NTNU

Department of Engineering Cybernetics

Examination paper for TTK4190 Guidance and Control of Vehicles

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Examination time (from-to): 09:00-13:00

Permitted examination support material: Code C

- Textbooks (or printed versions) of Fossen (2011) and Beard & McLain (2012)
- Printed lecture notes/slides.
- Printed assignments, problems and examination sheets.
- All handwritten materials are allowed.

Other information: All type of calculators is approved

Language: English

Number of pages (front page excluded): 5

Number of pages enclosed:

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig ☐ **2-sidig** ☐

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Date Signature

Figure 1: The Interdictor Star Destroyer from the movie Star Wars Rebels (2014).

Problem 1: Spacecraft Control System (25%)

The Interdictor Star Destroyer shown in Figure 1 is symmetrical about the xz- and xy-planes. The rigid-body equations of motion are:

$$\begin{aligned} m(\dot{u} - vr + wq) &= \tau_1 \\ m(\dot{v} - wp + ur) &= \tau_2 \\ m(\dot{w} - uq + vp) &= \tau_3 \end{aligned} \tag{1}$$

and

$$\begin{aligned} I_x\dot{p} + (I_z - I_y)qr + I_{yz}(r^2 - q^2) &= \tau_4 \\ I_y\dot{q} + (I_x - I_z)rp + (qp - \dot{r})I_{yz} &= \tau_5 \\ I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} &= \tau_6 \end{aligned} \tag{2}$$

1a (4%) Under which assumptions are (1)–(2) valid?

1b (4%) Eqs. (1)–(2) can be written in matrix-vector form according to:

$$\mathbf{M}_1\dot{\boldsymbol{\nu}}_1 + \mathbf{C}_1(\boldsymbol{\nu}_2)\boldsymbol{\nu}_1 = \boldsymbol{\tau}_1 \tag{3}$$

$$\mathbf{M}_2\dot{\boldsymbol{\nu}}_2 + \mathbf{C}_2(\boldsymbol{\nu}_2)\boldsymbol{\nu}_2 = \boldsymbol{\tau}_2 \tag{4}$$

Write down the expressions for the matrices $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{C}_1(\boldsymbol{\nu}_2)$ and $\mathbf{C}_2(\boldsymbol{\nu}_2)$. Also write down the expressions for the vectors $\boldsymbol{\nu}_1$,$\boldsymbol{\nu}_2$, $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$.

1c (4%) Explain why the matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ are positive definite and verify that the matrices $\mathbf{C}_1(\boldsymbol{\nu}_2)$ and $\mathbf{C}_2(\boldsymbol{\nu}_2)$ are skew-symmetric.

1d (4%) Derive a feedback linearizing controller for (3) such that the error dynamics become:

$$\dot{\tilde{\boldsymbol{\nu}}}_1 + 2\,\lambda\tilde{\boldsymbol{\nu}}_1 + \lambda^2 \int_0^t \tilde{\boldsymbol{\nu}}_1(\tau)\,\mathrm{d}\tau = \mathbf{0} \tag{5}$$

where $\lambda > 0$, $\tilde{\boldsymbol{\nu}}_1 = \boldsymbol{\nu}_1 - \boldsymbol{\nu}_{1d}$ is the tracking error and $\boldsymbol{\nu}_{1d}$ is the desired velocity.

1e (4%) Consider (3) and let:

$$\boldsymbol{\tau}_1 = -k_p\, \boldsymbol{\nu}_1 \tag{6}$$

where $k_p > 0$. Show by using a Laypunov function candidate that the equilibrium point $\boldsymbol{\nu}_1 = \mathbf{0}$ is globally exponentially stable.

1f (5%) Propose an attitude controller for the spacecraft and give an explicit formula for $\boldsymbol{\tau}_2$. You must show that the control law stabilizes the system.

Problem 2: Ship Control by Successive Loop Closure (35%)

Figure 2 shows a ship moving on a straight line. The yaw dynamics of the ship is modeled using a Nomoto model:

$$\dot{\psi} = r \tag{7}$$
$$T\dot{r} + r = K\delta + w \tag{8}$$

where ψ and r are the states, δ is the rudder angle, and w represents unmodeled dynamics and disturbances. The Nomoto time and gain constants are $T = 100$ s and $K = 0.1$ s^{-1}, respectively.

