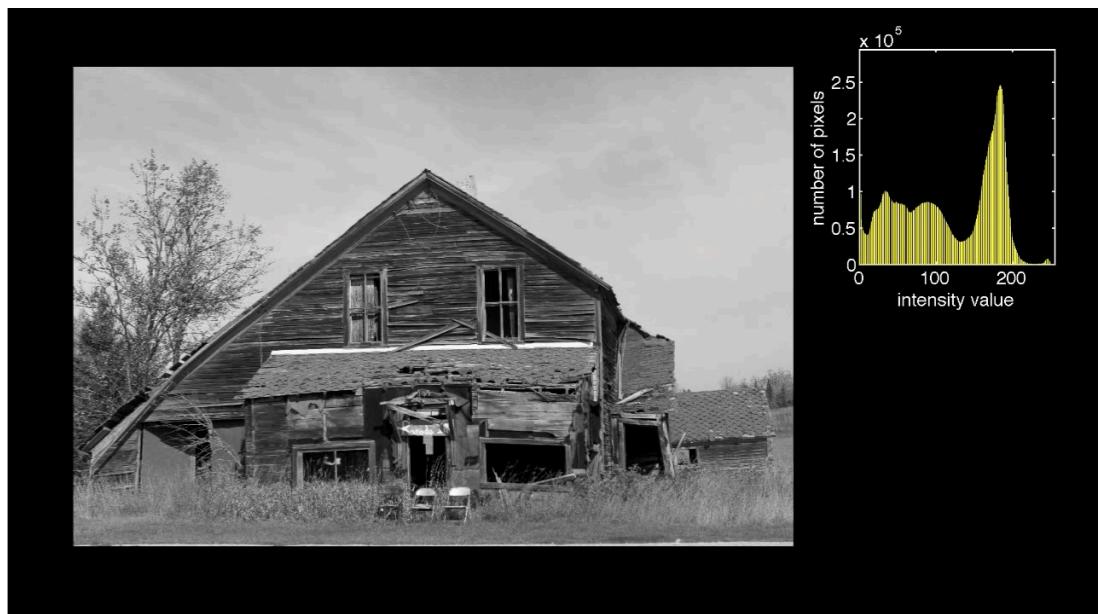
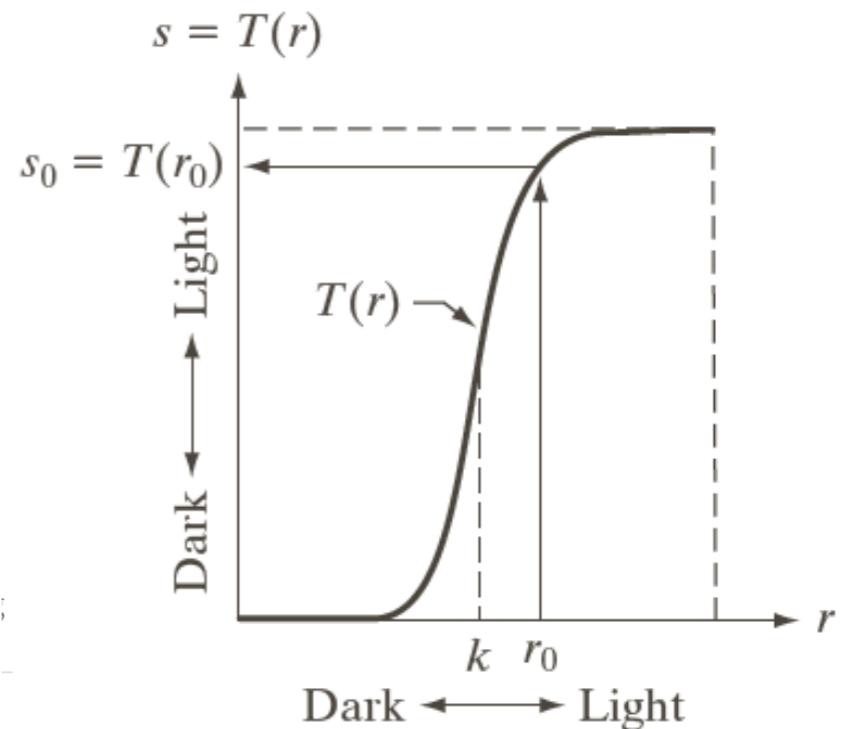


Spatial Neighborhood Filtering

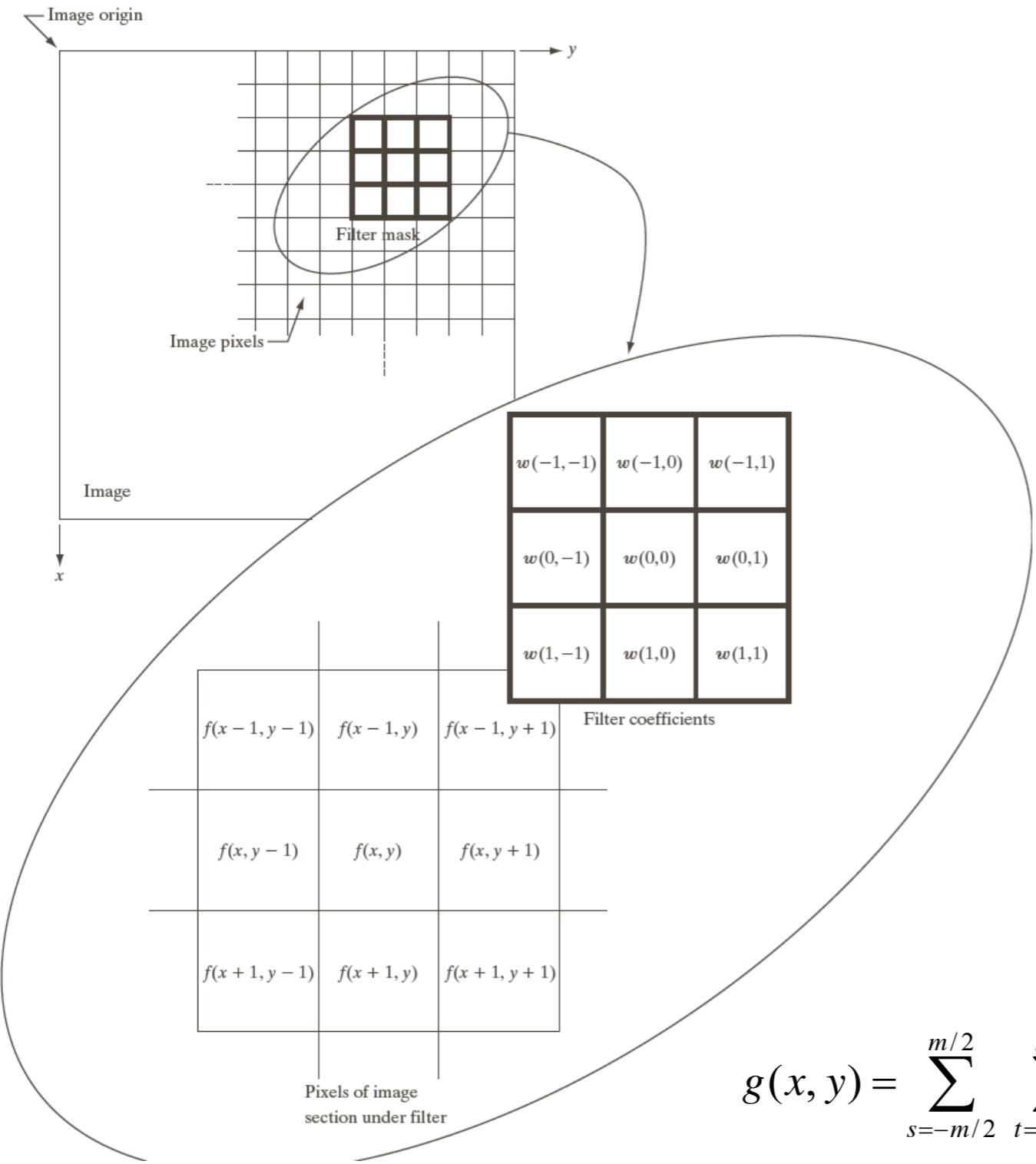
for Smoothing and Sharpening

Previously..

- Intensity transformations
(point processing)
- Histogram-based methods
(histogram equalization)



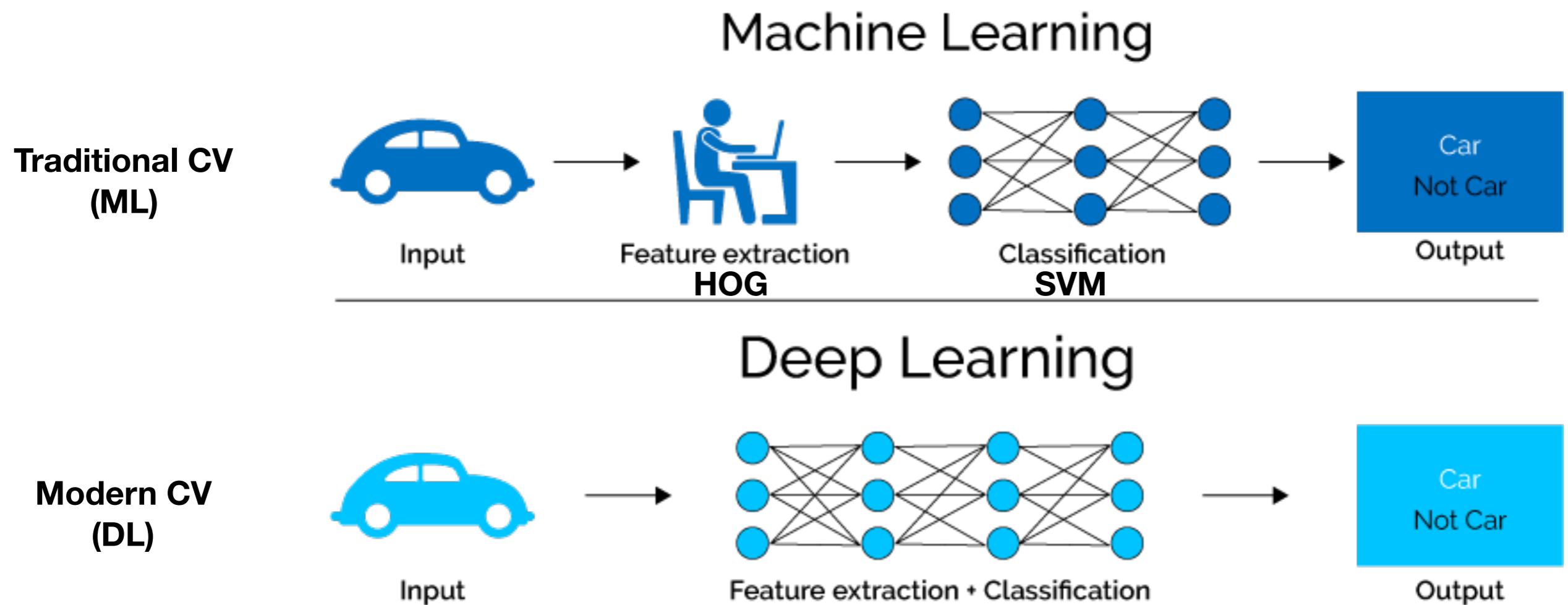
Today..



- Neighborhood processing /filtering:
 - Smoothing
 - Sharpening

$$g(x, y) = \sum_{s=-m/2}^{m/2} \sum_{t=-n/2}^{n/2} w(s, t) f(x+s, y+t)$$

Old vs. New approach..



Smoothing

Image Enhancement in the Spatial Domain

Gaussian noise

```
noise =  
    randn(size(im)).*sigma;  
output = im + noise;
```

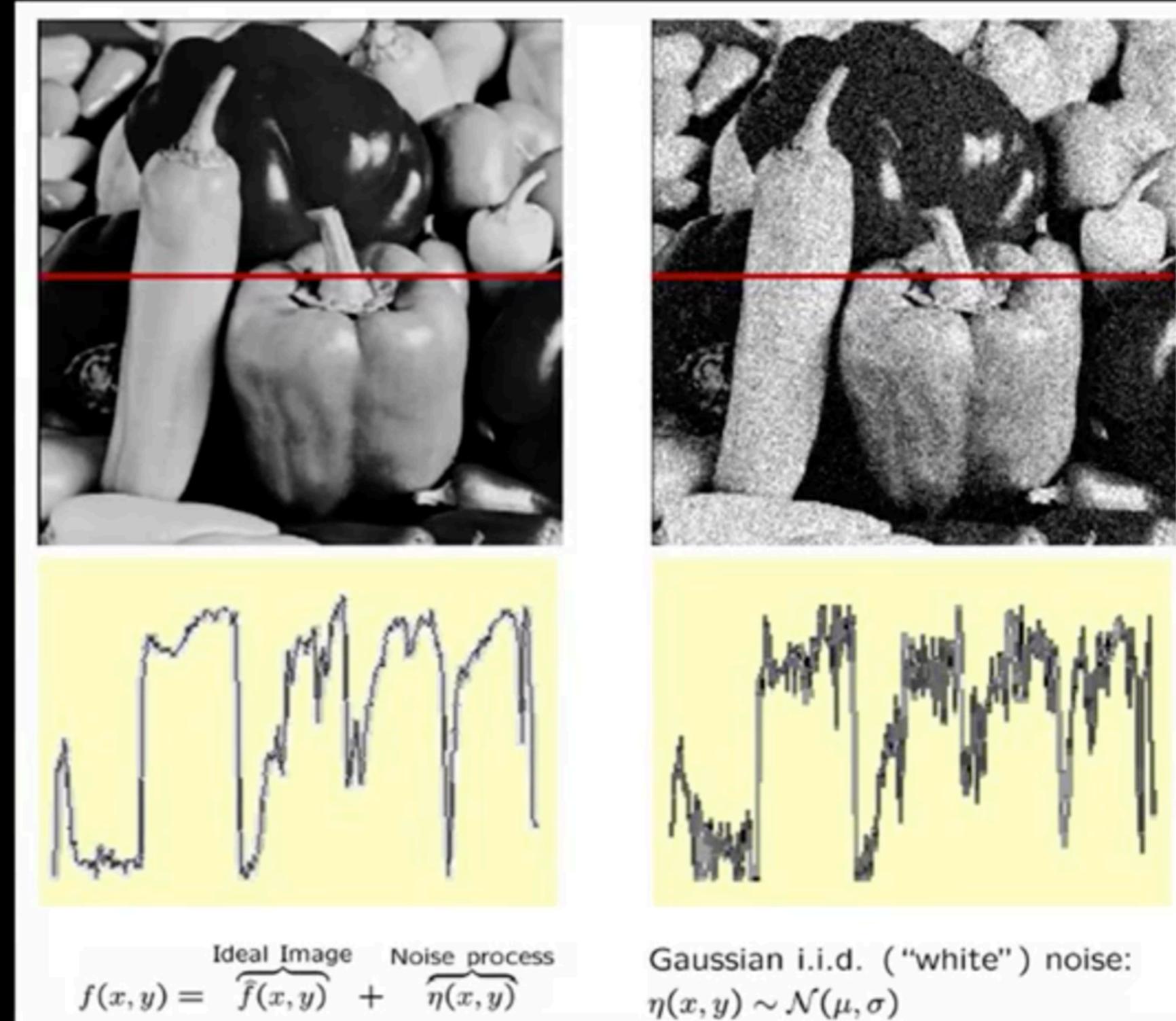


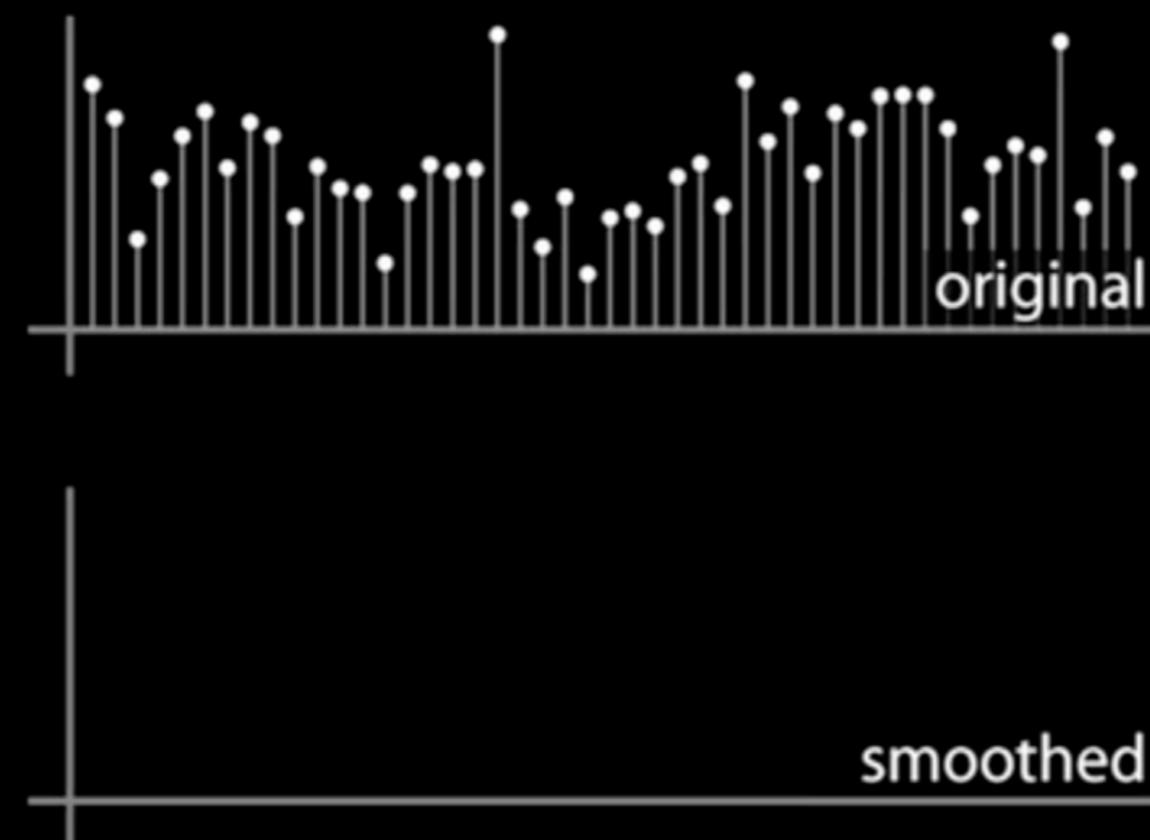
Fig: M. Hebert

$$f(x, y) = \widehat{f(x, y)} + \widehat{\eta(x, y)}$$

Gaussian i.i.d. ("white") noise:
 $\eta(x, y) \sim \mathcal{N}(\mu, \sigma)$

First attempt at a solution – 1D

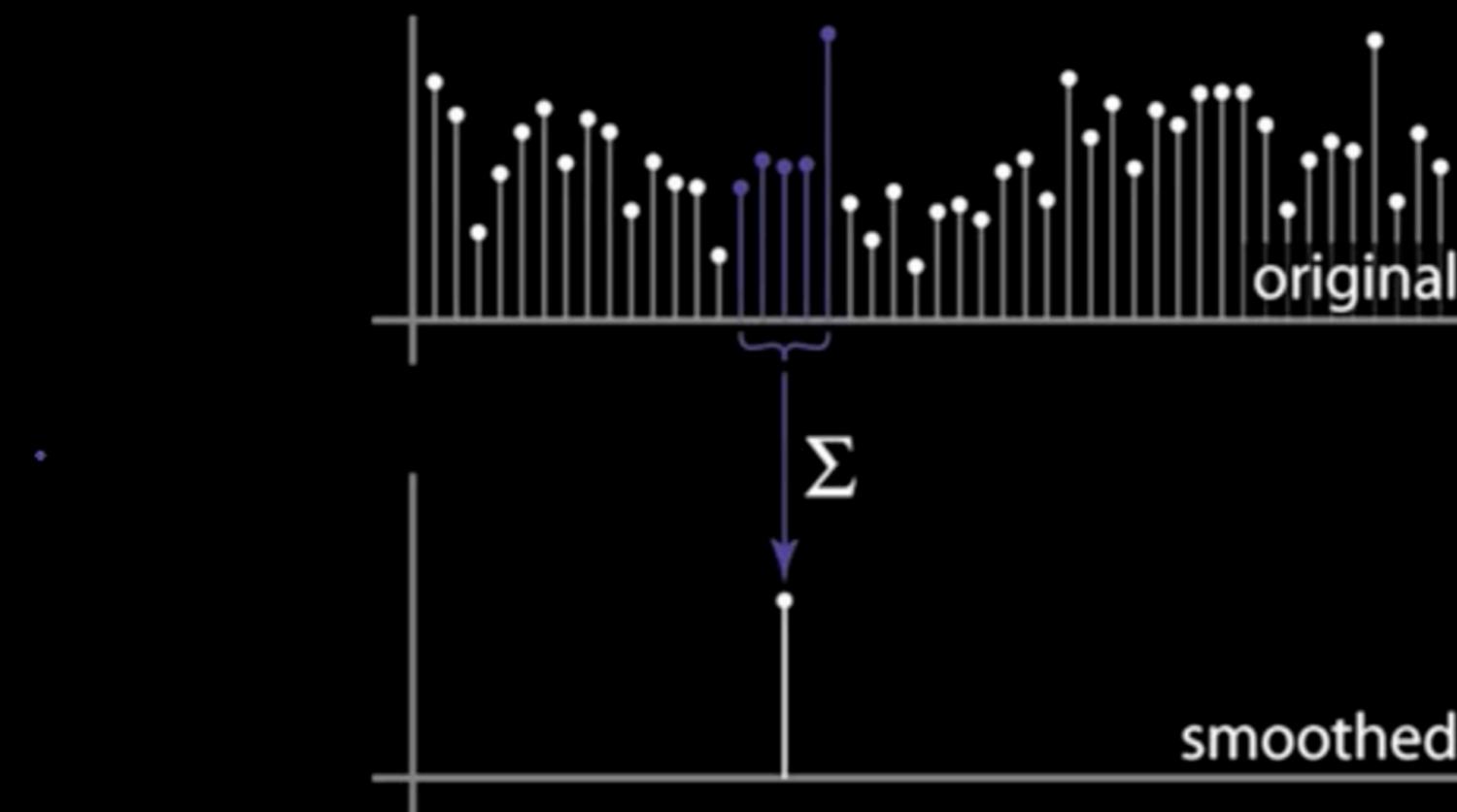
Replace each pixel with an average of all the values in its neighborhood – a *moving average*:



Source: S. Marschner

First attempt at a solution – 1D

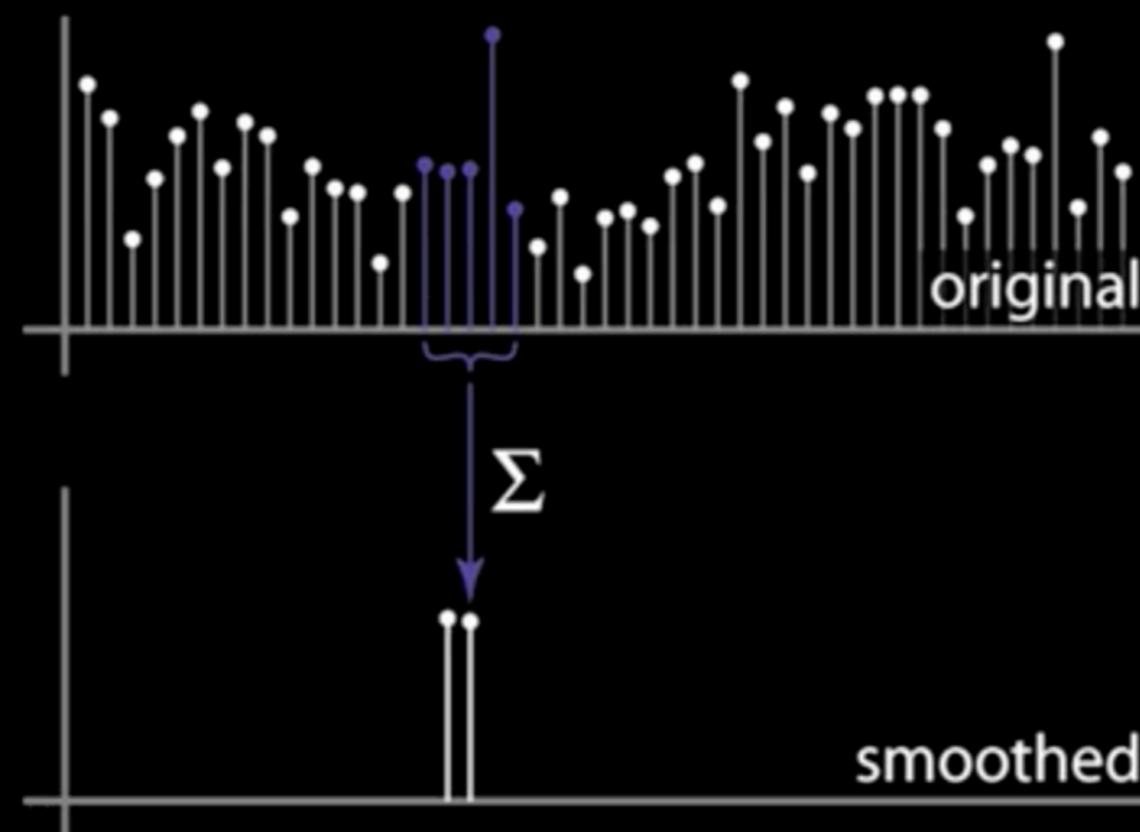
Replace each pixel with an average of all the values in its neighborhood – a *moving average*:



