - A perfect mirror will only specularly reflect in the direction of reflection $\hat{\mathbf{r}}$



The Phong Illumination Model (6)

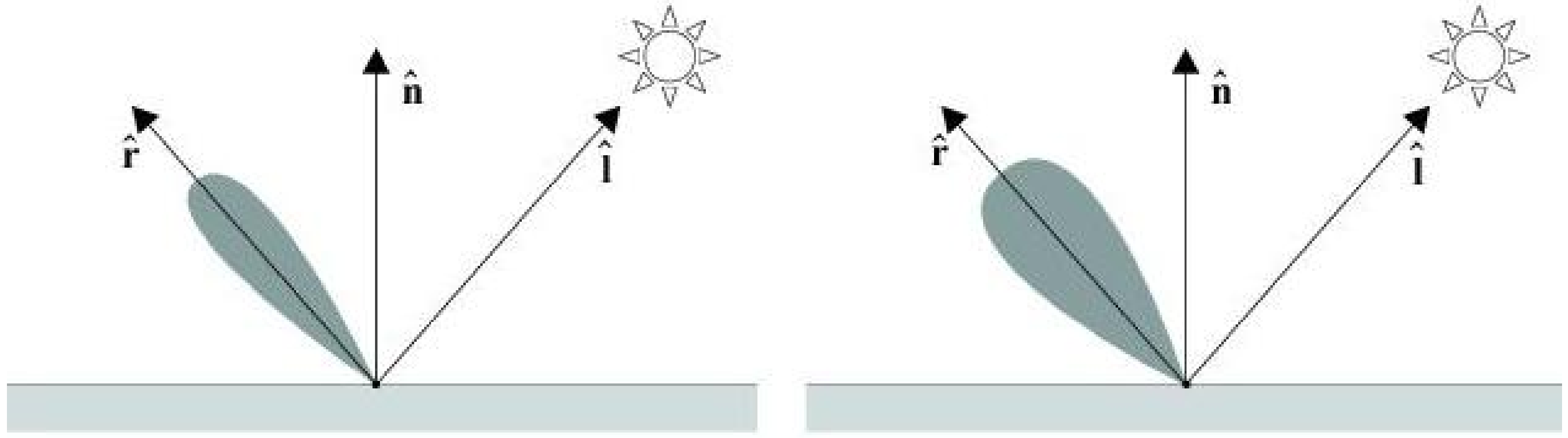
- Most surfaces have a diminishing function of specular reflection that attains its max value when the viewing direction $\hat{\mathbf{v}}$ coincides with $\hat{\mathbf{r}}$:

$$I_s = I_i k_s \cos^n \alpha = I_i k_s (\hat{\mathbf{r}} \cdot \hat{\mathbf{v}})^n \qquad (12.22)$$

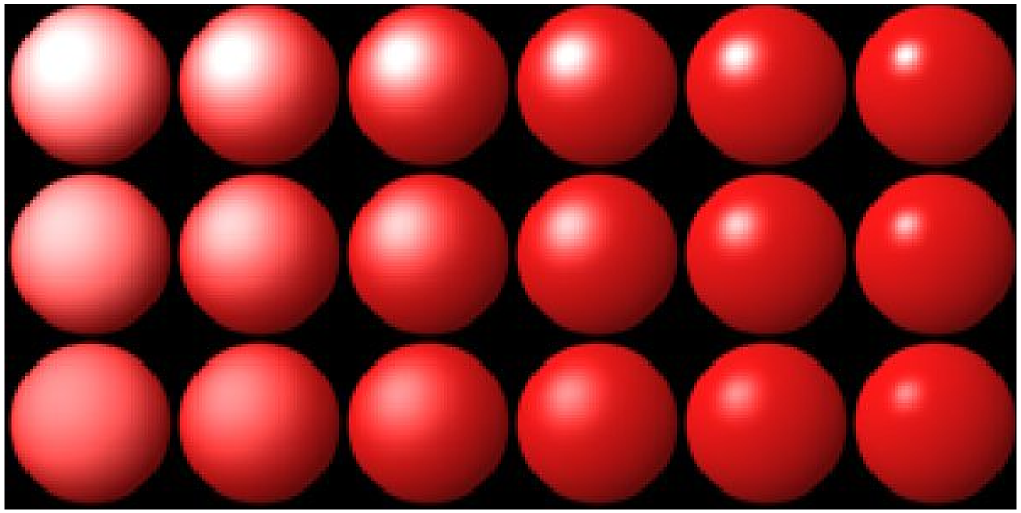
  where $\hat{\mathbf{r}}, \hat{\mathbf{v}}$ are unit vectors & *n* an empirical value that corresponds to surface shininess
- Specular reflection is responsible for the *highlights* that are visible on shiny objects
- The $\cos^n\alpha$ term intuitively approximates the spatial distribution of the specularly reflected light

The Phong Illumination Model (7)

- Effect of the material exponent *n* :



- *n* increases to the right, k_s increases upwards:



The Phong Illumination Model (8)

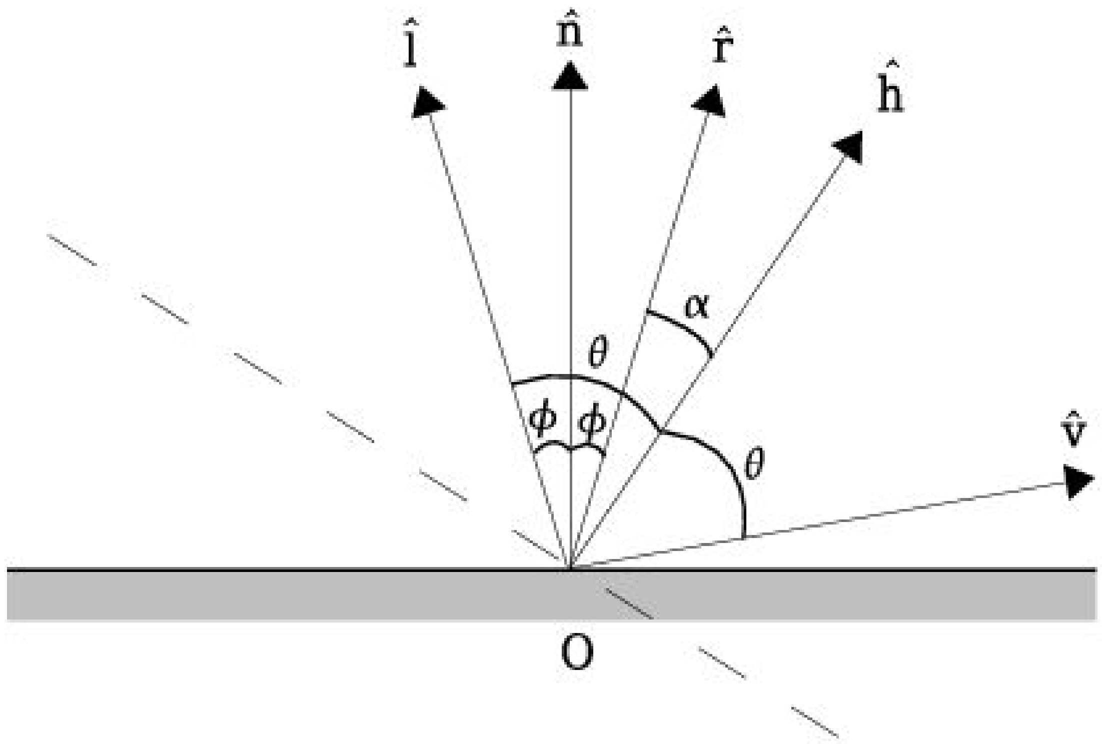
- Small values of *n* correspond to coarse materials where the size of the highlight is relatively large and scattered
- Large values of *n* correspond to shiny objects with a small and crisp highlight
- Specular reflection takes the color of the light source
- Example: If a blue object is illuminated by a white light source, the color of the diffuse reflection will be blue but that of the specular reflection will be white
- The value of the specular factor $\cos^n\alpha$ should not take on negative values, so we can replace it by $max(0, cos^n\alpha)$

- The **Phong model** computes the illumination value as:

$$I = I_e + I_a k_a + I_i (k_d (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) + k_s (\hat{\mathbf{r}} \cdot \hat{\mathbf{v}})^n) \qquad (12.23)$$

The Phong Illumination Model (9)

- To simplify computations: Light source & observation point can be assumed to be at infinity → constant values for $\hat{\mathbf{l}}, \hat{\mathbf{v}}$ vectors over the area of planar objects
- Efficient variant of the specular reflection calculation uses the *halfway vector* $\hat{\mathbf{h}}$ which is the average of $\hat{\mathbf{l}}, \hat{\mathbf{v}}$:



$$\hat{\mathbf{h}} = \frac{(\hat{\mathbf{l}} + \hat{\mathbf{v}})/2}{|(\hat{\mathbf{l}} + \hat{\mathbf{v}})/2|} \quad (12.24)$$

The Phong Illumination Model (10)

- Angle $\widehat{\mathbf{nh}} = \varphi + \alpha$, angle $\widehat{\mathbf{rv}} = \theta + \alpha$, and since θ = 2ϕ +α, we deduce that

$$\widehat{\mathbf{rv}} = 2\widehat{\mathbf{nh}}$$

- We can thus replace $\hat{\mathbf{r}}\cdot\hat{\mathbf{v}}$ by $\hat{\mathbf{n}}\cdot\hat{\mathbf{h}}$, and adjust *n:*

$$I = I_e + I_a k_a + I_i(k_d(\hat{\mathbf{n}}\cdot\hat{\mathbf{l}}) + k_s(\hat{\mathbf{n}}\cdot\hat{\mathbf{h}})^n) \qquad (12.25)$$

- The $\hat{\mathbf{h}}$ vector is much cheaper to compute than $\hat{\mathbf{r}}$
- If $\hat{\mathbf{l}}, \hat{\mathbf{v}}$ are constant then $\hat{\mathbf{h}}$ is also constant
- $\hat{\mathbf{h}}$ can be thought of as the normal vector to the plane for which the observer at $\hat{\mathbf{v}}$ would see the max value of the specular reflection from the light source at $\hat{\mathbf{l}}$
- Since we assumed the light source at infinity, the contribution of the specular and diffuse terms depends on the intensity of the light source and the ambient term is constant

The Phong Illumination Model (11)

- Objects with same properties & orientation but different distances from light will, wrongly, have the same intensity of illumination
- Can be corrected by including a factor dependent on the distance of the object point from the light source
  - Distance factor (physically correct square term often ignored for efficiency):

$$f(d) = 1/(c_1 + c_2 d + c_3 d^2)$$

- The model thus becomes

$$I = I_e + I_a k_a + f(d) I_i (k_d (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) + k_s (\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})^n)$$

- Multiple point light sources can also be handled:

$$I = I_e + I_a k_a + \sum_j (f(d) I_{i,j} (k_d (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}_j) + k_s (\hat{\mathbf{n}} \cdot \hat{\mathbf{h}}_j)^n))$$

- For monochromatic light, the original gray level value *v* of an object point ***p*** is thus modified by the result *I* of the intensity computation:

$$v' = v\, I$$

The Phong Illumination Model (12)

- Color can be handled by giving the color of the light source to the specular reflection
- The color of the ambient & diffuse components depends on the color coefficients of the object material
- 3 intensity values, one for each of the 3 primary colors, are then computed:

$$I_r = I_{er} + I_a k_{ar} + (f(d) I_i (k_{dr} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) + k_s (\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})^n))$$

$$I_g = I_{eg} + I_a k_{ag} + (f(d) I_i (k_{dg} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) + k_s (\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})^n)) \quad (12.28)$$

$$I_b = I_{eb} + I_a k_{ab} + (f(d) I_i (k_{db} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) + k_s (\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})^n))$$

- Specular reflection contributes equally to the 3 equations, simulating a white light source
  - If (r,g,b) is the original color of an object at point $\boldsymbol{p}$, this is modified by the result of the color intensity computation as: $(r', g', b')=(r\,I_r, g\,I_g, b\,I_b)$

Numerical Example

- Let us assume that we want to estimate the intensity value for a point ***p*** which, for ease of calculations, lies at the origin of the coordinate system ***p***$=[0,0,0]^T$. Let the normal to the object at ***p***, the light and the viewing vectors respectively be:

$$\vec{n} = [0,2,0]^T, \quad \vec{l} = [1,1,0]^T, \quad \vec{v} = [0,1,1]^T$$

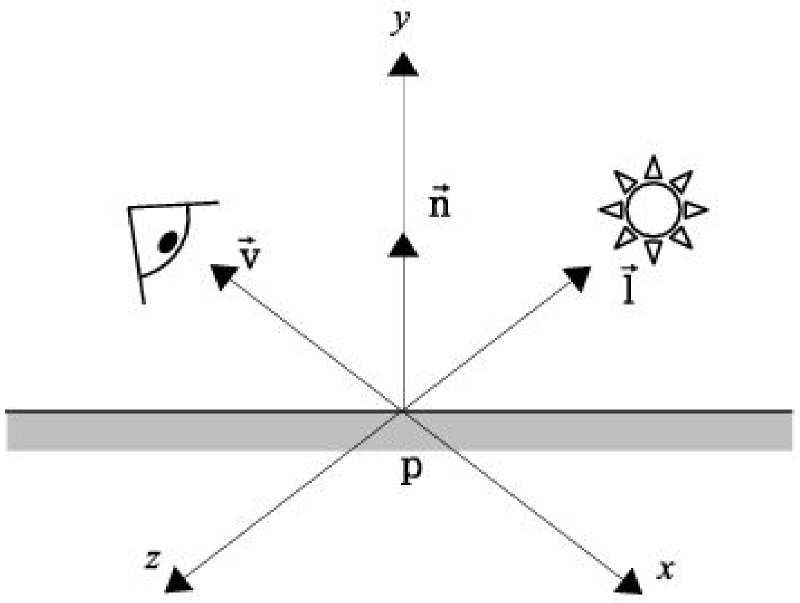
- Suppose the values of the emitted, ambient and incident intensity from the light source are:

  $I_e = 2$, $I_a = 1$, $I_i = 12$

- and that the constant values are:

  $k_a = 0.3$, $k_d = 0.3$, $k_s = 0.6$

  and $n = 3$



Numerical Example (2)

- Light source is twelve times more intense than the ambient light and the object is self-illuminated and emits twice the ambient intensity
- $k_d + k_s = 0.9 \rightarrow$ 10% of the incident light is absorbed by the object
- Before applying the Phong formula we must compute the halfway vector and normalize all the vectors involved:

$$\hat{\mathbf{l}} = \frac{\vec{l}}{|\vec{l}\,|} = \frac{[1,1,0]^T}{\sqrt{1^2+1^2}} = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]^T, \hat{\mathbf{v}} = \frac{\vec{v}}{|\vec{v}\,|} = \frac{[0,1,1]^T}{\sqrt{1^2+1^2}} = [0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^T$$

$$\vec{h} = (\hat{\mathbf{l}} + \hat{\mathbf{v}})/2 = [\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}]^T, \hat{\mathbf{h}} = \frac{\vec{h}}{|\vec{h}\,|} = \frac{[\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}]^T}{\sqrt{3/2}} = [\frac{1}{\sqrt{2}\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{2}\sqrt{3}}]^T,$$

$$\hat{\mathbf{n}} = \frac{[0,2,0]^T}{\sqrt{2^2}} = [0,1,0]^T$$

Numerical Example (3)

- We can now apply equation (12.25):

$$I = 2 + 1 \cdot 0.3 + 12 \cdot (0.3 \cdot (\frac{1}{\sqrt{2}}) + 0.6 \cdot (\frac{\sqrt{2}}{\sqrt{3}})^3) = 8.76$$

  - Final intensity value corresponds to the specified viewing angle and is related to the input intensities
  - The angle between the directions of reflection and viewing is:

$$\widehat{\mathbf{rv}} = 2\widehat{\mathbf{nh}} = 2arccos(\frac{\sqrt{2}}{\sqrt{3}}) = 70^\circ$$

  - If the viewing direction coincided with the direction of reflection (i.e. $\hat{\mathbf{v}} = [-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]^T$) then the specular reflection would attain its max value

since $\widehat{\mathbf{rv}} = 2\widehat{\mathbf{nh}} = 2arccos(1) = 0^\circ$:

$$\hat{\mathbf{h}} = [0,1,0]^T, \qquad I = 2 + 1 \cdot 0.3 + 12 \cdot (0.3 \cdot (\frac{1}{\sqrt{2}}) + 0.6 \cdot 1^3) = 12.05$$

Phong Model Vectors

- The Phong model requires a number of vectors for the computation of the illumination value at a surface point, namely:

$$\vec{\mathbf{n}},$$
$$\vec{\mathbf{l}},$$
$$\vec{\mathbf{v}} \;\; \textit{and}$$
$$\vec{\mathbf{r}} \;\; \textit{or} \;\; \vec{\mathbf{h}}$$

