Lecture 16: Friction and Electromechanical systems

- Dynamic friction models

- Electrical motors
- DC motor with constant field
- Some network modeling, passivity, ...

Book: 3.2, 3.3, 5

Static friction models

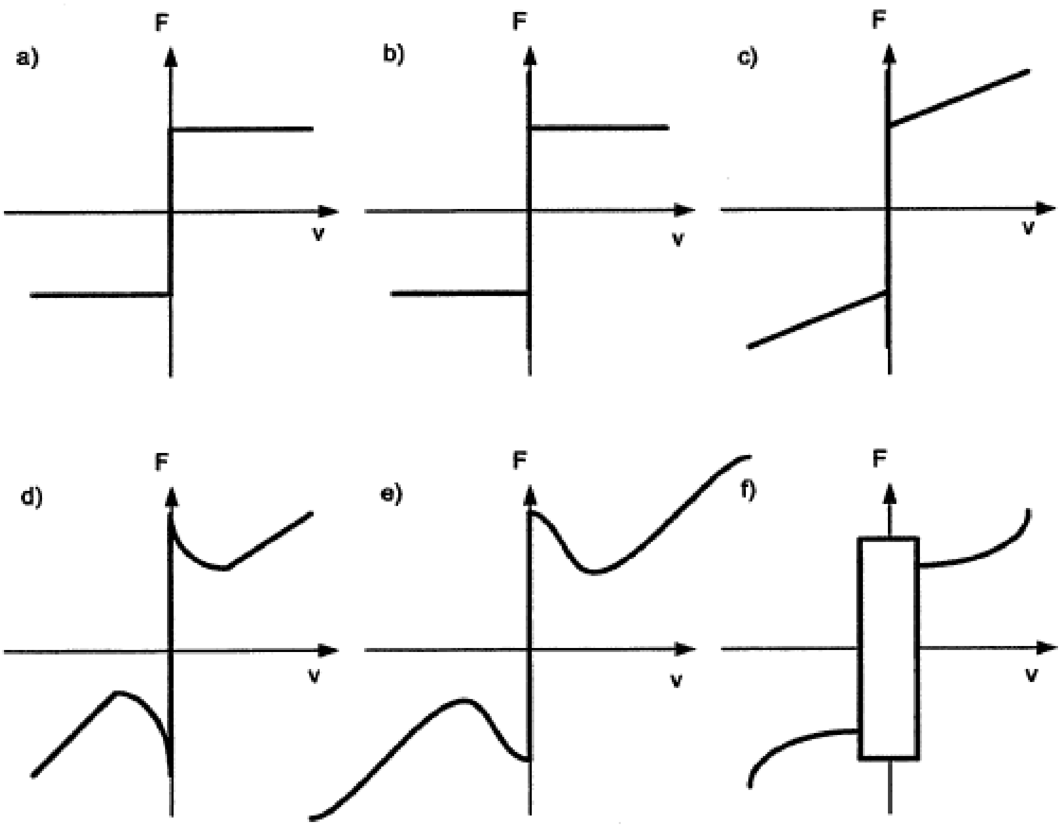
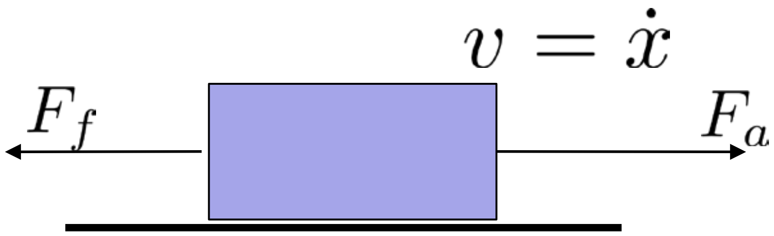
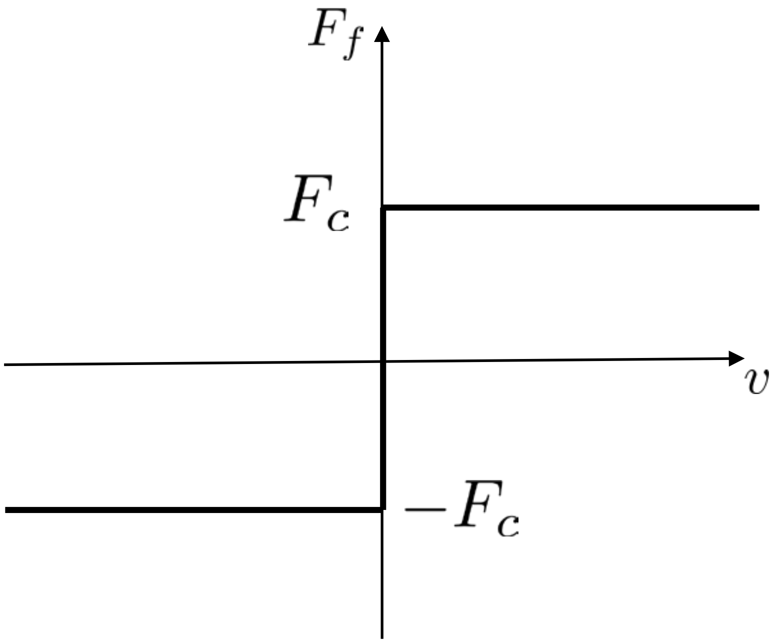


Figure 5.3: Static friction models: a) Colomb friction b) Coulomb+stiction c) Coulomb+stiction+viscous d) Stribeck effect e) Hess and Soom; Armstrong f) Karnopp model

Problems with the signum terms at zero velocity I





Newton's law:

$$m\dot{v} = F_a - F_f$$

$$F_f = F_c \,\mathrm{sign}(v) = \begin{cases} -F_c, & v < 0 \\ 0, & v = 0 \\ F_c, & v > 0 \end{cases}$$

Karnopp's model

$$m\dot{v} = F_a - F_f \qquad \begin{cases} F_a + F_c & v < 0 \\ F_a - F_c & v > 0 \end{cases}$$

We want $m\dot{v} = 0$ if $v = 0$ and $|F_a| < F_c$

Therefore $F_f = F_a$ if $v = 0$ and $|F_a| < F_c$

$$F_f = \begin{cases} \mathrm{sat}(F_a, F_c) \; ; & v = 0 \\ F_c \, \mathrm{sgn}(v) \; ; & v \neq 0 \end{cases}$$

Dynamic friction models

The Dahl model

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \sigma \left(v - |v| \frac{F}{F_c} \right)$$

The LuGre model

$$F = \sigma_0 z + \sigma_1 \frac{\mathrm{d}z}{\mathrm{d}t} + \sigma_2 v$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = v - \sigma_0 \frac{|v|}{g(v)} z$$

$$g(v) = F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2}$$

Why dynamic friction models?