Source: S. Marschner

First attempt at a solution – 1D

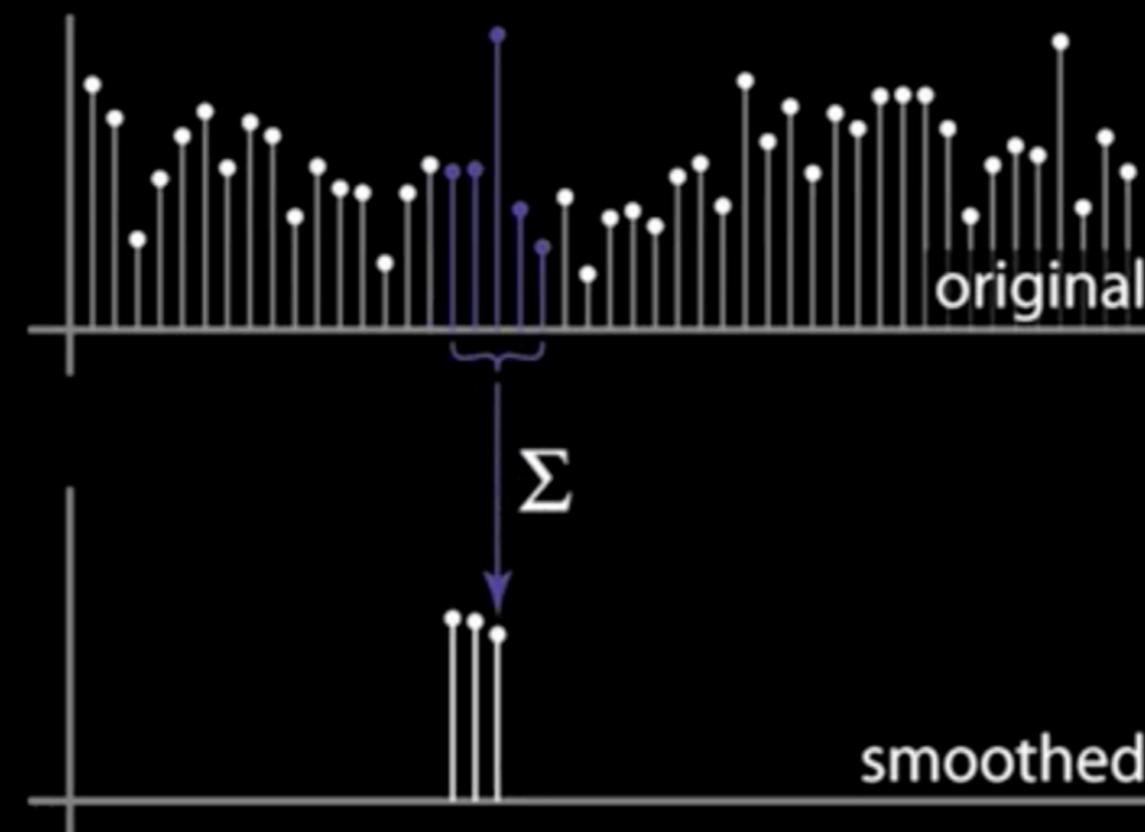
Replace each pixel with an average of all the values in its neighborhood – a *moving average*:



Source: S. Marschner

First attempt at a solution – 1D

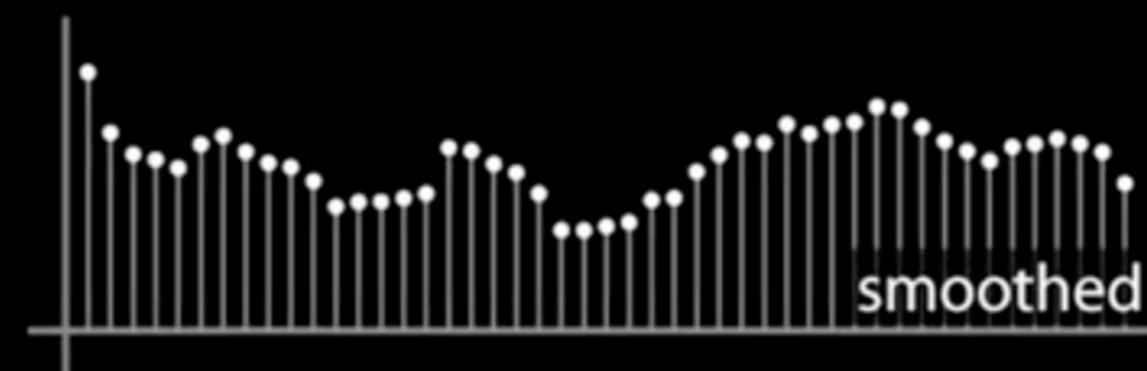
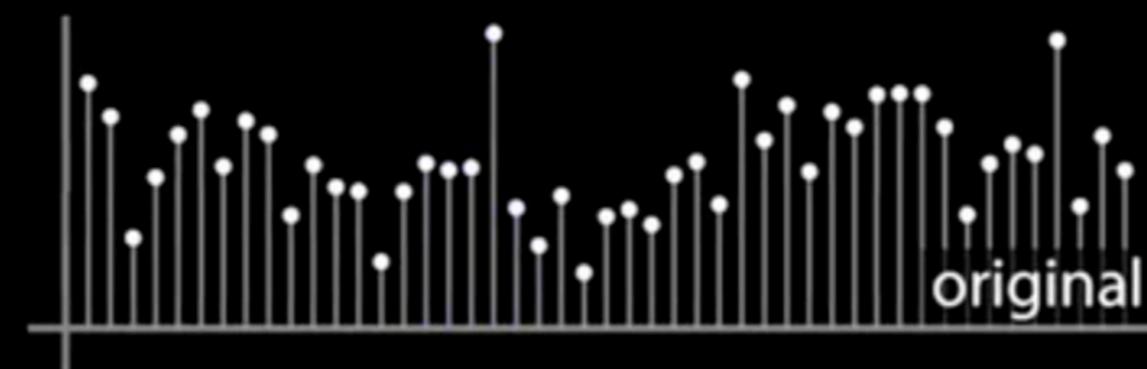
Replace each pixel with an average of all the values in its neighborhood – a *moving average*:



Source: S. Marschner

First attempt at a solution – 1D

Replace each pixel with an average of all the values in its neighborhood – a *moving average*:



Source: S. Marschner

Averaging assumptions

1. The “true” value of pixels are similar to the true value of pixels nearby.
2. The noise added to each pixel is done independently.

Q&A 01

Quiz

If noise is just a function added to an images, we could remove the noise by subtracting the noise again - that is the operation is reversible:

- a) True
- b) True but we don't know the noise function so we can't actually do the subtraction.
- c) False. Additive noise destroys information in the image and there is no way to recover it.

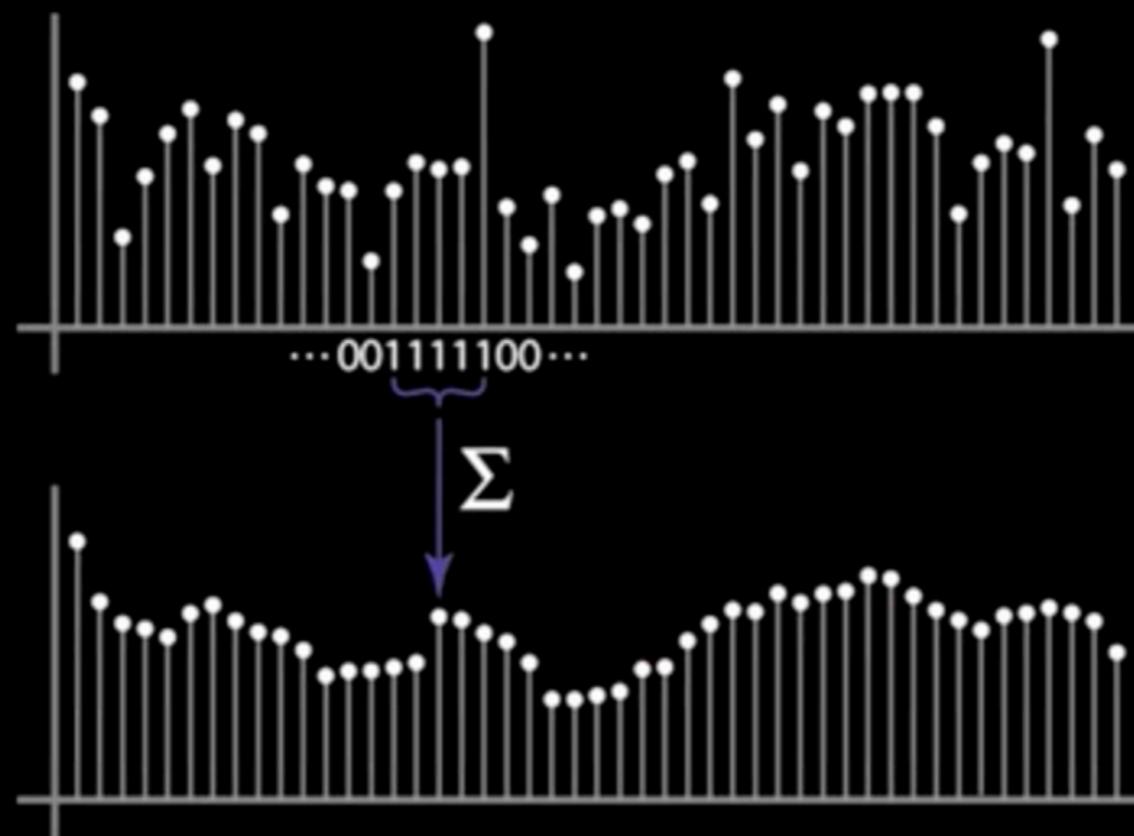
Quiz

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- a) True
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Weighted Moving Average

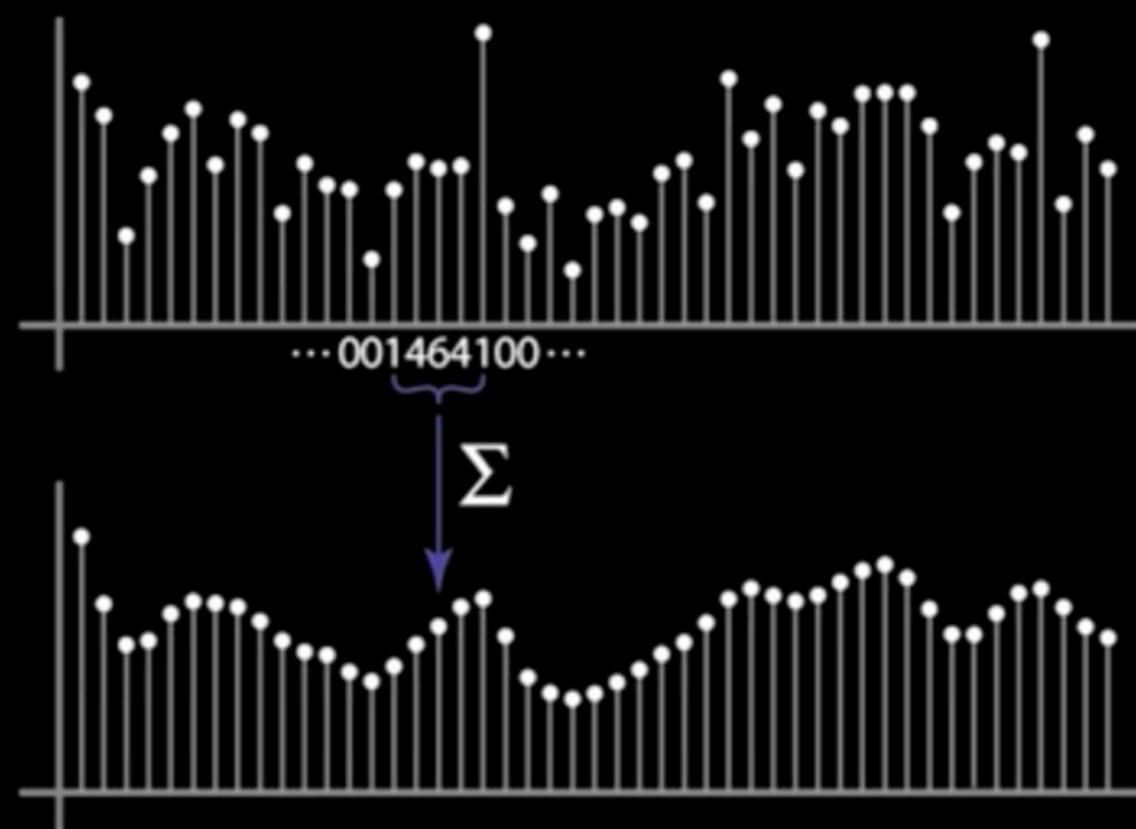
- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Source: S. Marschner

Weighted Moving Average

- Non-uniform weights $[1, 4, 6, 4, 1] / 16$



Source: S. Marschner

Q&A 02

Quiz

To do the moving average computation the number of weights should be

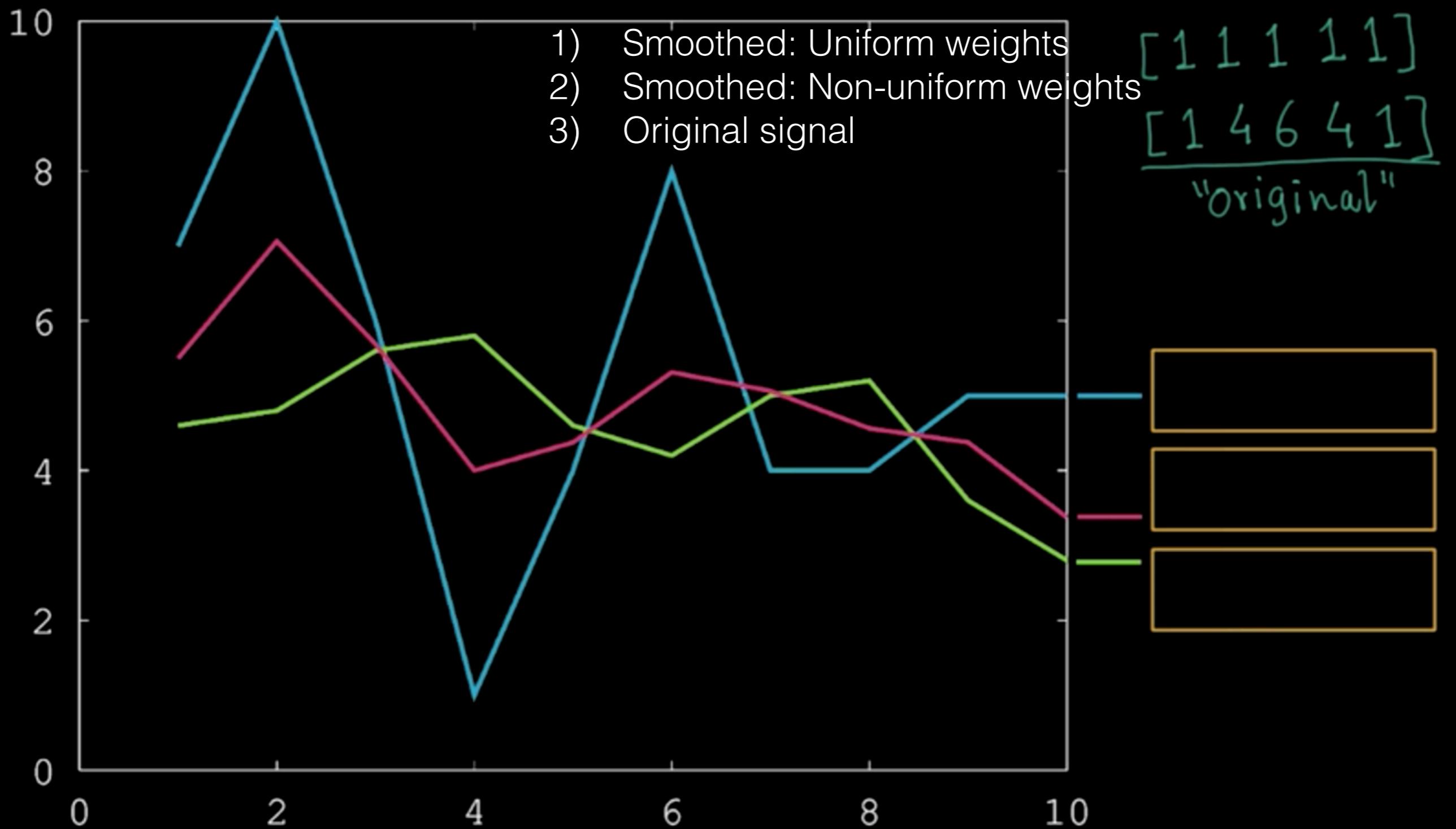
- a) Odd – makes it easier to have a middle pixel
- b) Even – that way the filter can be exactly symmetric around a pixel.
- c) Either even or odd.

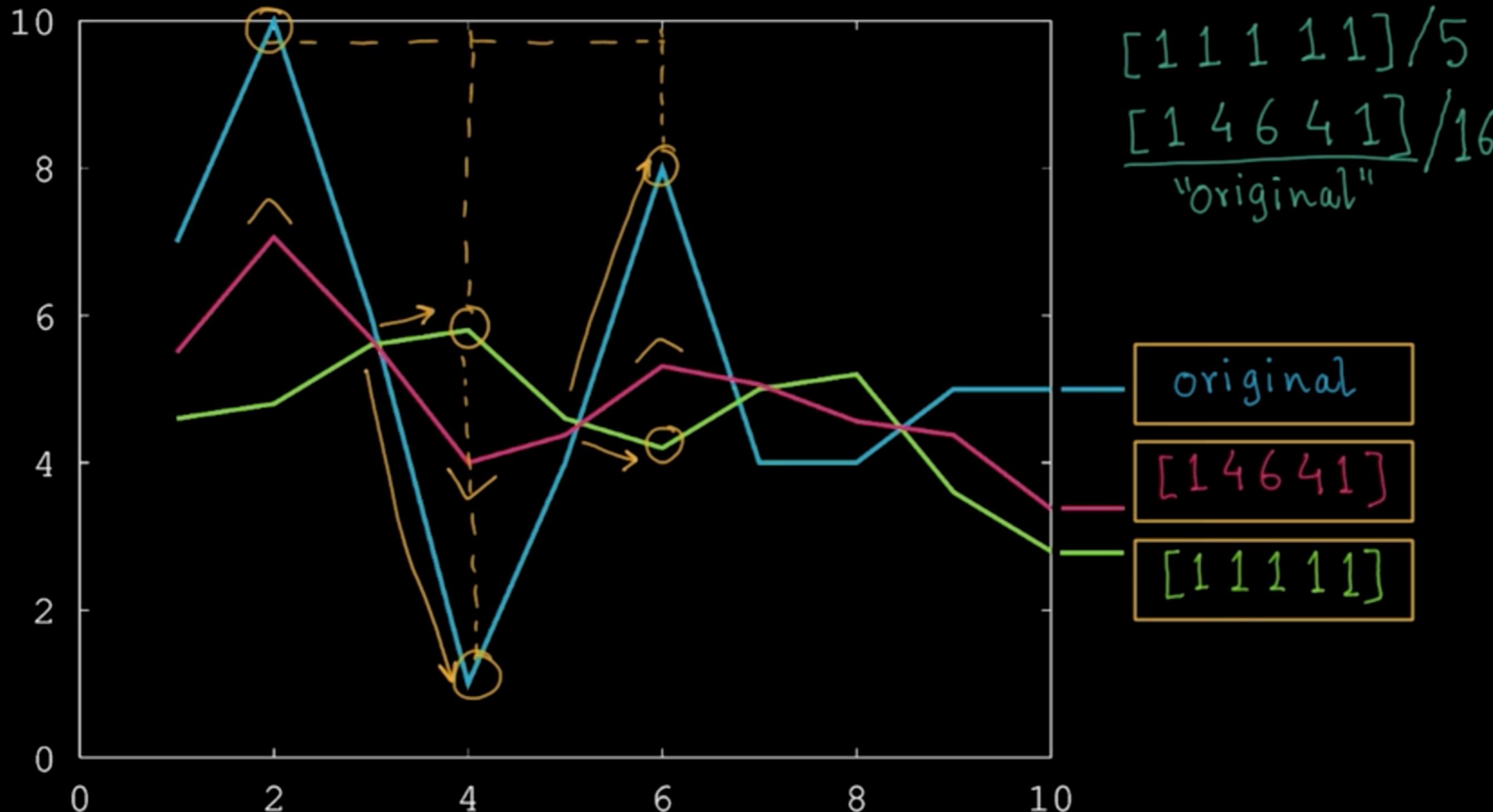
Quiz

To do the moving average computation the number of weights should be

- a) Odd – makes it easier to have a middle pixel
- b) Even – that way the filter can be exactly symmetric around a pixel.
- c) Either even or odd.