- Important to use efficient formulae for the computation of these vectors, since such computation is repeated for every point where the model is applied

The Normal Vector

- Defined as a vector perpendicular to a surface at a certain point
- Direction of normal vector defines the orientation of the surface
- Very useful in computer graphics

Normal vector for implicit surfaces:

- Implicit surfaces are defined by an equation of the form:

$$f(x, y, z) = 0$$

- The normal vector at a point $\mathbf{p} = [a,b,c]^T$ of such a surface is given by the gradient vector in the vicinity of $\mathbf{p}$:

$$\vec{\mathbf{n}} = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \\ \partial f / \partial z \end{bmatrix}$$

The Normal Vector (2)

- In the case of a planar surface defined by:

$$f(x, y, z) = ax+by+cz+d = 0$$

- The normal vector, which is constant over the entire planar surface, is:

$$\vec{\mathbf{n}} = [a, b, c]^T$$

Normal vector for parametric surfaces:

- Surfaces are often represented parametrically
- In 3D, a surface is represented by 3 parametric equations in terms of 2 parameters *u* and *v:*

$$x = f_x(u,v)$$

$$y = f_y(u,v)$$

$$z = f_z(u,v)$$

The Normal Vector (3)

- The normal vector is then:

$$\vec{\mathbf{n}} = \overrightarrow{\frac{\partial \mathbf{f}}{\partial \mathbf{u}}} \times \overrightarrow{\frac{\partial \mathbf{f}}{\partial \mathbf{v}}} \qquad (12.29)$$

where

$$\vec{\mathbf{f}} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}, \overrightarrow{\frac{\partial \mathbf{f}}{\partial \mathbf{u}}} = \begin{bmatrix} \partial f_x / \partial u \\ \partial f_y / \partial u \\ \partial f_z / \partial u \end{bmatrix}, \overrightarrow{\frac{\partial \mathbf{f}}{\partial \mathbf{v}}} = \begin{bmatrix} \partial f_x / \partial v \\ \partial f_y / \partial v \\ \partial f_z / \partial v \end{bmatrix}$$

Normal vector for polygons:

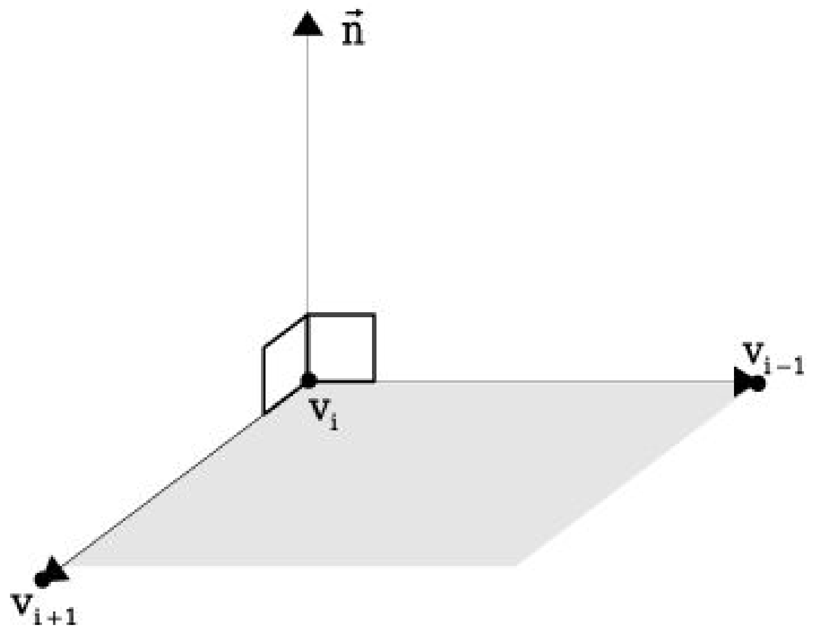
- Polygons are the usual building element for model composition
- In practice the equation of a polygon's plane is not known and the polygon is given in terms of a list of its vertices

The Normal Vector (4)

- Given 3 consecutive, non-collinear vertices of a polygon $\mathbf{v}_{i-1}$, $\mathbf{v}_i$, and $\mathbf{v}_{i+1}$, we can compute the normal vector:

$$\vec{\mathbf{n}} = (\mathbf{v}_{i+1} - \mathbf{v}_i) \times (\mathbf{v}_{i-1} - \mathbf{v}_i)$$

- Cross product is **not** commutative
- The above computation follows the right-hand rule:



The Normal Vector (5)

- Must select correct orientation, otherwise normal computations will be reversed & objects will take an “inside-out” look
- For polygons with more than 3 vertices, a slight non-planarity may exist
  - We may compute the polygon normal as the average of the normal vectors given by each pair of consecutive polygon edges
  - Another technique, due to Martin-Newell:

if $[x_i, y_i, z_i]^T$, $i = 1,2, \ldots, n$ are the n vertices of a polygon, then the coefficients a,b,c of an approximating plane are computed as:

$$a = \sum_{i=1}^{n} (y_i - y_{i\oplus 1})(z_i + z_{i\oplus 1}), \quad b = \sum_{i=1}^{n} (z_i - z_{i\oplus 1})(x_i + x_{i\oplus 1}), \quad c = \sum_{i=1}^{n} (x_i - x_{i\oplus 1})(y_i + y_{i\oplus 1})$$

where $\oplus$ represents addition modulo n

The d (constant) coefficient of the plane equation can be computed using the coordinates of a polygon vertex:

$$d = -(ax_1 + by_1 + cz_1)$$

The Normal Vector (6)

- Another way of computing the normal vector of a polygon:

  if $[x_1, y_1, z_1]^T$, $[x_2, y_2, z_2]^T$, and $[x_3, y_3, z_3]^T$ are 3 non-collinear points, then they must satisfy the plane equation:

$$ax_1 + by_1 + cz_1 = -1$$
$$ax_2 + by_2 + cz_2 = -1$$
$$ax_3 + by_3 + cz_3 = -1$$

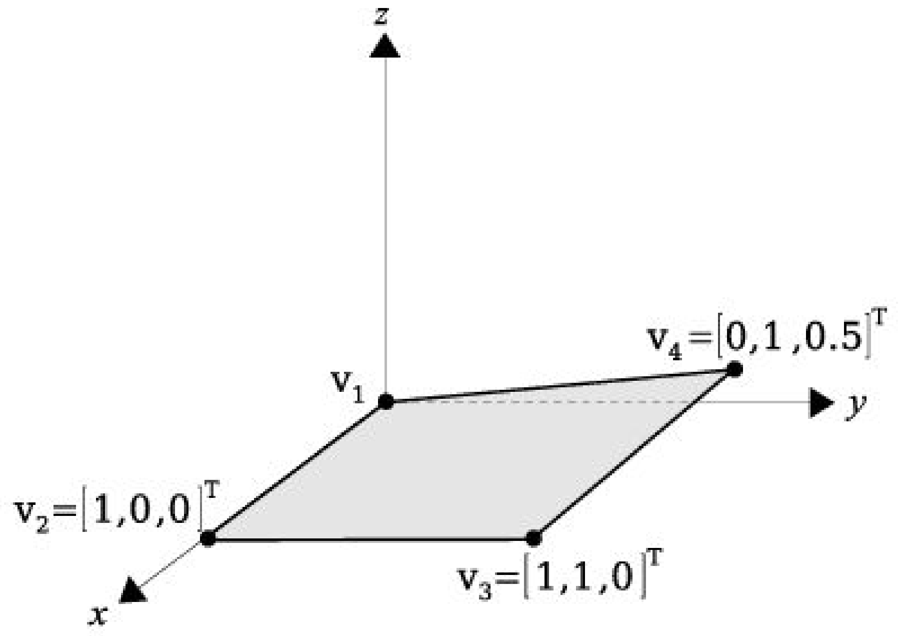
- Or

$$\begin{bmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

- Or $\mathbf{XC} = \mathbf{D}$
- So $\mathbf{C} = \mathbf{X}^{-1}\boldsymbol{D}$

Numerical Example

- Given polygon with vertices $\mathbf{v_1} = [0,0,0]^T$, $\mathbf{v_2} = [1,0,0]^T$, $\mathbf{v_3} = [1,1,0]^T$, and $\mathbf{v_4} = [0,1,0.5]^T$ (slightly non-planar). Compute its normal vector



- We shall consider two suitable methods:
  - 1) Average of the normals for each pair of successive edges
  - 2) Martin-Newell's technique

Numerical Example (2)

Method 1:

- We first compute 4 normal vectors
- Normals are indexed by the vertex onto which both edges are incident

$$\vec{\mathbf{n}}_{\mathbf{v}_1} = [1,0,0]^T \times [0,1,0.5]^T = [0,-0.5,1]^T$$

$$\vec{\mathbf{n}}_{\mathbf{v}_2} = [0,1,0]^T \times [-1,0,0]^T = [0,0,1]^T$$

$$\vec{\mathbf{n}}_{\mathbf{v}_3} = [-1,0,0.5]^T \times [0,-1,0]^T = [0.5,0,1]^T$$

$$\vec{\mathbf{n}}_{\mathbf{v}_4} = [0,-1,-0.5]^T \times [1,0,-0.5]^T = [0.25,-0.5,1]^T$$

- Next, compute the polygon normal by averaging the above
- Normalize vectors before summation → give equal weight to all edges:

$$\vec{\mathbf{n}} = \frac{\hat{\mathbf{n}}_{\mathbf{v}_1} + \hat{\mathbf{n}}_{\mathbf{v}_2} + \hat{\mathbf{n}}_{\mathbf{v}_3} + \hat{\mathbf{n}}_{\mathbf{v}_4}}{4} = [0.17,-0.22,0.91]^T,$$

$$\hat{\mathbf{n}} = [0.18,-0.23,0.96]^T$$

Numerical Example (3)

Method 2:

- Using Martin-Newell's technique ,we get:

$$a = 0 \cdot 0 + (-1) \cdot 0 + 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$$

$$b = 0 \cdot 1 + 0 \cdot 2 + (-0.5) \cdot 1 + 0.5 \cdot 0 = -0.5$$

$$c = (-1) \cdot 0 + 0 \cdot 1 + 1 \cdot 2 + 0 \cdot 0.5 = 2$$

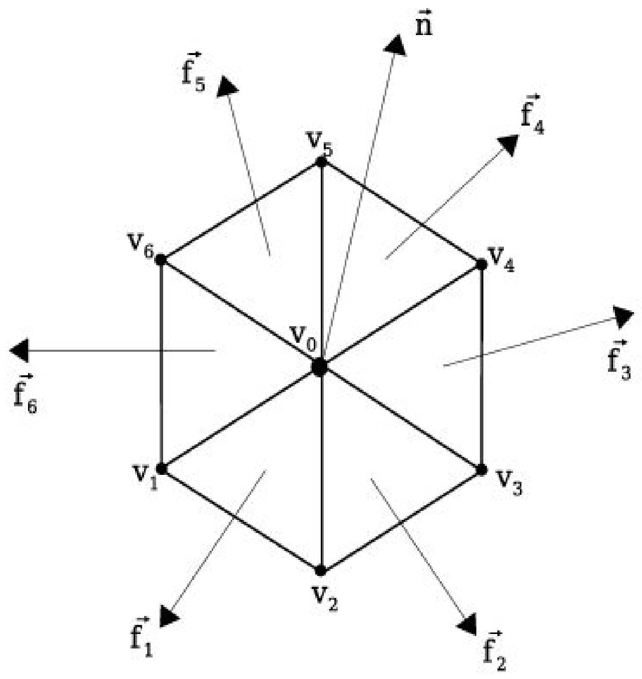
- Thus:

$$\vec{\mathbf{n}} = [0.5, -0.5, 2]^T ,$$

$$\hat{\mathbf{n}} = [0.24, -0.24, 0.94]^T$$

Vertex Normal Vector for Polygonal Meshes

- Polygonal meshes are often used to approximate objects with smooth change of their surface normal vector
- Assume objects that consist of a single manifold surface
- In illumination, we need the normal vector to an object’s surface at a discrete set of points covered by the surface :
  - Determine the normal at the vertices of the polygonal mesh as a weighted average of the normals to the adjacent faces to the vertex
  - Use this normal to perform bilinear interpolation along edges and finally across edges, on points of the underlying grid
- The polygons adjacent to a vertex are the *1-ring neighbors* or the *star* of the vertex



Vertex Normal Vector for Polygonal Meshes (2)

- *Vertex normal* refers to a weighted average of the normals to the faces of the vertex's star
- 3 common approaches for computing the unit vertex normal $\hat{\mathbf{n}}$

Approach 1:

- Weights can be taken to be equal
- Achieved by normalizing the normals of the faces of the star $\vec{\mathbf{f}}_{\mathrm{i}}$ before averaging**:**

$$\hat{\mathbf{n}} = \frac{\sum_{i=1}^{m} \hat{\mathbf{f}}_{\mathbf{i}}}{|\sum_{i=1}^{m} \hat{\mathbf{f}}_{\mathbf{i}}|} \qquad (12.31)$$

where $\hat{\mathbf{f}}_{\mathrm{i}} = \vec{\mathbf{f}}_{\mathrm{i}} / |\vec{\mathbf{f}}_{\mathrm{i}}|$

m: number of faces in the star

Vertex Normal Vector for Polygonal Meshes (3)

Approach 2:

- This approach observes that larger polygons should contribute more than smaller ones
  - Face normals are thus weighted by the area of the corresponding polygons
- For triangular faces, this amounts to taking the face normals as computed by the outer product of the vectors represented by 2 of the triangle’s edges
  - This is because the outer product is equal to twice the area of the triangle

$$\hat{\mathbf{n}} = \frac{\sum_{i=1}^{m} \overrightarrow{\mathbf{f_i}}}{|\sum_{i=1}^{m} \overrightarrow{\mathbf{f_i}}|} \qquad (12.32)$$

Vertex Normal Vector for Polygonal Meshes (4)

Approach 3:

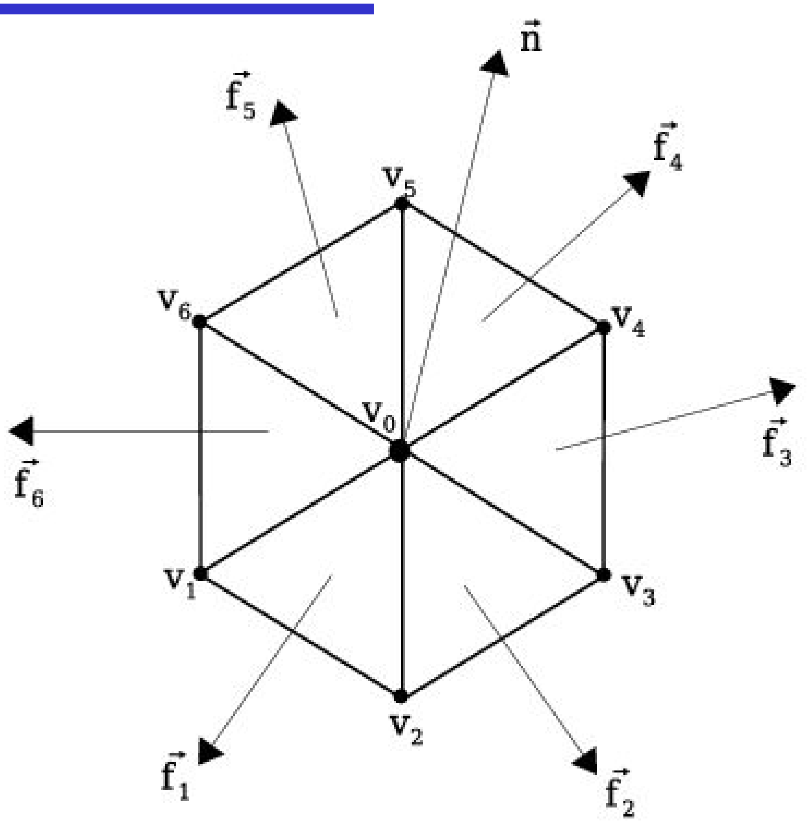
- This approach observes that in order to ensure that vertex normals are invariant to mesh restructuring, a good weight is the incident angle θ of the faces of the star
- Angle θ can be computed by taking the *arccos* of the dot product of the vectors defined by the incident edges that form it:

$$\hat{\mathbf{n}} = \frac{\sum_{i=1}^{m} \theta_i \hat{\mathbf{f}}_{\mathbf{i}}}{\left| \sum_{i=1}^{m} \theta_i \hat{\mathbf{f}}_{\mathbf{i}} \right|} \qquad (12.33)$$

- Note: Vertex normals should be computed before the perspective division (projection)

