$K = \frac{1}{2} m v^2$ $\quad (ab)^T = b^T a^T \quad a^T = a$ (a is scalar)

$$V = \frac{1}{2} \nu^T M_{RB} \nu > 0, \quad \forall \nu \neq 0$$

$$\dot{V} = \left(\frac{1}{2}\dot{\nu}^T M_{RB} \nu\right)^T + \frac{1}{2}\nu^T \underset{0}{\underset{\shortparallel}{\dot{M}_{RB}}} \nu + \frac{1}{2}\nu^T M_{RB} \dot{\nu}$$

$\dot{M}_{RB} = 0$

$M_{RB} = M_{RB}^T > 0$

$$= \frac{1}{2}\nu^T \underset{M_{RB}}{\underset{\shortparallel}{M_{RB}^T}} \dot{\nu} + \frac{1}{2}\nu^T M_{RB} \dot{\nu}$$

$$= \nu^T M_{RB} \dot{\nu} \quad \longleftarrow \quad M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_{RB}$$

$$= \nu^T \left(\tau_{RB} - \underset{0}{\underset{\shortparallel}{C_{RB}(\nu)\nu}}\right)$$

$C_{RB}(\nu) = -C_{RB}^T(\nu)$

$\nu^T C_{RB}(\nu)\, \nu \equiv 0$

$$= \nu^T \tau_{RB}$$

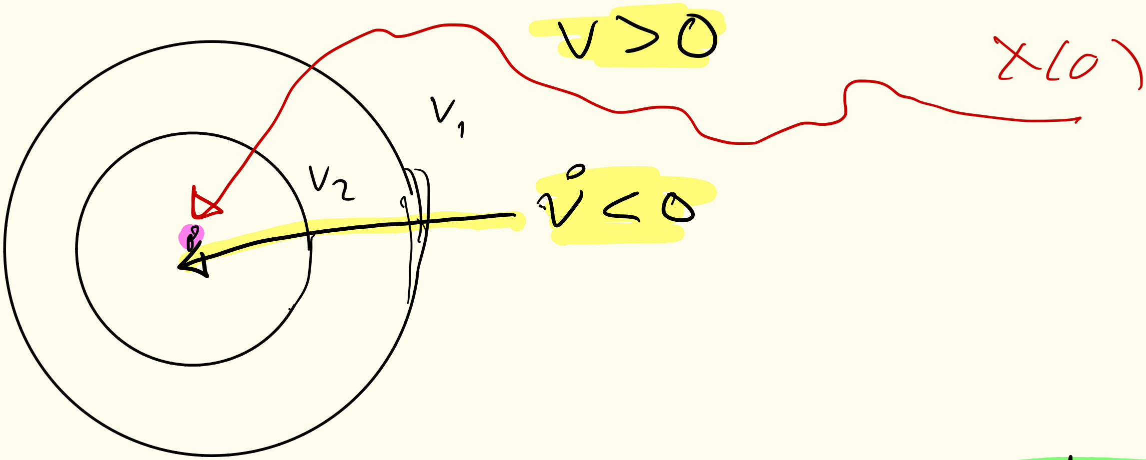
$$= -\nu^T K_D \nu < 0, \quad \forall \nu \neq 0$$

D controller

$\tau_{RB} = -K_D \nu$

$K_D > 0$

$\Longrightarrow$ Lyapunov's direct method $\Longrightarrow$ Eq. Point $\nu = 0$ is GAS