Q&A 03





Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0									

NB: The filter / mask / kernel is a 3x3 matrix with «1 divided by 9» weights

Source: S. Seitz

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0	10									

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0	10	20							

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

	0	10	20	30					

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0	10	20	30	30

Q&A 04

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0	10	20	30	30	30	20	10		
0	20	40	60	60	60	40	20		
0	30	60	90	90	60	30			
0	30	50	80	80	90	60	30		
0	30	50	80	80	90	60	30		
0	20	30	50	50	60	40	20		
10	20	30	30	30	30	20	10		
10	10	10	0	0	0	0	0		

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0	10	20	30	30	30	20	10		
0	20	40	60	60	60	40	20		
0	30	60	90	90	90	60	30		
0	30	50	80	80	90	60	30		
0	30	50	80	80	90	60	30		
0	20	30	50	50	60	40	20		
10	20	30	30	30	30	20	10		
10	10	10	0	0	0	0	0		

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

0	10	20	30	30	30	20	10	
0	20	40	60	60	60	40	20	
0	30	60	90	90	90	60	30	
0	30	50	80	80	90	60	30	
0	30	50	80	80	90	60	30	
0	20	30	50	50	60	40	20	
10	20	30	30	30	30	20	10	
10	10	10	0	0	0	0	0	

Correlation filtering – uniform weights

Say the averaging window size is $2k+1 \times 2k+1$:

$$G[i, j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k F[i+u, j+v]$$

Correlation filtering – uniform weights

Say the averaging window size is $2k+1 \times 2k+1$:

$$G[i, j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k F[i+u, j+v]$$

*Uniform
weight for
each pixel*

*Loop over all pixels in
neighborhood around
image pixel $F[i,j]$*

Correlation filtering – nonuniform weights

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

Non-uniform weights

This is called **cross-correlation**, denoted $G = H \otimes F$

$k=1$

Indices
for H :

-1,-1	-1,0	-1,1
1,-1		1,1

Indices
for F :

i-1, j-1	i-1,j	i-1, j+1		
	i,j			
i+1, j-1		i+1,j+1		

Averaging filter

$$F(x, y) \otimes H(u, v) = G(x, y)$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$\frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

“box filter”

0	10	20	30	30	30	20	10
0	20	40	60	60	60	40	20
0	30	60	90	90	90	60	30
0	30	50	80	80	90	60	30
0	30	50	80	80	90	60	30
0	20	30	50	50	60	40	20
10	20	30	30	30	30	20	10
10	10	10	0	0	0	0	0

$$G = F \otimes H$$

Smoothing by box averaging



original

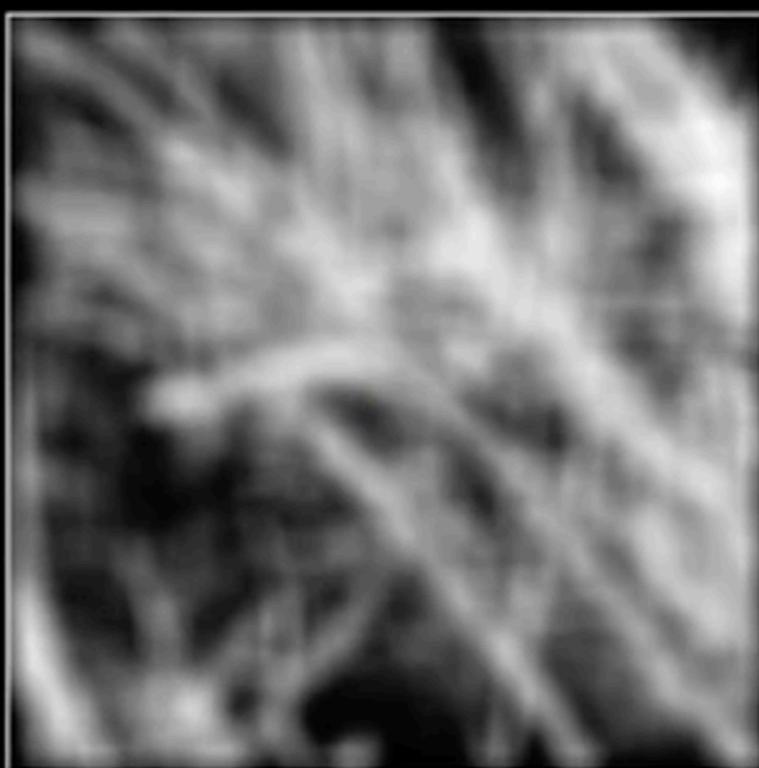


?

Smoothing by box averaging

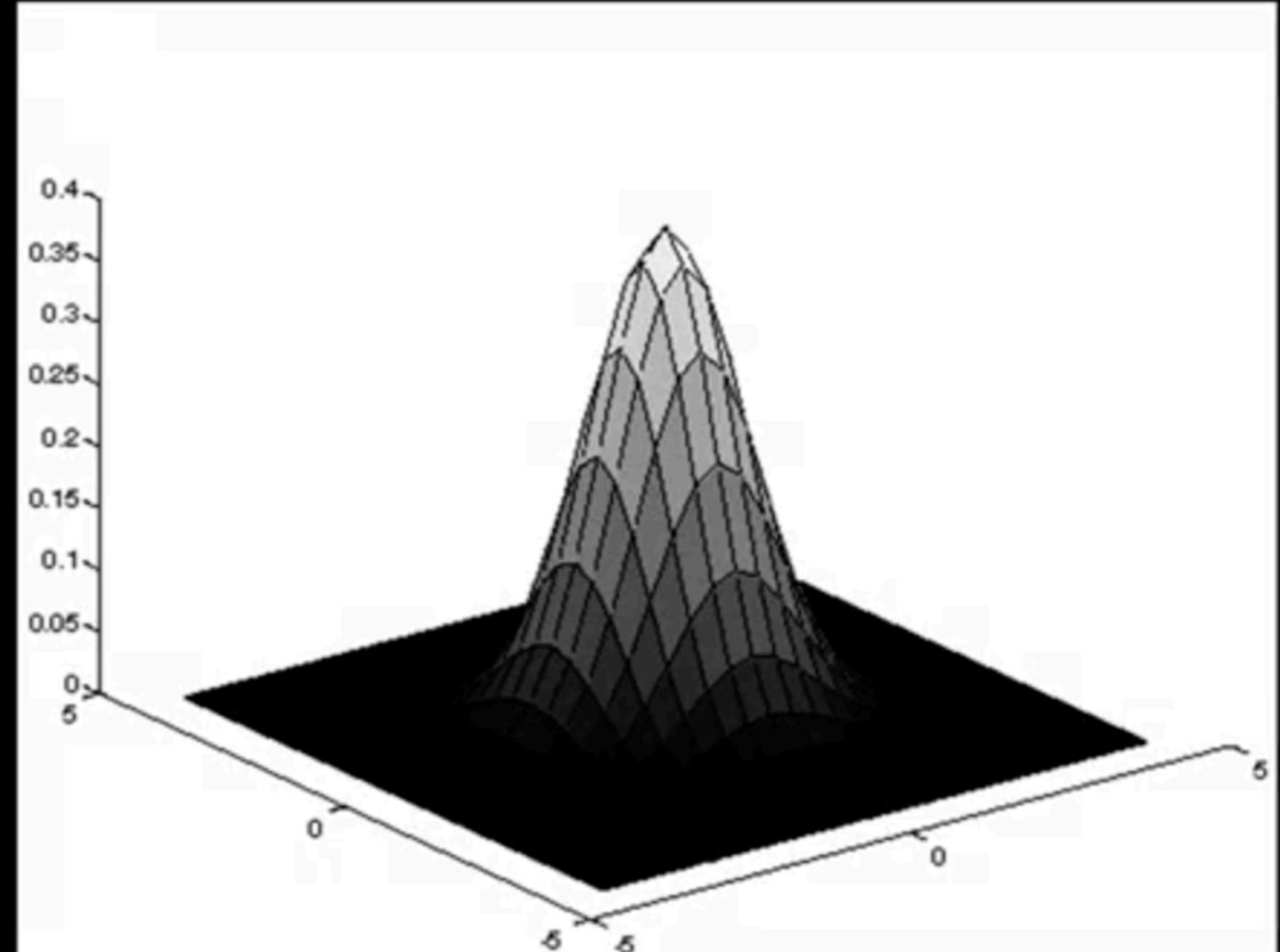
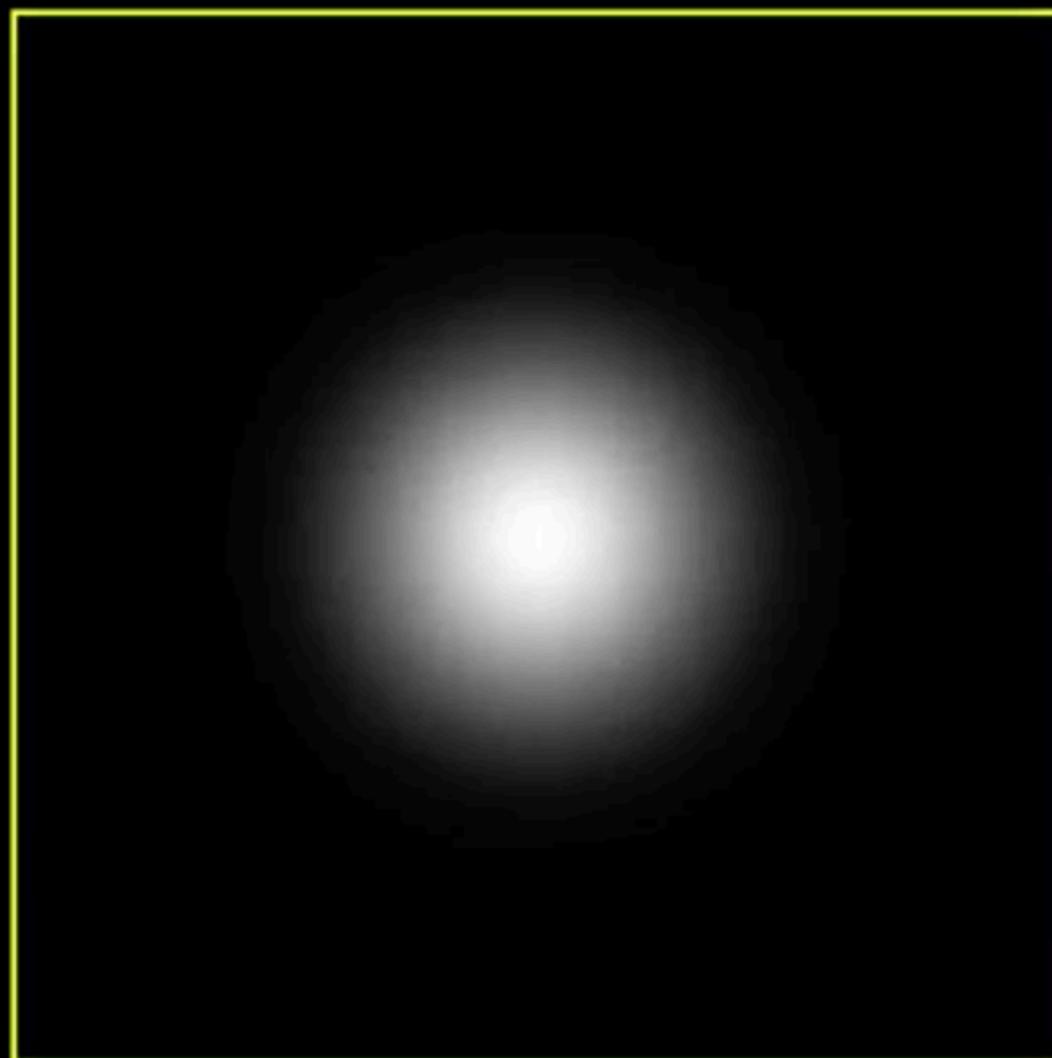


original



filtered

Blurry spot as a function



D. Forsyth

Q&A 05

Quiz

To blur a single pixel into a “blurry” spot, we would need to filter the spot with a

- a) 3x3 square of uniform weights
- b) 11x11 square of uniform weights would be better since it's bigger
- c) Something that looks like a blurry spot – higher values in the middle, falling off to the edges.

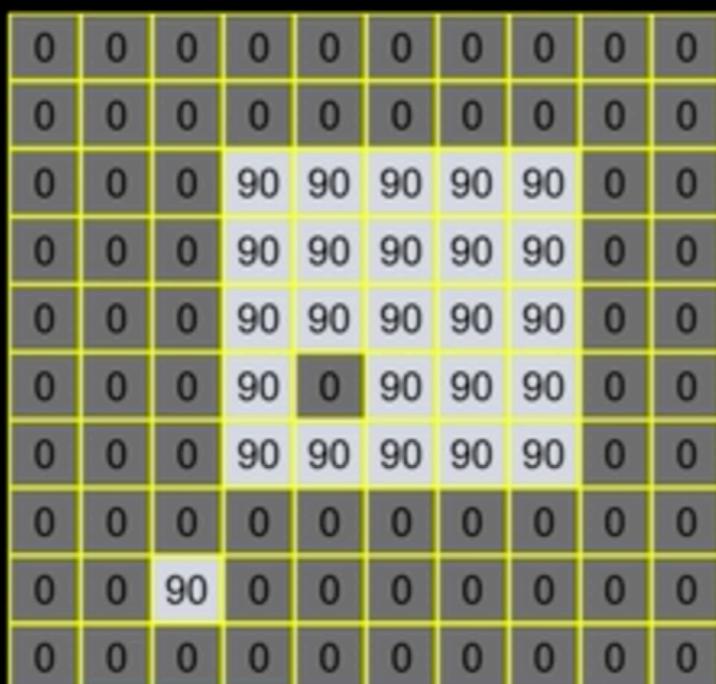
Quiz

To blur a single pixel into a “blurry” spot, we would need to filter the spot with a

- a) 3x3 square of uniform weights
- b) 11x11 square of uniform weights would be better since it's bigger
- c) Something that looks like a blurry spot – higher values in the middle, falling off to the edges.

Gaussian filter

- Nearest neighboring pixels have the most influence

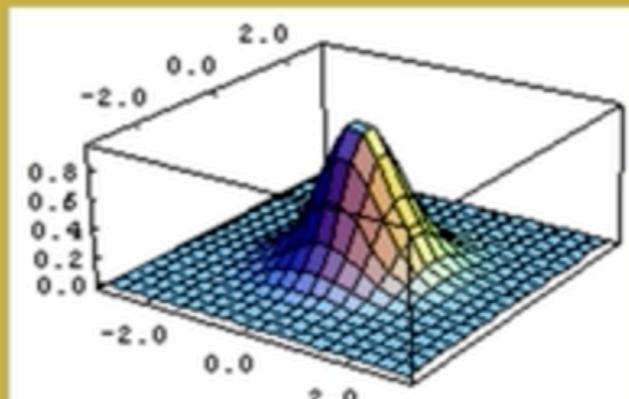


$$F(x, y)$$

$$\frac{1}{16} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$
$$H(u, v)$$

This kernel is an approximation of a Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

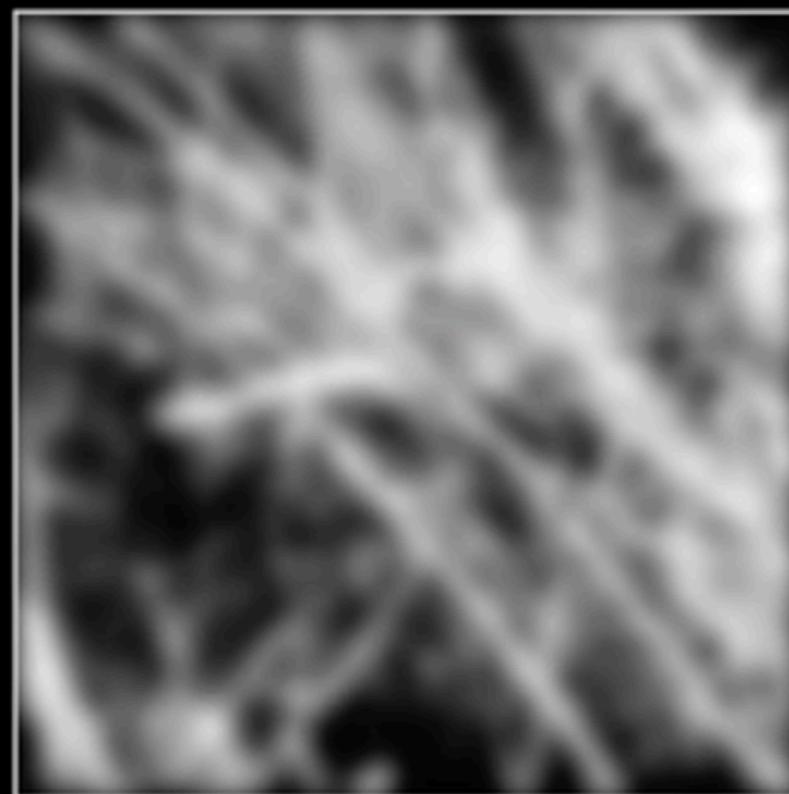


Source: S. Seitz

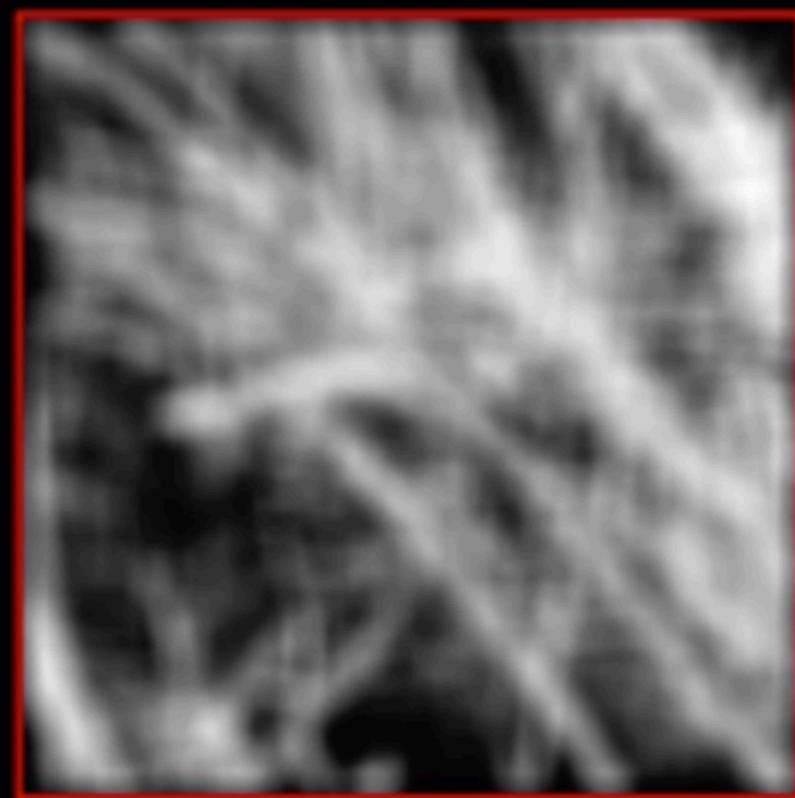
Smoothing with a Gaussian



Smoothing with a Gaussian



Smoothing with not a Gaussian



Q&A 06

Normalization constant

Quiz

Gaussian filters are referred to as exponentials. The complete formula is:

- a) is
$$h(u,v) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$
.
- b) Or
$$h(u,v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$
- c) Or even:
$$h(u,v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

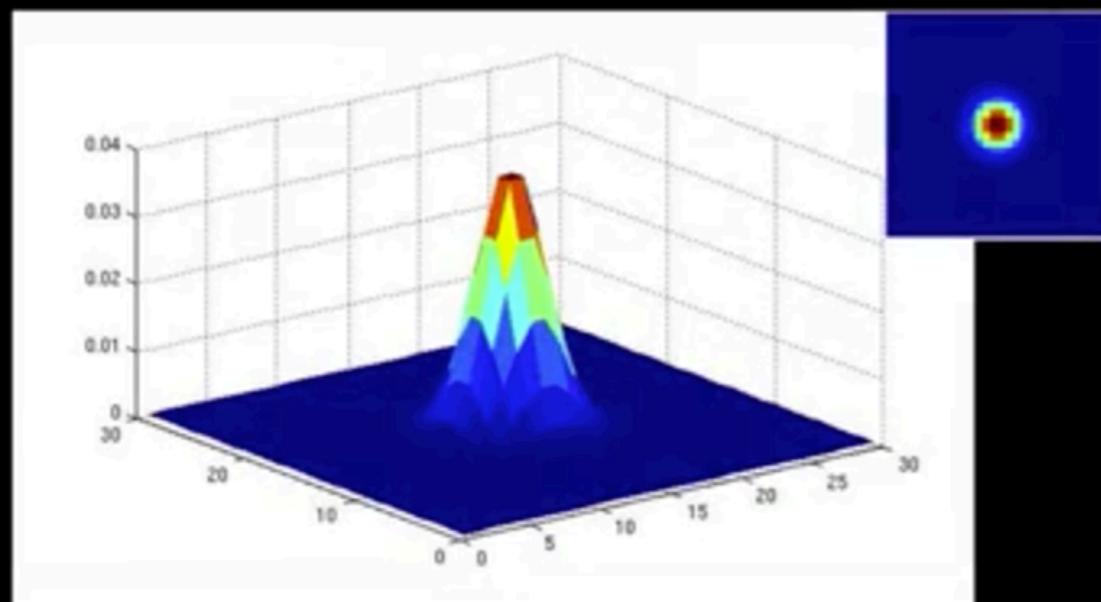
Quiz

Gaussian filters are referred to as exponentials. The complete formula is:

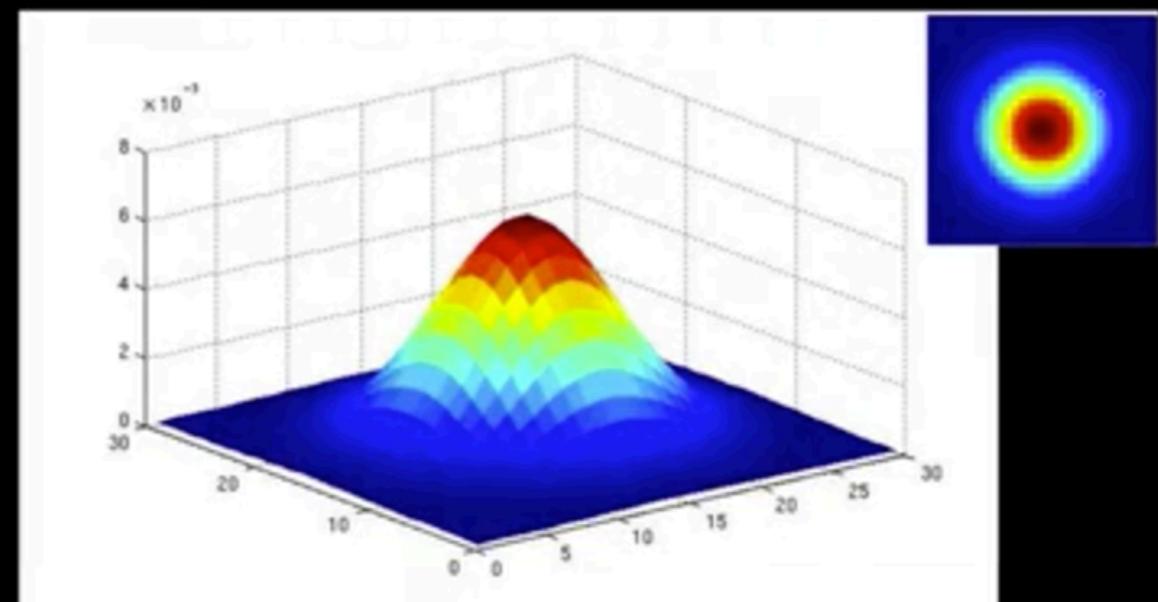
- a) is
$$h(u,v) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$
- b) Or
$$h(u,v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$
- c) Or even:
$$h(u,v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

Gaussian filters

Variance (σ^2) or standard deviation (σ) –
determines extent of smoothing



$\sigma = 2$ with 30×30 kernel



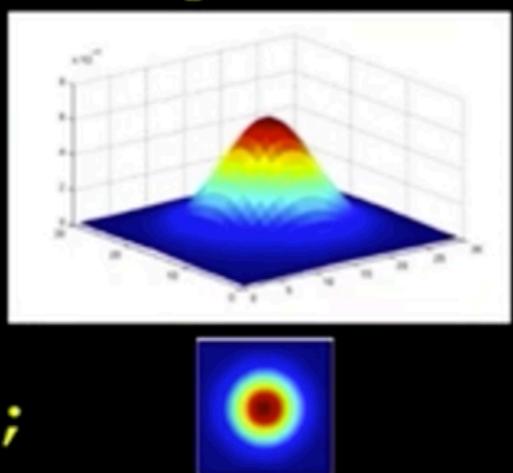
$\sigma = 5$ with 30×30 kernel

K. Grauman

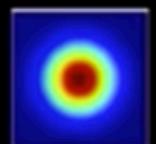
Matlab

```
>> hsize = 31;  
>> sigma = 5;  
>> h = fspecial('gaussian', hsize, sigma);
```

```
>> surf(h);
```



```
>> imagesc(h);
```



```
>> outim = imfilter(im, h);  
>> imshow(outim);
```