Vertex Normal: Symbolic Example

- In the following figure *m* is 6 as there are 6 polygons in the star
- In order to evaluate all the vertex normal expressions above, we need to compute the $\vec{\mathbf{f}}_i, \hat{\mathbf{f}}_i, \theta_i$
- Take the first triangle ***v₀v₁v₂***:



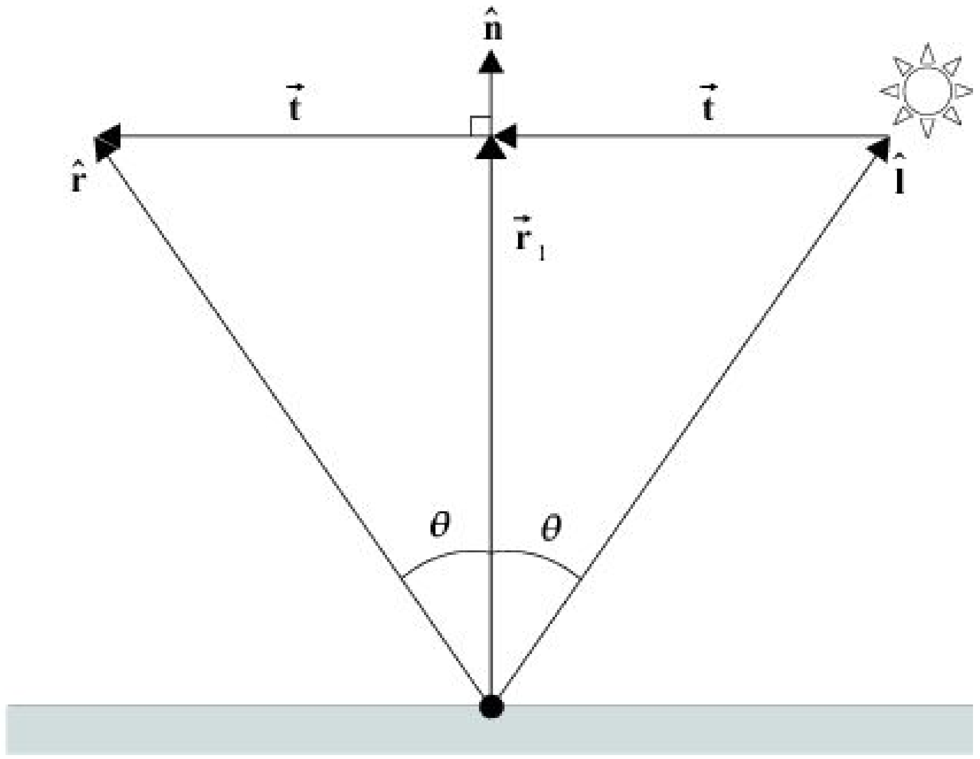
$$\vec{\mathbf{f}}_1 = \overrightarrow{\mathbf{v}_0\mathbf{v}_1} \times \overrightarrow{\mathbf{v}_0\mathbf{v}_2}, \quad \hat{\mathbf{f}}_1 = \frac{\vec{\mathbf{f}}_1}{|\vec{\mathbf{f}}_1|}$$

$$\theta_1 = \arccos\left(\frac{\overrightarrow{\mathbf{v}_0\mathbf{v}_1}}{|\overrightarrow{\mathbf{v}_0\mathbf{v}_1}|} \cdot \frac{\overrightarrow{\mathbf{v}_0\mathbf{v}_2}}{|\overrightarrow{\mathbf{v}_0\mathbf{v}_2}|}\right)$$

- Similar computations are performed for the other five triangles in the star and expressions (12.31) – (12.33) can then be evaluated

Reflection Vector

- *Reflection vector* $\vec{\mathbf{r}}$ is computed by noticing that:
  - Angles between pairs of vectors $(\hat{\mathbf{l}},\hat{\mathbf{n}})$, $(\hat{\mathbf{n}},\vec{\mathbf{r}})$ are equal
  - $\hat{\mathbf{l}}, \hat{\mathbf{n}}, \vec{\mathbf{r}}$ are coplanar



Reflection Vector (2)

- Let $\vec{\mathbf{r}}_1$ be the vector defined by the projection of $\hat{\mathbf{l}}$ onto the axis of $\hat{\mathbf{n}}$:

$$|\overrightarrow{\mathbf{r}_1}| = |\hat{\mathbf{l}}| \cos\theta = |\hat{\mathbf{l}}| (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) = \hat{\mathbf{n}} \cdot \hat{\mathbf{l}},$$

since $\hat{\mathbf{l}}$ is a unit vector, so:

$$\overrightarrow{\mathbf{r}_1} = \hat{\mathbf{n}} |\overrightarrow{\mathbf{r}_1}| = \hat{\mathbf{n}} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})$$

We also have:

$$\vec{\mathbf{r}} = \overrightarrow{\mathbf{r}_1} + \vec{\mathbf{t}} \qquad \vec{\mathbf{t}} = \overrightarrow{\mathbf{r}_1} - \hat{\mathbf{l}}$$

- Thus:

$$\vec{\mathbf{r}} = 2\overrightarrow{\mathbf{r}_1} - \hat{\mathbf{l}} = 2\hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) - \hat{\mathbf{l}}$$

- 6 multiplications and 5 additions are required
- When performance is an issue, the reflection vector is replaced by the halfway vector

Light, View & Halfway Vectors

- *Light* and *View* vectors $\vec{\mathbf{l}}, \vec{\mathbf{v}}$ are:
  - either given constant vectors, if the light & view points are placed at infinity,
  - or simply computed as:

$$\vec{\mathbf{l}} = \mathbf{l} - \mathbf{p} \qquad (12.35)$$

$$\vec{\mathbf{v}} = \mathbf{v} - \mathbf{p} \qquad (12.36)$$

where **p**: object point

l: light point

v: view point

- *Halfway* vector $\vec{\mathbf{h}}$, useful for specular reflection, is then computed as:

$$\vec{\mathbf{h}} = (\hat{\mathbf{l}} + \hat{\mathbf{v}}) / 2 \qquad (12.37)$$

Illumination Algorithms History

- Illumination is applied to produce realistic synthetic images
- Warnock (1969):
  - intensity diminishes according to depth
  - objects were illuminated according to their distance from the light source
- Gouraud (1971):
  - interpolation of intensity values within polygons from intensity values computed at the vertices
- Phong :
  - compute intensity values at every pixel by linearly interpolating vertex normals
  - using the model he introduced in 1975
  - there are instances where the linear interpolation of the vertex normals does not work well
- Overveld (1997) :
  - proposed a quadratic interpolation scheme

Illumination Algorithms based on the Phong Model

- Constant shading
- Gouraud shading
- Phong shading

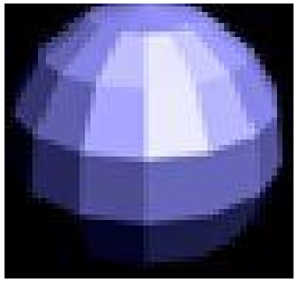
Constant Shading

- Is the simplest algorithm for polygonal objects
- Applies a constant illumination value to each polygonal facet
- Incorporated:
  - Constant ambient lighting
  - Diffuse reflection
- No Specular reflection
- The light & view points:
  - Are both placed at infinity and coincide, $\vec{\mathbf{l}} = \vec{\mathbf{v}}$
  - Eliminates shadows
  - $(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})$ is constant for each polygon

Constant Shading (2)

- If the light & view points are on the +z-axis:
  - $\hat{\mathbf{l}} = \hat{\mathbf{v}} = [0,0,1]^T$
  - $(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) = n_z$ for $\hat{\mathbf{n}} = [n_x, n_y, n_z]^T$
- Illumination equation:

$$I = I_e + I_a k_a + I_i k_d n_z$$

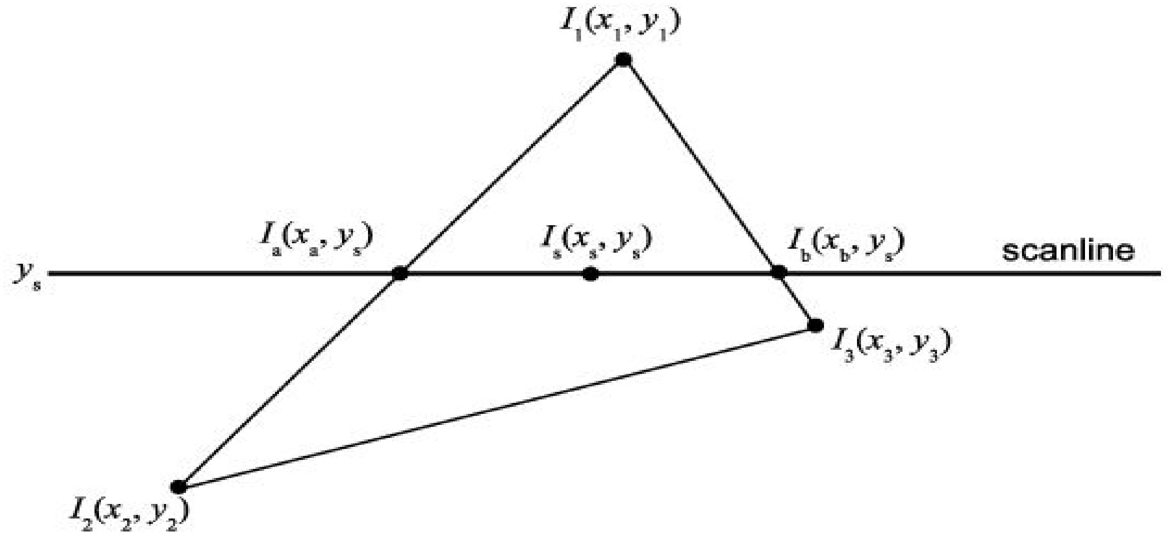


- I is computed once for each polygon
  - used for all pixels that the polygon covers
- Problem:
  - A polygon mesh often samples a curved surface
  - The human eye is sensitive to intensity discontinuities
  - Polygon silhouettes stand out → objects have a “polygonal look”
- One solution:
  - Use some form of illumination interpolation

Gouraud Shading

- Is a simple illumination interpolation algorithm
- If the sampling density is sufficiently high, it can capture local maxima (highlights) and minima of shading distribution over the polygon mesh
- It computes intensity values for pixels inside a polygon:
  1. Interpolate the intensity values at its vertices
     - Intensity values at vertices estimated using the Phong model
     - Use vertex normals to evaluate the Phong equation at the vertices
  2. Bi-linearly interpolate the vertex intensities along the polygon edges & between the edges (along the scanlines)
     - Scalar interpolation

Gouraud Shading (2)



- Intensities I_1, I_2, I_3 are computed using the Phong model
- I_a, I_b : using interpolation between (I_1, I_2, I_3):

$$I_{\mathbf{a}} = I_1 \frac{y_{\mathbf{s}} - y_2}{y_1 - y_2} + I_2 \frac{y_1 - y_{\mathbf{s}}}{y_1 - y_2} = \frac{1}{y_1 - y_2}(I_1(y_{\mathbf{s}} - y_2) + I_2(y_1 - y_{\mathbf{s}}))$$

$$I_{\mathbf{b}} = \frac{1}{y_1 - y_3}(I_1(y_{\mathbf{s}} - y_3) + I_3(y_1 - y_{\mathbf{s}}))$$

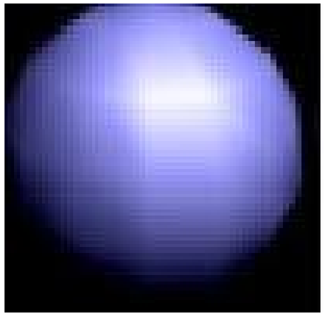
- I_s : using interpolation between I_a, I_b : $I_{\mathbf{s}} = \frac{1}{x_{\mathbf{b}} - x_{\mathbf{a}}}(I_{\mathbf{a}}(x_{\mathbf{b}} - x_{\mathbf{s}}) + I_{\mathbf{b}}(x_{\mathbf{s}} - x_{\mathbf{a}}))$

Gouraud Shading (3)

- Intensity values are computed **incrementally** within a scanline:
- If $\mathbf{s}_1$ and $\mathbf{s_2}$ are the indices of 2 pixels on the same scanline, then:

$$I_{\mathbf{s}_1} = \frac{1}{x_{\mathbf{b}} - x_{\mathbf{a}}}(I_{\mathbf{a}}(x_{\mathbf{b}} - x_{\mathbf{s}_1}) + I_{\mathbf{b}}(x_{\mathbf{s}_1} - x_{\mathbf{a}}))$$

$$I_{\mathbf{s}_2} = \frac{1}{x_{\mathbf{b}} - x_{\mathbf{a}}}(I_{\mathbf{a}}(x_{\mathbf{b}} - x_{\mathbf{s}_2}) + I_{\mathbf{b}}(x_{\mathbf{s}_2} - x_{\mathbf{a}}))$$



- Subtracting the above equations:

$$\Delta I_{\mathbf{s}} = I_{\mathbf{s}_2} - I_{\mathbf{s}_1} = \frac{x_{\mathbf{s}_2} - x_{\mathbf{s}_1}}{x_{\mathbf{b}} - x_{\mathbf{a}}}(I_{\mathbf{b}} - I_{\mathbf{a}}) = \frac{\Delta x}{x_{\mathbf{b}} - x_{\mathbf{a}}}(I_{\mathbf{b}} - I_{\mathbf{a}})$$

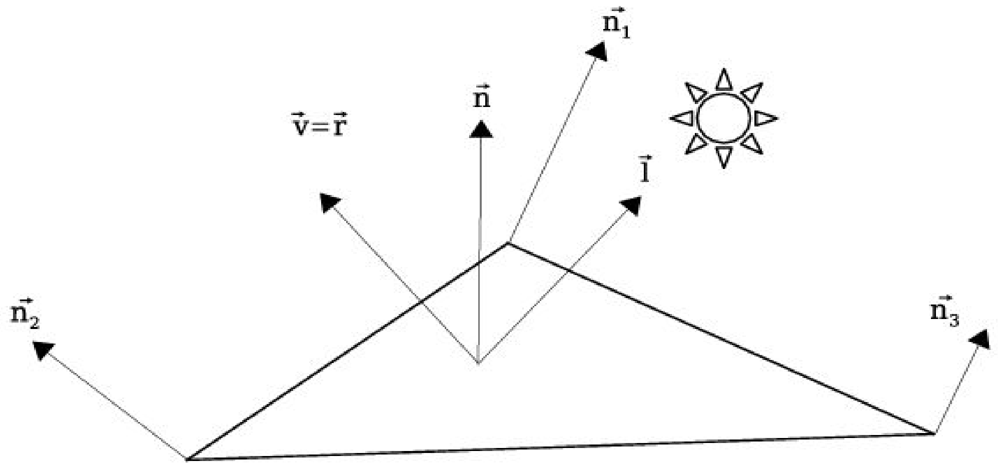
- In the case of neighboring pixels (Δx=1): $\Delta I_{\mathbf{s}} = \frac{I_{\mathbf{b}} - I_{\mathbf{a}}}{x_{\mathbf{b}} - x_{\mathbf{a}}}$

- Incremental intensity computation:

$$I_{\mathbf{s},n} = I_{\mathbf{s},n-1} + \Delta I_{\mathbf{s}}$$

Phong Shading

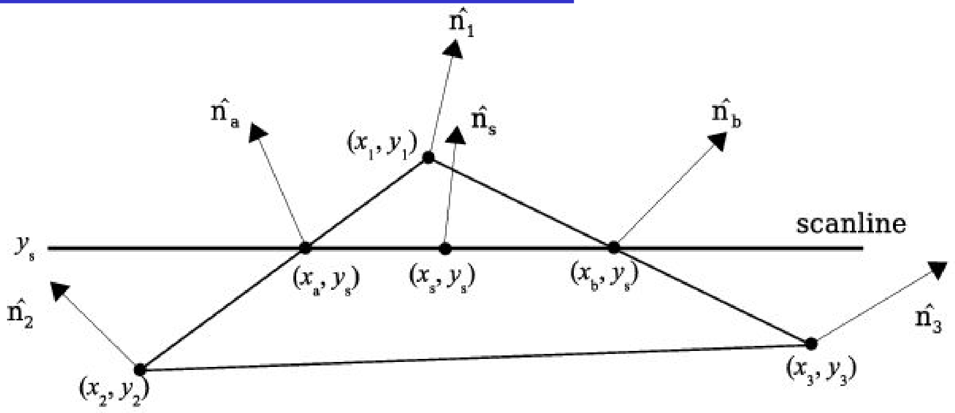
- Problems of Gouraud Shading:
  - The sampling density is rarely sufficient to capture highlights
    - The shading vectors are not interpolated within the polygon but are used to capture intensities at the vertices only
  - No elimination of mach-bands:
    - the linear intensity interpolation leaves second-order intensity discontinuities



- The Phong algorithm solves the problems of Gouraud shading by applying the Phong model to each pixel covered by a polygon