- Easier to simulate
- Easier to analyze
- They reproduce (to some extent) dynamic friction phenomena
  - Presliding displacement
    - friction force act as a spring in sticking region
  - Frictional lag
    - Dynamic friction force depends on direction of velocity
  - Varying break-away force
    - Break-away force depends on rate-of-change of applied force

Dahl's model $\dfrac{\mathrm{d}F}{\mathrm{d}t} = \sigma\left(v - |v|\dfrac{F}{F_c}\right)$

steady-state (constant velocity)

$$\frac{dF}{dt} = 0 \rightarrow v - |v|\frac{F}{F_c} = 0$$

Coulomb

$$\Rightarrow F = F_c \frac{v}{|v|} = F_c \,\mathrm{sgn}(v)$$

dynamics: $\dfrac{dF}{dt} = \sigma\left(v - |v|\dfrac{F}{F_c}\right)$

$$\dot{y} = -1/T\, y + u \rightarrow \frac{y}{u} = \frac{1}{1+Ts}$$

Friction models have "time constant" $T = \dfrac{F_c}{\sigma |v|}$; $T \to \infty$ if $v \to 0$

+ good to simulate (no discont.)

- can drift in "stick" area ($v=0$)

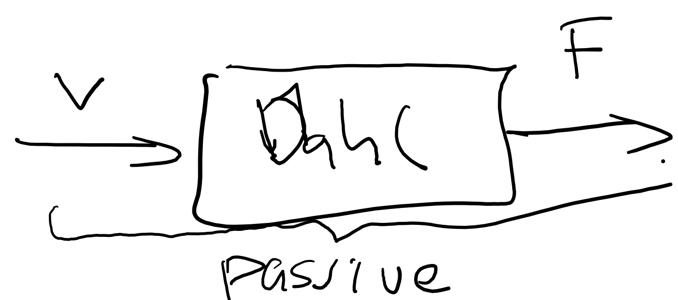
Passivity of Dahl's model $\frac{\mathrm{d}F}{\mathrm{d}t} = \sigma\left(v - |v|\frac{F}{F_c}\right)$

storage function: $V = \frac{1}{2\sigma} F^2$

$$\dot{V} = \frac{1}{\sigma} F \dot{F} = \frac{1}{\sigma} F \sigma \left(v - |v| \frac{F}{F_c}\right)$$

$$= F \cdot v - \frac{F^2}{F_c} |v|$$

$$= u \cdot y - g(x)$$



LuGre model I

$$F = \sigma_0 z + \underbrace{\sigma_1 \dot{z}}_{\text{often } \sigma_1 = 0} + \underbrace{\sigma_2 v}_{\text{viscous function}}$$

$$\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z \qquad g(v) = F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2}$$

steady-state (constant velocity)

$$\dot{z} = 0 \quad \rightarrow \quad v - \sigma_0 \frac{|v|}{g(v)} z_{ss} = 0$$

$$\rightarrow \quad z_{ss} = \frac{g(v)}{\sigma_0} \frac{v}{|v|} = \frac{g(v)}{\sigma} \operatorname{sgn}(v)$$

$$F_{ss} = \underbrace{\left[F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2} \right] \operatorname{sgn}(v)}_{\text{Stribeck}} + \underbrace{\sigma_2 v}_{\text{viscous}}$$

LuGre model II

- «time constant»:

$$T = \frac{g(v)}{\sigma_0 |v|} \to \infty, \quad \text{if} \quad |v| \to 0$$

- Same advantageous/disadvantegous as Dahl's model
- Possible more realistic dynamic behaviour
- LuGre-model is passive from v to F if σ_1 is small enough

Control systems with friction, II

Friction can be used to control motion

- Electronic stability control (ESC), ”anti-skidding”

Without ESC:

With ESC:

- Also ABS systems exploits friction characteristics

ABS-system – blokkeringsfrie bremser

- Hva er det som gjør at **bremsing**, **gass**, **styring** får bilen til å endre hastighet?

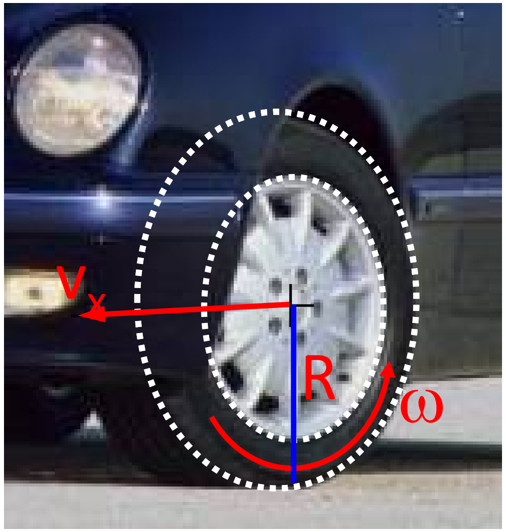
Friksjon mellom hjul og vei

- Hva bestemmer friksjon?
  - Tyngde
  - Underlag og egenskaper ved dekk
    - tørr asfalt, våt asfalt, snø, is
  - **Relativ hastighetsforskjell** mellom bil og hjul
    - langsgående (longitudinal) slipp, side- (lateral) slipp

Slipp – relativ hastighetsforskjell

- I langsretning:

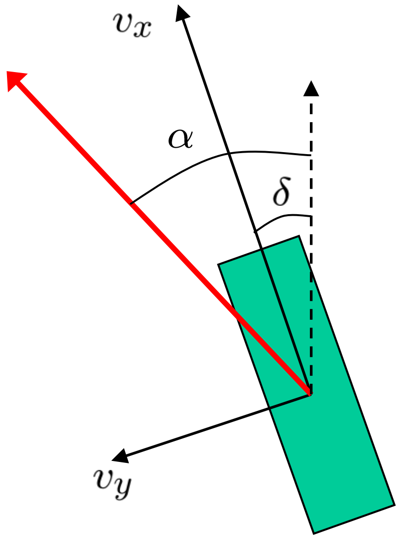
$$\lambda_x := \frac{v_x - R\omega}{v_x}$$