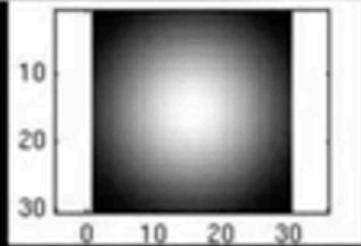
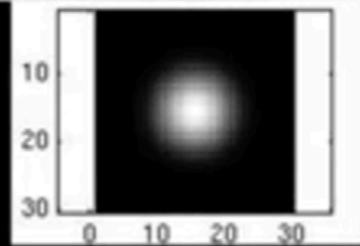
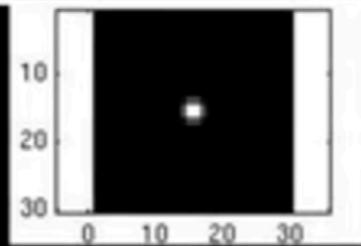


im

K. Grauman

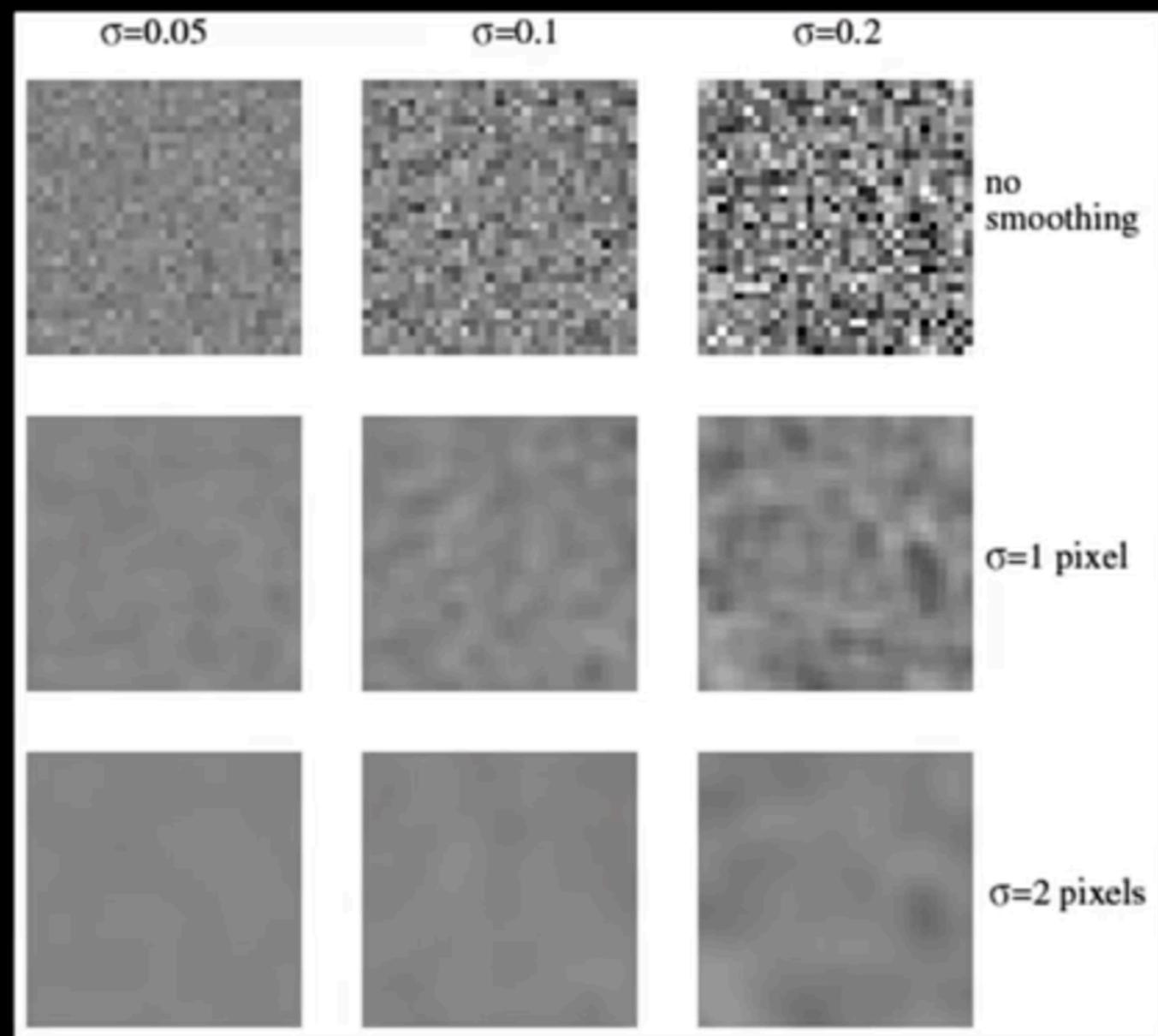
Smoothing with a Gaussian

```
for sigma=1:3:10  
    h = fspecial('gaussian',  
    fsize, sigma);  
    out = imfilter(im, h);  
    imshow(out);  
    pause;  
end
```



Keeping the two Gaussians straight...

More Gaussian noise (like earlier) $\sigma \rightarrow$



Wider Gaussian smoothing kernel $\sigma \rightarrow$

Q&A 07

Gaussian:
effect of sigma, size and normalization coefficient

Quiz

When filtering with a Gaussian, which is true:

- a) The sigma is most important – it defines the blur kernel's scale with respect to the image.
- b) The kernel size is most important – because it defines the scale.
- c) Altering the normalization coefficient does not effect the blur, only the brightness.
- d) A and C.

Quiz

When filtering with a Gaussian, which is true:

- a) The sigma is most important – it defines the blur kernel's scale with respect to the image.
- b) The kernel size is most important – because it defines the scale.
- c) Altering the normalization coefficient does not effect the blur, only the brightness.
- d) A and C.

Linearity and convolution

And now some linear intuition...

- An operator H (or system) is linear if two properties hold (f_1 and f_2 are some functions, a is a constant):
- Additivity (things sum):
 - $H(f_1 + f_2) = H(f_1) + H(f_2)$ (looks like distributive law)
- Multiplicative scaling (Homogeneity of degree 1) (constant scales):
 - $H(a \cdot f_1) = a \cdot H(f_1)$

And now some linear intuition...

- An operator H (or system) is linear if two properties hold (f_1 and f_2 are some functions, a is a constant):
 - Additivity (things sum):
 - $H(f_1 + f_2) = H(f_1) + H(f_2)$ (looks like distributive law)
 - Multiplicative scaling (Homogeneity of degree 1) (constant scales):
 - $H(a \cdot f_1) = a \cdot H(f_1)$

Because it is sums and multiplies, the “filtering” operation we were doing are linear.

Q&A 08

Quiz

Which of these operators are not linear:

- a) Sum
- b) Max
- c) Average
- d) Square root
- e) (b) and (d)

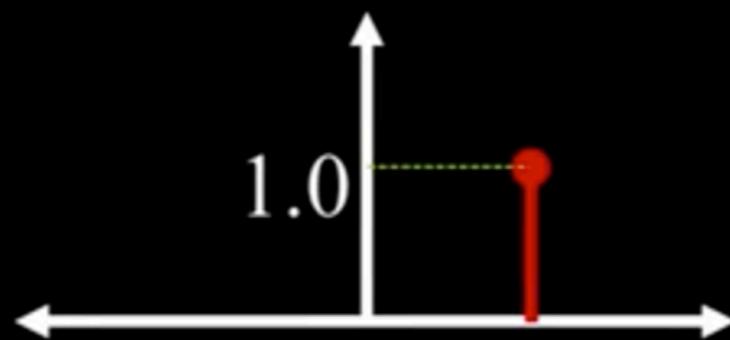
Quiz

Which of these operators are not linear:

- a) Sum
- b) Max
- c) Average
- d) Square root
- e) (b) and (d)

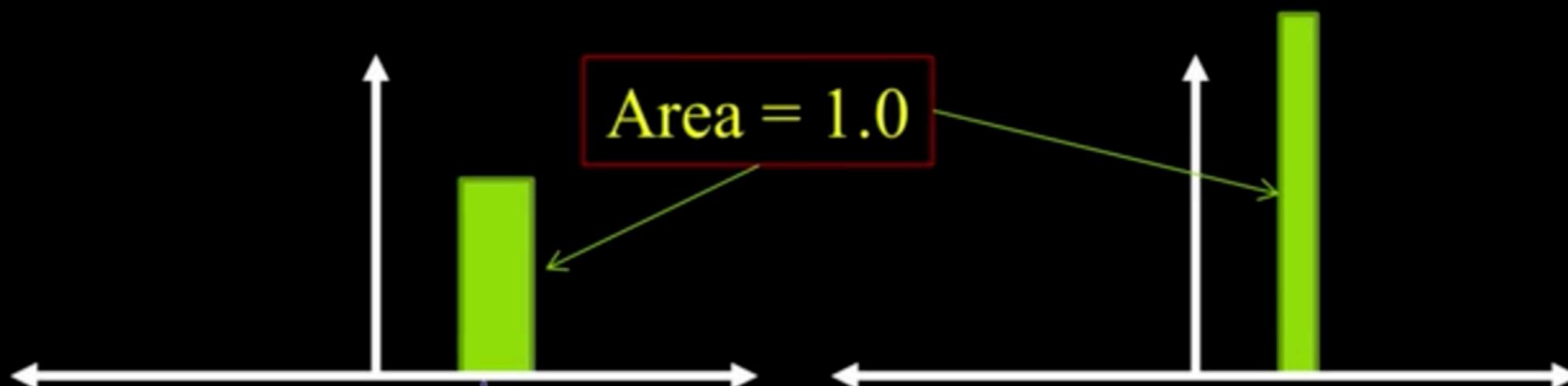
An impulse function...

- In the discrete world, an *impulse* is a very easy signal to understand: it's just a value of 1 at a single location.



An impulse function...

- In the continuous world, an *impulse* is an idealized function that is very narrow and very tall so that it has a unit area. In the limit:



An impulse response

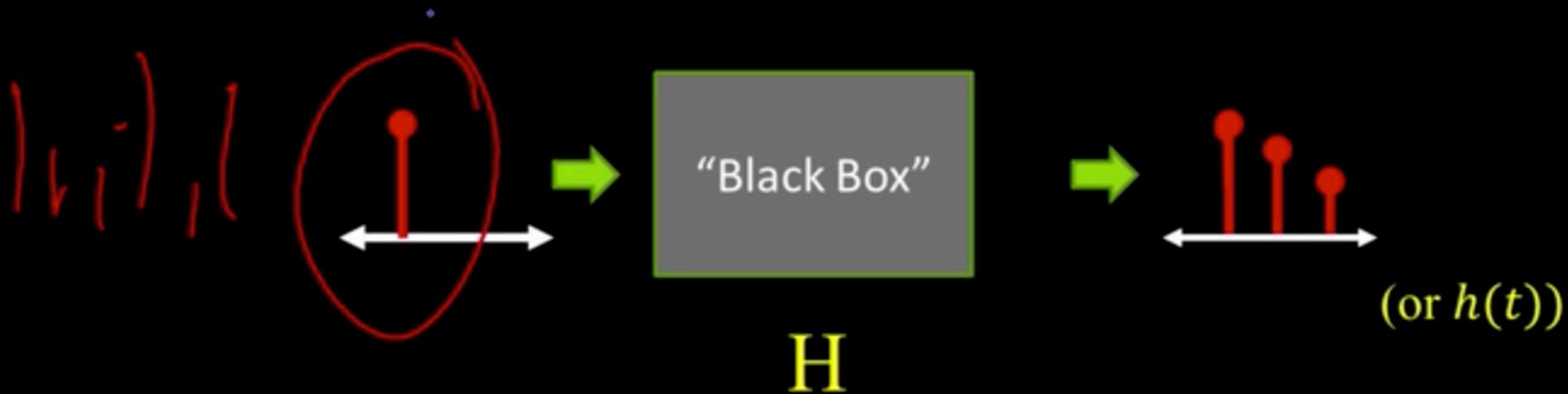
- If I have an unknown system and I “put in” an impulse, the response is called the impulse response. (Duh?)



- So if the black box is linear you can describe H by $\mathbf{h}(\mathbf{x})$
- Why?

An impulse response

- If I have an unknown system and I “put in” an impulse, the response is called the impulse response. (Duh?)



- So if the black box is linear you can describe H by $\mathbf{h}(\mathbf{x})$
- Why?

Filtering an impulse signal

What is the result of filtering the impulse signal (image) F with the arbitrary kernel H ?

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

$F(x,y)$

$H(u,v)$

$G(x,y)$

Filtering an impulse signal

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

$F(x,y)$

$H(u,v)$

$G(x,y)$

Filtering an impulse signal

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	f

$F(x,y)$

$H(u,v)$

$G(x,y)$

Filtering an impulse signal

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	f	?

$F(x,y)$

$H(u,v)$

$G(x,y)$

Filtering an impulse signal

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	f	e	d

$F(x,y)$

$H(u,v)$

$G(x,y)$

Filtering an impulse signal

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	i	h	g
0	f	e	d
0	c	b	a

$F(x,y)$

$H(u,v)$

$G(x,y)$

Filtering an impulse signal

Assuming center coordinate is “reference point”.

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	i	h	g
0	f	e	d
0	c	b	a

$F(x,y)$

$H(u,v)$

$G(x,y)$

Q&A 09

Quiz

Suppose our kernel was size $M \times M$ and our image was $N \times N$. How many multiples would it take to filter the whole image with the filter?

- a) $M \times N \times 2$
- b) $M \times M \times N \times 2$
- c) $M \times N \times N$
- d) $M \times M \times N \times N$

Quiz

Suppose our kernel was size $M \times M$ and our image was $N \times N$. How many multiples would it take to filter the whole image with the filter?

- a) $M \times N \times 2$
- b) $M \times M \times N \times 2$
- c) $M \times N \times N$
- d) $M \times M \times N \times N$ $M^2 N^2$.

Correlation vs Convolution

Cross-correlation

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

$$G = H \otimes F$$

.

K. Grauman

Correlation vs Convolution

Cross-correlation

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

$$G = H \otimes F$$

Convolution

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

$$G = H \star F$$

Correlation vs Convolution

Cross-correlation

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

$$G = H \otimes F$$

Flip in both dimensions
(bottom to top, right to left)

Convolution

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

$$G = H \star F$$

Convolution

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*

Correlation:

Indices in F corresponding to the cells in H above

i-1, j-1	i-1,j	i-1, j+1		
	i,j			
i+1, j-1		i+1,j+1		

k=1
Indices for H:

-1,-1	-1,0	-1,1
	0,0	
1,-1		1,1

Convolution:

Indices in F corresponding to the cells in H above

i+1,j+1	i+1,j	i+1, j-1		
	i,j			
i-1, j+1		i-1, j-1		

Convolution

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*

H*

F

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Convolution

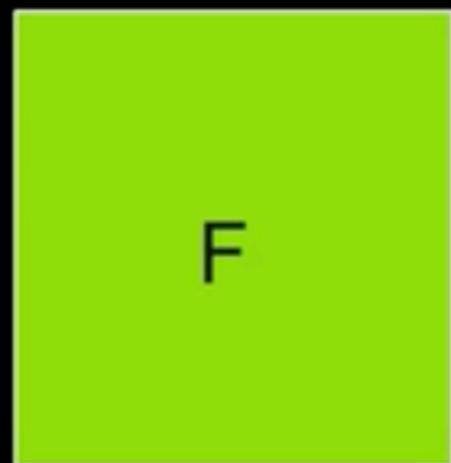
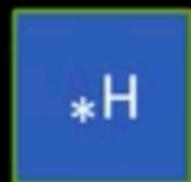
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*



K. Grauman

Convolution

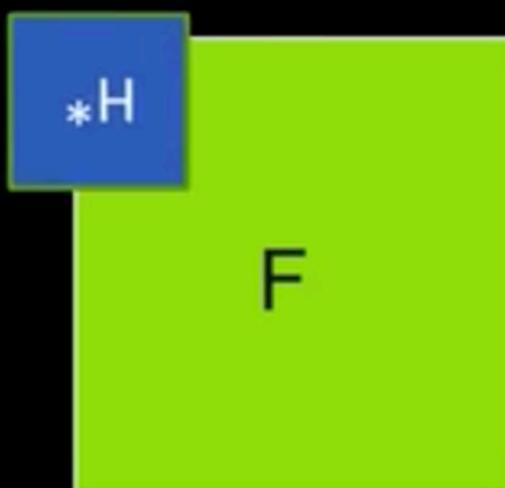
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*



K. Grauman

Convolution

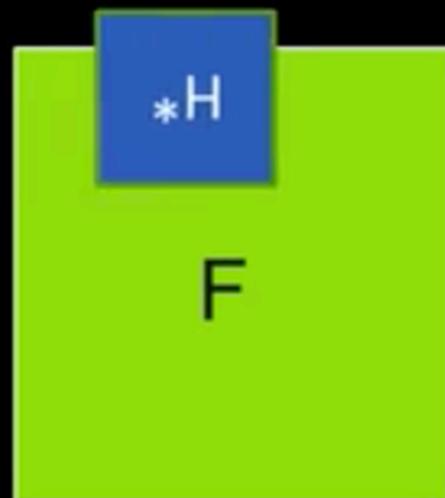
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*



K. Grauman

Convolution

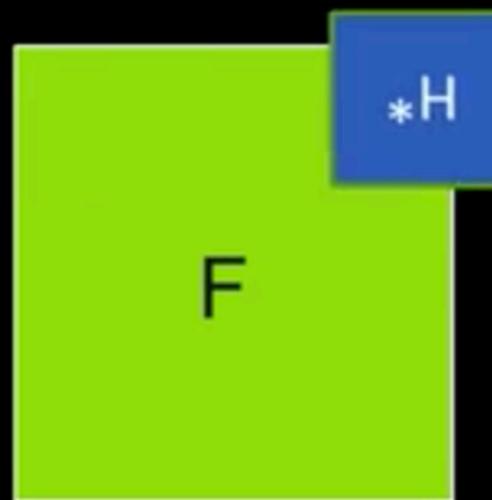
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*



K. Grauman

Convolution

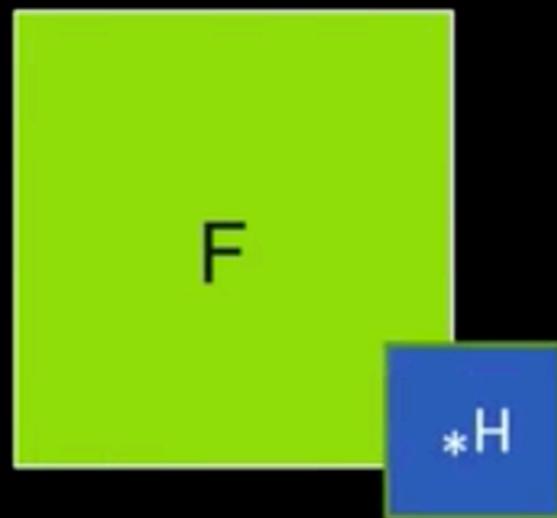
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

Centered at zero!

$$G = H * F$$



*Notation for
convolution
operator*



K. Grauman

Correlation:

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	i	h	g	
0	f	e	d	
0	c	b	a	

$F(x,y)$

$H(u,v)$

$G(x,y)$

Convolution:

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

=

0	a	b	c	
0	d	e	f	
0	g	h	i	

$F(x,y)$

$H(u,v)$

$G(x,y)$

Q&A 10

Quiz

When convolving a filter with an impulse image, we get the filter back as a result.

So if we convolve an image with an impulse we get:

- a) A blurred version of the image
- b) The original image
- c) A shifted version of the original image.
- d) No idea

Quiz

When convolving a filter with an impulse image, we get the filter back as a result.

So if we convolve an image with an impulse we get:

- a) A blurred version of the image
- b) The original image
- c) A shifted version of the original image.
- d) No idea

One more thing...

Shift invariant:

- Operator behaves the same everywhere, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.

Properties of convolution

- Linear & shift invariant

- Commutative:

$$f * g = g * f$$

- Associative

$$(f * g) * h = f * (g * h)$$

- Identity:

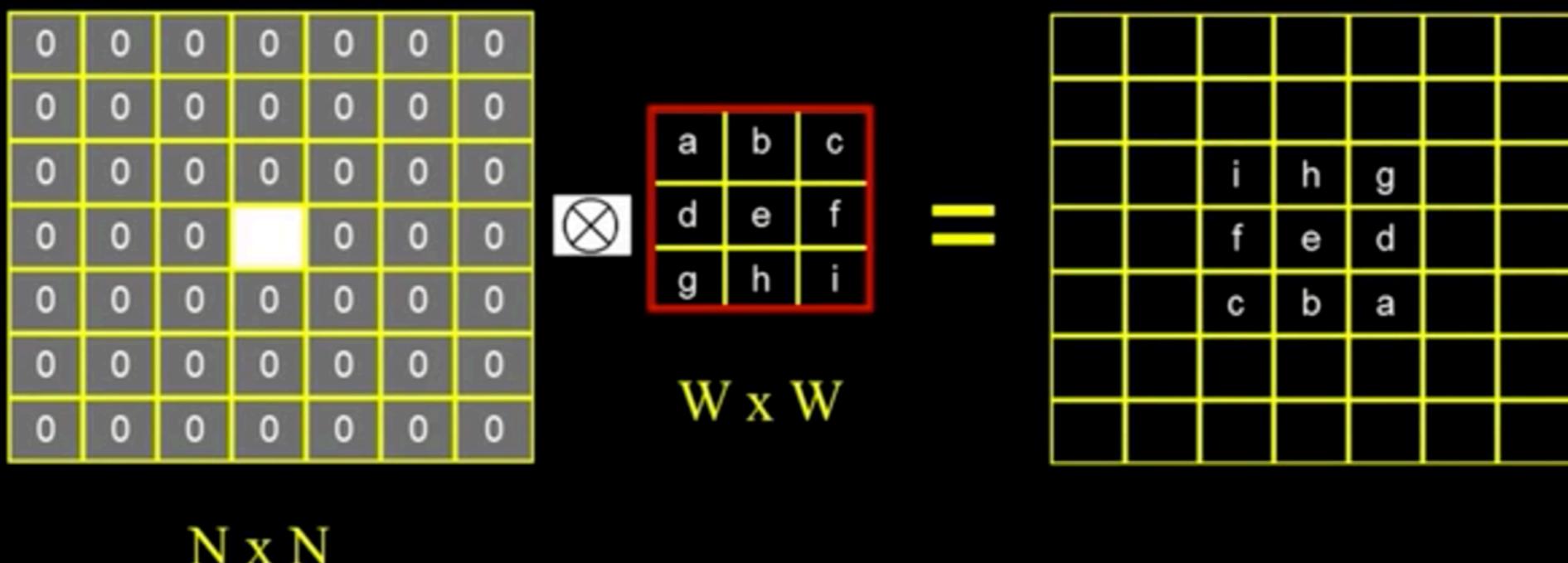
unit impulse $e = [..., 0, 0, 1, 0, 0, ...]$. $f * e = f$

- Differentiation: $\frac{\partial}{\partial x}(f * g) = \frac{\partial f}{\partial x} * g$

Separability

Computational Complexity

- If an image is $N \times N$ and a kernel (filter) is $W \times W$, how many multiplies do you need to compute a convolution?



- You need $N \times N \times W \times W = N^2 W^2$
 - which can get big (ish)

Separability

- In some cases, filter is separable, meaning you can get the square kernel H by convolving a single column vector by some row vector:

$$\mathbf{c} \quad \mathbf{r} \quad \mathbf{H}$$
$$\begin{matrix} 1 \\ 2 \\ 1 \end{matrix} \times \begin{matrix} 1 & 2 & 1 \end{matrix} = \begin{matrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{matrix}$$

•

Separability

- In some cases, filter is separable, meaning you can get the square kernel H by convolving a single column vector by some row vector:

$$\mathbf{c} \quad \mathbf{r}$$
$$\begin{matrix} 1 \\ 2 \\ 1 \end{matrix} \times \begin{matrix} 1 & 2 & 1 \end{matrix} = \begin{matrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{matrix} \quad \textcircled{\mathbf{H}}$$

$$\mathbf{c} * \mathbf{r} = \mathbf{H}$$

Separability

$$\mathbf{c} \quad \begin{matrix} 1 \\ 2 \\ 1 \end{matrix} \quad \times \quad \begin{matrix} 1 & 2 & 1 \end{matrix} \quad = \quad \begin{matrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{matrix} \quad \mathbf{H}$$

r

$$G = H * F = (C * R) * F = C * (R * F)$$

- So we do two convolutions but each is $W \cdot N \cdot N$. So this is useful if W is big enough such that $2 \cdot W \cdot N^2 \ll W^2 \cdot N^2$
- Used to be **very** important. Still, if $W=31$, save a factor of 15.