Phong Shading (2)

- Phong algorithm computations:



  - The unit normal vectors: bi-linear interpolation from the unit vertex normals

$$\vec{\mathbf{n}}_{\mathbf{a}} = \frac{1}{y_1 - y_2}(\hat{\mathbf{n}}_1(y_{\mathbf{s}} - y_2) + \hat{\mathbf{n}}_2(y_1 - y_{\mathbf{s}}))$$

$$\vec{\mathbf{n}}_{\mathbf{b}} = \frac{1}{y_1 - y_3}(\hat{\mathbf{n}}_1(y_{\mathbf{s}} - y_3) + \hat{\mathbf{n}}_3(y_1 - y_{\mathbf{s}}))$$

$$\hat{\mathbf{n}}_{\mathbf{s}} = \frac{1}{x_{\mathbf{b}} - x_{\mathbf{a}}}(\hat{\mathbf{n}}_{\mathbf{a}}(x_{\mathbf{b}} - x_{\mathbf{s}}) + \hat{\mathbf{n}}_{\mathbf{b}}(x_{\mathbf{s}} - x_{\mathbf{a}}))$$

  - For neighboring pixels on the same scanline, use incremental computation

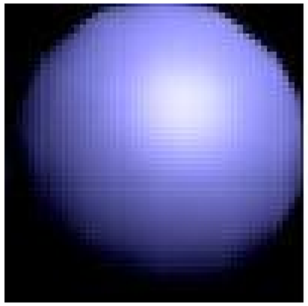
$$n_{sx,n} = n_{sx,n-1} + \Delta n_{sx} \qquad \Delta n_{\mathbf{s}x} = \frac{n_{\mathbf{b}x} - n_{\mathbf{a}x}}{x_{\mathbf{b}} - x_{\mathbf{a}}}$$

$$n_{sy,n} = n_{sy,n-1} + \Delta n_{sy} \qquad \text{where} \qquad \Delta n_{\mathbf{s}y} = \frac{n_{\mathbf{b}y} - n_{\mathbf{a}y}}{x_{\mathbf{b}} - x_{\mathbf{a}}}$$

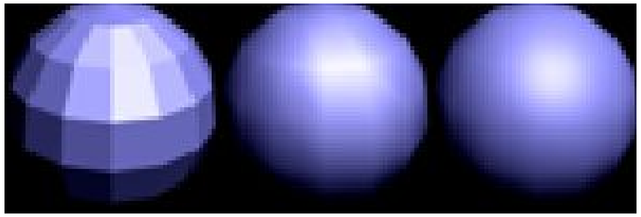
$$n_{sz,n} = n_{sz,n-1} + \Delta n_{sz} \qquad \Delta n_{\mathbf{s}z} = \frac{n_{\mathbf{b}z} - n_{\mathbf{a}z}}{x_{\mathbf{b}} - x_{\mathbf{a}}}$$

Phong Shading (3)

- The Phong algorithm:
  - Is a significant improvement over Gouraud
  - Requires considerably more computations
  - Is implemented on graphics accelerators

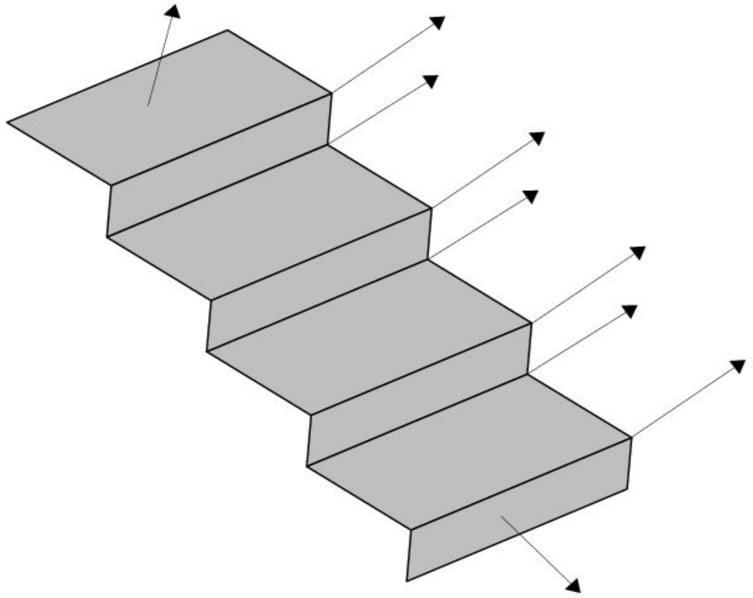


Constant – Gouraud – Phong shading example:



Quadratic Interpolation of Vertex Normals

- Phong shading algorithm
  - polygonal mesh sufficiently dense → acceptable quality
  - large polygons → shading artifacts
- The *silhouette edge problem:*
  - In e.g. below, normal vectors do not vary at all over the surface
  - Completely flat illumination appearance
  - This is at odds with the appearance of the silhouette



Quadratic Interpolation of Vertex Normals (2)

- Vertex normal interpolation essentially aims to reconstruct a surface from discrete samples
- Reconstruction cannot add information
- Tries to come up with a reconstructed surface consistent with the sampled data:
  - interpolates the vertex normals
  - is perpendicular to the face normals
- The linear interpolation of vertex normals in Phong shading is not consistent in this sense (previous e.g.)

Quadratic Interpolation of Vertex Normals (3)

- Overveld and Wyvill showed that the quadratic interpolation of normals achieves better results
- If $\hat{\mathbf{n}}_0, \hat{\mathbf{n}}_1$: the normal vectors to be interpolated and
  $\vec{\delta}$: the vector defined by the subtraction of the 1st from the last interpolation point
- Then the interpolated vector $\vec{\mathbf{n}}(s)$ is:

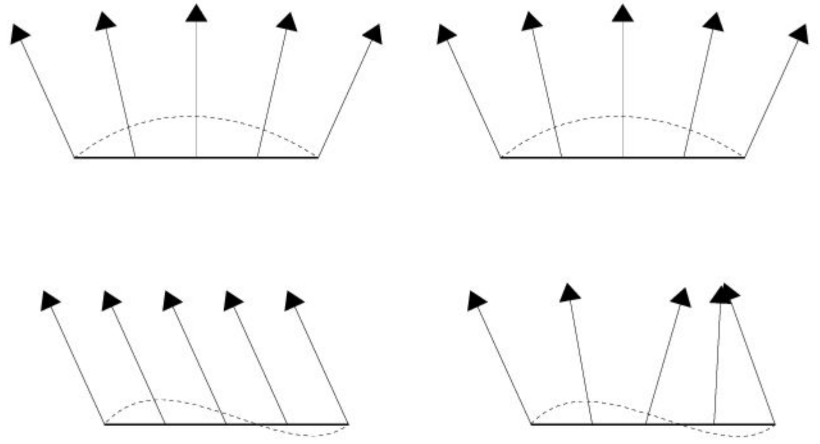
$$\vec{\mathbf{n}}(s) = \hat{\mathbf{n}}_0 + s\vec{\mathbf{a}} + s^2\vec{\mathbf{b}} \quad \text{with } s \in [0..1] \text{ and}$$

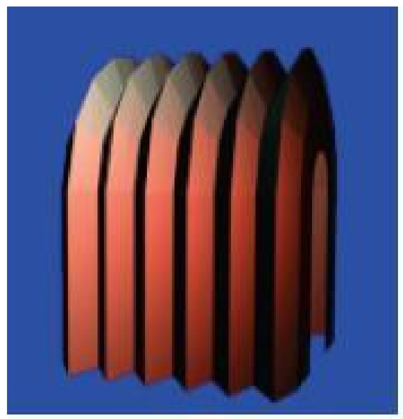
$$\vec{\mathbf{a}} = \hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_0 - \vec{\mathbf{b}}\ , \quad \vec{\mathbf{b}} = 3\left(\frac{(\hat{\mathbf{n}}_0 + \hat{\mathbf{n}}_1)\cdot\vec{\delta}}{\vec{\delta}^{\,2}}\right)\vec{\delta}$$

- As expected: $\vec{\mathbf{n}}(0) = \hat{\mathbf{n}}_0$ and $\vec{\mathbf{n}}(1) = \hat{\mathbf{n}}_1$
- Implemented by taking the forward differences of the quadratic function, at a cost of 2 vector additions per pixel

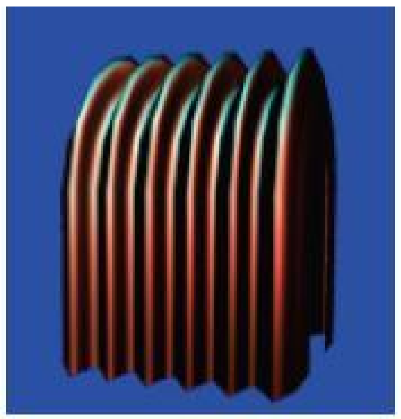
Quadratic Interpolation of Vertex Normals (4)

- Linear (left) vs Quadratic (right) vector interpolation:

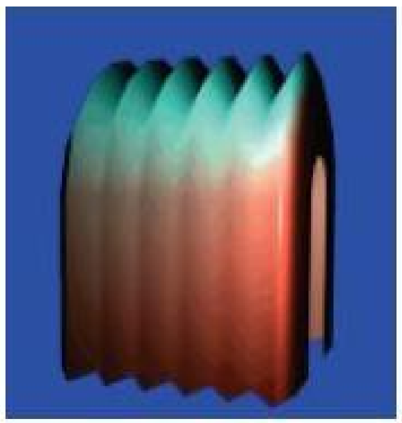



(a)

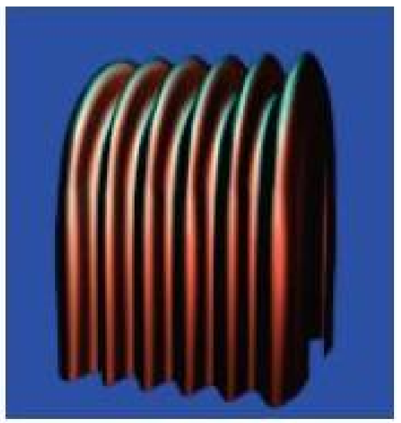
flat


(b)

quadratic


(c)

linear


(d)

linear/dense mesh

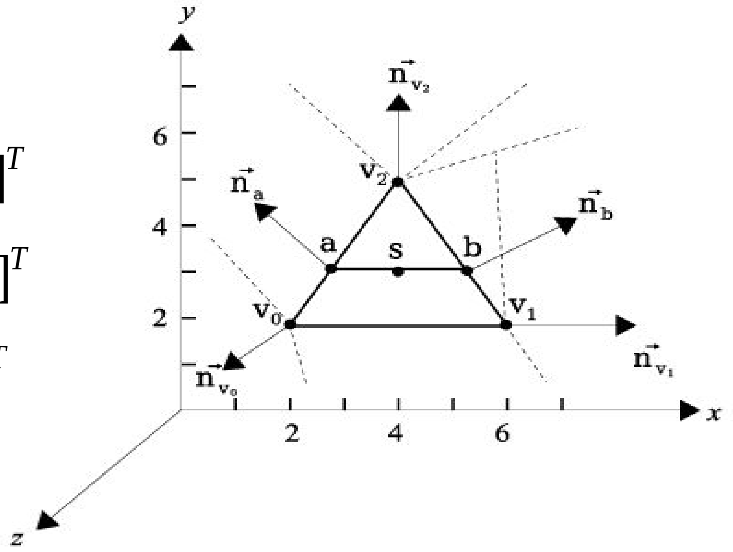
Numerical Example

- Given the triangle mesh:

$$\mathbf{v_0} = [2,2,1]^T \quad \mathbf{v_1} = [6,2,1]^T \quad \mathbf{v_2} = [4,5,1]^T$$

$$\overrightarrow{\mathbf{n}_{\mathbf{v}_0}} = [-1,-1,1]^T \quad \overrightarrow{\mathbf{n}_{\mathbf{v}_1}} = [1,0,0]^T \quad \overrightarrow{\mathbf{n}_{\mathbf{v}_2}} = [0,1,1]^T$$

$$\mathbf{a} = [2.66,3,1]^T \quad \mathbf{b} = [5.33,3,1]^T \quad \mathbf{s} = [4,3,1]^T$$



- Assume emitted, ambient & incident intensities from light source:

$$I_e = 2, \quad I_a = 1, \quad I_i = 12$$

- And constant values:

$$k_a = 0.3\,, \quad k_d = 0.3, \quad k_s = 0.6, \quad n = 3$$

- Also assume the light and view points at infinity on the +*z*-axis :

$$\hat{\mathbf{l}} = \hat{\mathbf{v}} = [0,0,1]^T$$

Numerical Example – Constant Shading

- Compute the polygon normal:

$$\vec{\mathbf{n}} = (\mathbf{v_1} - \mathbf{v_0}) \times (\mathbf{v_2} - \mathbf{v_0}) = [0, 0, 12]^T$$

$$\text{or} \quad \hat{\mathbf{n}} = [0, 0, 1]^T$$

- From equation $I = I_e + I_a\, k_a + I_i\, k_d\, n_z$:

$$I = 2 + 1 \cdot 0.3 + 12 \cdot 0.3 \cdot 1 = \mathbf{5.9}$$

Numerical Example – Gouraud Shading

- Normalize the vertex normals:

$$\hat{\mathbf{n}}_{\mathbf{v}_0} = [-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]^T \quad \hat{\mathbf{n}}_{\mathbf{v}_1} = [1,0,0]^T \quad \hat{\mathbf{n}}_{\mathbf{v}_2} = [0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^T$$

- Use the Phong model to compute the intensities at the vertices:

$$I_{\mathbf{v}_0} = 2 + 1 \cdot 0.3 + 12(0.3(\hat{\mathbf{n}}_{\mathbf{v}_0} \cdot \hat{\mathbf{l}}) + 0.6(\hat{\mathbf{n}}_{\mathbf{v}_0} \cdot \hat{\mathbf{h}})^3) = 5.76$$

$$I_{\mathbf{v}_1} = 2 + 1 \cdot 0.3 + 12(0.3(\hat{\mathbf{n}}_{\mathbf{v}_1} \cdot \hat{\mathbf{l}}) + 0.6(\hat{\mathbf{n}}_{\mathbf{v}_1} \cdot \hat{\mathbf{h}})^3) = 2.3$$

$$I_{\mathbf{v}_2} = 2 + 1 \cdot 0.3 + 12(0.3(\hat{\mathbf{n}}_{\mathbf{v}_2} \cdot \hat{\mathbf{l}}) + 0.6(\hat{\mathbf{n}}_{\mathbf{v}_2} \cdot \hat{\mathbf{h}})^3) = 7.39$$

- From Gouraud shading equation:

$$I_{\mathbf{a}} = \frac{1}{3}(1 \cdot I_{\mathbf{v}_2} + 2 \cdot I_{\mathbf{v}_0}) = 6.3$$

$$I_{\mathbf{b}} = \frac{1}{3}(1 \cdot I_{\mathbf{v}_2} + 2 \cdot I_{\mathbf{v}_1}) = 4.0$$

$$I_{\mathbf{s}} = \frac{1}{2.67}(1.33 \cdot I_{\mathbf{a}} + 1.33 \cdot I_{\mathbf{b}}) = \mathbf{5.13}$$

Numerical Example – Phong Shading

- Compute the normals at the edge points **a**, **b** from the unit vertex normals (by linear interpolation):

$$\overrightarrow{\mathbf{n}_\mathbf{a}} = \frac{1}{3}(1 \cdot \hat{\mathbf{n}}_{v_2} + 2 \cdot \hat{\mathbf{n}}_{\mathbf{v}_0}) = [-0.39, 0.15, 0.62]^T$$

$$\overrightarrow{\mathbf{n}_\mathbf{b}} = \frac{1}{3}(1 \cdot \hat{\mathbf{n}}_{\mathbf{v}_2} + 2 \cdot \hat{\mathbf{n}}_{\mathbf{v}_1}) = [0.67, 0.71, 0.71]^T$$

- Convert them to unit vectors:

$$\hat{\mathbf{n}}_\mathbf{a} = [-0.52, 0.2, 0.83]^T \quad , \quad \hat{\mathbf{n}}_\mathbf{b} = [0.55, 0.59, 0.59]^T$$

- Compute the unit normal vector at scanline point **s**

$$\overrightarrow{\mathbf{n}_\mathbf{s}} = \frac{1}{2.67}(1.33 \cdot \hat{\mathbf{n}}_\mathbf{a} + 1.33 \cdot \hat{\mathbf{n}}_\mathbf{b}) = [0.02, 0.4, 0.71]^T = [0.02, 0.49, 0.87]^T$$