- I sideretning:

$$\lambda_y := \sin\alpha$$

$$\alpha := \arctan\frac{v_y}{v_x}$$

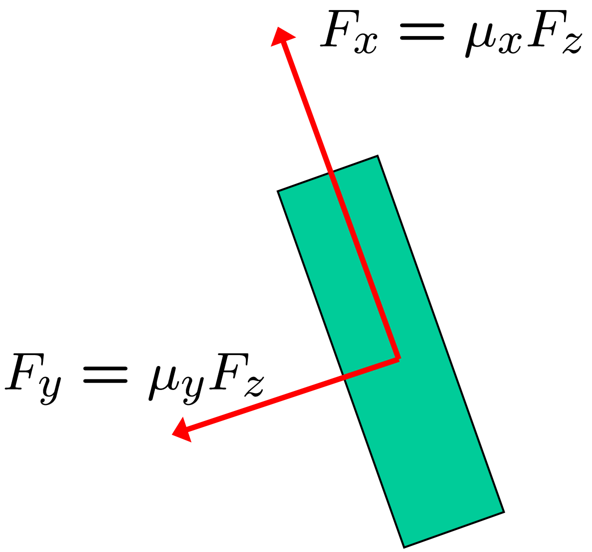


Friksjonskrefter

Coloumbs lov:

- Friksjonskrefter gitt av vertikale krefter og friksjonskoeffisient

- Friksjonskoeffisient gitt av slipp og underlag

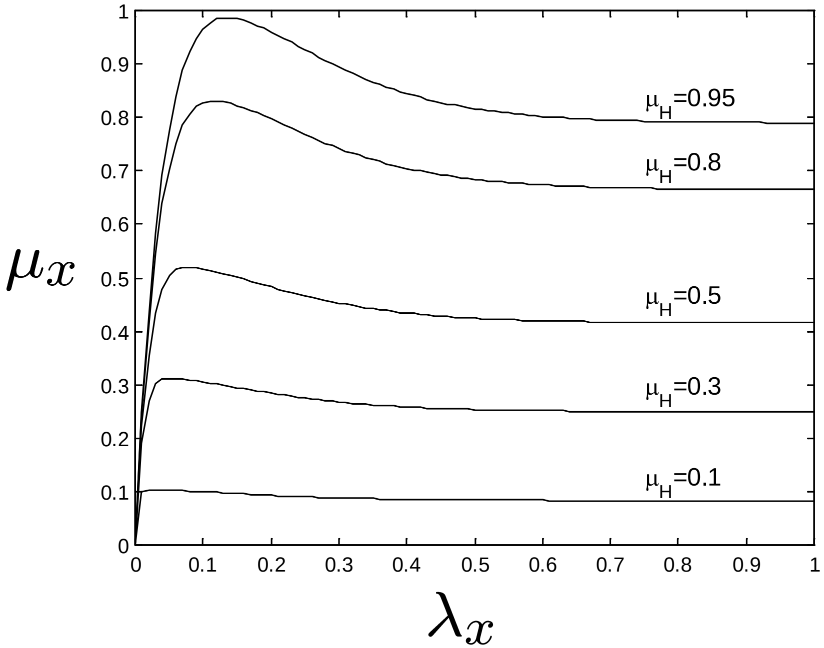


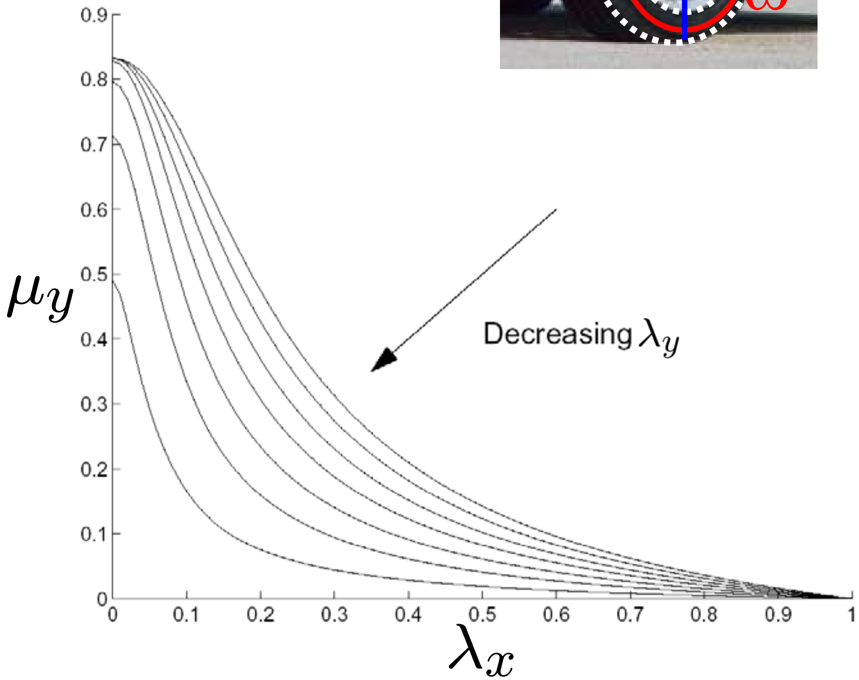
$$\mu_x \approx \mu_x(\lambda_x, \lambda_y, \mu_H)$$
$$\mu_y \approx \mu_y(\lambda_y, \lambda_x, \mu_H)$$