Figure 2: Ship on straight course.

2a (4%) Assume that the heading autopilot is a PD controller

$$\delta = -K_p\, e_\psi - K_d\, r, \qquad e_\psi = \psi - \psi^c \tag{9}$$

where ψ^c is the heading angle command. Derive the expressions for the two transfer functions in the expression:

$$\psi(s) = H_{\psi/\psi^c}(s)\, \psi^c(s) + H_{\psi/w}(s)\, w(s) \tag{10}$$

2b (8%) The heading autopilot should satisfy the following specifications:

i) Maximum rudder angle: $|\delta| \leq 10$ deg

ii) Maximum tracking error: $|e_\psi| \leq 1$ deg

iii) Relative damping ratio: $\zeta_\psi = 1.0$

Find the numerical values for K_p and K_d satisfying these requirements based on the method by Beard & McLain (2012).

2c (2%) What is the heading loop DC gain $K_{\psi_{DC}}$ corresponding to:

$$\frac{\psi}{\psi^c} \approx K_{\psi_{DC}} \tag{11}$$

2d (3%) Assume that the motions in heave, roll and pitch can be neglected. The ship is moving North on a straight line at constant forward speed $U = 10$ m/s. The x-axis is pointing North and the y-axis points East. Show that and specify under which assumption:

$$\dot{y} = U\,\psi \tag{12}$$

is a good approximation for the cross-track error.

2e (8%) Assume that heading autopilot (10) is working satisfactory such that $\psi/\psi^c = K_{\psi_{DC}}$. Consider the path-following controller:

$$\psi^c = -K_{p_y}\,y - K_{i_y}\int_0^t y(\tau)\,\mathrm{d}\tau \tag{13}$$

The control objective is to regulate the cross-track error to zero. Compute the controller gains K_{p_y} and K_{i_y} such that the bandwidth (natural frequencies) of the two control loops satisfies:

$$\omega_{n_y} = \frac{1}{10}\,\omega_{n_\psi} \tag{14}$$

with relative damping ratio $\zeta_y = 1.0$.

2f (3%) Set up a bullet list of sensors you need to implement the path-following controller.

2g (4%) Explain what physical effects (bullet list), which contributes to w in (8). What is key assumption when using integral action to cancel w.

2h (3%) The proposed solution is for a ship moving North. Explain how you can use vessel-parallel coordinates (x_p, y_p) instead of (x, y) to handle the case when the ship moves on a straight-line with arbitrarily direction.

Problem 3: Nonlinear Control of Autonomous Rotorcraft (15%)

Figure 3 shows a small unmanned aircraft in automatic hover. The simplified altitude error dynamics of the aircraft is given by:

$$\dot{\mathbf{\Theta}} = \mathbf{T}_\Theta(\mathbf{\Theta})\boldsymbol{\omega} \tag{15}$$

$$\mathbf{I}\,\dot{\boldsymbol{\omega}} = \boldsymbol{\tau} + \mathbf{\Delta}(\mathbf{\Theta}, \boldsymbol{\tau}) \tag{16}$$

Figure 3: Small unmanned aircraft in automatic hover.

where $\boldsymbol{\Theta} = [\phi, \theta, \psi]^\top$, $\boldsymbol{\omega} = [p, q, r]^\top$ and $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^\top$ is a vector of control inputs. The inertia matrix is denoted $\mathbf{I}$ and $\boldsymbol{\Delta}(\boldsymbol{\Theta}, \boldsymbol{\tau})$ represents the model uncertainty caused by the highly nonlinear and destabilizing effect of four rotor downwashes interacting, blade flex, and battery dynamics. We assume that $\boldsymbol{\Delta}(\boldsymbol{\Theta}, \boldsymbol{\tau})$ satisfies $\|\boldsymbol{\Delta}(\boldsymbol{\Theta}, \boldsymbol{\tau})\| \leq \delta$, where δ is known. Physically, δ represents the worst possible vertical disturbance force acting on the aircraft.