- Apply the Phong model using the unit normal vector $\hat{\mathbf{n}}_\mathbf{s}$:

$$I_\mathbf{s} = 2 + 1 \cdot 0.3 + 12(0.3(\hat{\mathbf{n}}_\mathbf{s} \cdot \hat{\mathbf{l}}) + 0.6(\hat{\mathbf{n}}_\mathbf{s} \cdot \hat{\mathbf{h}})^3) = \mathbf{10.25}$$

Numerical Example

- Phong shading gives significantly higher intensity value compared to Constant or Gouraud shading
- This is explained by the existence of a highlight at **s**
- The quadratic interpolation scheme computes I_s similarly to Phong
  - The only difference is the quadratic formulae used for the computation of $\hat{\mathbf{n}}_{\mathbf{a}}, \hat{\mathbf{n}}_{\mathbf{b}}$ and $\hat{\mathbf{n}}_{\mathbf{s}}$

The Cook-Torrance Illumination Model*

- Problems with the Phong model:
  - Objects often appear too plastic
  - The metallic shine or the off-specular-direction highlights are not captured correctly for many shiny materials
  - The reflected light scattering distribution due to the geometric variation of a rough surface cannot be captured
- Cook – Torrance Model:
  - Extension of the Phong model
  - General illumination model for rough surfaces
  - Takes into account the directional distribution and the wavelength dependence of the reflected light

The Cook-Torrance Illumination Model (2)

- Distinguishes the reflected light into :
  - The ambient term
  - The diffuse scattering
  - The specular highlight
- Provides a modeling & parameterization of the BRDF f_r of a material
- The BRDF f_r is linearly composed of :
  - a pure diffuse term
  - a pure specular term

$$f_r = k_d f_d + k_s f_s, \qquad k_d + k_s = 1$$

The Cook-Torrance Illumination Model (3)

- The Cook-Torrance reflectance model for N_L light sources:

$$I_r = I_a f_a + \sum_{l=1}^{N_L} I_i^{(l)} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}^{(l)}) \, [k_s f_s + k_d f_d] \, d\vec{\omega}_i^{(l)}$$

  where:
  - $I_i^{(l)}$: the incident light intensity from light source *l* located at a direction $\hat{\mathbf{l}}^{(l)}$ through a solid angle $\vec{\omega}_i^{(l)}$
  - $\hat{\mathbf{n}}$: the normal vector at the given surface location
- $I_a f_a$ is the ambient term & I_a can be regarded as constant
- f_d is the diffuse BRDF of a Lambertian surface
- f_a uses the same distribution as f_d
- The specular part of the BRDF depends on:
  - the relative location of the observer
  - the properties of the material

The Cook-Torrance Illumination Model (4)

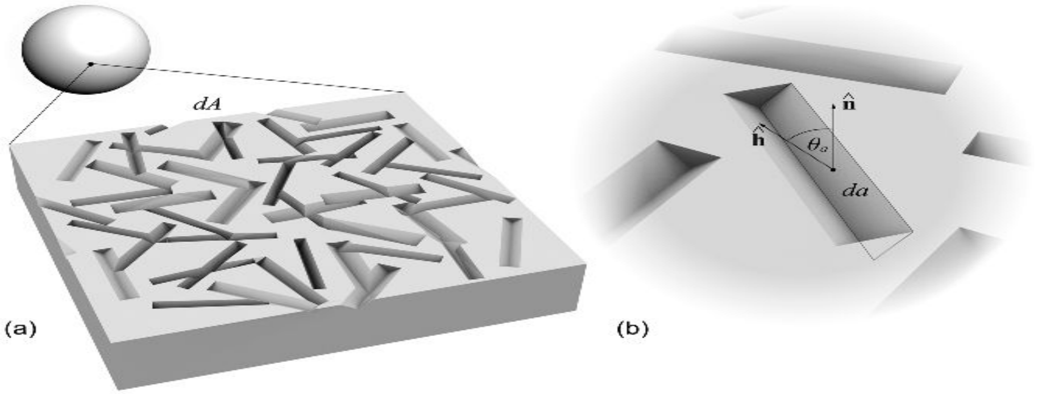
- In the original paper I_a was multiplied by a visibility factor f:
  - f is the amount of incoming ambient light that was not blocked by the surrounding environment
  - A distant uniformly luminous hemisphere radiates light toward the inspected surface point **p**
  - Uses a binary visibility function $V(\mathbf{p},\hat{\mathbf{l}})$ with max value 1 when there is a clear line of sight between point **p** and the surrounding distant hemisphere in direction $\hat{\mathbf{l}}$:

$$f = \int_{unblocked\ \Omega} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) d\vec{\omega} = \int_{\Omega} (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) V(\mathbf{p},\hat{\mathbf{l}}) d\vec{\omega}$$

The Cook-Torrance Illumination Model (5)

- The micro-facet model of Torrance and Sparrow is used for the derivation of the specular term f_s
  - a surface is assumed to be composed of long symmetric V-shaped grooves
  - each groove consists of two planar facets
- The facets:
  - Are tilted at equal but opposite angles to the surface normal at *dA*
  - Are considered perfect mirrors
  - Reflect light only in the direction of perfect reflection
  - The slope of the facets (polar angle) θ_a
  - The orientation of the cavities (azimuth) φ_a

} determined by a statistical distribution for the material



The Cook-Torrance Illumination Model (6)

- *da* : the area of a micro-facet
- *dA* : the inspected area, where reflectance is calculated
- In order for the Torrance-Sparrow model to work:
  - $da << dA$
  - wavelength λ of the incident light << micro-facet dimensions $\rightarrow$
    - avoid interference phenomena
    - be able to work with geometrical optics
    - dispense with wave theory
- Cook-Torrance model depends on
  - Micro-facet distribution term *D*
  - Geometric term *G*
  - Fresnel term *F*

The Cook-Torrance Illumination Model (7)

The Micro-facet distribution D

- Is the fraction of micro-facets aligned with direction $\hat{\mathbf{h}}$
- The contribution of each facet is binary :
  - light reflected fullyfrom direction$\hat{\mathbf{l}}$ to $\hat{\mathbf{v}}$
  - or, no light reflected at all
- Determines the fraction of incident light reflected back to the environment in the direction of view. Several possibilities:
- Gaussian distribution (easier to compute)

$$D_{(Gaussian)} = c \cdot e^{-(\theta_a/m)^2}$$

- Beckmann distribution (more physically correct)

$$D_{(Beckmann)} = \frac{1}{m^2 \cos^4 \theta_a} \cdot e^{-(\tan\theta_a/m)^2}$$

where *m*: the RMS slope of the surface

θ_a : the angle between the normal $\hat{\mathbf{n}}$ of *dA* and $\hat{\mathbf{h}}$ of *da*

The Cook-Torrance Illumination Model (8)

- The larger *m* is:
  - the more rough the surface
  - the specular highlight is spread out
- Small *m* :
  - Micro-facets with normal vectors closer to $\hat{\mathbf{n}}$
  - The material has a polished look
  - Specular highlight is tighter

The Cook-Torrance Illumination Model (9)

Incoming light interception

- Some of the outgoing light in the direction of $\vec{\mathbf{v}}$ is attenuated due to the interception by blocking geometry
- The amount of blocking depends on
  - the outgoing direction
  - the slope of the micro-facet relative to $\hat{\mathbf{n}}$
- The amount of light blocked due to light interception $G_{intercept} \in [0,1]$ is:

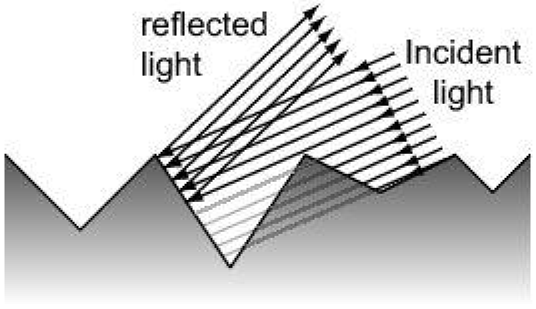
$$G_{intercept} = \frac{2(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}})}{\hat{\mathbf{v}} \cdot \hat{\mathbf{h}}}$$

The Cook-Torrance Illumination Model (10)

Shadow

- Some of the light incoming from a direction $\hat{\mathbf{l}}$ on a facet *da* is blocked by the opposite facet of the groove
- This leaves the lower part of the micro-facet in shadow

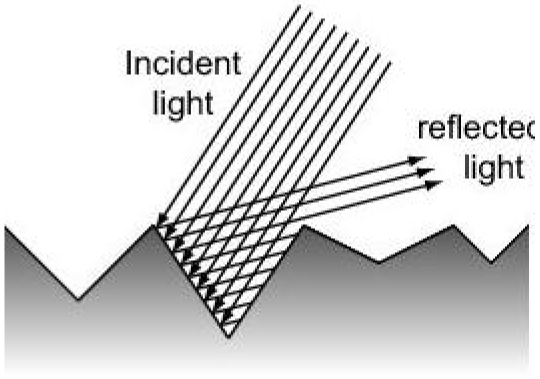
$$G_{shadow} = \frac{2(\hat{\mathbf{n}}\cdot\hat{\mathbf{h}})(\hat{\mathbf{n}}\cdot\hat{\mathbf{l}})}{\hat{\mathbf{l}}\cdot\hat{\mathbf{h}}} = \frac{2(\hat{\mathbf{n}}\cdot\hat{\mathbf{h}})(\hat{\mathbf{n}}\cdot\hat{\mathbf{l}})}{\hat{\mathbf{v}}\cdot\hat{\mathbf{h}}}$$



masking (shadow)

Combining, Geometric attenuation factor G

$$G = \min\{1, \frac{2(\hat{\mathbf{n}}\cdot\hat{\mathbf{h}})(\hat{\mathbf{n}}\cdot\hat{\mathbf{v}})}{\hat{\mathbf{v}}\cdot\hat{\mathbf{h}}}, \frac{2(\hat{\mathbf{n}}\cdot\hat{\mathbf{h}})(\hat{\mathbf{n}}\cdot\hat{\mathbf{l}})}{\hat{\mathbf{v}}\cdot\hat{\mathbf{h}}}\}$$



interception

The Cook-Torrance Illumination Model (11)

- Spectral composition for Cook-Torrance model:

1. Diffuse part of BRDF
   - Is constant
   - Equal to the reflectance at normal incidence
2. Specular part of BRDF
   - Is associated with the angle of incidence
   - It leads to a color shift when the direction of incidence and reflection are at grazing angles

The Cook-Torrance Illumination Model (12)

The Fresnel term *F*

- Describes how a single micro-facet reflects light
- Implements the dependence on:
  - *n*: the relative index of refraction of the material
  - *k*: the extinction coefficient In the Cook-Torrance model
- For *k = 0* and unpolarized light, the Fresnel equation is:

$$F = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1+\frac{[c(g+c)-1]^2}{[c(g-c)+1]^2}\right)$$

where

$$c = \hat{\mathbf{v}}\cdot\hat{\mathbf{h}}$$

$$g = \sqrt{n^2 + c^2 - 1}$$

The Cook-Torrance Illumination Model (13)

The Fresnel term *F*

- $F \rightarrow 1$
  - The angle between $\hat{\mathbf{v}}$ and $\hat{\mathbf{h}}$ tends to $\pi / 2$
  - When we look at the direction of the light source from a very low position with respect to the surface (*grazing angle*)
  - Is independent of the *n* and *k* values
  - At a grazing angle, the spectral composition of the reflected light is the same as that of the light source
- $F \neq 1$ for other angles
- $k = 0$ is also true for non-metals
- The Fresnel equation produces a good approximation for metals

The Cook-Torrance Illumination Model (14)

- The specular part of the BRDF:
  - Gathering *D*, *G*, *F* in a single equation:

$$f_s = \frac{1}{\pi}\frac{DGF}{(\hat{\mathbf{n}}\cdot\hat{\mathbf{l}})(\hat{\mathbf{n}}\cdot\hat{\mathbf{v}})}$$

  - $(\hat{\mathbf{n}}\cdot\hat{\mathbf{l}})(\hat{\mathbf{n}}\cdot\hat{\mathbf{v}})$ maximizes the specular highlight when viewing the light source from a grazing angle

Approximation of Cook and Torrance:

  - Since the calculation of the Fresnel term is expensive

1. Measure/estimate the reflected color at normal incidence F_0 via the Fresnel equation

The Cook-Torrance Illumination Model (15)

2. At grazing angle ($F_{\pi/2} = 1$) for all wavelengths →
   - (R, G, B) of the reflected light = (R,G,B) of the incident light
   - The reflected specular color component at an angle $\theta = \widehat{\hat{\mathbf{v}}\hat{\mathbf{h}}}$ may be interpolated:

$$c_i = c_{i,0} + (c_{i,\frac{\pi}{2}} - c_{i,0}) \frac{\max(0, F_\theta(\lambda) - F_0(\lambda))}{F_{\frac{\pi}{2}} - F_0(\lambda)}$$

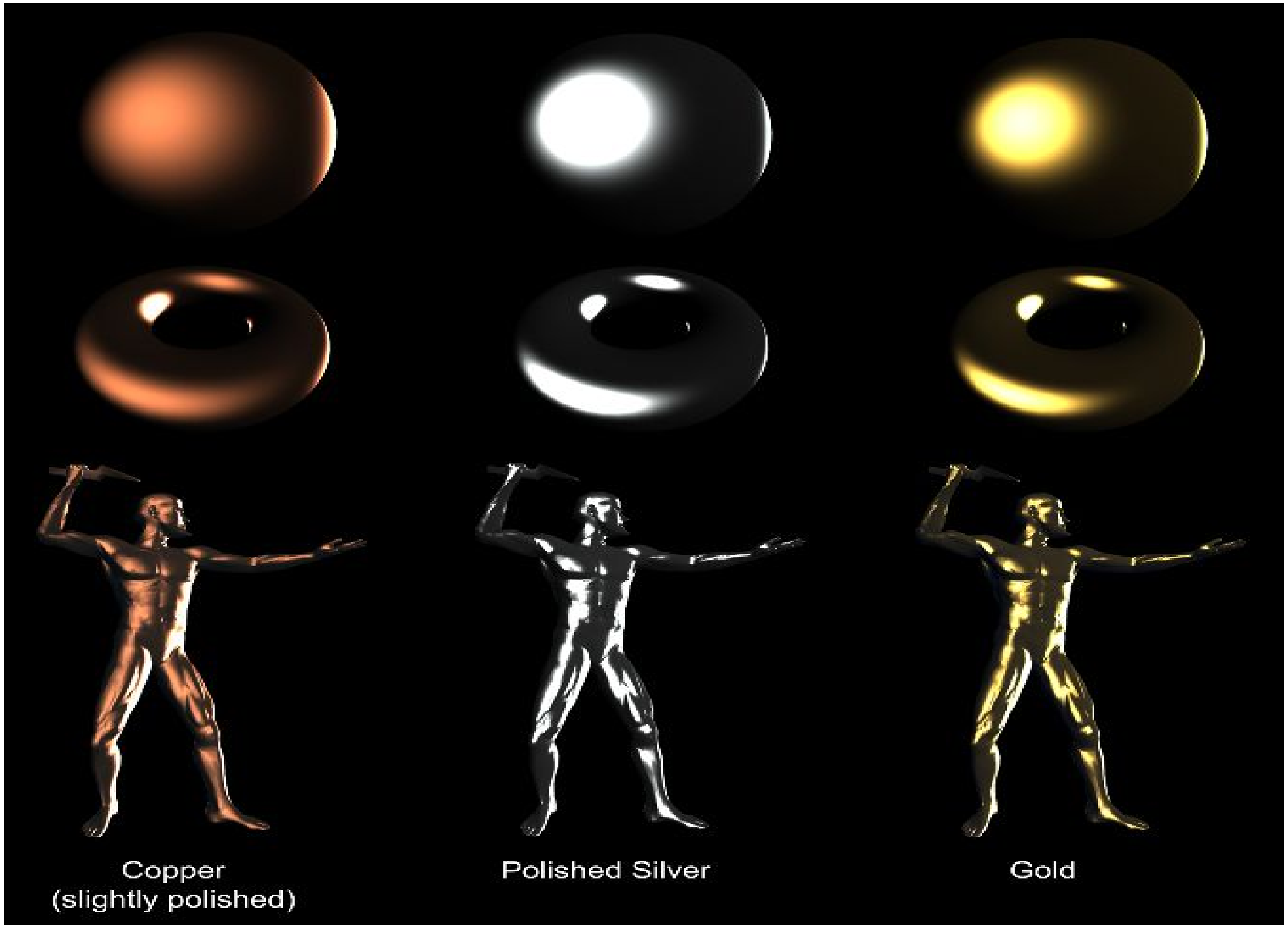
where

- c_i : the color components (i=R,G,B) of the resulting color
- $c_{i,\pi/2}$: the color components of the material color at normal incidence
- $c_{i,0}$: the color components of the incident light color