Friksjonskoeffisienter under bremsing

- Bremsing reduserer hjulhastighet i forhold til bilhastighet

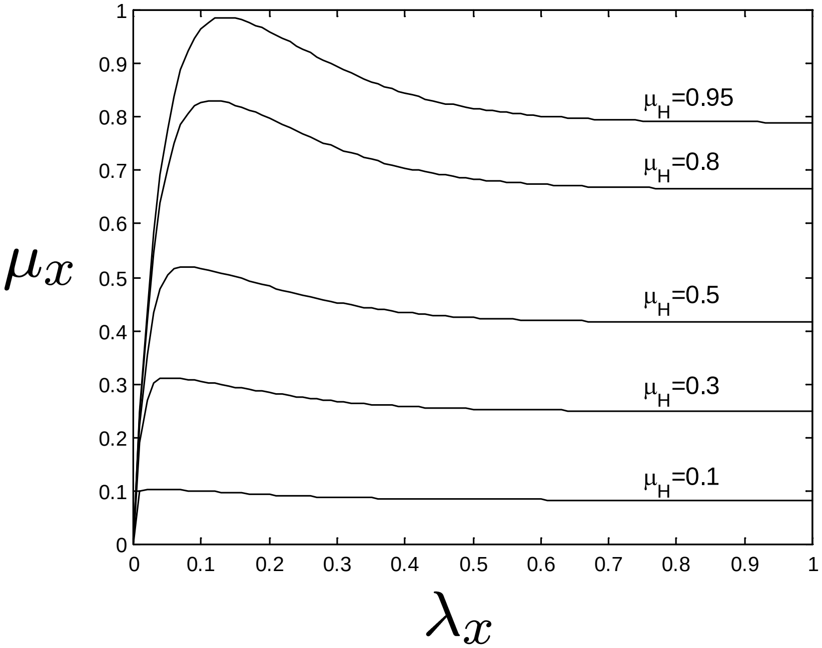
$$\lambda_x := \frac{v_x - R\omega}{v_x}$$

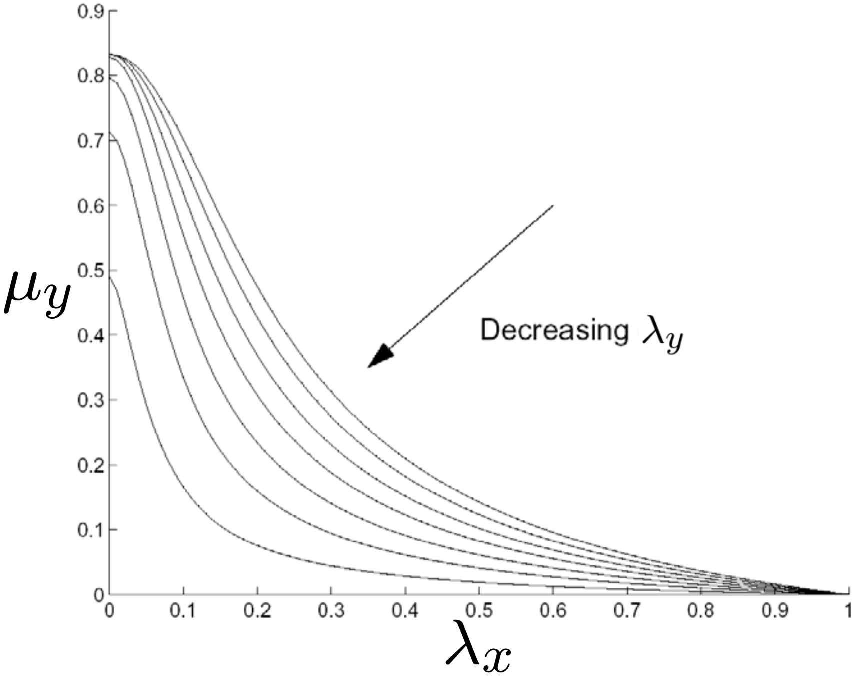




Blokkeringsfrie bremser – ABS

- Ønsker konstant lav slipp under bremsing fordi
  - Det gjør bremsing mest effektivt
  - Kan styre bilen under bremsing





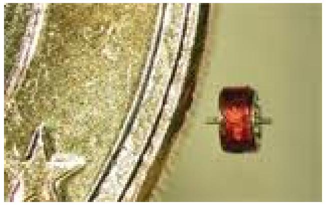
ABS i praksis

Bremselengde:

Unnamanøver:

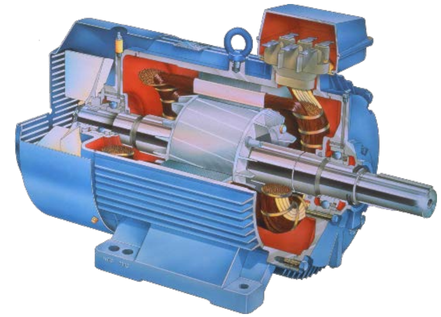
Why modeling of electrical motors?

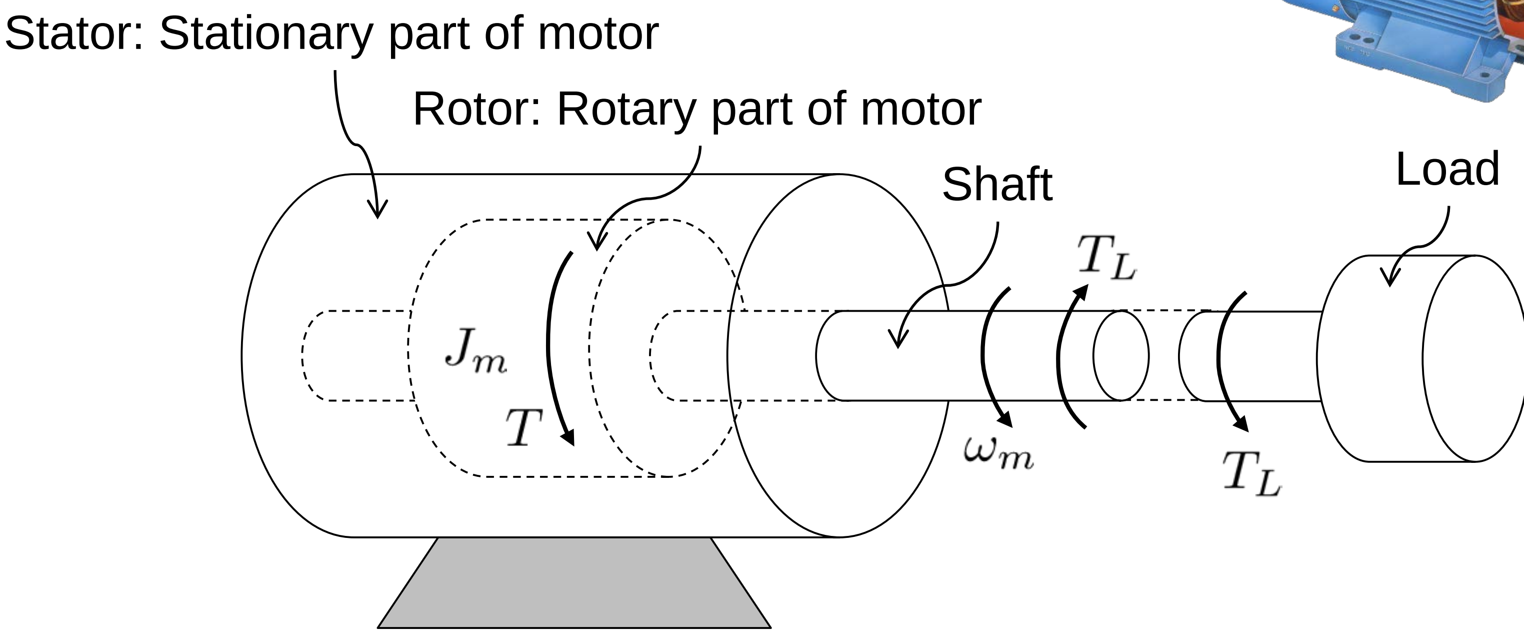
- Electrical (and hydraulic) motors are used when something should move
  - Used everywhere: Process industries, offshore oil&gas production, electromechanical systems, cars ...
  - Large and small
  - Often actuator (e.g. in a valve, in a compressor, ...)