3a (6%) Show that the nonlinear controller

$$\boldsymbol{\tau} = \mathbf{I}\,\mathbf{T}_\Theta(\boldsymbol{\Theta})^{-1}\left(\ddot{\boldsymbol{\Theta}}_d - \lambda\,\dot{\tilde{\boldsymbol{\Theta}}} - \dot{\mathbf{T}}_\Theta(\boldsymbol{\Theta})\boldsymbol{\omega} - \mathbf{K}_d\,\mathrm{s}\right) \tag{17}$$

where $\mathrm{s} = \dot{\tilde{\boldsymbol{\Theta}}} + \lambda\,\tilde{\boldsymbol{\Theta}}$ is a sliding surface and $\tilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta} - \boldsymbol{\Theta}_d$ is the attitude tracking error, $\mathbf{K}_d > 0$ and $\lambda > 0$, gives the error dynamics

$$\dot{\mathrm{s}} + \mathbf{K}_d\,\mathrm{s} = \mathbf{T}_\Theta(\boldsymbol{\Theta})\,\mathbf{I}^{-1}\,\boldsymbol{\Delta}(\boldsymbol{\Theta}, \boldsymbol{\tau}) \tag{18}$$

3b (7%) Modify the control law to include a discontinuous feedback term $k_s\mathrm{sgn}(\mathrm{s})$, where $\mathrm{sgn}(\mathrm{s}) = [\mathrm{sgn}(s_1), \mathrm{sgn}(s_2), \mathrm{sgn}(s_3)]^\top$ is the vectorial (element-by-element) signum function, and find the lower bound k_s must satisfy in order to stabilize the system for a time-varying unknown term $\boldsymbol{\Delta}(\boldsymbol{\Theta}, \boldsymbol{\tau})$.

Hint: use the Lyapunov function candidate $V(\mathrm{s}) = (1/2)\,\mathrm{s}^\top\mathrm{s}$ to prove convergence of s to zero. Also notice that $\|\mathrm{s}\| = \mathrm{s}^\top\mathrm{sgn}(\mathrm{s})$ where $\|\mathrm{s}\| = |s_1| + |s_2| + |s_3|$ is the L^1 norm.

3c (2%) Explain how you can modify the control law to avoid chattering when the s elements are close to zero.

Problem 4: Ship Maneuvering (25%)

Figure 4 shows a ship maneuvering in waves. The ship is exposed to a 2-D current in the horizontal plane, which is expressed in the NED coordinate frame according to:

$$\mathbf{v}_c^n = \begin{bmatrix} V_c\cos(\beta_c) \\ V_c\sin(\beta_c) \\ 0 \end{bmatrix} \tag{19}$$

Figure 4: Ship maneuvering in waves.

where V_c and β_c denote the current speed and direction, respectively. Assume that the roll and pitch angles, ϕ and θ, are zero.

4a (2%) Find an expression for the current velocities $\mathbf{v}_c^b$ in the body-fixed coordinate system.

4b (5%) Assume that $\mathbf{v}_c^n$ = *constant* and show that the current velocities in the body-fixed coordinate system satisfies:

$$\dot{\mathbf{v}}_c^b = -\mathbf{S}(\boldsymbol{\omega}_{b/n}^b)\,\mathbf{v}_c^b \tag{20}$$

where $\boldsymbol{\omega}_{b/n}^b = [p, q, r]^\top$ and

$$\mathbf{S}(\boldsymbol{\omega}_{b/n}^b) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \tag{21}$$

Hint: $\dot{\mathbf{R}}_b^n = \mathbf{R}_b^n \mathbf{S}(\boldsymbol{\omega}_{b/n}^b)$.

4c (8%) Consider the linear maneuvering model:

$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} \delta \tag{22}$$

where v is the sway velocity, r is the yaw rate, ψ is the yaw angle and δ is the rudder angle. Modify the model (22) to include current, wind and wave forces (write down the new equation).

4d (5%) Under which assumptions is the maneuvering model including environmental disturbances valid? Set up a bullet list.

4e (5%) When simulating ocean currents the assumption $\mathbf{v}_c^n$ = *constant* can be relaxed. Propose a mathematical model, which can be used to simulate a realistic time-varying current $\mathbf{v}_c^n$ in a ship simulator.