3. The final color c_i is:

$$c_i = \frac{1}{\pi}[c_{i,0} + (c_{i,\frac{\pi}{2}} - c_{i,0}) \frac{\max(0, F_\theta(\lambda) - F_0(\lambda))}{F_{\frac{\pi}{2}} - F_0(\lambda)}] \frac{DGF_\theta(\lambda)}{(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}})}$$

The Cook-Torrance Illumination Model (16)



The Oren-Nayar Illumination Model

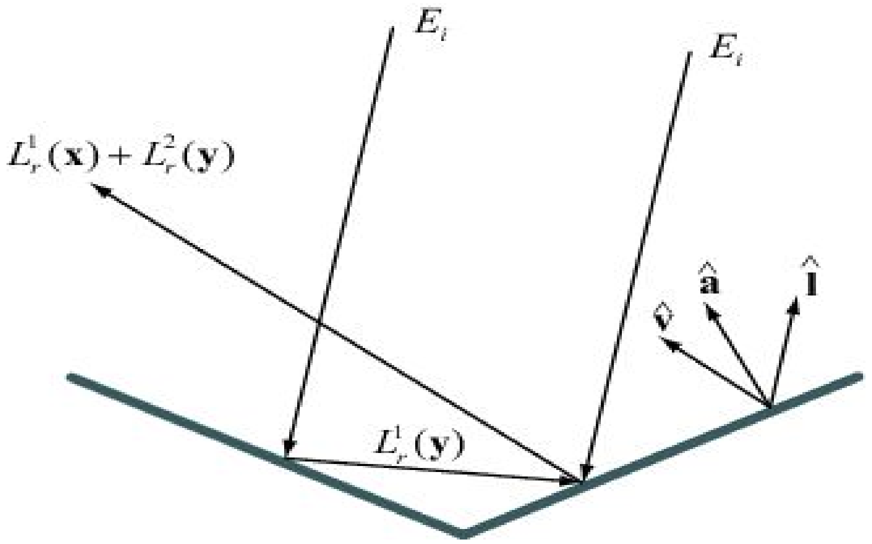
- So far the **diffuse** component of illumination was based on the Lambertian principle, i.e. equal brightness from all view directions
  - Works well for smooth surfaces
  - **Rough** surfaces are not in general Lambertian
  - E.g. Full Moon, clay, cement and sand
- A rough surface exhibits phenomena such as
  - Light masking and shadows
  - Secondary reflections of light on the walls of the irregular microscopic structures
  - => Brightness of the reflected light increases as the viewing direction approaches the light direction
- Oren and Nayar model:
  - Incorporates the above factors to predict the diffuse behavior of rough materials
  - Adopts the micro-facet model of Torrance-Sparrow
  - A rough surface consists of long V-shaped grooves

The Oren-Nayar Illumination Model (2)

- The facets are Lambertian surfaces (not perfect mirrors)
  - The reflected light in direction (θ_r, φ_r) from incident direction $\hat{\mathbf{l}}$ is computed as:

i. The 1st order reflected radiance $L_r^1(\theta_r, \phi_r, \theta_i, \phi_i)$
   - The light directly reflected in direction $\hat{\mathbf{v}}$ from a micro-facet

ii. The 2nd reflected radiance $L_r^2(\theta_r, \phi_r, \theta_i, \phi_i)$
   - The light reflected in the same direction after having bounced off the opposite facet of the groove



The Oren-Nayar Illumination Model (3)

- Projected Radiance the contribution of a facet to the total radiance of patch *dA*

$$L_{rp}(\theta_a,\phi_a)=\frac{d\Phi_r(\theta_a,\phi_a)}{(da\cos\theta_a)\cos\theta_r d\vec{\omega}_r} \qquad (12.53)$$

where θ_a : the slope of the facet wrt surface tangent plane

- From the relation between *L*, *E* and *Φ*:

$$\left.\begin{array}{c} dE_r(\theta_r,\phi_r)=L_r(\theta_r,\phi_r)\cos\theta_r d\vec{\omega}_r = L_r(\theta_r,\phi_r)(\hat{\mathbf{a}}\cdot\hat{\mathbf{v}})d\vec{\omega}_r \\ d\Phi_r(\theta_r,\phi_r)=dE_r(\theta_r,\phi_r)da \end{array}\right\}\Leftrightarrow$$

$$d\Phi_r(\theta_r,\phi_r)=L_r(\theta_r,\phi_r)(\hat{\mathbf{a}}\cdot\hat{\mathbf{v}})d\vec{\omega}_r da$$

- Substituting radiant flux in (12.53), $L_{rp}(\theta_a,\phi_a)$ becomes

$$L_{rp}(\theta_a,\phi_a)=\frac{L_r(\theta_r,\phi_r)(\hat{\mathbf{a}}\cdot\hat{\mathbf{v}})d\vec{\omega}_r da}{(da\cos\theta_a)\cos\theta_r d\vec{\omega}_r}=\frac{L_r(\theta_r,\phi_r)(\hat{\mathbf{a}}\cdot\hat{\mathbf{v}})}{(\hat{\mathbf{a}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{v}}\cdot\hat{\mathbf{n}})} \qquad (12.55)$$

The Oren-Nayar Illumination Model (4)

- Micro-facets are Lambertian → BRDF is constant & equal to $1/\pi$
- From the definition of BRDF:

$$L_r(\theta_r,\phi_r) = \rho f_d E_i(\theta_i,\phi_i) = \rho f_d E_0 \cos\theta_i = \rho f_d E_0 (\hat{\mathbf{l}}\cdot\hat{\mathbf{a}}) = \frac{\rho}{\pi} E_0 (\hat{\mathbf{l}}\cdot\hat{\mathbf{a}})$$

where ρ = the surface albedo

E_0 = the irradiance from the source at normal incidence

- Replacing the radiance in (12.55):

$$L_{rp}(\theta_a,\phi_a) = \frac{\rho}{\pi} E_0 \frac{(\hat{\mathbf{l}}\cdot\hat{\mathbf{a}})(\hat{\mathbf{a}}\cdot\hat{\mathbf{v}})}{(\hat{\mathbf{a}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{v}}\cdot\hat{\mathbf{n}})}$$

The Oren-Nayar Illumination Model (5)

- The contribution of all facets facing in the direction of $\hat{\mathbf{a}}$:

$$L_r^1(\theta_r,\phi_r,\theta_i,\phi_i) = \int_{\theta_a=0}^{\pi/2}\int_{\phi_a=0}^{2\pi} P(\theta_a,\phi_a)L_{rp}^1(\theta_a,\phi_a)\sin\theta_a d\phi_a d\theta_a \quad (12.58)$$

- Geometric factor is a generalization of the Cook-Torrance factor *G*
  - works for any facet normal $\hat{\mathbf{a}}$
  - not necessarily the halfway vector $\hat{\mathbf{h}}$ between the viewing and the incident direction

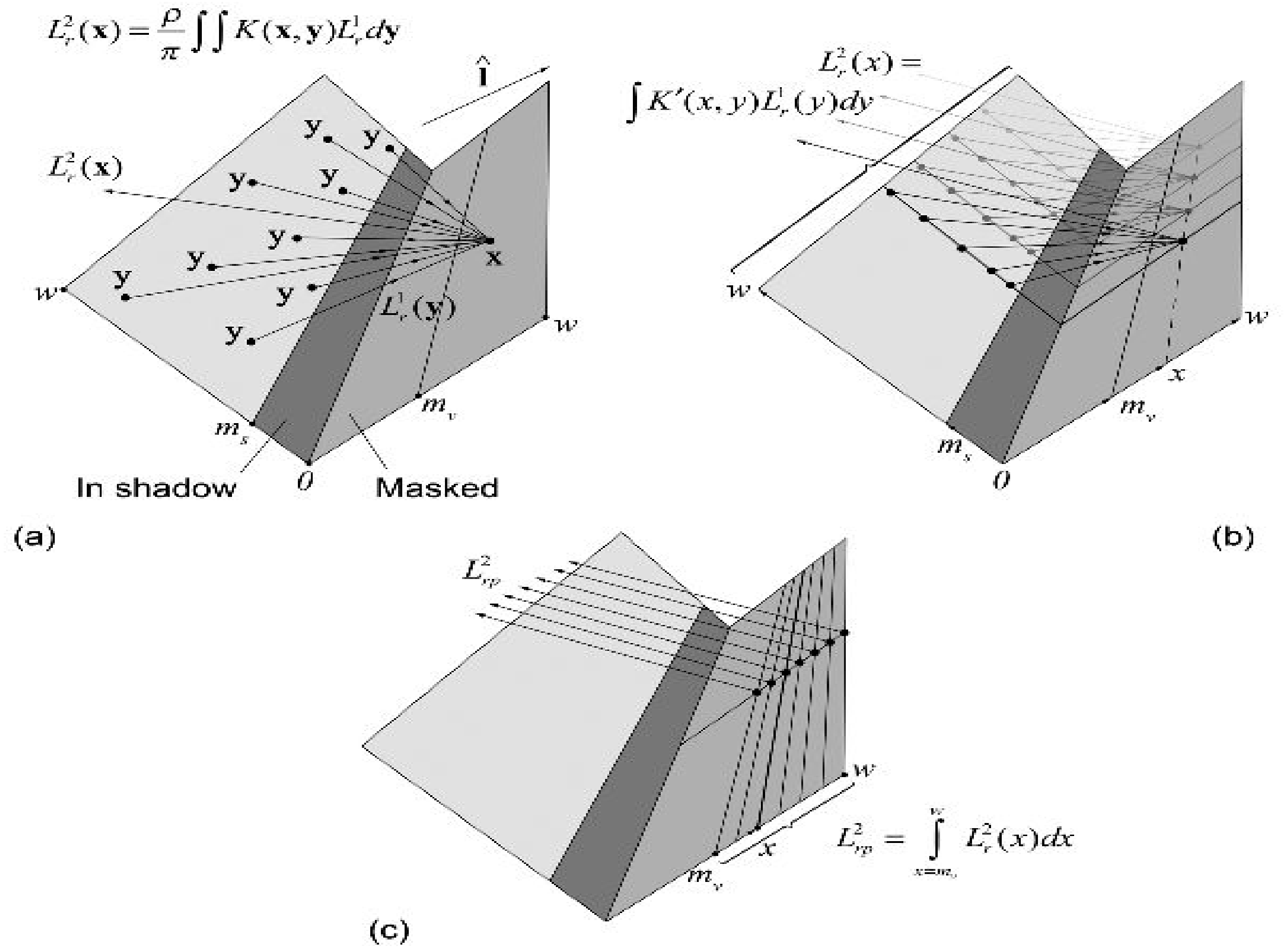
$$GAF = \min\left\{1, \max\left\{0, \frac{2(\hat{\mathbf{l}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{a}}\cdot\hat{\mathbf{n}})}{\hat{\mathbf{l}}\cdot\hat{\mathbf{a}}}, \frac{2(\hat{\mathbf{v}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{a}}\cdot\hat{\mathbf{n}})}{\hat{\mathbf{v}}\cdot\hat{\mathbf{a}}}\right\}\right\}$$

- Taking also into account the blocked incident and reflected light, (12.58) becomes:

$$L_r^1(\theta_r,\phi_r,\theta_i,\phi_i) = \int_{\theta_a=0}^{\pi/2}\int_{\phi_a=0}^{2\pi} P(\theta_a,\phi_a)L_{rp}^1(\theta_a,\phi_a)GAF\sin\theta_a d\phi_a d\theta_a$$

The Oren-Nayar Illumination Model (6)

- Radiance from second-order reflections :



The Oren-Nayar Illumination Model (7)

- The overall radiance leaving patch *dA* in the direction $\hat{\mathbf{v}}\ (\theta_r, \phi_r)$:

$$L_r(\theta_r,\phi_r,\theta_i,\phi_i) = L_r^1(\theta_r,\phi_r,\theta_i,\phi_i) + L_r^2(\theta_r,\phi_r,\theta_i,\phi_i) \qquad (12.61)$$

- Simplification of the original model:

$$L_r^1(\theta_r,\phi_r,\theta_i,\phi_i) = \frac{\rho}{\pi} E_0 \cos\theta_i [C_1 + \cos(\phi_r - \phi_i) C_2 \tan\beta + (1 - |\cos(\phi_r - \phi_i)|) C_3 \tan(\frac{\alpha+\beta}{2})]$$

$$L_r^2(\theta_r,\phi_r,\theta_i,\phi_i) = 0.17 \frac{\rho}{\pi} E_0 \frac{\sigma^2}{\sigma^2 + 0.13} \cos\theta_i [1 - (\frac{2\beta}{\pi})^2 \cos(\phi_r - \phi_i)]$$

where

$$C_1 = 1 - 0.5 \frac{\sigma^2}{\sigma^2 + 0.3}$$

$$C_2 = \begin{cases} 0.45 \dfrac{\sigma^2}{\sigma^2 + 0.09} \sin\alpha, & \cos(\phi_r - \phi_i) \geq 0 \\ 0.45 \dfrac{\sigma^2}{\sigma^2 + 0.09} (\sin\alpha - (\dfrac{2\beta}{\pi})^3), & \text{otherwise} \end{cases}$$

$$C_3 = 0.125 \frac{\sigma^2}{\sigma^2 + 0.09} (\frac{4\alpha\beta}{\pi^2})^2$$

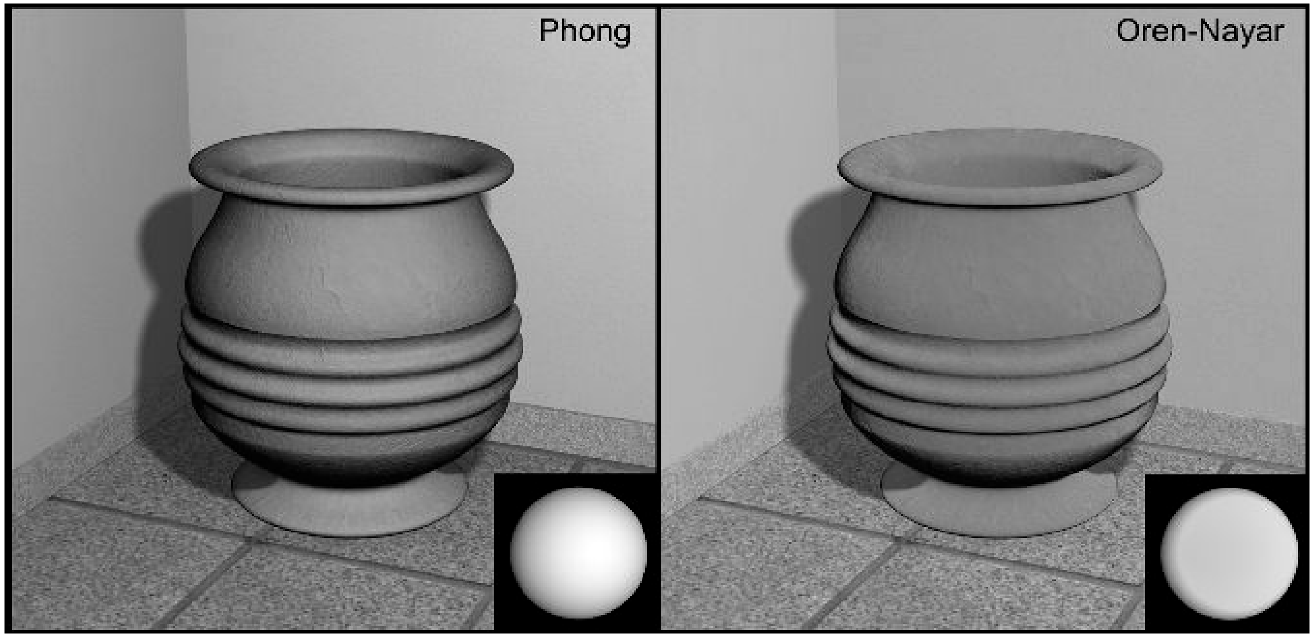
$$\alpha = \max(\theta_r, \theta_i), \beta = \min(\theta_r, \theta_i)$$

The Oren-Nayar Illumination Model (8)

- The BRDF is acquired by applying the BRDF definition to (12.61):

$$f_{Oren-Nayar} = \frac{L(\theta_r, \phi_r, \theta_i, \phi_i)}{E_i} = \frac{L(\theta_r, \phi_r, \theta_i, \phi_i)}{E_0 \cos\theta_i} =$$

$$\frac{L_r^1(\theta_r, \phi_r, \theta_i, \phi_i) + L_r^2(\theta_r, \phi_r, \theta_i, \phi_i)}{E_0 \cos\theta_i}$$



The Strauss Illumination Model

- Illumination models based on geometrical optics (Blinn, Cook-Torrance and Oren-Nayar)
  - Produce very realistic shading
  - But, use actual physical parameters found in material science which are generally unintuitive for users (e.g. artists)
- The Phong model
  - Cannot effectively capture the appearance of metallic surfaces
  - The specular exponent is specified as an unbounded positive number →
    - Cannot easily produce a balanced shininess between a dull surface & a fully reflective one by adjusting its value between two limits
  - The shininess adjustment is complex
    - Two seemingly independent parameters (the exponent and the specular coefficient) control the same material attribute