- Example of modeling across domains (electrical + mechanical), and network modeling
  - Hydraulic motors another example (Ch. 4)
- Example of control-relevant modeling
  - Linear (transfer function) modeling

Motors





- Equation of motion for motor shaft:

$$J_m \dot{\omega}_m = T - T_L$$

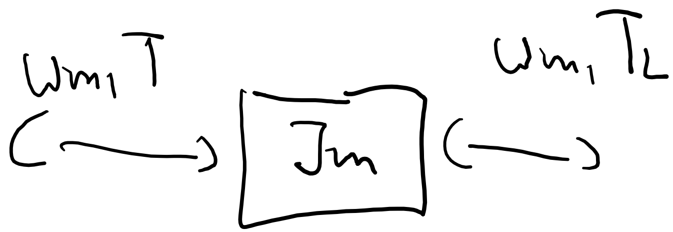
where

- T : Motor torque (set up by some device, e.g. DC motor)
- T_L : Load torque
- J_m : Moment of inertia for rotor and shaft
- ω_m: Angular velocity/motor speed [rad/s, or rev./min]

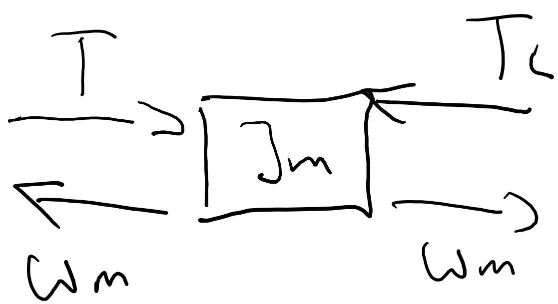
Mechanical Power

$$P_m = T \cdot \omega_m$$ (motor → shaft)

$$P_L = T_L \cdot \omega_m$$ (rotor → load)



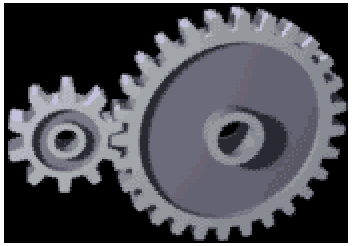
object-oriented



signal flow

Gears

Rotational gear (cogwheel)



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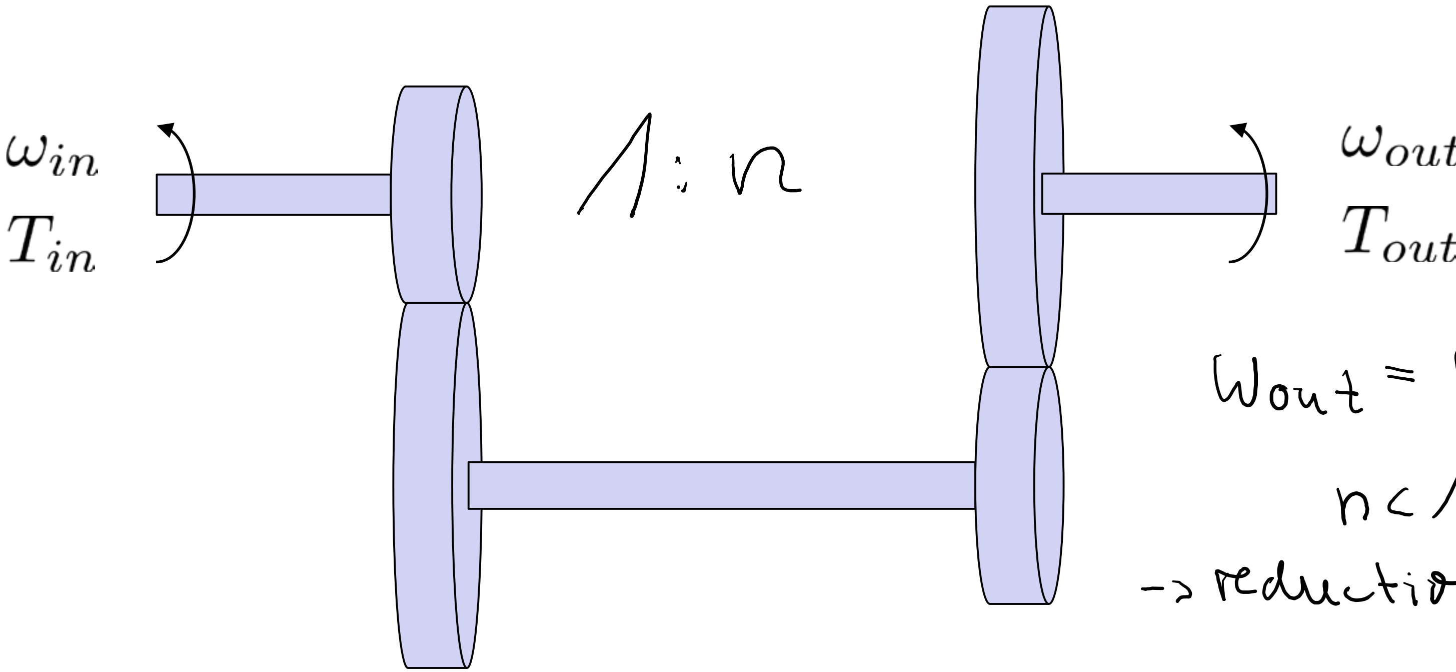
Gear model