Q&A 11

Quiz?

True or false: Division is a linear operation.

- a) False because $X/(Y + Z) \neq X/Y + X/Z$
- b) True because $(X + Y)/Z = X/Z + Y/Z$
- c) I have no idea

Quiz?

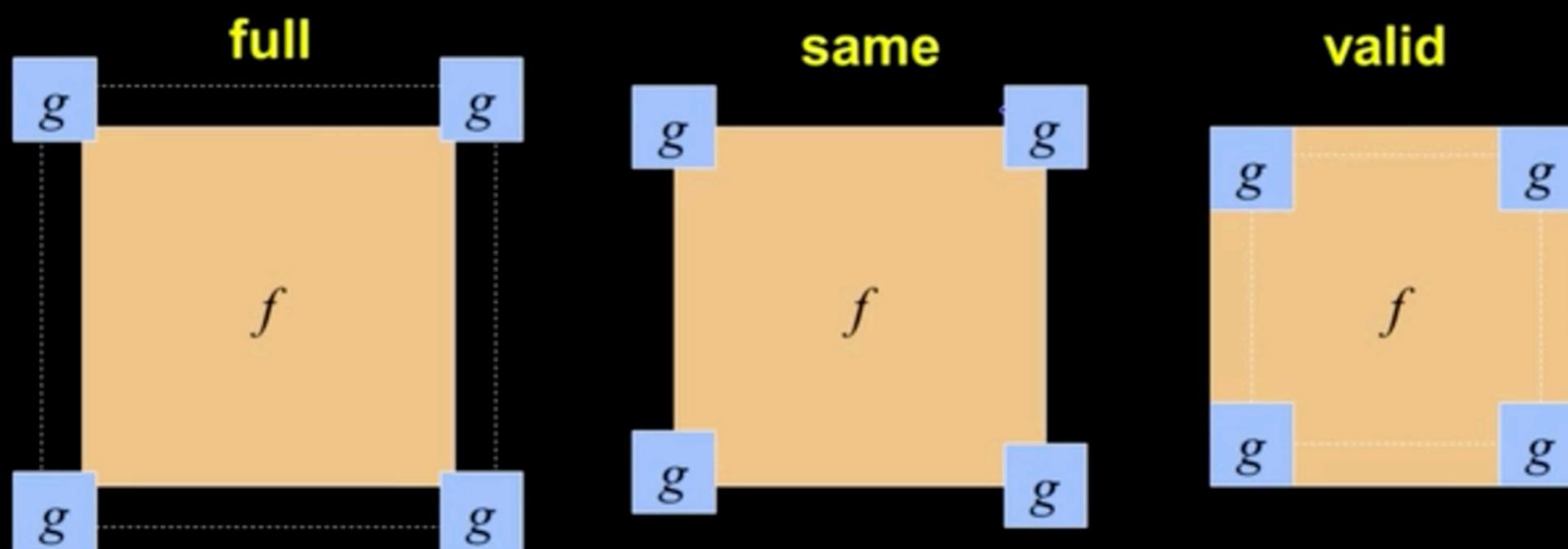
c) ~~com~~A

True or false: Division is a linear operation.

- a) False because $X/(Y + Z) \neq X/Y + X/Z$
- b) True because $(X + Y)/Z = X/Z + Y/Z$
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Boundary issues

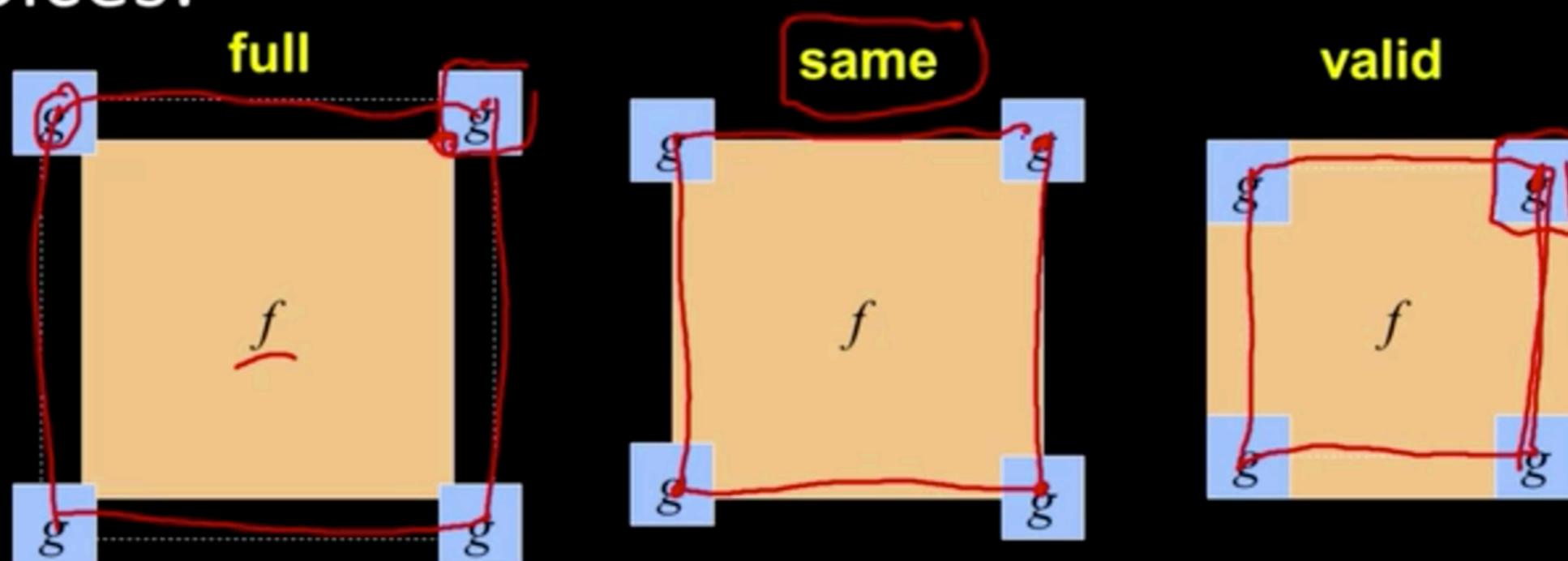
- What is the size of the output?
- Using old Matlab nomenclature we have three choices:



Source: S. Lazebnik

Boundary issues

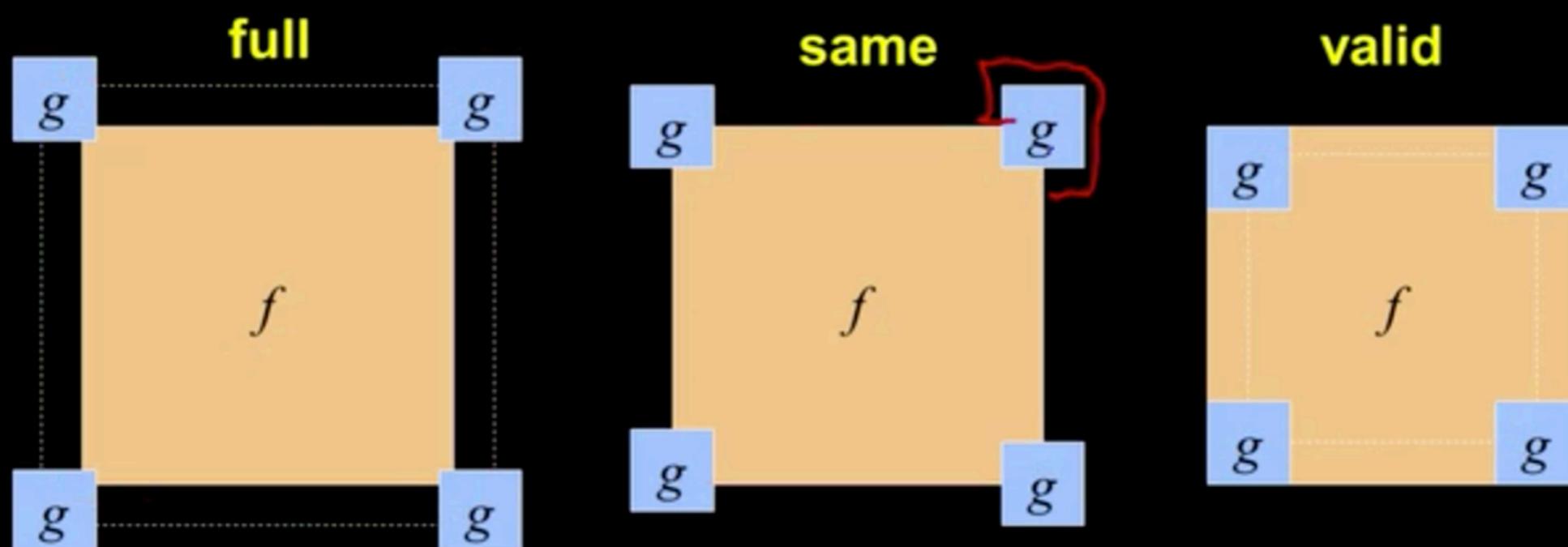
- What is the size of the output?
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Source: S. Lazebnik

Boundary issues

- What is the size of the output?
- Using old Matlab nomenclature we have three choices:



Source: S. Lazebnik

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)

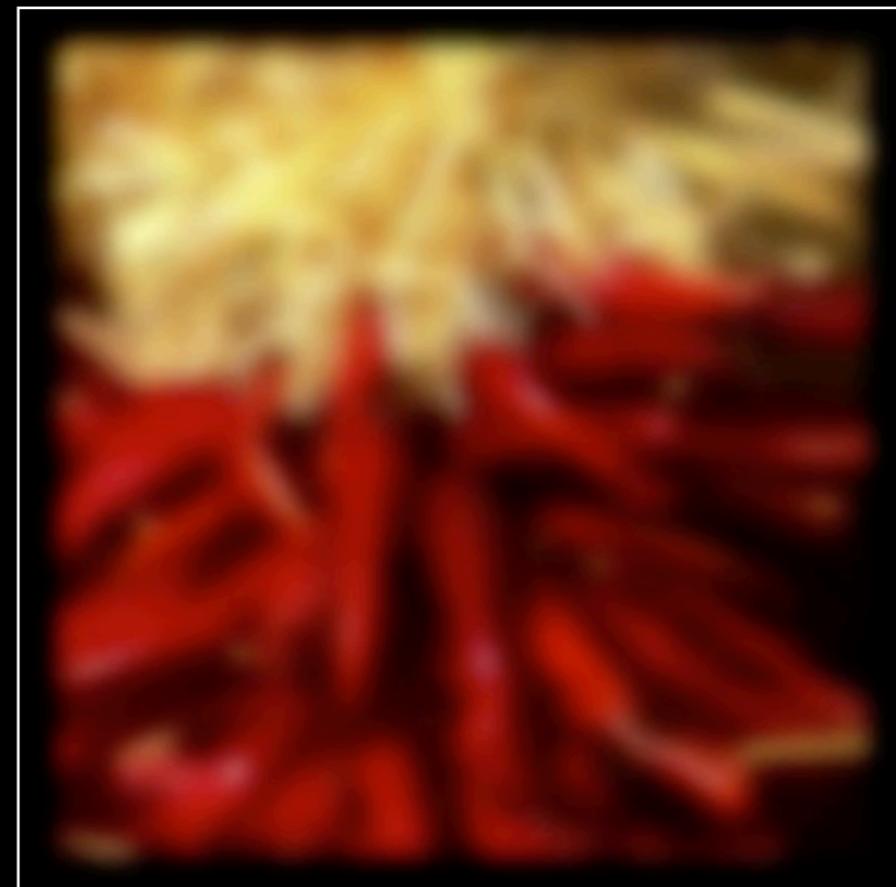


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)

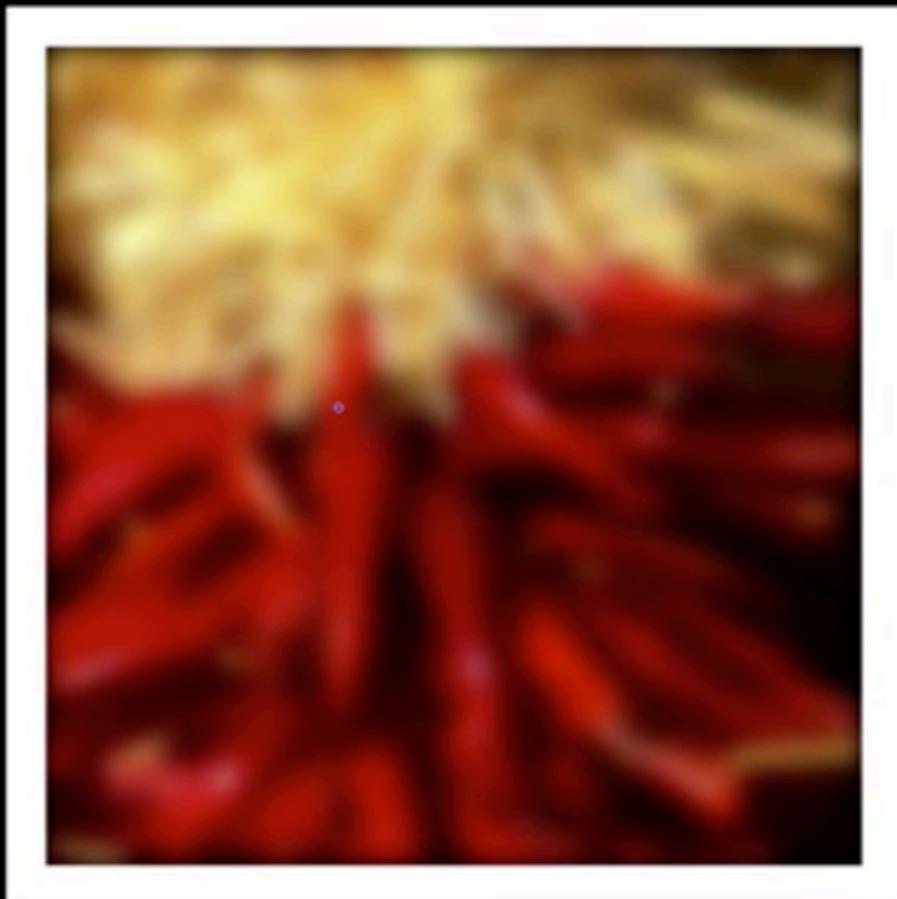


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)

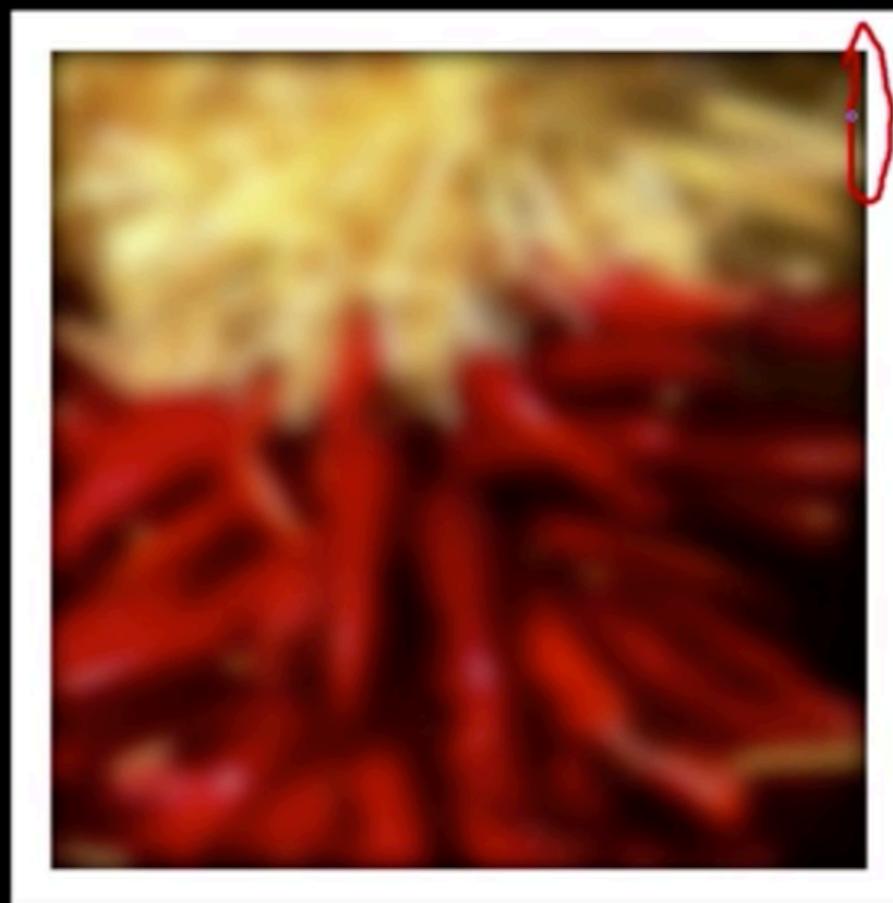


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around

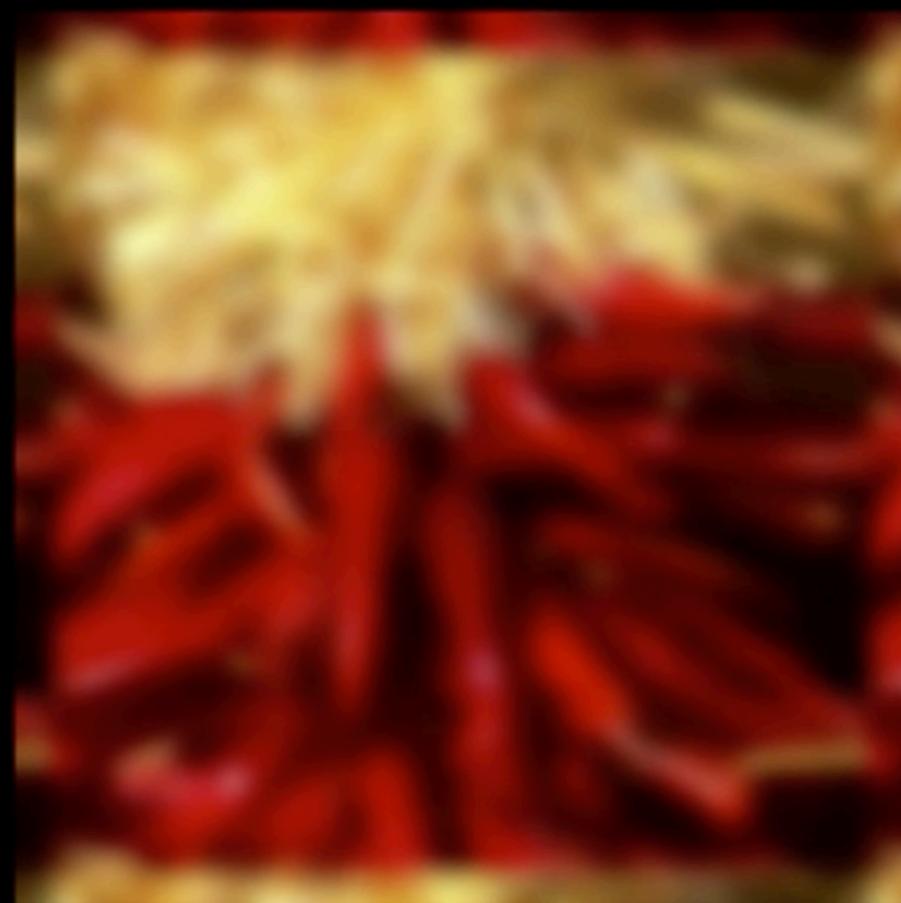


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around

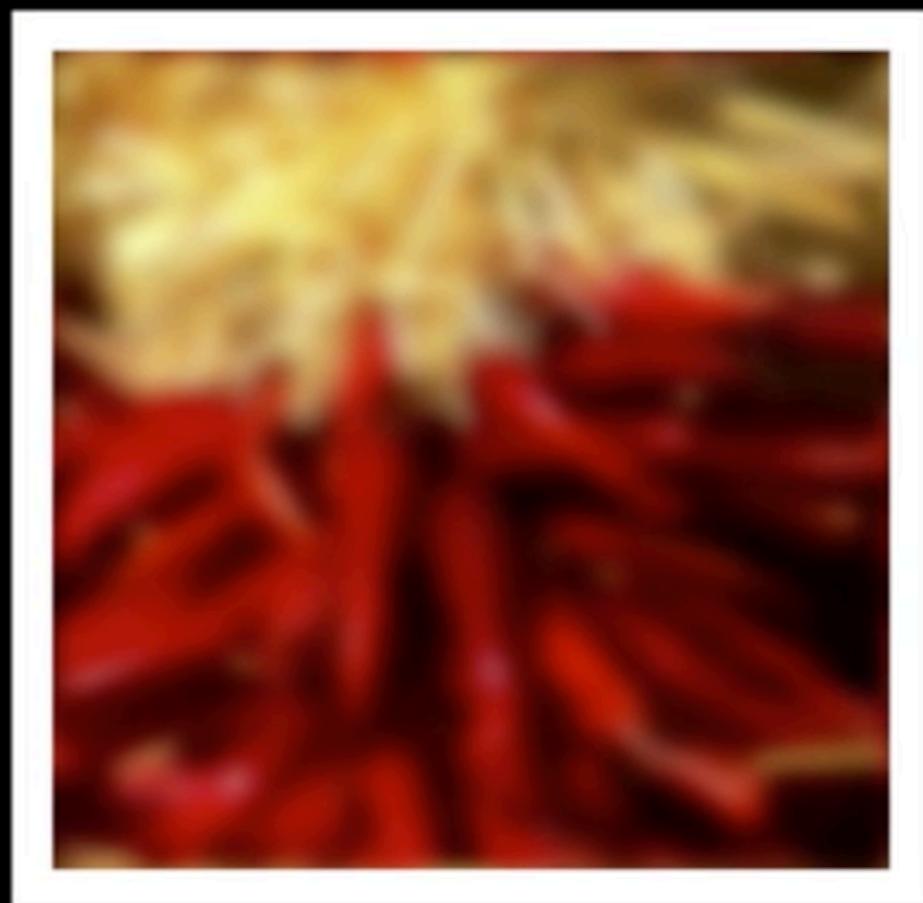


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around

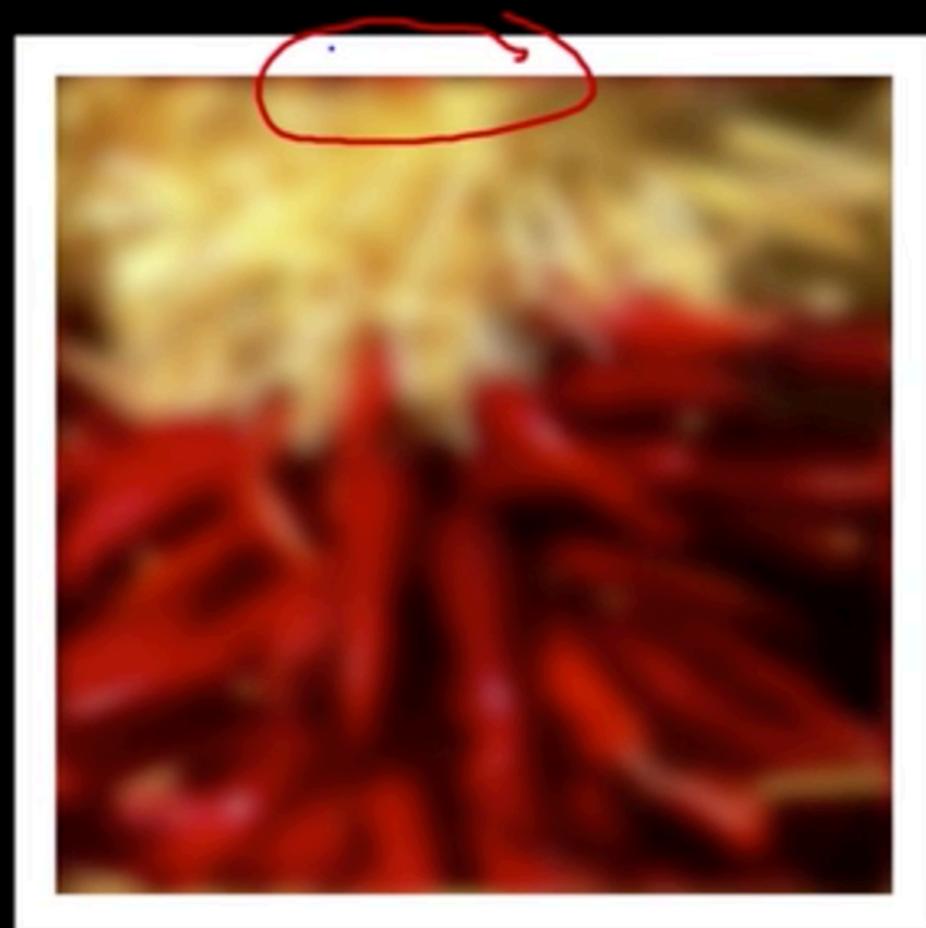


Source: S. Marschner

Boundary issues

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- methods:
 - clip filter (black)
 - wrap around