App A1.2 Lyapunov's direct method

i) $V>0$ and $V(0)=0$

ii) $\dot{V}<0$

iii) $V\to\infty$ as $\|x\|\to\infty$

then the equilibrium point $x=0$ is GAS

$$V = \frac{1}{2} x^T P x > 0 \qquad P = \begin{bmatrix} p_{11} & \\ & p_{22} \end{bmatrix} > 0$$

$$\dot{V} = -x^T Q x \le 0 \qquad Q = \begin{bmatrix} q_{11} & \\ & 0 \end{bmatrix} \ge 0$$

pos. definite

pos. semidefinite

Use

La Salle-Krasovskii theorem

$$\boxed{\begin{aligned}\dot{\eta} &= J(\eta)\, \nu \\ M_{RB}\dot{\nu} + C_{RB}(\nu)\nu &= \tau_{RB}\end{aligned}}$$

$$V = \tfrac{1}{2}\nu^T M_{RB}\nu + \tfrac{1}{2}\eta^T K_p \eta > 0 \qquad K_p = K_p^T > 0$$

$$K_d = K_d^T > 0$$

$$\dot{V} = \nu^T M_{RB}\dot{\nu} + \eta^T K_p \dot{\eta}$$

$$= \nu^T(\tau_{RB} - \underbrace{C_{RB}(\nu)\nu}_{=0}) + (\eta^T K_p J(\eta)\nu)^T$$

$$= \nu^T(\tau_{RB} + J^T(\eta) K_p \eta)$$

$$= -\nu^T K_D \nu \leq 0$$

$$\boxed{\begin{aligned}\tau_{RB} &= -J^T(\eta) K_p \eta \\ &= -K_D \nu\end{aligned}}$$

PD control law

$$V = \frac{1}{2}\begin{bmatrix}\nu & \eta\end{bmatrix}\begin{bmatrix} M_{RB} & 0 \\ 0 & K_p\end{bmatrix}\begin{bmatrix}\nu \\ \eta\end{bmatrix}$$ pos. definite

$$\dot{V} = -\begin{bmatrix}\nu & \eta\end{bmatrix}\begin{bmatrix} K_D & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}\nu \\ \eta\end{bmatrix}$$ neg. semidefinite

Krasovskii – La Salle $V>0$, $\dot V \le 0$

i) $V \to \infty$ as $\|x\| \to \infty$

ii) $\dot V \le 0$

"Get stuck analysis"

$V>0$ and $\dot V \le 0$ but will we stop when $\dot v = 0$?

$$\dot V = -v^T K_D v$$

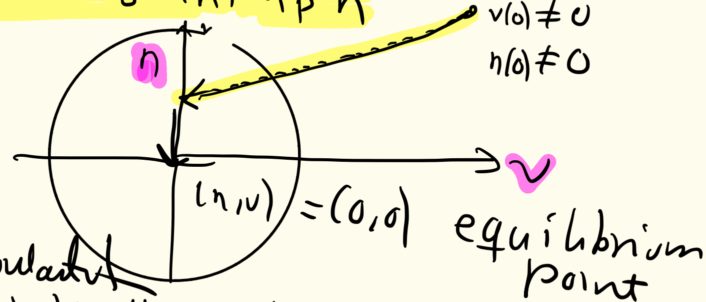
What happens if $v = 0$ then $\dot v = 0$

$$M_{RB}\dot v + C_{RB}(v)\,v = -J^T(\eta) K_p \eta - K_D v$$

$$v = 0 \Rightarrow M_{RB}\dot v = -J^T(\eta) K_p \eta$$

$$\Downarrow$$

GAS



but I have a singularity
so it is only asymptotically stable

POLE PLACEMENT PD CONTROL

$$m\ddot{x} + d\dot{x} + kx = \tau \leftarrow \tau = -k_p x - k_d\dot{x} \cancel{- k_i \int_0^t x\,dt}$$

$$m\ddot{x} + (d+k_d)\dot{x} + (k+k_p)x = 0$$

$$m(\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x) = 0$$

$$\boxed{2\zeta\omega_n \cdot m = d + k_d}, \quad \boxed{\omega_n^2 m = k + k_p}$$

$$\Rightarrow \boxed{\begin{aligned} k_p &= m\omega_n^2 - k \\ k_d &= m(2\zeta\omega_n) - d \end{aligned}} \leftarrow \omega_n, \zeta \text{ specified by user}$$