$W_{out} = n \cdot W_{in}$

$n < 1$

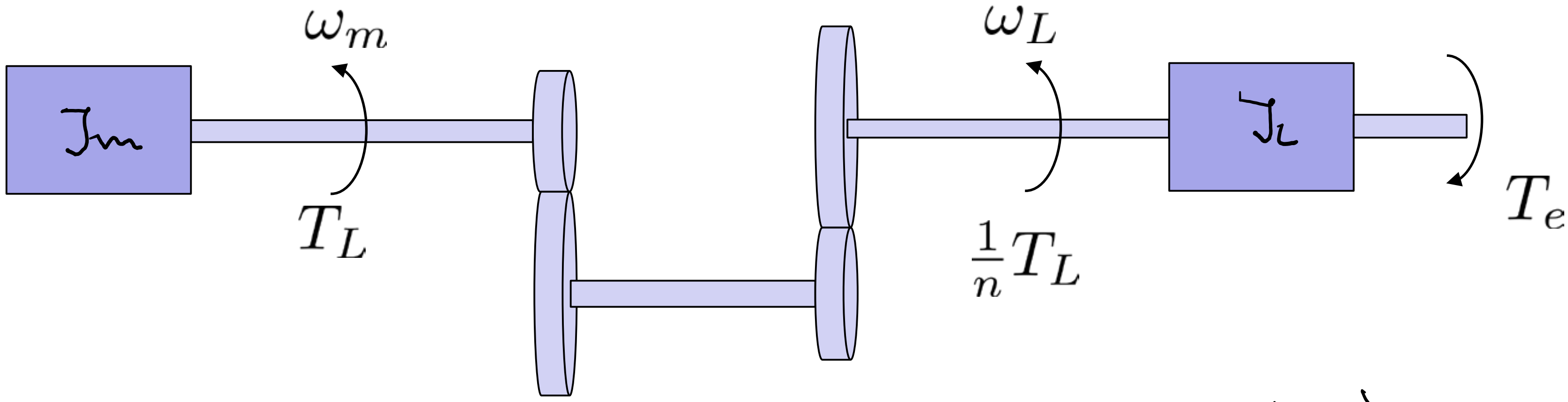
-> reduction

loseless: $\text{Power}_{in} = \text{Power}_{out}$

$[Nm \cdot \frac{1}{s} = \frac{J}{s} = W]$ $T_{in} \cdot \omega_{in} = T_{out} \cdot \omega_{out}$

$T_{out} = \frac{1}{n} T_{in}$

Motor + Gear I



$J_m \cdot \dot{\omega}_m = T - T_L \quad (1) \quad$ (motor)

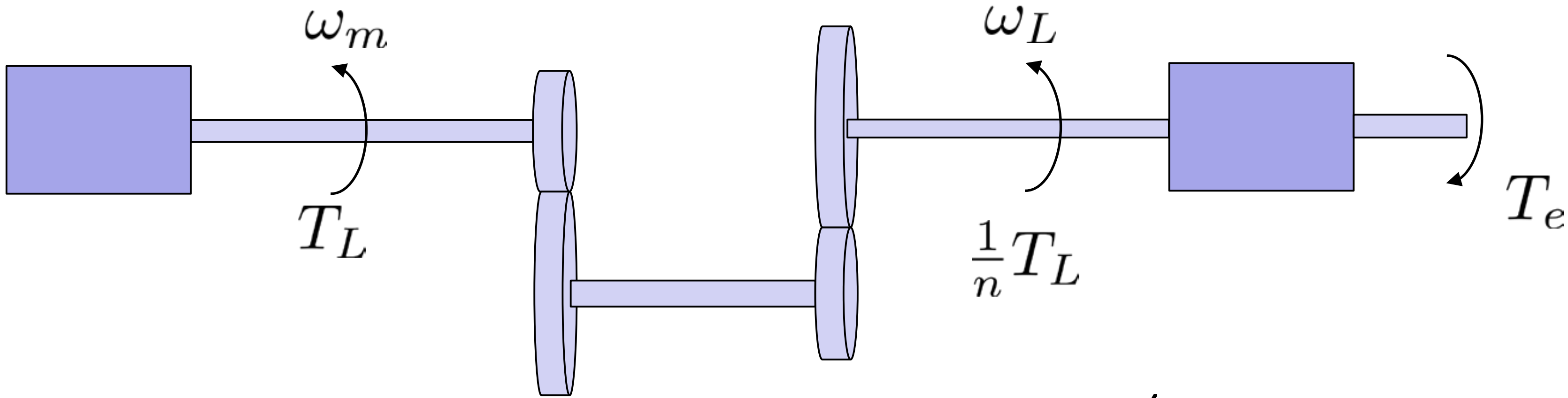
$J_L \cdot \dot{\omega}_L = \frac{1}{n} T_L - T_e \quad (2) \quad$ (load)

algebraic connection $\quad \omega_L = n \cdot \omega_m$

(difficult in simulate)

Motor + Gear II

$$\xleftrightarrow{\omega_m,\,T} \boxed{J_m} \xleftrightarrow{\omega_m,\,T_L} \boxed{\text{Gear}} \xleftrightarrow{\omega_L,\,\frac{1}{n}T_L} \boxed{J_L} \xleftrightarrow{\omega_L,\,T_e}$$



rewrite (2) : $J_L\, n\, \dot{\omega}_m = \frac{1}{n} T_L - T_e$ (3)

(1) + (3) $(J_m + J_L n^2)\,\dot{\omega}_m = T - T_e \cdot n$

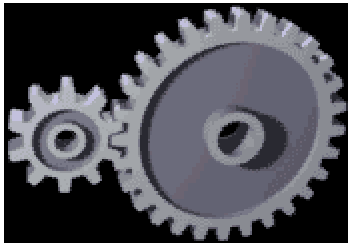
[motor side]

or

$$\left(\frac{1}{n^2} J_m + J_L\right)\dot{\omega}_L = \frac{1}{n} T - T_e$$

Gears

Rotational gear
(cogwheel)