The Strauss Illumination Model (2)

- Strauss Illumination Model:
  - Borrows many lighting calculations from Phong
  - Incorporates features like
    - metallic appearance
    - off-specular reflections
    - unified shininess control

    through intuitive normalized parameters
  - Empirical model (targeting animators and 3D modelers)
- Normalized parameters that control surface appearance:

i. The material color **c** = (*r*, *g*, *b*) : represents the albedo of the surface

ii. The smoothness *s* : – ranges from 0 (dull surface) to 1 (perfect mirror)

– controls : the specular/diffuse contribution ratio

the size of the highlight

i. The metalness *m* : – is ranges from 0 to 1 (metallic surface)

– affects the color of the specularly reflected light

The Strauss Illumination Model (3)

- The intensity of the reflected light per color channel c_r :

$$c_r = c_i(Q_d + Q_s + Q_a)$$

  where

  - c_i : the corresponding incident light component
  - Q_d, Q_s, Q_a : the diffuse, specular and ambient components of the Strauss model

- The amount of diffuse illumination Q_d :
  - Depends on the shininess of the surface *s*
    - The more shiny the surface, the less it behaves as a Lambertian reflector
  - Decreases with the increase of the metalness *m*
  - Depends on the angle of incidence

The Strauss Illumination Model (4)

- The Strauss diffuse and ambient components are:

$$Q_d = (\hat{\mathbf{n}} \cdot \hat{\mathbf{l}}) r_d dc \qquad Q_a = r_d c$$

$$r_d = (1 - s^3)(1 - t)$$

$$d = (1 - ms)$$

where :

- *t:* the transparency of the surface (0 (fully opaque) → 1)
- *c:* one of the red, green or blue components of the surface color
- (*1-s*^{3}) is experimentally chosen to account for a linear perceptual transition from a dull surface to a perfect mirror, with a corresponding linear change in the *s* parameter

The Strauss Illumination Model (5)

- The specular component Q_s is:

$$Q_s = r_s \; c_s$$

where

  - r_s: the specular reflectivity, defines the shape of the highlight
  - c_s: the specular color is interpolated for metallic surfaces between the surface color and the light color

- The specular reflectivity r_s:
  - Depends on the angle between the mirror reflection direction & the view vector
  - Is raised to a power to tighten the highlight

$$r_s = (\hat{\mathbf{r}} \cdot \hat{\mathbf{v}})^h r_j$$

$$h = \frac{3}{1-s}$$

The Strauss Illumination Model (6)

- The <u>adjusted reflectivity</u> r_j encapsulates
  - the specular attenuation due to the Fresnel term
  - the geometric attenuation factor
- r_j depends on the reflectivity of the surface at normal incidence

$r_n = 1 - t - r_d$ so: $r_j = \min[1,\; r_n + (r_n + k_j)\, F(\theta_i)\, G(\theta_i)\, G(\theta_r)]$

where

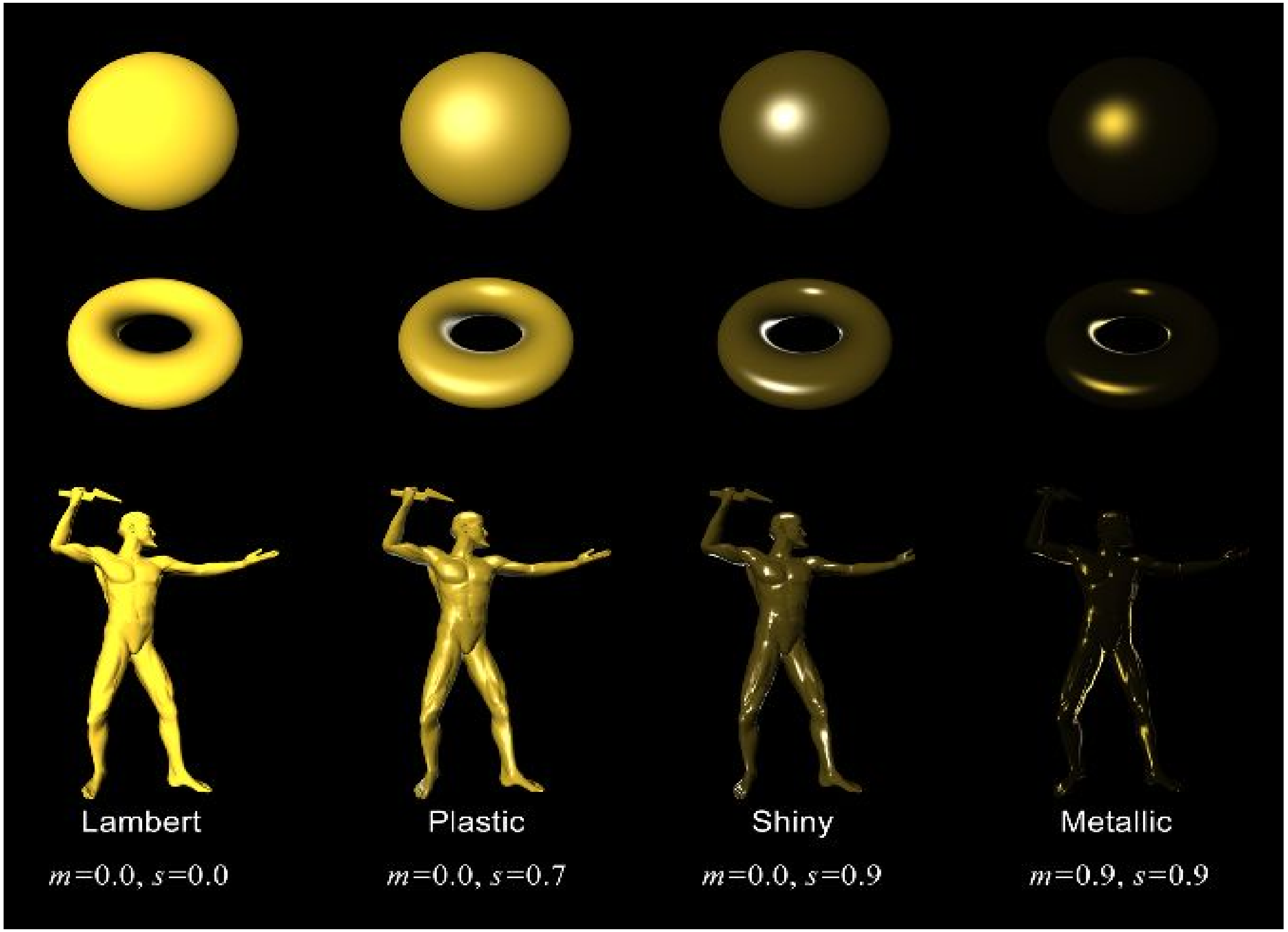
- $F(x)$: an empirical Fresnel-like function
- $G(x)$: a geometric attenuation function

$$F(x) = [\frac{1}{(x-k_f)^2} - \frac{1}{k_f^2}] / [\frac{1}{(1-k_f)^2} - \frac{1}{k_f^2}]$$

$$G(x) = [\frac{1}{(1-k_g)^2} - \frac{1}{(x-k_g)^2}] / [\frac{1}{(1-k_g)^2} - \frac{1}{k_g^2}]$$

- The constants k_j , k_f, k_g are experimentally chosen
- Strauss suggests the values $k_j = 0.1$, $k_f = 1.12$, $k_g = 1.01$

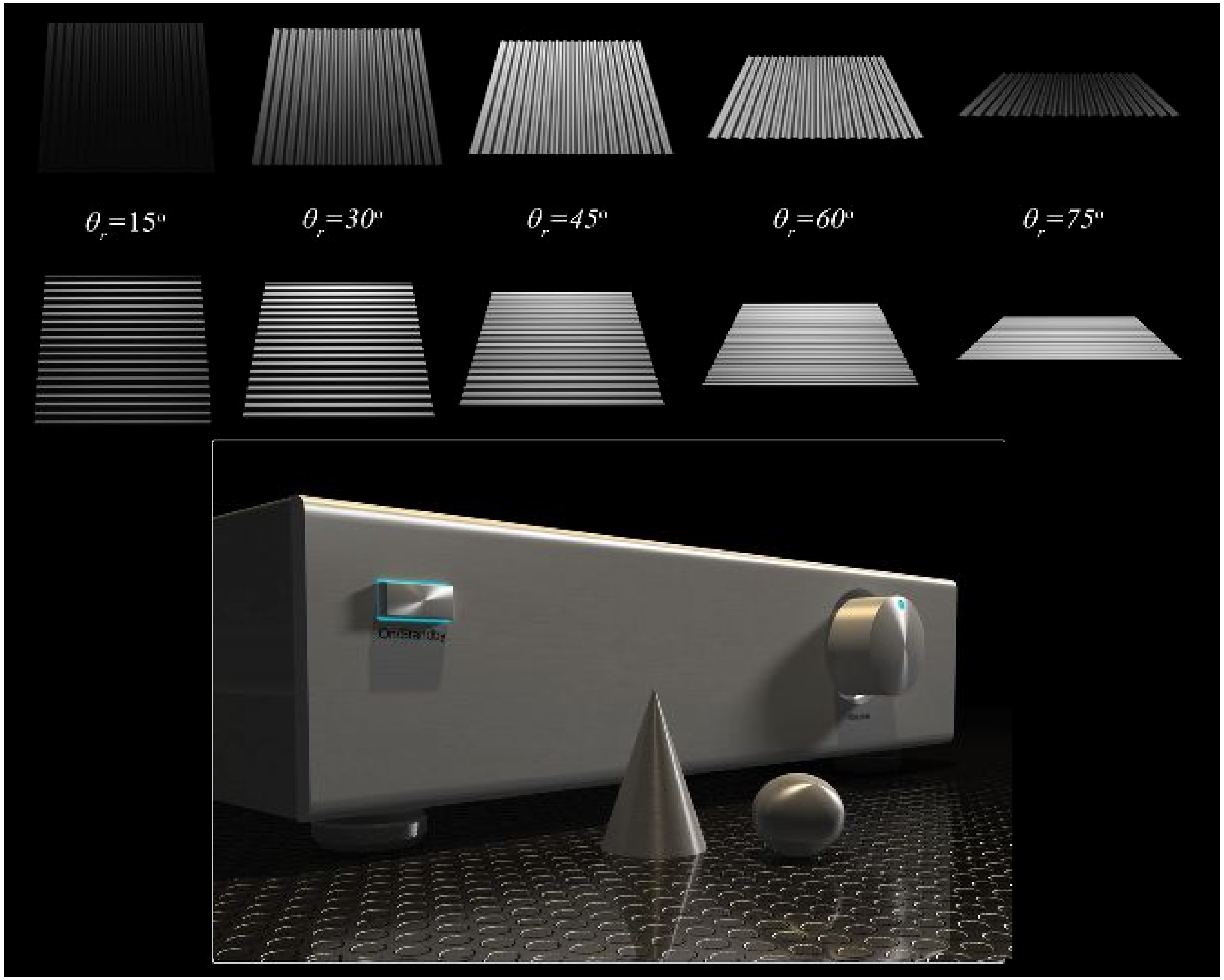
The Strauss Illumination Model (7)



Anisotropic Reflectance

- All previous lighting models possessed an isotropic BRDF
  - Reflected light did not depend on the azimuth angle of incidence
- Many real materials and treated surfaces exhibit a distinctive directional bias
- Anisotropic specular reflection is caused by the microscopic geometric structures of the surface
- Most anisotropic reflective materials possess a characteristic grain or a set of very small grooves which are roughly oriented in a specific direction
- The grooves appear parallel within a magnified surface area
- Good examples of anisotropic reflectors
  - brushed metals (e.g. brushed aluminum)
  - varnished wood
  - vinyl music records

Anisotropic Reflectance (2)

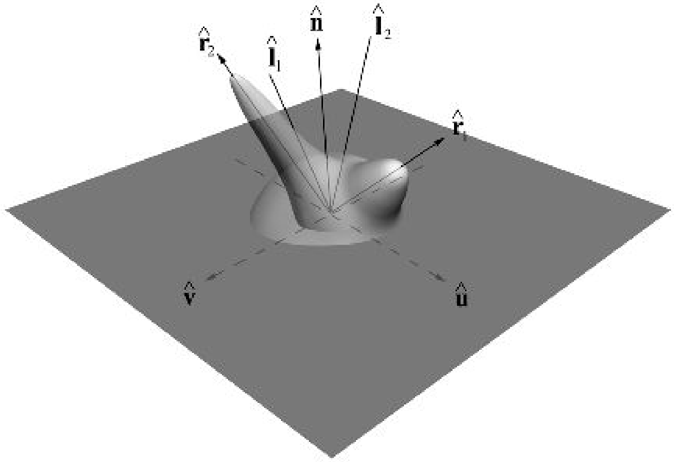


Anisotropic Reflectance (3)

- Model the surface according to the micro-facet approach
  - Assume that the surface grain lays on a longitude direction φ_g
  - The distribution of the facets *da* with respect to their normal direction $\hat{\mathbf{a}} = (\theta_a, \phi_a)$ is clearly directional
    - $\theta_a = 0$ for $\varphi_a = \varphi_g, \ \varphi_g + \pi$
    - θ_a ranges from $-\theta_s$ to θ_s for $\varphi_a = \varphi_g \pm \pi/2$
    - θ_s : the maximum slope
- Observing the surface from a macroscopic level
  - Incident light coming from (θ_i, φ_i)
  - In the extreme case where all grooves are aligned with φ_g
    - The surface becomes a perfect mirror when $\varphi_i = \varphi_g, \ \varphi_g + \pi$
    - The surface has a wider spread of the highlight as $\varphi_i \rightarrow \varphi_g \pm \pi/2$ (maximum anisotropy)

Anisotropic Reflectance (4)

- Several models to deal with anisotropy
  - Kajiya
  - Poulin-Fournier
- For arbitrary geometry it is difficult to represent the direction of maximum (and minimum) reflectance on the surface, which is dependent on the azimuth angle φ_g
  - This direction is a local attribute of the model
  - Cannot in general be expressed relative to the object or world reference frame
  - Most implementations rely on local tangent space (e.g. using texture mapping)

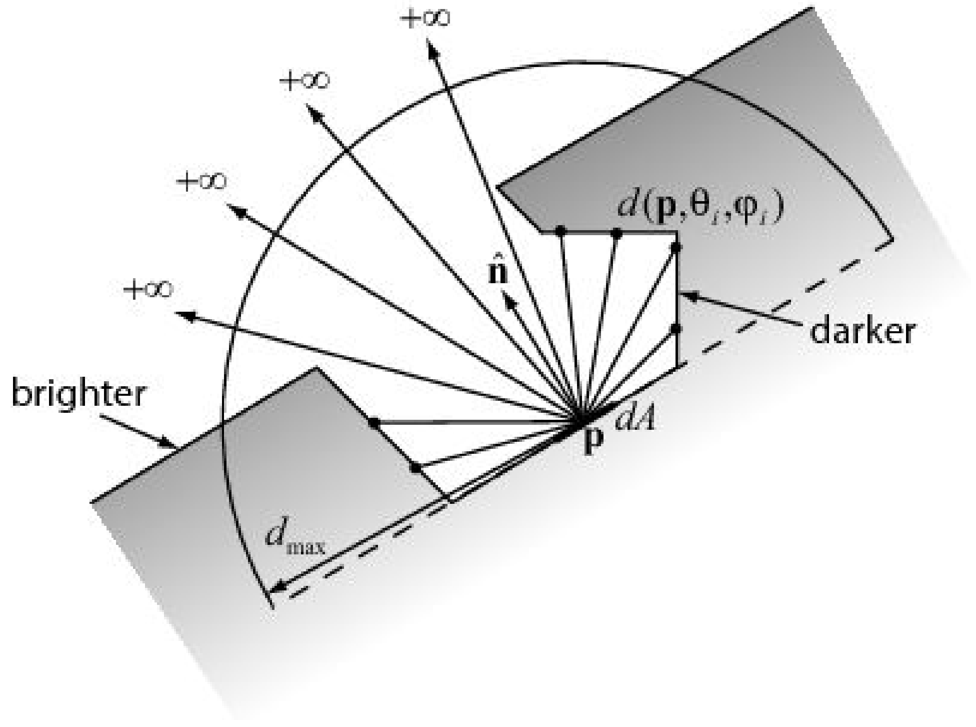


Ambient Occlusion

- Local illumination models regard ambient illumination as constant
  - Ambient term is the irradiance that reaches a surface as the summed contribution of the emitted or reflected light from the environment and
  - Accounts for the exchange of energy between a patch *dA* and all other possibly contributing patches in a scene
- A constant ambient illumination is clearly a very rough approximation, even for simple scenes
- Exchange of energy in a closed environment is simulated via a *global illumination* method (Chapter 16)
- One aspect of the global energy exchange that affects the ambient term, **the darkening effect in obscured parts of a scene**, can be simulated in a more efficient manner
- <u>**Ambient occlusion:**</u> Assuming a uniform (ambient) distant environment irradiance from every direction, estimates the portion of it that finally reaches a small patch *dA*