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge

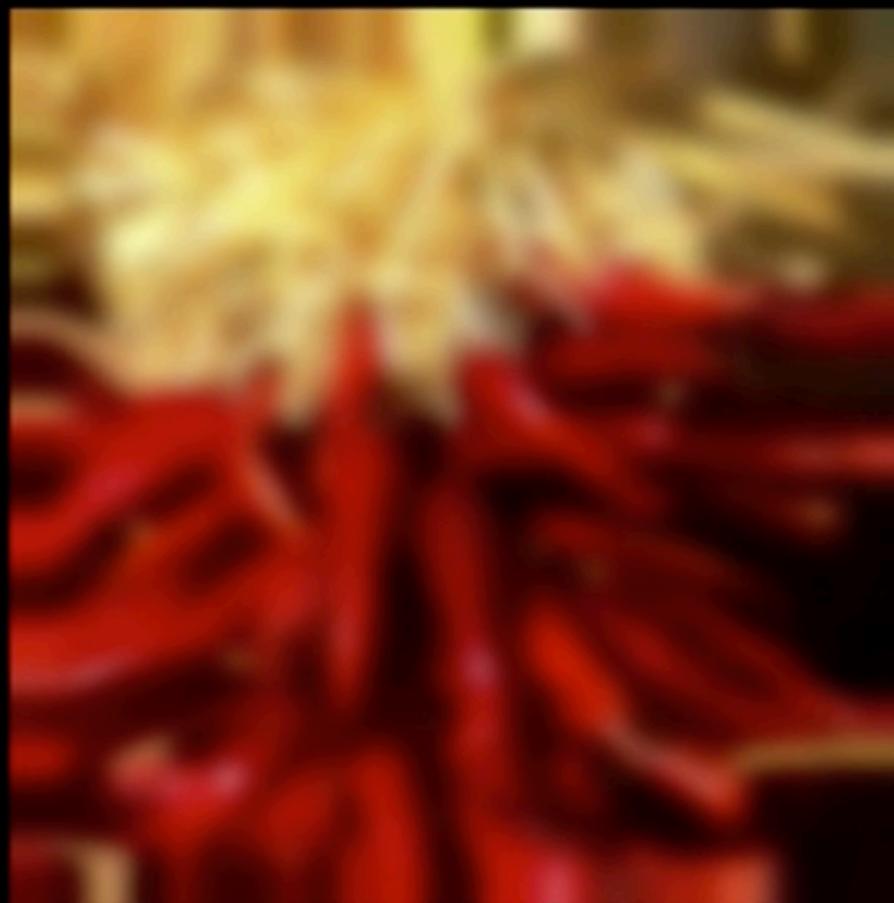


Source: S. Mars

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge

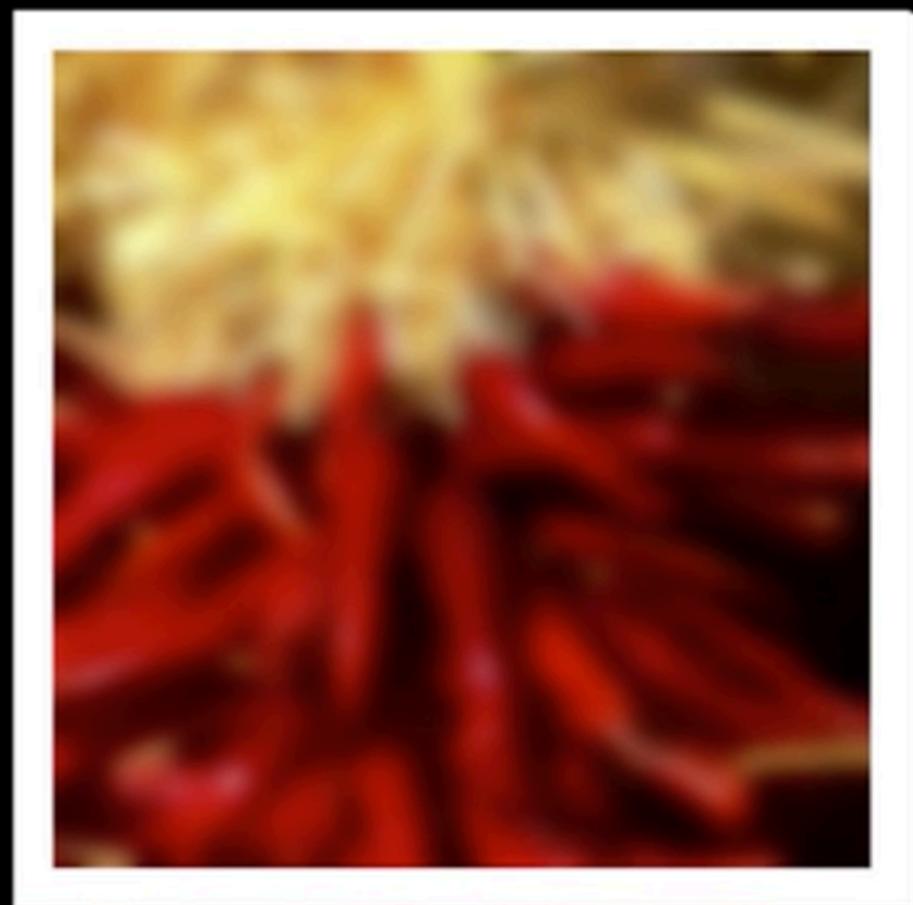


Source: S. Mars

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge



Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge

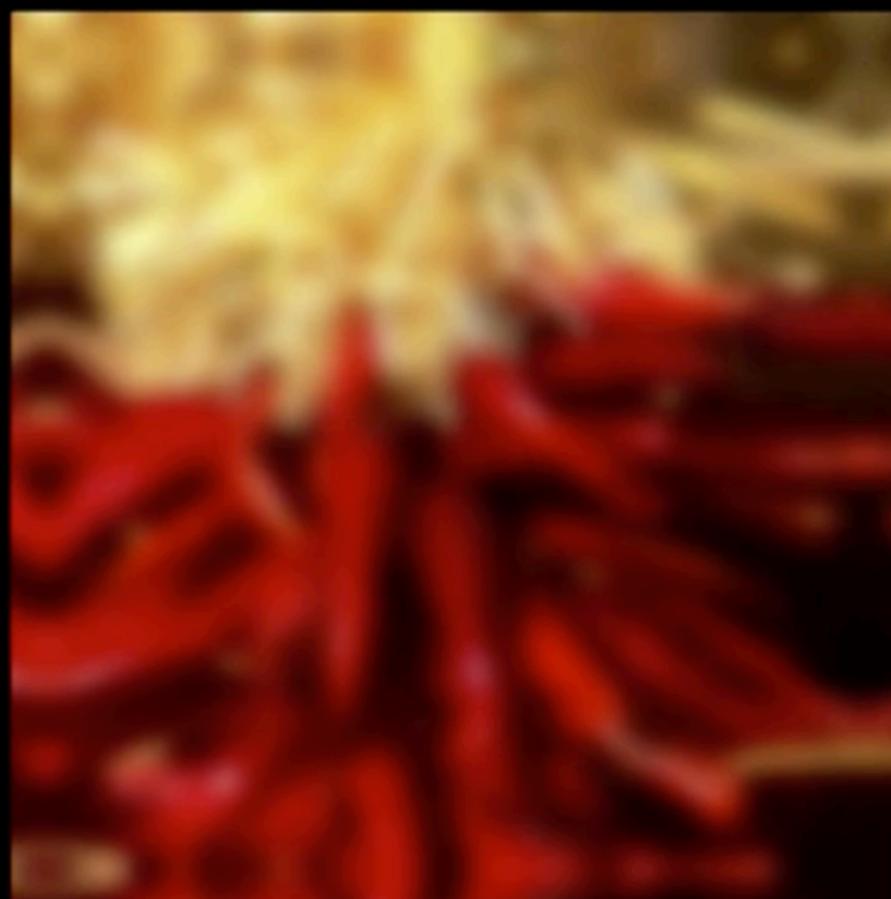


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge

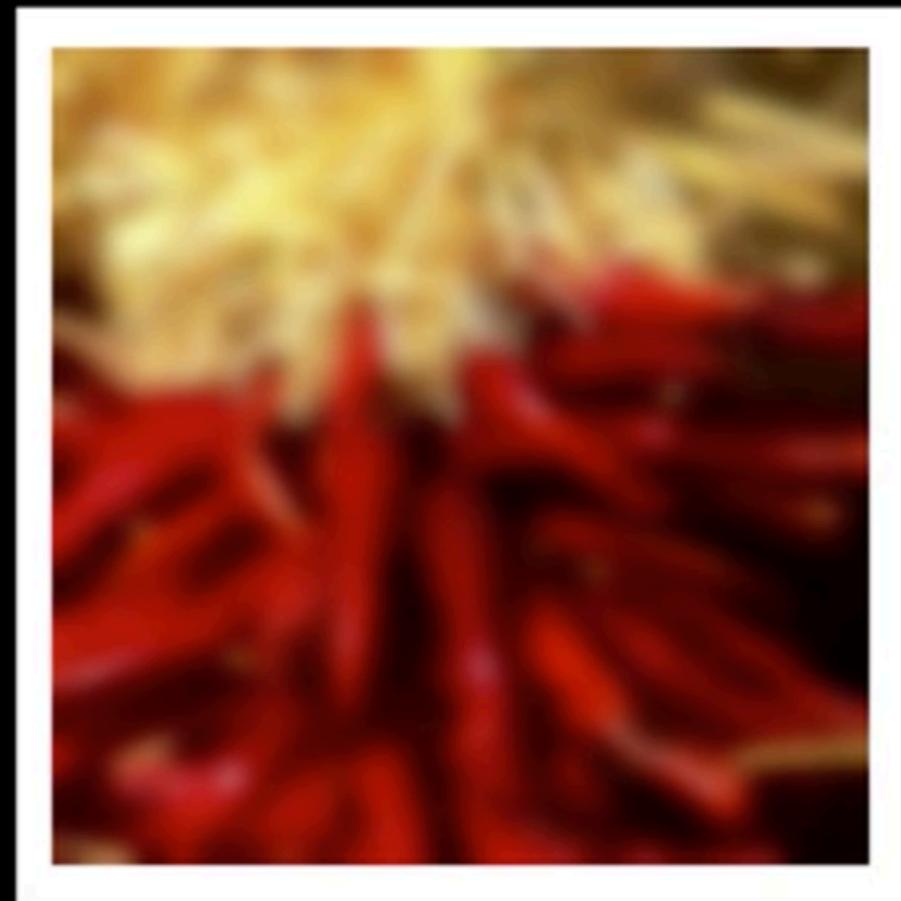


Source: S. Marschner

Boundary issues

What about near the edge?

- methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge



Source: S. Marschner

Boundary issues

What about near the edge?

- methods (new MATLAB):
 - clip filter (black): `imfilter(f, g, 0)`
 - wrap around: `imfilter(f, g, 'circular')`
 - copy edge: `imfilter(f, g, 'replicate')`
 - reflect across edge: `imfilter(f, g, 'symmetric')`

Q&A 12

Quiz

The reflection method of handling boundary conditions in filtering is good because:

- a) The created imagery has the same statistics as the original image
- b) The computation is the least expensive of the possibilities.
- c) Setting pixels to zero is fast.
- d) None of the above.

Quiz

The reflection method of handling boundary conditions in filtering is good because:

- a) The created imagery has the same statistics as the original image
- b) The computation is the least expensive of the possibilities.
- c) Setting pixels to zero is fast.
- d) None of the above.

Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters



Original

0	0	0
0	1	0
0	0	0



Filtered
(no
change)

Source: D. Lowe

Practice with linear filters



0	0	0
0	0	1
0	0	0

?

Original

Source: D. Lowe

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0



Shifted left
by 1 pixel with
correlation

Practice with linear filters



Original

0	0	0
0	0	1
0	0	0



Center coordinate is 0,0!



Shifted left
by 1 pixel with
correlation

Practice with linear filters



$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

?

Original

Source: D. Lowe

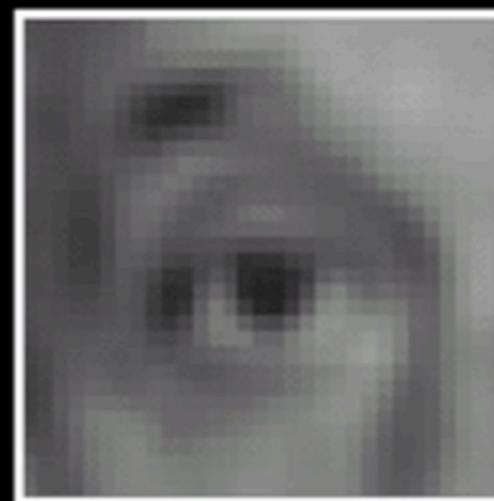
Practice with linear filters



Original

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1



Blur (with a
box filter)

Practice with linear filters



Original

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{matrix} - \cdot \frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

?

Practice with linear filters



$$\begin{matrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{matrix}$$

$$- \frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

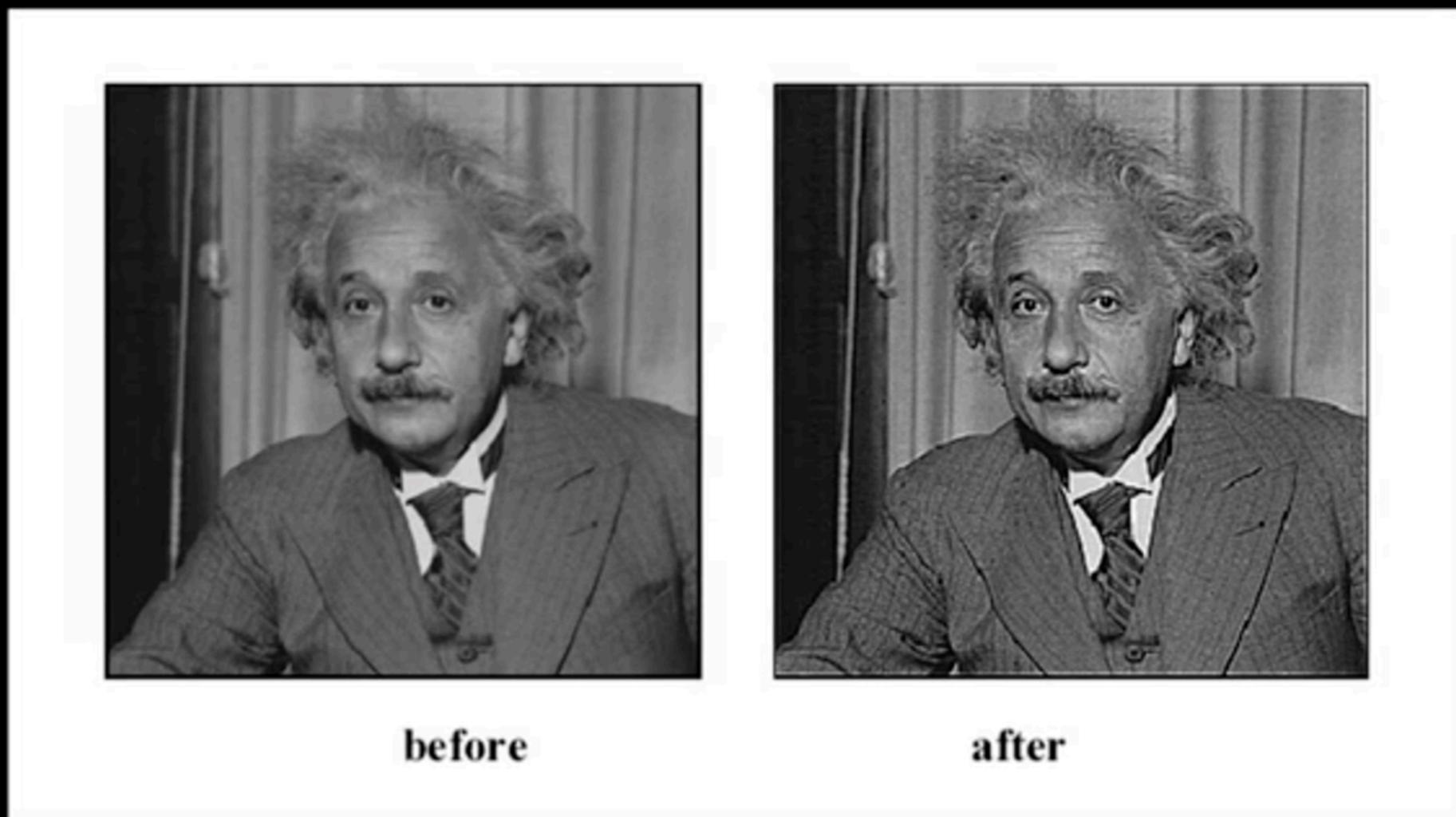


Original

Sharpening Filter:

Accentuates
differences with
local average

Filtering examples: sharpening



K. Grauman

Different kinds of noise

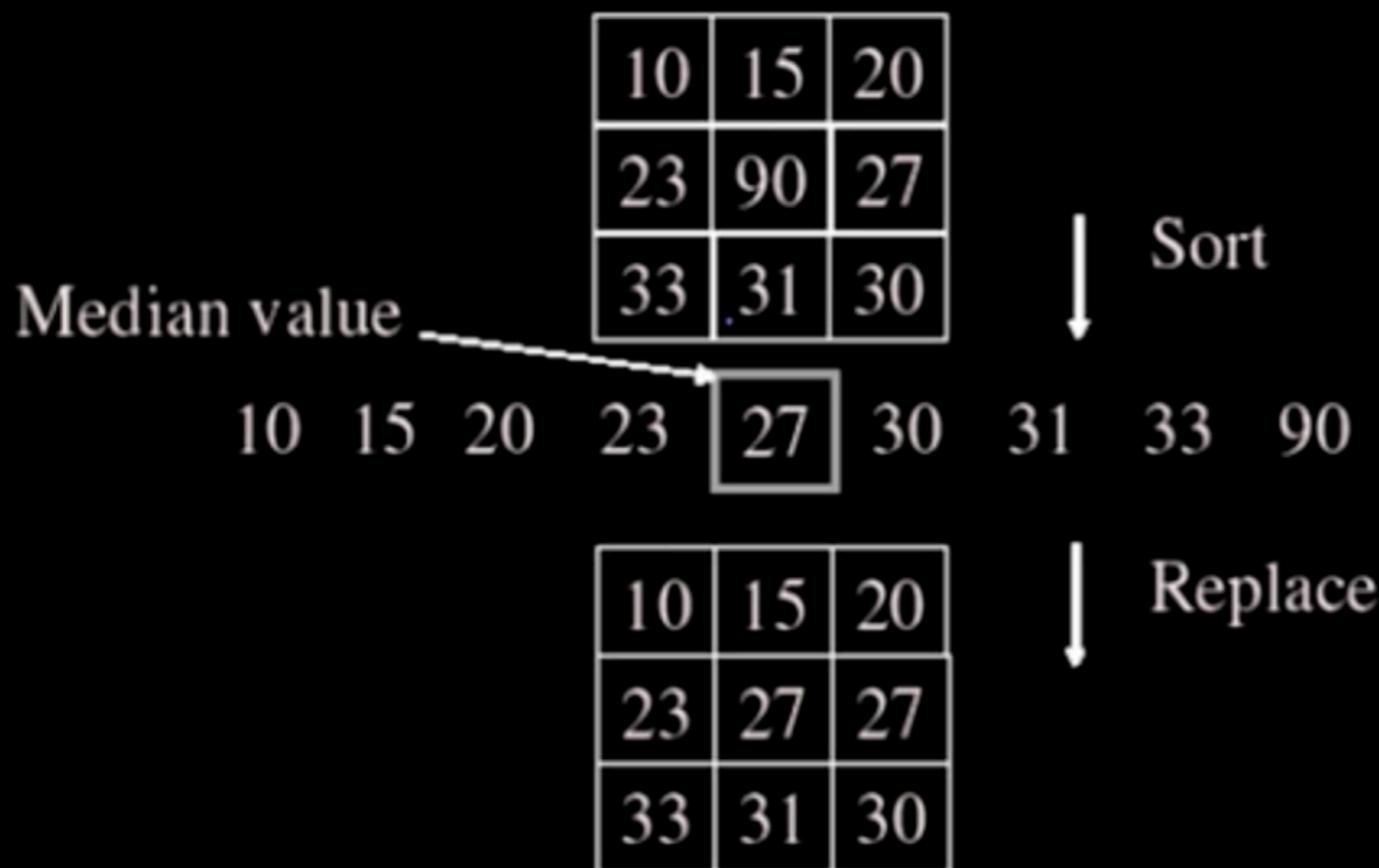


Additive Gaussian noise



Salt and pepper noise

Median filter



K. Grauman

Median filter



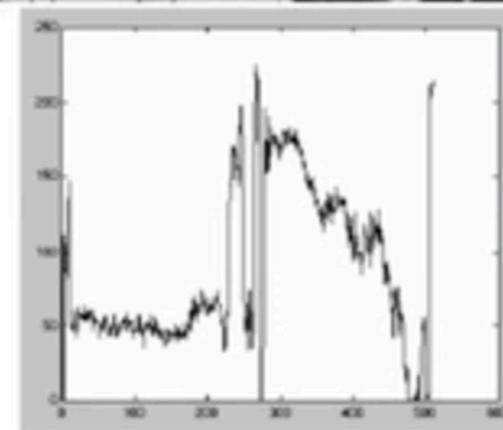
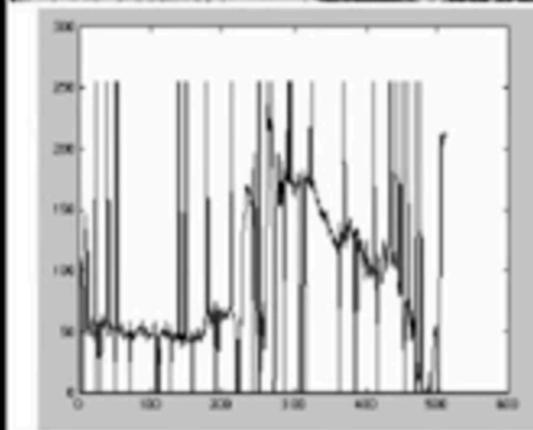
K. Grauman