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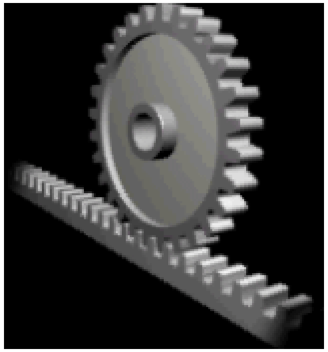
Translational gear
(rack and pinion)



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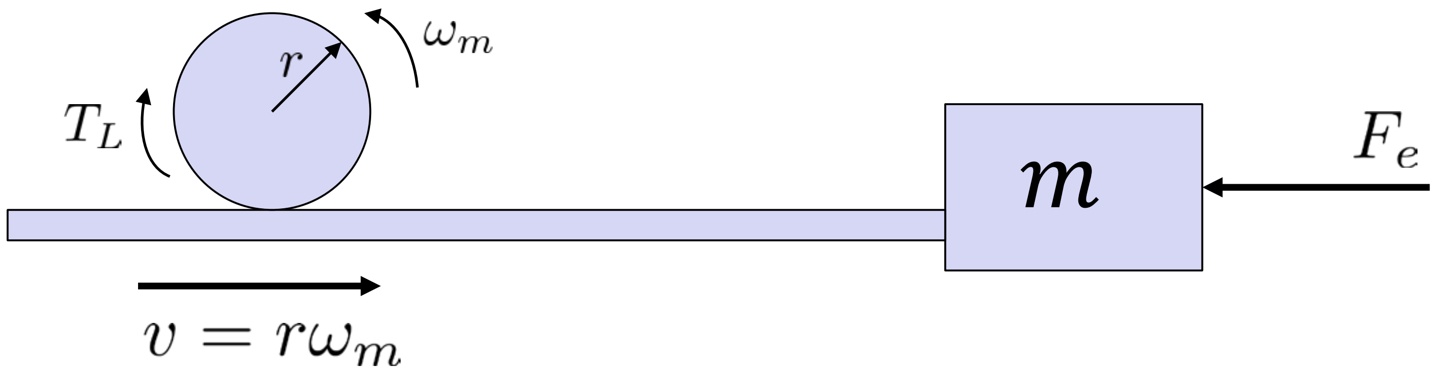
Translational gear I

T, ω_m ⟷ $\boxed{J_m}$ ⟷ T_L, ω_m ⟷ $\boxed{\text{"gear"}}$ ⟷ F, v ⟷ $\boxed{m}$ ⟷ F_e, v



lossless:

$$\omega_m \cdot T_L = v \cdot F \qquad \left[\frac{m}{s} \cdot N = \frac{Nm}{s} = W \right]$$

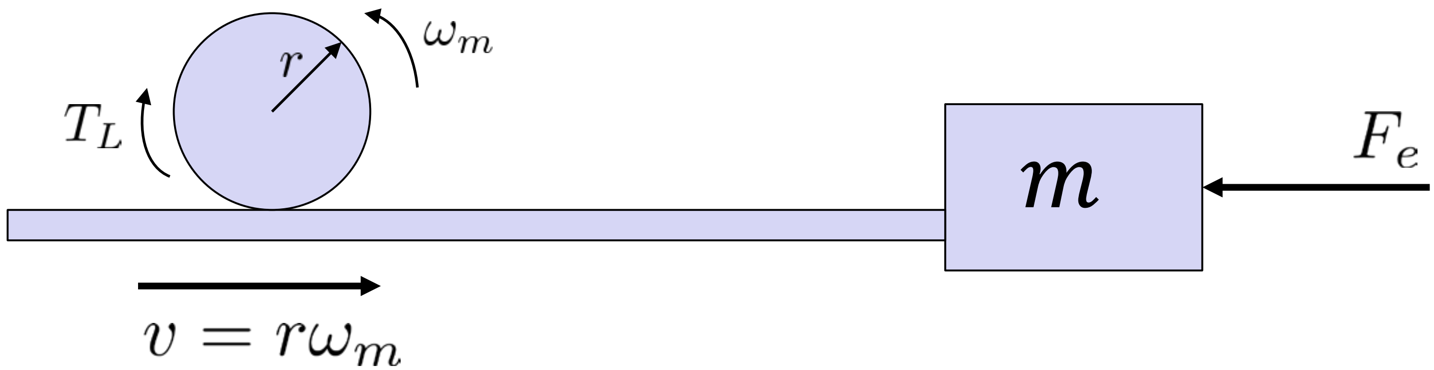
$$= r \cdot \omega_m \cdot F$$

$$F = \frac{1}{r} T_L$$

$$J_m \cdot \dot{\omega}_m = T - T_L$$

$$m \cdot \dot{v} = F - F_e$$

Translational gear II



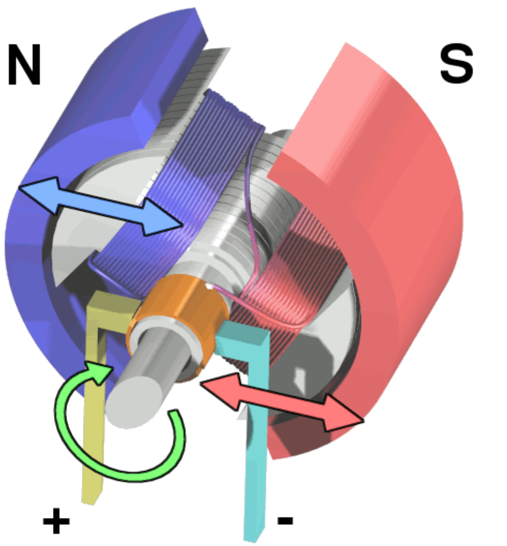
$$m r \dot{\omega}_m = \frac{1}{r} T_L - F_e$$

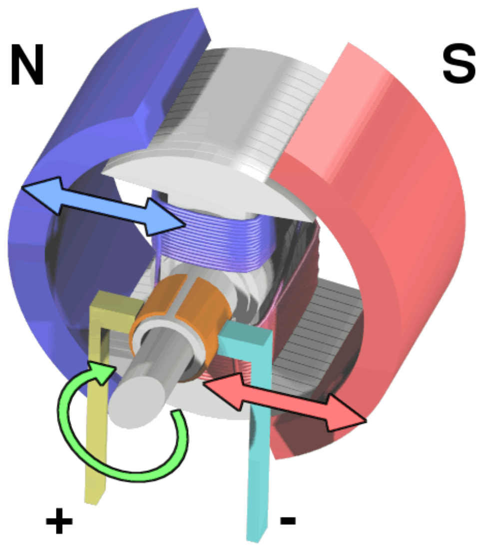
$$\Rightarrow \quad (J_m + m r^2)\dot{\omega}_m = T - r F_e$$

