NTNU – Trondheim
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Department of Engineering Cybernetics

Examination paper for TTK4190 Guidance and Control of Vehicles

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Examination time (from-to): 09:00 – 13:00

Permitted examination support material: All printed and handwritten materials are allowed. All type of calculators is approved*.

* For *all type of calculators* the following applies:
- Calculators must not communicate with other electronic units/computers.
- The calculator must not be attached to the power outlet.
- The calculator must not make noise.
- The unit's display must be the only printing device.
- The calculator must only be one unit.
- The calculator must be in pocket size.

Language: English

Number of pages (front page excluded): 4

Number of pages enclosed:

Checked by:

Date Signature

Problem 1: Kinematics (25 %)

Consider a ship that moves on a straight line at constant forward speed and heading. The ship is exposed to a uniform constant ocean current in the horizontal plane. The GPS shows that the speed over ground is U = 10 m/s, while the gyrocompass reads 20°. A hydroacoustic position reference (HPR) system is used to measure the current speed and direction with respect to the sea floor. The current speed is V_c = 1 m/s and the direction is β_c = 40° (going to the North-East).

A. (2 %) Compute the North-East current velocities V_x and V_y and let $\dot{\boldsymbol{\eta}}_c = [V_x, V_y, 0]^T$ denote the NED current velocity vector.

B. (2 %) Compute the body-fixed current velocities u_c and v_c corresponding to the body-fixed current velocity vector $\boldsymbol{\nu}_c = [u_c, v_c, 0]^T$.

C. (2 %) Explain why the body-fixed ship velocities are $u = U$ and $v = 0$.

D. (4 %) Compute the sideslip angle β when the ship is exposed to the current. Also compute the sideslip angle for the case $u_c = v_c = 0$ and comment upon the result.

E. (3 %) Compute the course angle χ. Is it possible to measure the course angle for a ship (explain your answer)?

F. (6 %) Show that the following expressions:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu}_r + \begin{bmatrix} V_x \\ V_y \\ 0 \end{bmatrix}$$

are equal when $\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$ and $\boldsymbol{R}(\psi)$ is the yaw angle rotation matrix.

G. (6 %) Assume that the ship dynamics satisfies the linear model

$$\boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{D}\boldsymbol{\nu}_r = \boldsymbol{\tau}_{prop}$$

where $\boldsymbol{M}$ and $\boldsymbol{D}$ are the mass and damping matrices, respectively. Assume that the propulsion system $\boldsymbol{\tau}_{prop}$ fails and explain how the ship will move when exposed to currents (no wind and waves).

Problem 2: LOS guidance (20 %)

Consider the cross-track error dynamics

$$\dot{y} = U \sin\left(\chi - \gamma_p\right)$$

where $U > 0$ is the forward speed, χ is the course angle and γ_p is the path-tangential angle with respect to NED.

A. (5 %) The desired course angle is chosen as a proportional line-of-sight (LOS) guidance law:

$$\chi = \gamma_p + \arctan\left(-\frac{y}{\Delta}\right)$$

where $\Delta > 0$ is the look-ahead distance. Show that:

$$\dot{y} = -\frac{U}{\sqrt{\Delta^2 + y^2}} y$$

Hint:

$$\sin\left(\arctan(x)\right) = \frac{x}{\sqrt{1 + x^2}}$$

B. (4 %) Linearize the cross-track error dynamics and show that the equilibrium point *y = 0* is uniformly (locally) asymptotically stable. Explain how the result is affected by the choice of Δ and *U*.

C. (4 %) Propose a Lyapunov function *V* for the cross-track error and show that $\dot{V} \leq 0$. What kind of stability (local/global, asymptotic/exponential etc.) can be guaranteed by the Lyapunov analysis?

D. (5 %) Assume that course angle χ is unknown but the heading angle ψ is measured using a gyrocompass. The sideslip angle is also unknown. Explain how the LOS guidance law can be modified to use heading angle commands and at the same time compensate the unknown sideslip. Include the modified equation for the LOS guidance law.

E. (2 %) Can the LOS guidance law be used to follow a curve instead of a straight-line path? Outline your answer. No equations are required.