Median filter

Salt and pepper noise →



← Median filtered

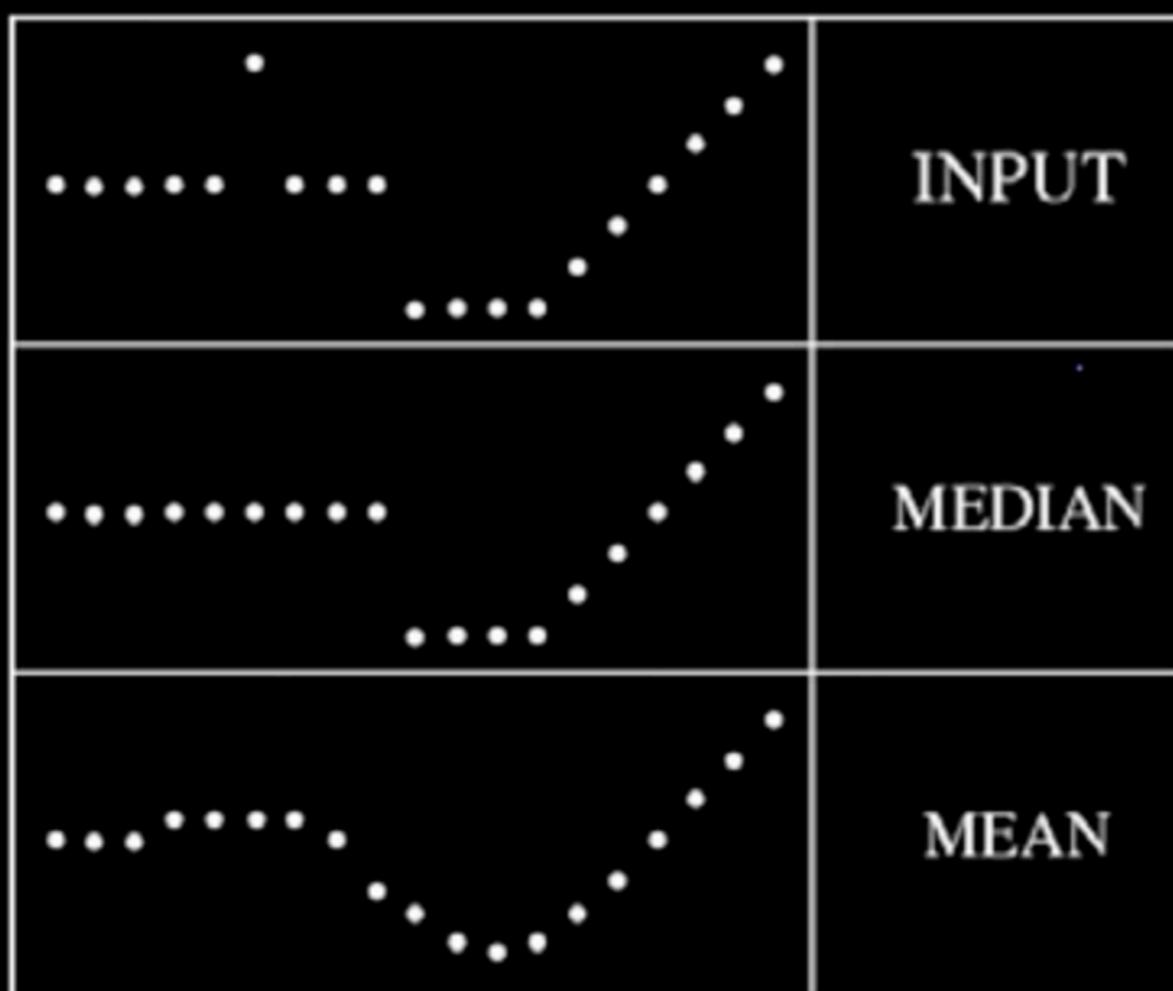


Plots of a row of the image

Source: M. Hebert

Median filter

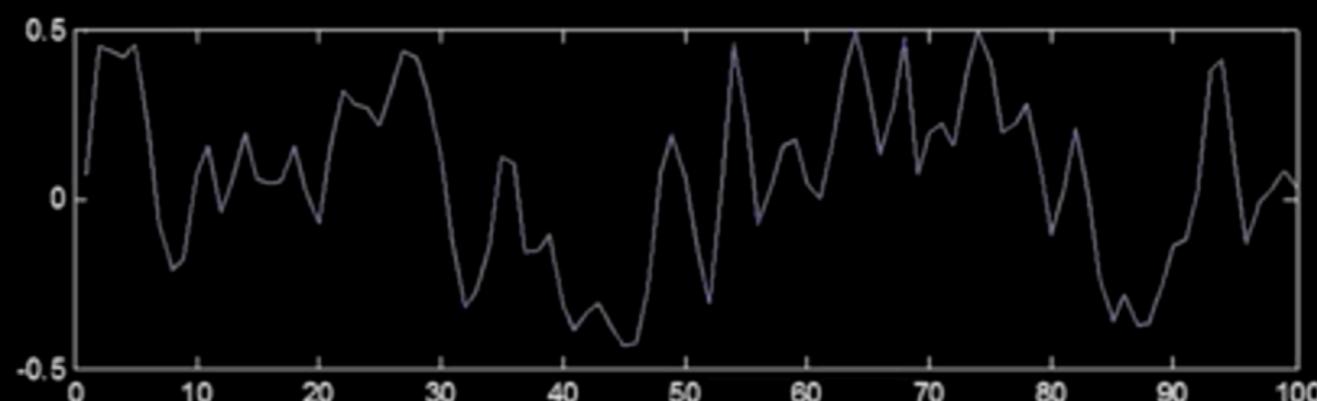
Median filter is edge preserving



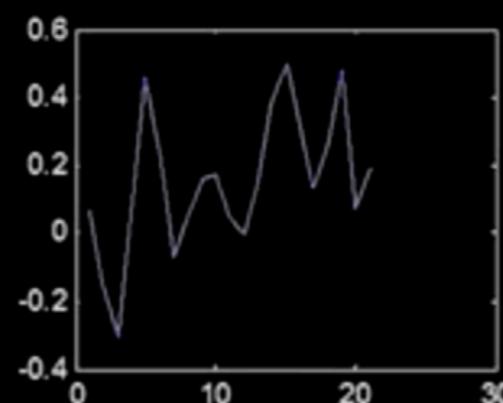
Filters as templates

1D (nx)correlation

Signal

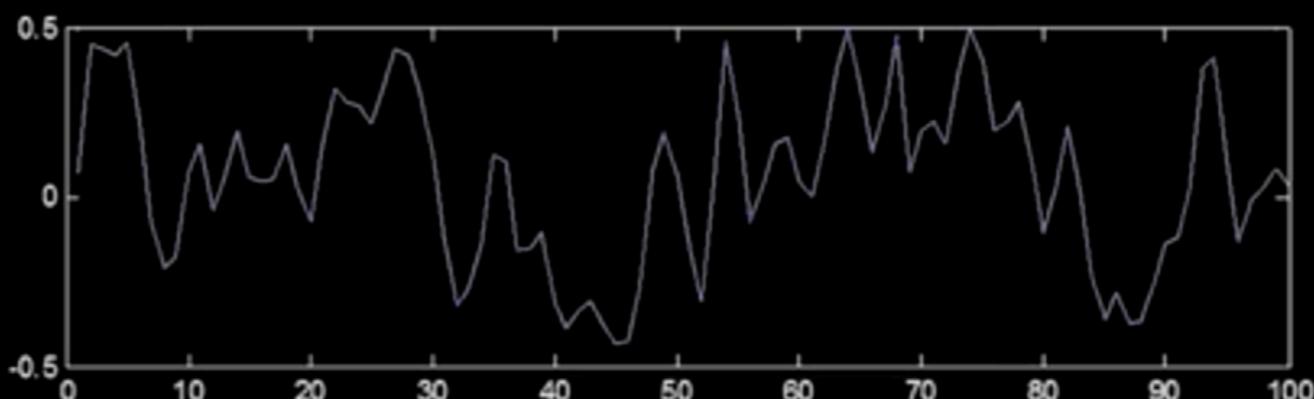


Filter

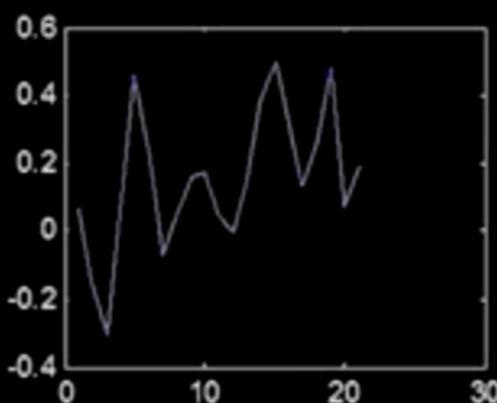


1D (nx)correlation

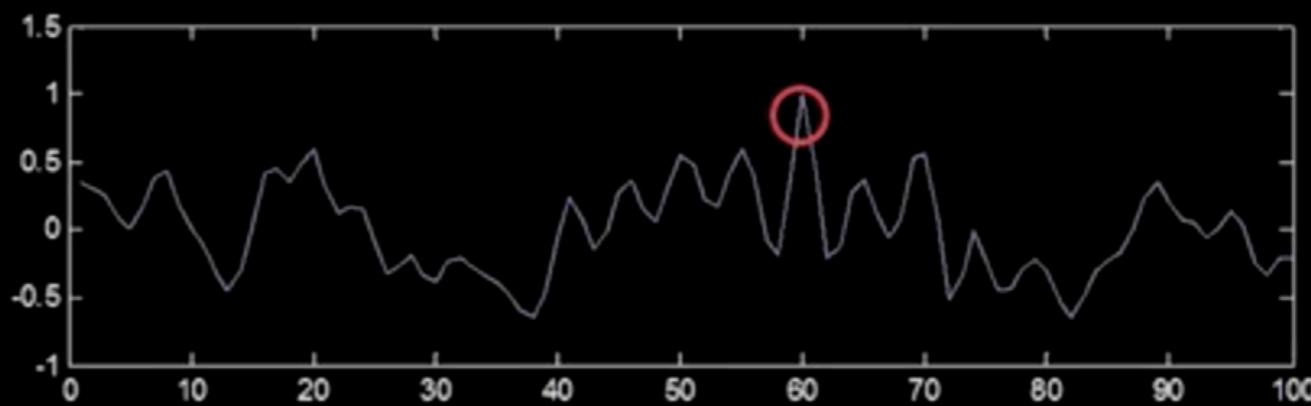
Signal



Filter

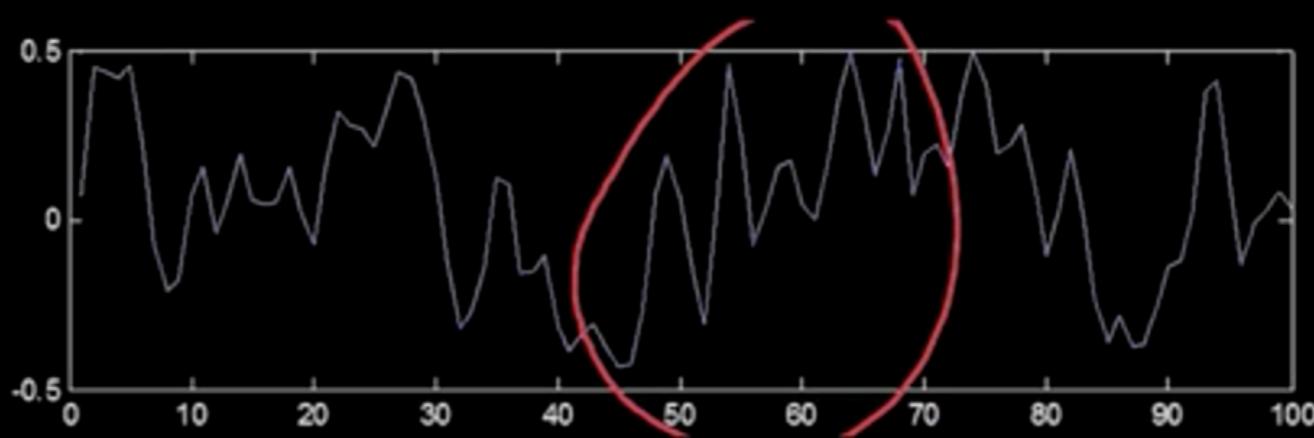


Normalized
cross-correlation

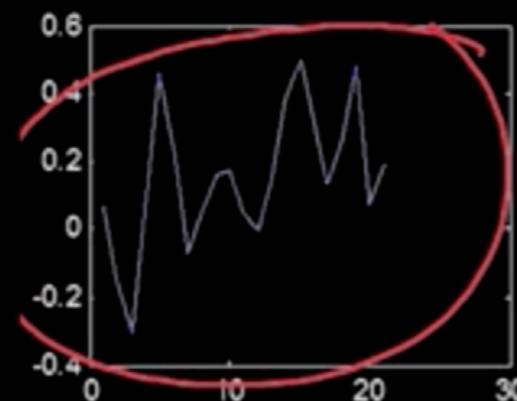


1D (nx)correlation

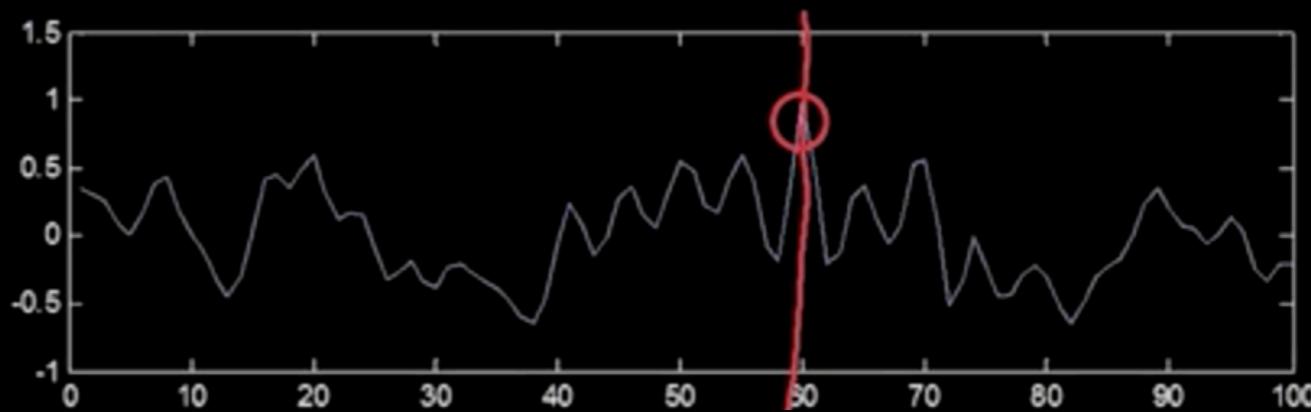
Signal



Filter

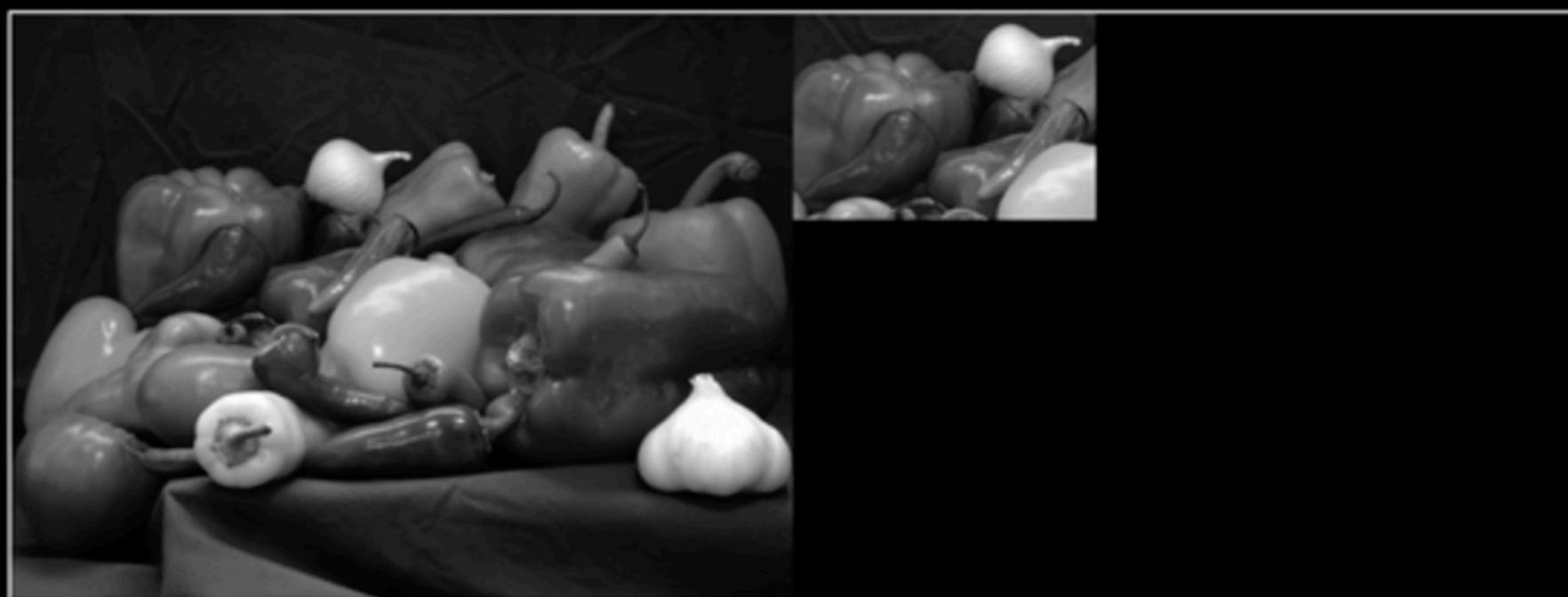


Normalized
cross-correlation



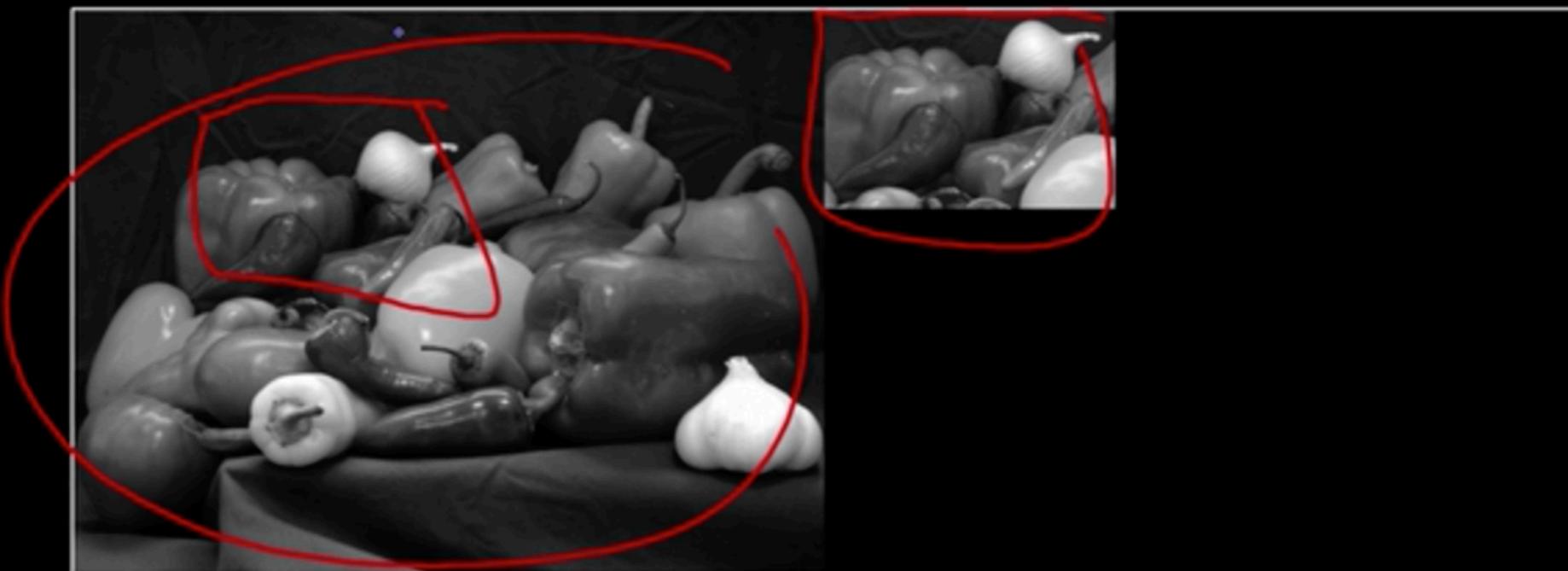
Matlab cross-correlation doc

```
onion      = rgb2gray(imread('onion.png'));  
peppers   = rgb2gray(imread('peppers.png'));  
imshowpair(peppers,onion,'montage')
```



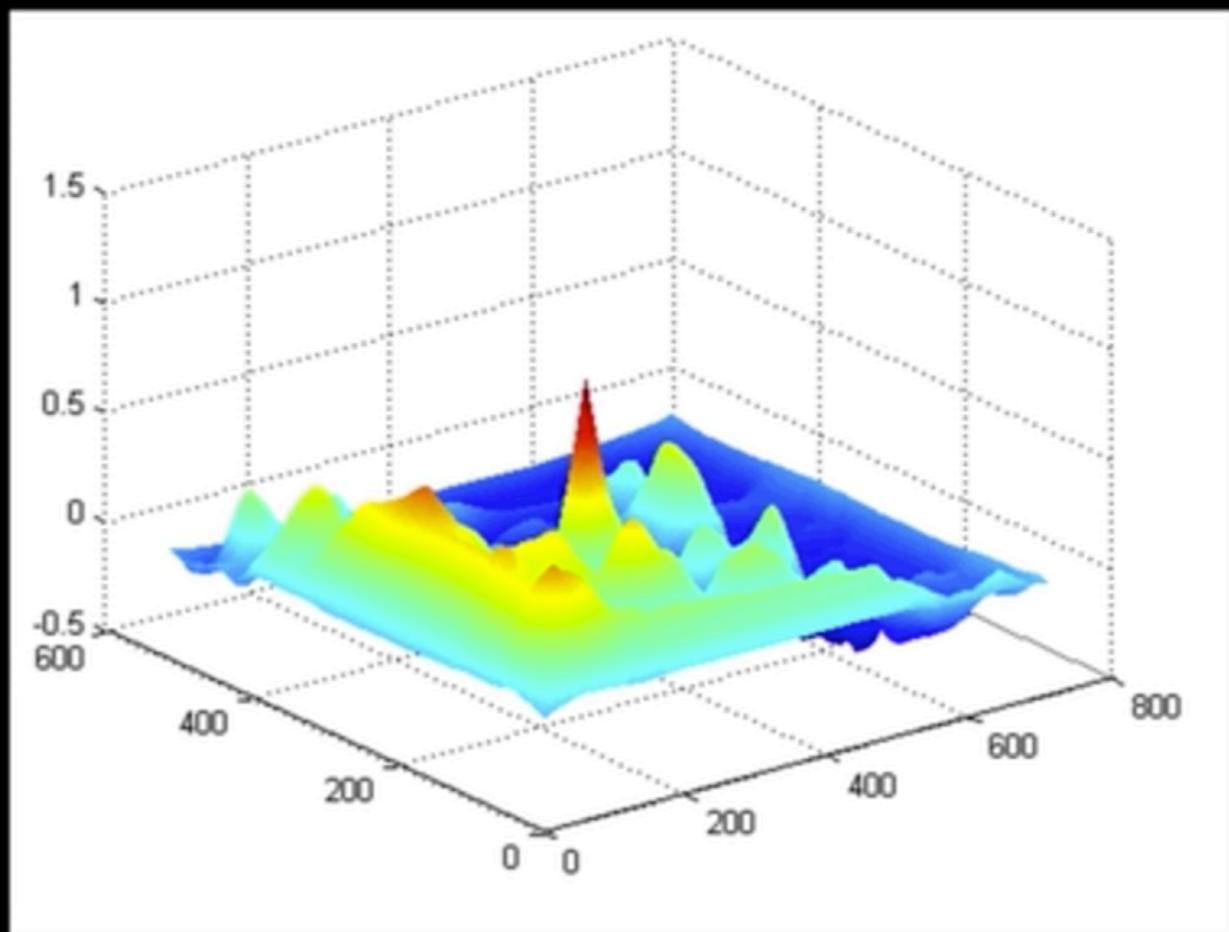
Matlab cross-correlation doc

```
onion      = rgb2gray(imread('onion.png'));  
peppers = rgb2gray(imread('peppers.png'));  
imshowpair(peppers,onion,'montage')
```