Ambient Occlusion (2)

- Equivalent to calculating the visibility of a patch due to the presence of the rest of the geometry
  - i.e. portion of the solid angle around the patch, from where *dA* is visible
- Inversely, the *obscurance* of a patch *dA* is the portion of the hemispherical solid angle around the patch that is blocked by other geometry



Ambient Occlusion (3)

- The higher the obscurance, the darker the patch
  - *dA* is blocked at many incident directions by other patches → less light from the environment can hit the surface
- *Obscurance* $w(\mathbf{p})$ reflects the “openness” of a patch *dA* centered at a point $\mathbf{p}$
  - A purely geometric property
  - Does not depend on any particular lighting conditions or viewing direction
  - Is usually pre-calculated and stored as vertex data
- $w(\mathbf{p})$ can be multiplied with a constant ambient term & provides a convincing estimate of the incident light from the environment

Ambient Occlusion (4)

- Note that ambient occlusion is not a physical simulation model and does not provide an accurate global illumination calculation:
  - Misses the high-order bounces of energy that eventually hit the surface
  - Regards irradiance to be constant in all incident directions
- Assumptions:
  - No specific light sources in the environment
  - The (uniform) incident ambient illumination can be modeled as a perfectly diffuse light that radiates from all directions towards *dA*
  - Light is not emitted from some infinite medium far from the scene itself, but the geometry is immersed in a radiating, non-absorbing, gaseous medium.
- Let $d(\mathbf{p},\theta_i,\varphi_i)$: distance between **p** and the closest surface point in direction (θ_i,φ_i):

$$d(\mathbf{p},\theta_i,\phi_i)=\begin{cases}|\mathbf{c}-\mathbf{p}|, & \mathbf{c}:\text{first intersection point in direction }(\theta_i,\phi_i)\\ +\infty, & \text{no intersections in direction }(\theta_i,\phi_i)\end{cases} \quad (12.71)$$

Ambient Occlusion (5)

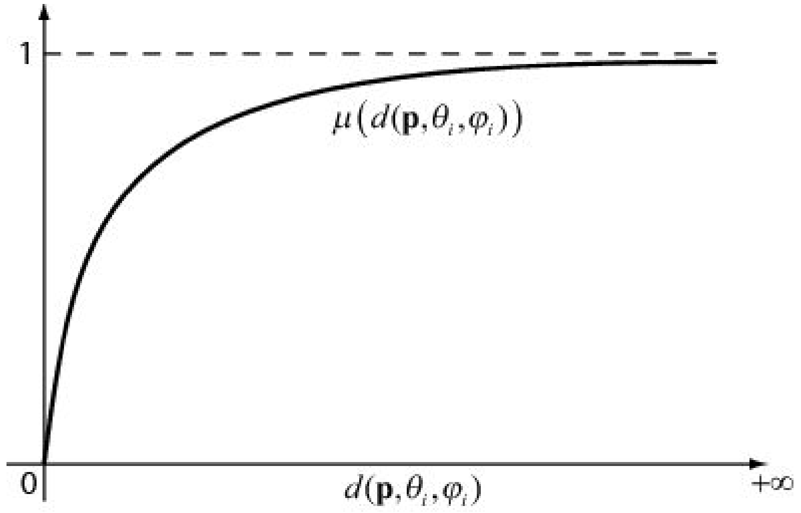
- The farther from **p** an intersection point is, the more light reaches the surface of the patch *dA*
- If the hemispherical solid angle above the patch is completely open up to a distance d_{max}, the obscurance $w(\mathbf{p})$ is set to 1
- Obscurance becomes 0 only in degenerate cases or where 2 surfaces firmly touch each other
- d_{max} is the maximum distance at which the contribution of the surrounding geometry is non-negligible
- Intensity of reflected light from patch *dA* centered at **p,** due to ambient illumination coming from hemisphere *Ω* above *dA* can be approximated as:

$$I_a(\mathbf{p}) = k_a I_a w(\mathbf{p})$$

$$w(\mathbf{p}) = \frac{1}{\pi}\int_{\Omega} \mu(d(\mathbf{p},\theta_i,\phi_i))\cos\theta_i d\vec{\omega} \quad (12.72)$$

Ambient Occlusion (6)

- Function $\mu(x)$ maps the distance $x = d(\mathbf{p},\theta_i,\varphi_i)$ to a normalized obscurance factor
  - It represents the energy emitted by the gaseous medium in the line of sight from **p** to the closest surface in the direction (θ_i,φ_i)
- $\mu(x)$ must be:
  - Monotonically increasing and smooth
  - 0 for zero distance and 1 at infinity with a decreasing slope



$$\mu(x)=\begin{cases}0, & x=0\\ 1, & x=+\infty\end{cases}, \quad \frac{d\mu(x)}{dx}=\begin{cases}0, & x=+\infty\\ >0, & \text{otherwise}\end{cases}, \quad \frac{d^2\mu(x)}{dx^2}<0$$

(12.73)

Ambient Occlusion (7)

- Common family of functions that conforms to the requirements:

$$\mu(x) = 1 - e^{-\tau x} \qquad (12.74)$$

- Parameter τ regulates the spread of the shadowed area
- Since d_{max} defines a range of distance from **p** beyond which no patch is taken into account, $\mu(x)$ has to be modified to normalize this input range

Ambient Occlusion (8)

- Let us now introduce N_L light sources with intensities:

$$I_L(j), j = 1...N_L,$$

at distance d_j from the patch dA & direction of incidence $\hat{\mathbf{l}}_j$

- Assuming Lambertian surfaces, these light sources contribute to the illumination of the patch both in the ambient & the diffuse term
- The resulting illumination for a point **p** of the patch is:

$$I(\mathbf{p}) = [k_a I_a + k_d I_d(\mathbf{p})]w(\mathbf{p}) + I_d(\mathbf{p})$$

$$I_d(\mathbf{p}) = \sum_{j=1}^{N_L} \delta(\mathbf{p}, j) \frac{I_L(j)}{d_j^2} (\hat{\mathbf{l}}_j \cdot \hat{\mathbf{n}}) \qquad (12.75)$$

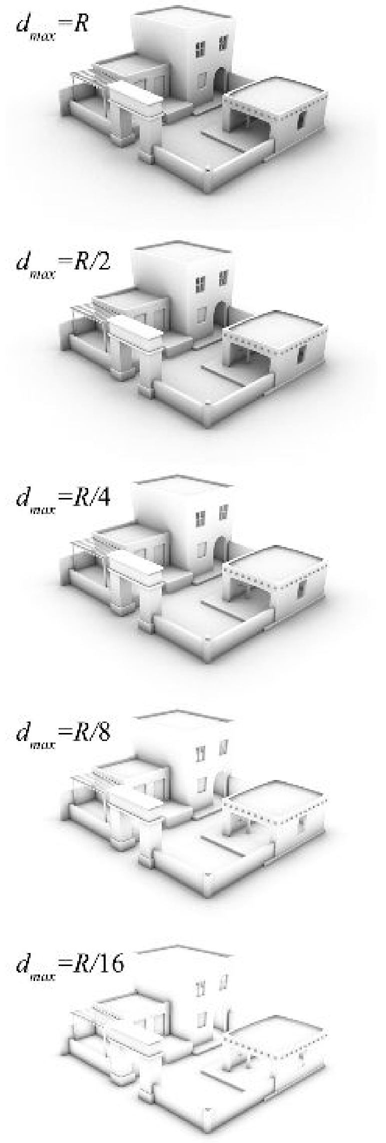
where

δ (**p,** j) is a visibility factor that:

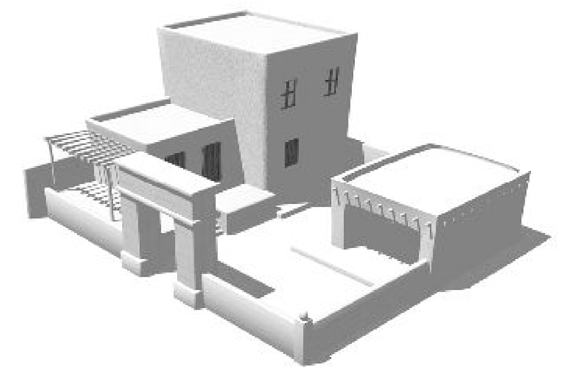
- becomes 1 if the *j*th light source is visible from the patch
- becomes 0 if the patch is in shadow for the specific light source

Ambient Occlusion (9)

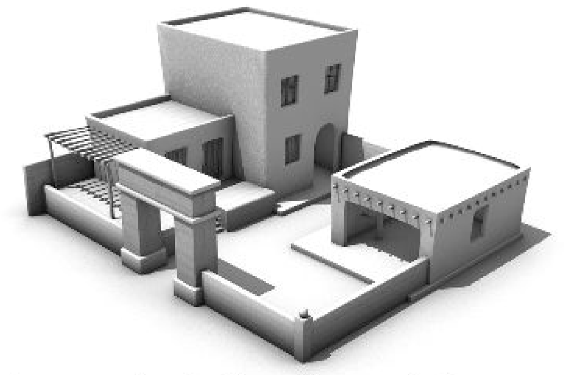
- EXAMPLE
- Obscurance estimation for various values of the distance limit d_{max} (left). R is the scene radius.
- Scene rendered with constant ambient illumination (top right) & with obscurance-weighted ambient-diffuse illumination (bottom right).



Obscurance



Scene rendered with constant ambient lighting



Scene rendered with ambient occlusion ($d_{max}=R/8$)

Shader Source Code - Cook-Torrance

```
//##### Cook-Torrance Model #####//
//##### Vertex program ##########//
varying vec3 N,P;
void main()
{
  gl_Position    =    gl_ModelViewProjectionMatrix    *
  gl_Vertex;
  N = normalize ( gl_NormalMatrix * gl_Normal );
  P = vec3 (gl_Position) / gl_Position.w;
}
```

Shader Source Code - Cook-Torrance (2)

```
//###### Cook-Torrance Model #####//
//###### Fragment program #########//
varying vec3 N, P; const float pi = 3.1415936;
const float e = 2.718282; const int numLights = 2;
uniform float Ka, Kd, Ks,        // ambient, diffuse, specular coefs.
                          m;     // RMS micro-facet slope
uniform vec3 n; // n(630nm) n(530nm) n(465nm) at normal incidence
uniform vec3 color;              // The material color
// The Beckmann distribution function
float Beckmann ( in float a ) {
      float tana = tan(a)/m; float cosa = cos(a); cosa *= cosa;
      return pow ( e, -tana*tana ) / (m*m*cosa*cosa); }
// The Fresnel term
float Fresnel( in float n, in float c ) {
      float g, gc, F; g = clamp ( n*n+c*c-1, 0.000001, 1.0); g = sqrt(g);
      gc = g+c; F = (g-c)*(g-c)/(2*gc*gc);
      return F * ( 1 + (c*gc-1)*(c*gc-1)/( (c*gc+1)*(c*gc+1) ) );
}
```

Shader Source Code - Cook-Torrance (3)

```
// The Cook-Torrance model for the specular reflectance
void CookTorrance (   in vec3 L,     // light direction
                      in vec3 V,     // view direction
                      in vec3 H,     // half-way vector
                      in float a,    // angle ( N, H )
                      in vec3 Il,    // incident illumination
                      in vec3 C0,    // material color
                      out vec3 Is_I  // resulting specular color
                   )
{
float NL, NV, VH, NH;         // dot products
float D, G;                   // D and G scalar terms
vec3 F0, F;                   // The tri-chromatic Fresnel terms
                              // for normal & arbitrary incidence
```

Shader Source Code - Cook-Torrance (4)

```
NL = dot(N,L); NV = dot(N,V); VH = dot(V,H); NH = dot(N,H);
D = Beckmann(a); G = min ( 1, min( 2*NH*NV/VH, 2*NH*NL/VH ) );
F0.r = Fresnel(n.r,1); F0.g = Fresnel(n.g,1); F0.b = Fresnel(n.b,1);
F.r = Fresnel(n.r,VH); F.g = Fresnel(n.g,VH); F.b = Fresnel(n.b,VH);
Is_i = (C0+(Il-C0)*(max(F-F0,0)/(1.0-F0)))*((F.r+F.g+F.b)/3) *D*G/
       pi*NL*NV);
}

void main() {
     vec3 Pl;                       // Light position
     vec3 L, H, V;                  // directions (unit vectors)
     vec3 Ia, Id, Is, Is_i, Il;     // Intensity values
     int i;
     float NL, a;
     V = vec3 (0.0, 0.0, 1.0);      // View direction
```

Shader Source Code - Cook-Torrance (5)

```
Ia = vec3 (0.0, 0.0, 0.0);          // Init. amb/dif/spec values
Id = vec3 (0.0, 0.0, 0.0);  Is = vec3 (0.0, 0.0, 0.0);
// Add the contribution of all light sources
for ( i = 0; i< numLights; i++ ) {
   Pl = vec3 (gl_LightSource[i].position); L = normalize( Pl - P );
   H = normalize( L + V ); NL = dot (N,L);
   // Diffuse
Id += gl_LightSource[i].diffuse * NL; a = acos( dot(N,H) );
   Il = vec3 (gl_LightSource[i].diffuse);
   CookTorrance ( L, V, H, a, Il, color, Is_i );
   // Specular
   Is += Is_i; }
// Ambient
Ia = Ka * gl_FrontLightModelProduct.sceneColor;
gl_FragColor = vec4(Ia,1)+Kd *vec4(Id,1)*vec4(color,1)+Ks*vec4(Is,1);
}
```

Shader Source Code - Strauss

```
//###### Strauss Model ######//
//###### Vertex program #####//
varying vec3 N; varying vec3 P;
void main() {
   gl_Position = gl_ModelViewProjectionMatrix * gl_Vertex;
   N = normalize ( gl_NormalMatrix * gl_Normal );
   P = vec3 (gl_Position) / gl_Position.w;
}
```

Shader Source Code - Strauss (2)

```
//###### Strauss Model #####//
//###### Fragment program ##//
varying vec3 N; varying vec3 P; const float pi = 3.1415936;
const int numLights = 2; uniform float m;       // metalness uniform
float s; // shininess uniform float t; //transparency
uniform vec3 C; // surface color
//----- Fresnel term -----
float F ( in float x ) {
   const float kf = 1.12f; const float kf2 = kf*kf;
   const float denom = ( 1.0/((1.0-kf)*(1.0-kf)) - 1.0/kf2 );
   return ( ( 1.0/((x-kf)*(x-kf)) - 1.0/kf2 ) / denom);
}
//----- Geometric Attenuation-----
float G ( in float x ) {
   const float kg = 1.01f; const float kg2 = kg*kg;
   const float denom = ( 1.0/((1.0-kg)*(1.0-kg)) - 1.0/kg2 );
   return ( 1.0/((1.0-kg)*(1.0-kg)) - 1.0/((x-kg)*(x-kg)) )/ denom;
}
```

Shader Source Code - Strauss (3)

```
void main() {
   vec3 Pl, L, V, H; vec3 Qa, Qd, Qs, Ir, Cs; int I; float NL, NV, f;
   float theta_i, theta_r; float rn, rj, rd, rs, d; const float kj = 0.1;
   // Note that conventions in the original paper differ from standard
   // normalized vector definitions: L & V face towards the local point P
   // View direction
   V = -normalize(P); NV = dot(N,V); Ir = vec3 (0.0, 0.0, 0.0);
   for ( i = 0; i< numLights; i++ ) {
      Pl = vec3 (gl_LightSource[i].position); L = normalize( P - Pl );
      NL = dot(N,L); H =normalize( L-2*NL*N ); theta_i = 2*acos(abs(NL))/pi;
      theta_r = 2*acos(abs(NV))/pi; rd = (1-s*s*s)*(1-t); d = 1-m*s;
      rn = 1-t-rd; f = F((theta_i+theta_r)/2);
      rj = min (1, rn+(rn+kj)*f*G(theta_i)*G(theta_r));
      rs = pow(-dot(H,V),3/(1.0001-s))*rj; Cs = 1 + m*(1-f)*(C-1);
      Qd = clamp (-NL*d*rd*C,0,1); Qs = clamp (rs*Cs,0,1);
      Ir += gl_LightSource[i].diffuse * (Qd+Qs) +gl_LightSource[i].ambient * Qa;}
   gl_FragColor = vec4(Ir,1-t);
}
```