TTT4275 Basic Statistics and Linear Algebra

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Lecturer: Magne Hallstein Johnsen,

IES, NTNU

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Basic statistics 1

- Joint and conditional densities/probabilites and Bayes law
  - Definitions
    - Prior probability $P(\omega_i)$
    - A posteriori probability $P(\omega_i/x)$
    - Class-independent density $p(x)$
    - Class-dependent density $p(x/\omega_i)$
  - Bayes law :

    $p(\omega_i, x) = P(\omega_i/x)p(x) = p(x/\omega_i)P(\omega_i) \Rightarrow$

    $P(\omega_i/x) = p(x/\omega_i)P(\omega_i)/p(x)$
  - $p(x) = \sum_i p(\omega_i, x)$
  - $P(\omega_i) = \int_{-\infty}^{\infty} p(\omega_i, x)dx$

Basic statistics 2 - moments

- Mean of a variable : $m_x = E[x] = \int_{-\infty}^{\infty} xp(x)dx$
- Mean of a sequence : $m_x = E[x(n)] = lim_{N\to\infty} \frac{!}{2N+1} \sum_{n=-N}^{N} x(n)$
- Correlation between two sequences :

$$\gamma_{xy}(l) = \sum_{-\infty}^{\infty} x(n)y(n+l) \quad l = -\infty, \infty$$

- Autocorrelation : $\gamma_{xx}(l)$
- Covariance matrix : $C_x \Leftrightarrow C[m,k] = \gamma_{xx}(m-k) - m_x^2$

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Linear Algebra

- Matrix vector equation : $y = Ax$

- If A is quadratic the inverse may exits $\Rightarrow$ $x = A^{-1}y$

- A general matrix A is rectangular with dimension $[M, N]$

  - The transposed A^T is defined by $A^T[m, k] = A[k, m]$ i.e. has dimension $[N, M]$

  - We define two quadric matrixes as follows :

$$B = A^T A \quad \text{with dimension } [N, N]$$
$$C = AA^T \quad \text{with dimension } [M, M]$$

  - The inverse of a rectangular matrix A does not exist. However the matrix with least square error (also called pseudo-inverse) $\hat{A}^{-1}$ have dimension $[N, M]$ and is given by either

$$\hat{A}^{-1} = \begin{cases} A^T C^{-1} & \text{for } M < N \\ B^{-1} A^T, & \text{for } N < M \end{cases}$$