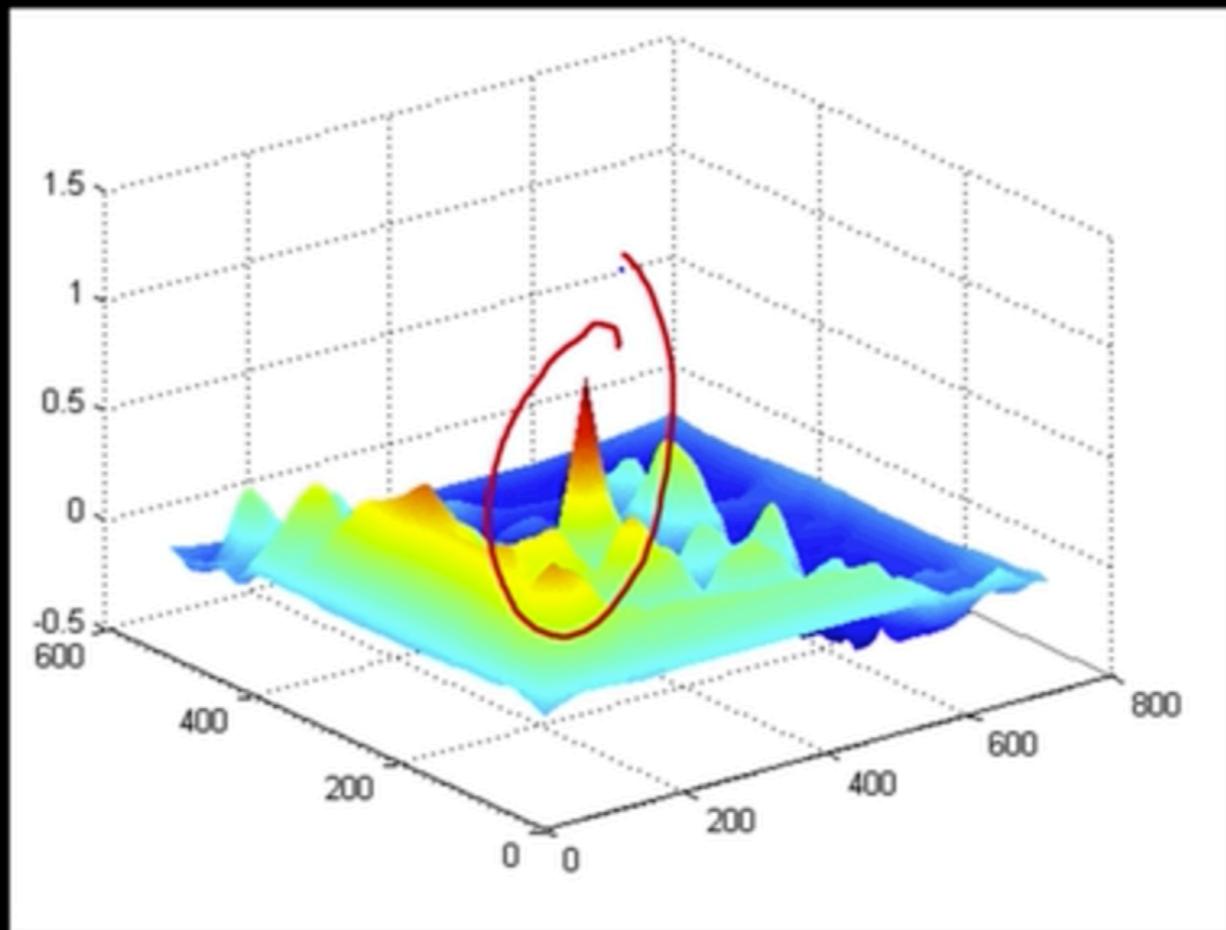
Matlab cross-correlation doc

```
c = normxcorr2(onion,peppers);  
figure, surf(c), shading flat;
```

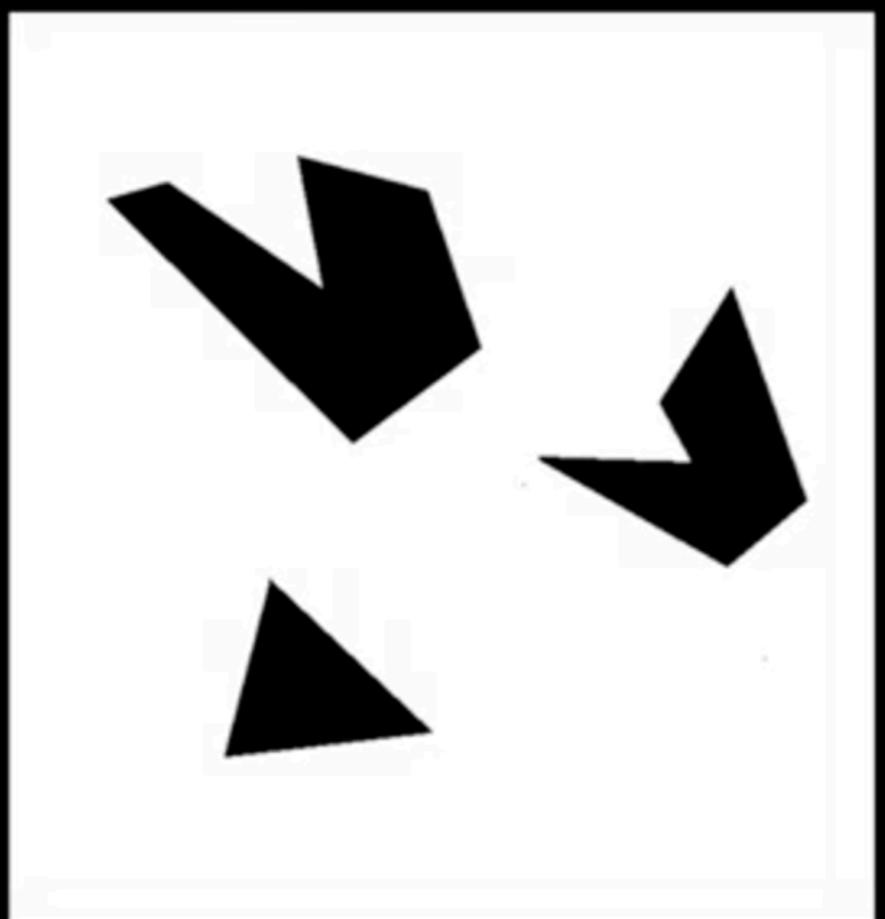


Matlab cross-correlation doc

```
c = normxcorr2(onion,peppers);  
figure, surf(c), shading flat;
```



Template matching



Scene

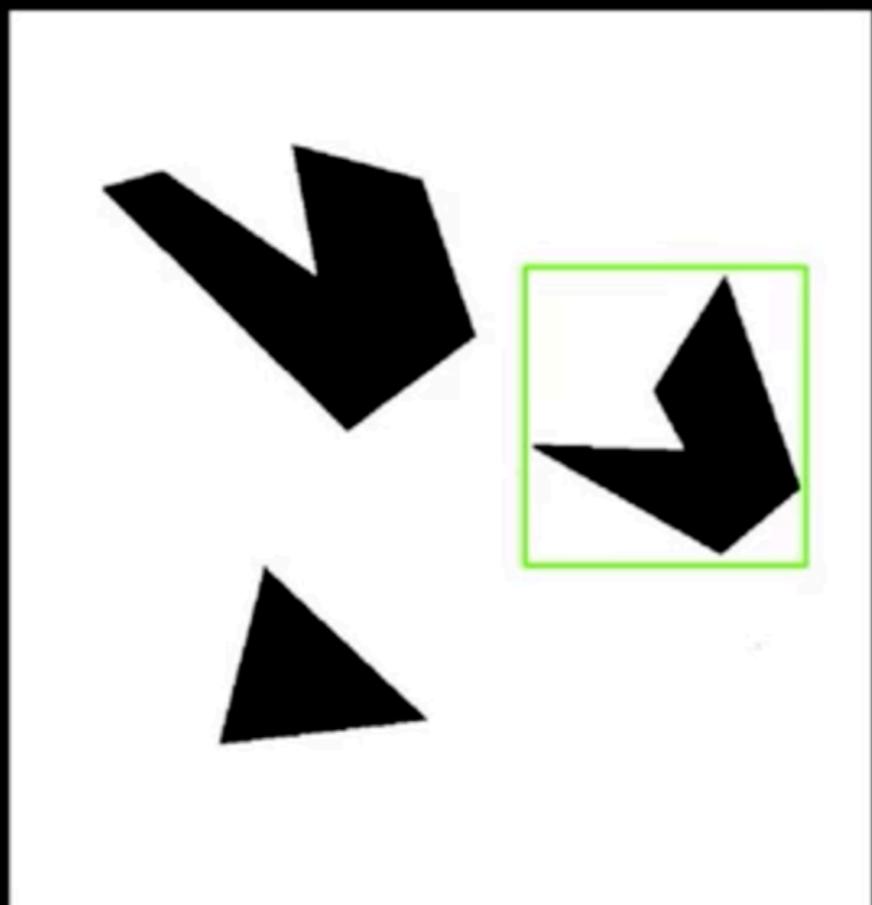
A toy example



Template (mask)

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Template matching



Detected template

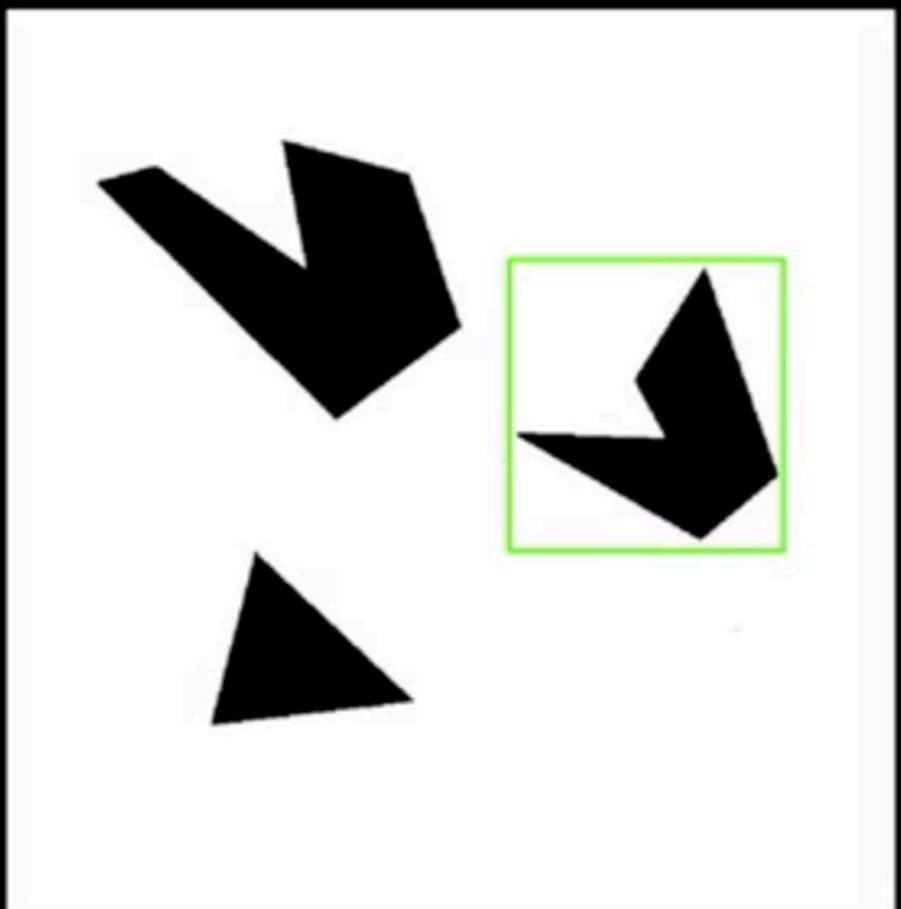
A toy example



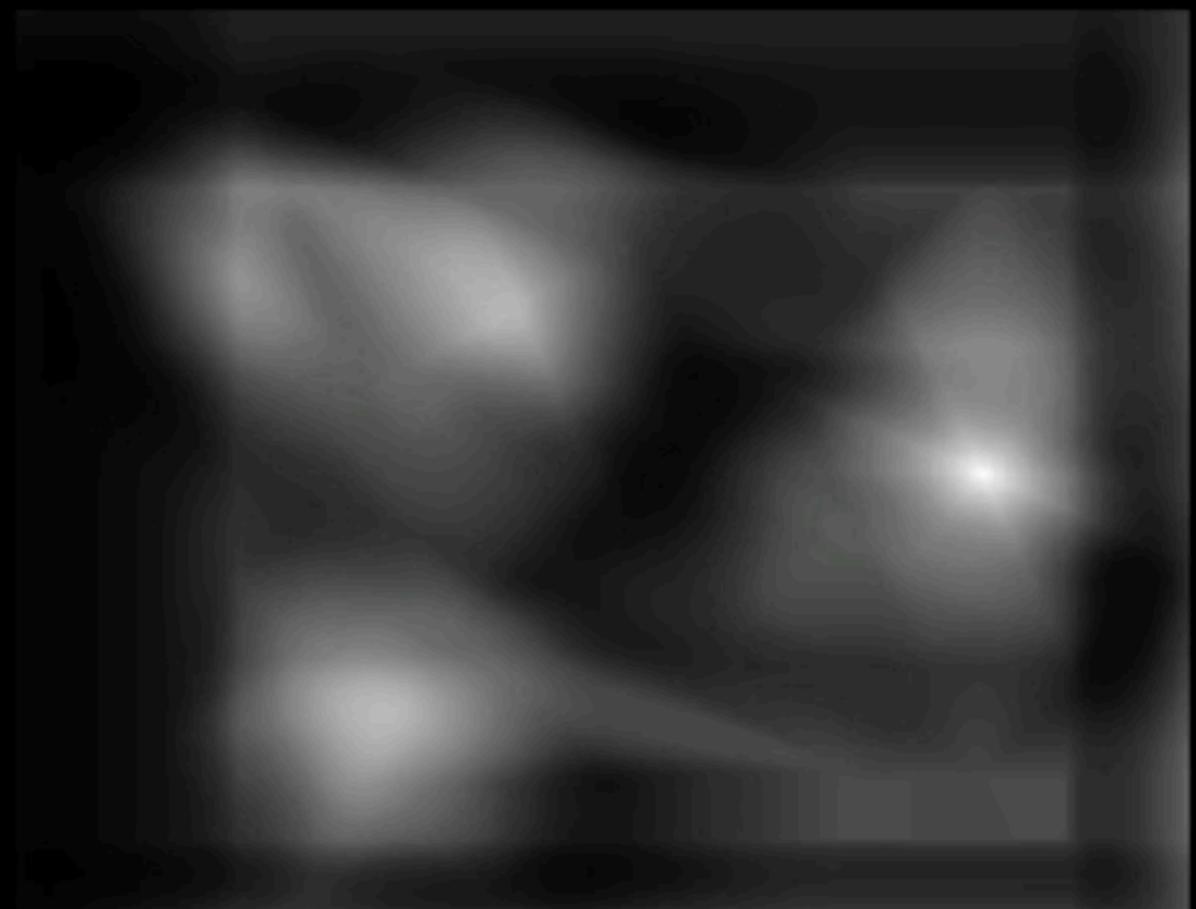
Template (mask)

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Template matching

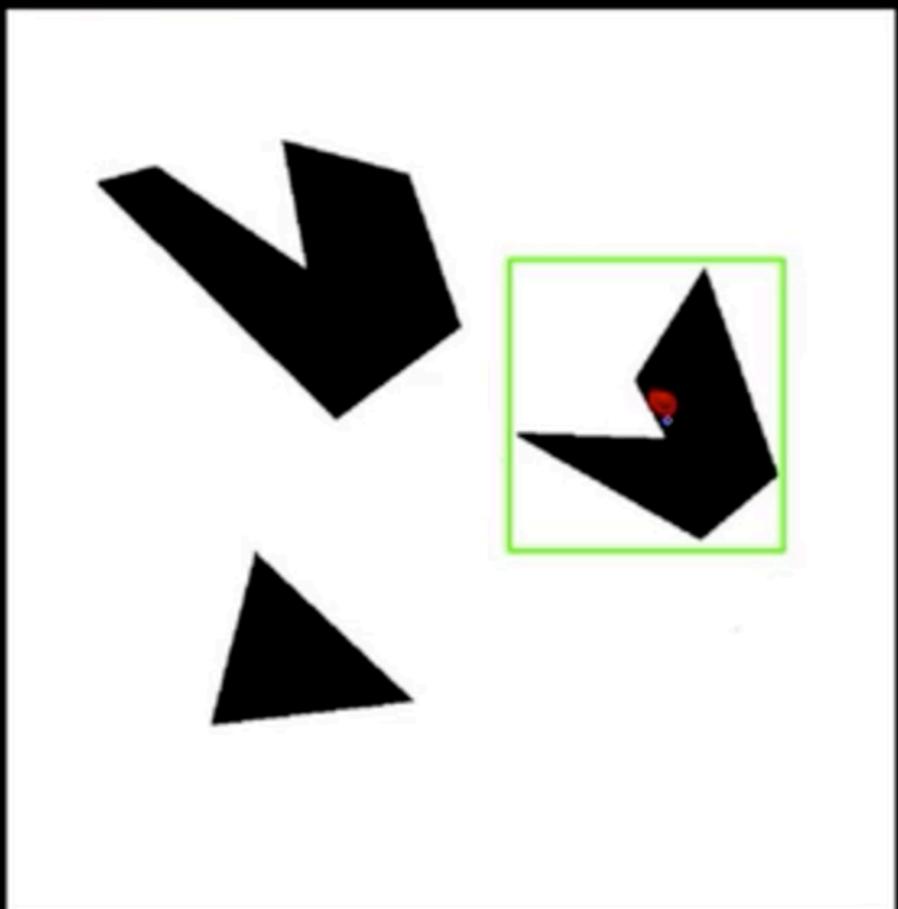


Detected template

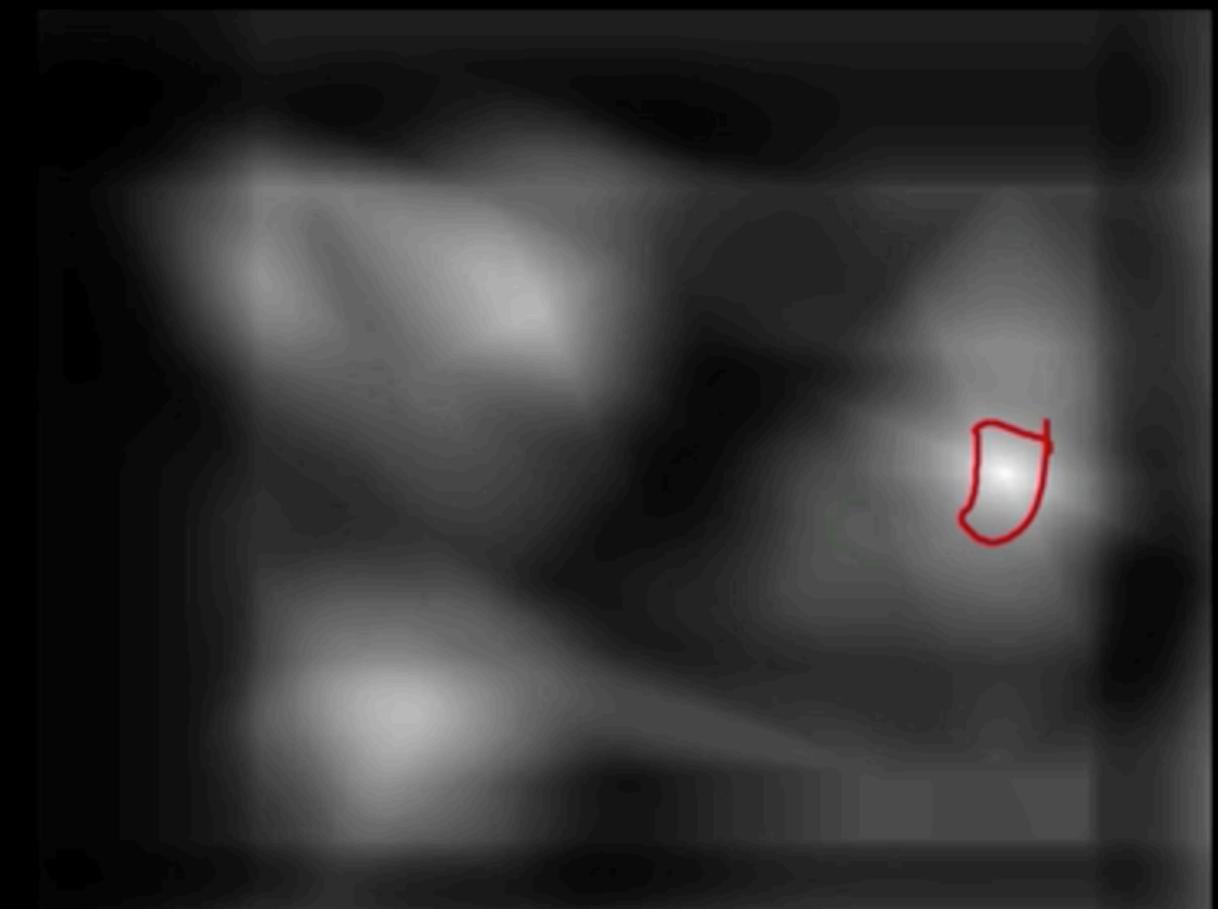


Correlation map

Template matching



Detected template



Correlation map

Template matching

What if the template is not identical to some sub-image in the scene?



Scene

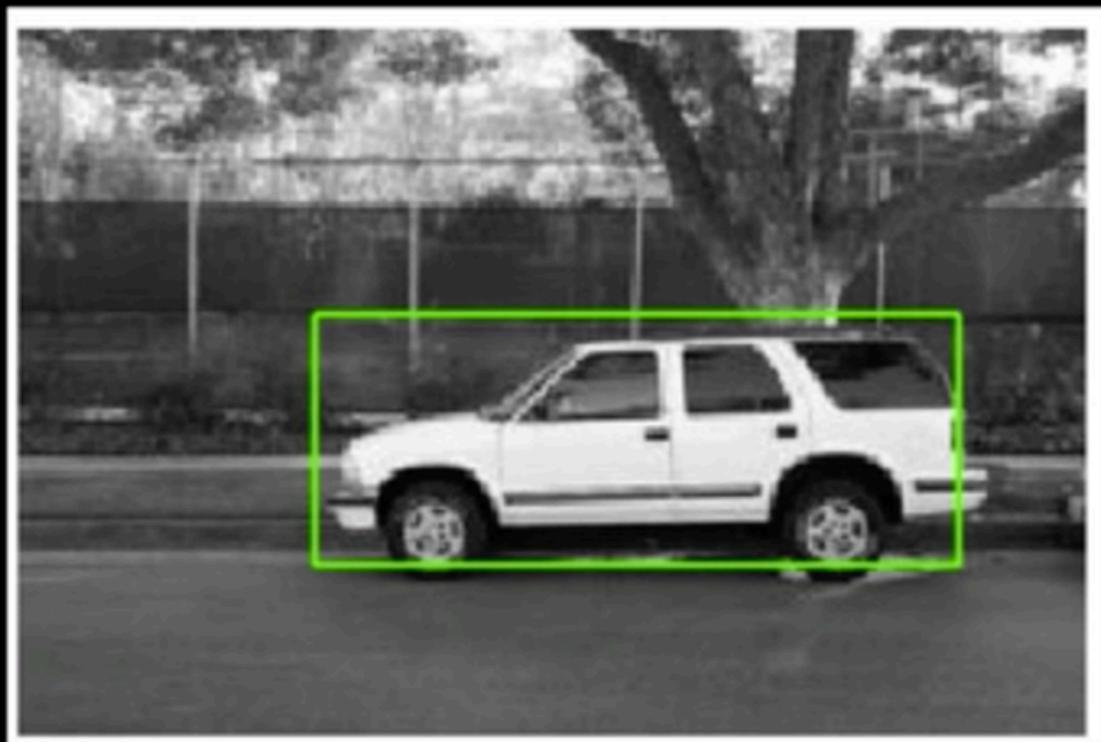


Template

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Template matching

Match can be meaningful, if scale, orientation, and general appearance is right.



Detected template



Template