Problem 3: Constant Bearing Guidance (25 %)

A ship equipped with radar measures the NED position $\boldsymbol{p}_t^n = [x_t, y_t]^T$ of a second ship. The control objective is to track the target ship using a constant bearing (CB) guidance law. The velocity of the target ship is denoted $\boldsymbol{v}_t^n = [\dot{x}_t, \dot{y}_t]^T$.

A. (5 %) Assume that:

$$\dot{\boldsymbol{p}}_t^n = \boldsymbol{v}_t^n$$

$$\dot{\boldsymbol{v}}_t^n = \boldsymbol{0}$$

Write down the state-space model in matrix-vector form and design a continuous-time Kalman filter (KF) for estimation of the target velocity $\boldsymbol{v}_t^n$ using $\boldsymbol{p}_t^n$ as the only measurement. Write down the KF differential equations and the matrices needed for implementation. Indicate which parameters that are tunable.

B. (2 %) Is the Kalman filter model valid for tracking of a target ship moving at constant and/or time-varying speed? Explain why/why not.

C. (5 %) Use CB guidance and show how the desired velocity $\boldsymbol{v}_d^n$ for the intercepting ship can be chosen to achieve the goal $\boldsymbol{p}^n = \boldsymbol{p}_t^n$. Explain all variables in the control law.

D. (8 %) The intercepting ship is modeled as:

$$(m - X_{\dot{u}})\dot{u} - X_{|u|u}|u|u = T$$
$$\dot{v} = 0$$

where T is the propeller thrust. Design a feedback linearizing controller in BODY coordinates such that $u = u_d$. Finally explain how the desired velocities $\boldsymbol{v}_d^n$ in NED can be computed.

E. (5 %) Draw a block diagram containing four blocks showing the guidance systems, feedback linearizing control law, ship model and kinematics.

Problem 4: Unmanned Aerial Vehicle (UAV) (30 %)

Consider the X8 fixed-wing UAV in Figure 2. The drag and lift coefficients are given by:

$$C_L(\alpha) = 0.18\alpha - 0.007\alpha^2$$
$$C_D(\alpha) = 0.003\alpha^2$$

Figure 2. The X8 fixed-wing UAV.

where α is the angle of attack in deg. The nonlinear *longitudinal* equations of motion are:

$$\begin{aligned}
\dot{u} &= vr - wq - g\sin(\theta) + \frac{T_{prop}}{m} + \frac{\bar{q}S}{m}C_X(\alpha) \\
\dot{w} &= uq - vp + g\cos(\theta)\cos(\varphi) + \frac{\bar{q}S}{m}C_Z(\alpha) + b_1\delta_e \\
\dot{q} &= \frac{I_x - I_z}{I_y}pr + \frac{\bar{q}Sc}{I_y}\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{cq}{2V_a}\right] + b_2\delta_e \\
\dot{\theta} &= q
\end{aligned}$$

where $\bar{q} = (1/2)\rho V_a^2$ is the dynamic pressure and

$$\begin{aligned}
C_X(\alpha) &= -\cos(\alpha)C_D(\alpha) + \sin(\alpha)C_L(\alpha) \\
C_Z(\alpha) &= -\sin(\alpha)C_D(\alpha) - \cos(\alpha)C_L(\alpha)
\end{aligned}$$

A. (2 %) Compute the *stall angle*.

B. (10 %) Show that the linear pitch dynamics can be written in the form:

$$\theta(s) = \frac{a_{\theta_3}}{s^2 + a_{\theta_1}s + a_{\theta_2}}\left(\frac{1}{a_{\theta_2}}d_{\theta_3} + \delta_e(s)\right)$$

by writing down the expressions for the parameters in the transfer function.

C. (10 %) Design a pitch attitude hold control system, which guarantees that

$$H_{\theta/\theta^c}(s) = K_{\theta_{DC}}$$

in steady state. You must include the equation for the controller and show how you specify the controller parameters. How is d_{θ_2} compensated?

D. (5 %) Draw a block diagram showing how altitude hold can be implemented using commanded pitch (successive loop closure). Explain how the bandwidth separation between the loops is specified.

E. (3 %) Explain what kind of sensors you need to implement the pitch and altitude control systems (give the answer as a bullet list).