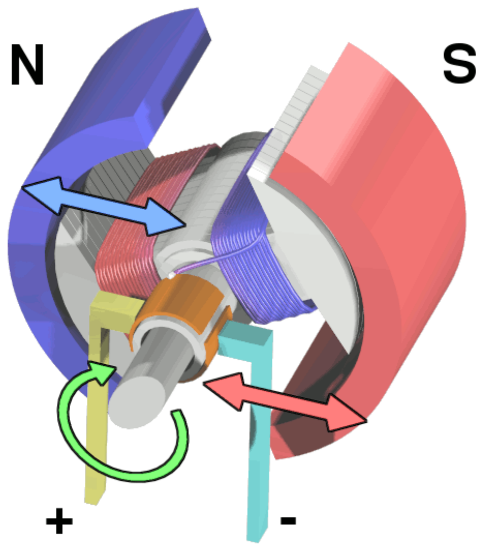
or

$$\left(\frac{1}{r^2} J_m + m\right)\dot{v} = \frac{1}{r} T - F_e$$

A simple DC electric motor







A simple DC electric motor. When the coil is powered, a magnetic field is generated around the armature. The left side of the armature is pushed away from the left magnet and drawn toward the right, causing rotation.

The armature continues to rotate.

When the armature becomes horizontally aligned, the commutator reverses the direction of current through the coil, reversing the magnetic field. The process then repeats.

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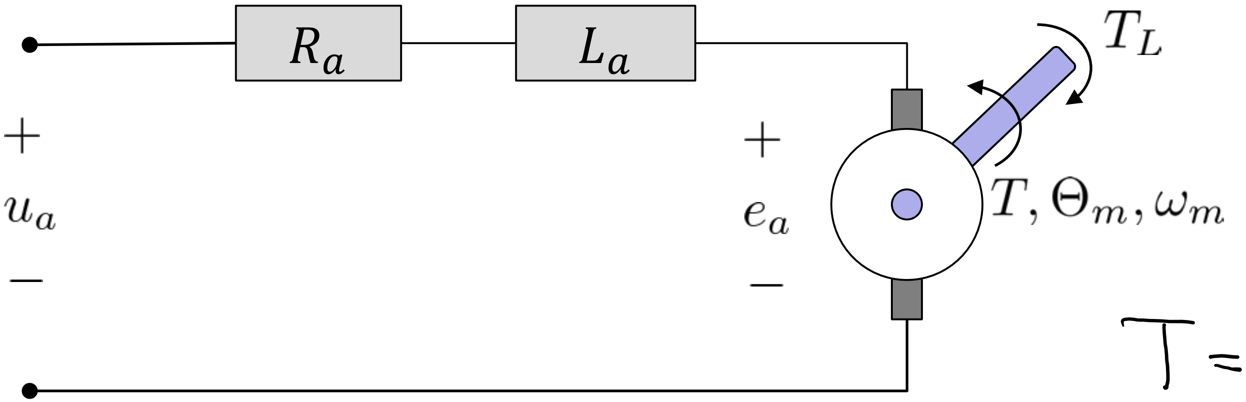
Armature circuit I



$$T = K_T \cdot i$$

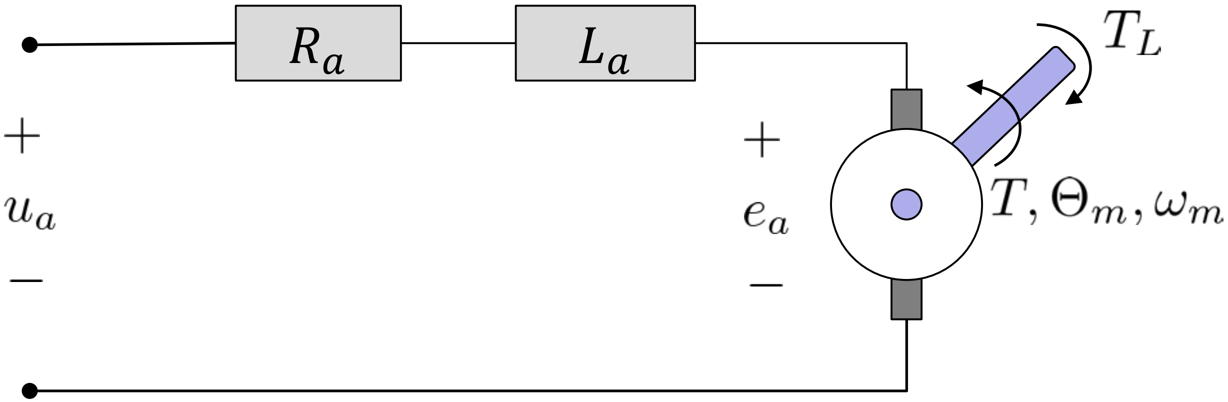
Kirchhoff's 2nd law

$$u_a = R_a \cdot i_a + L_a \frac{di_a}{dt} + e_a$$

motor equation

$$J_m \dot{\omega}_m = T - T_L$$

Armature circuit II



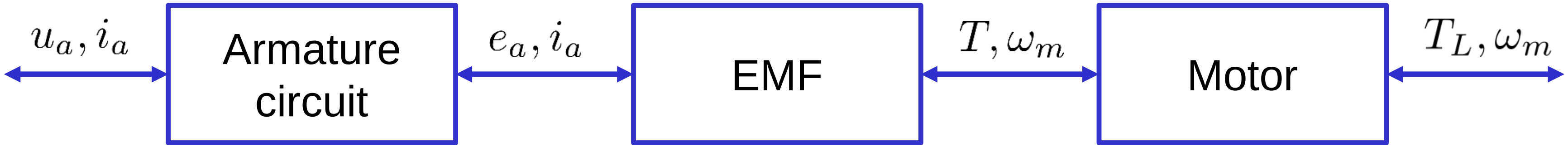
lossless / no energy storage in electro-mechanical energy units

$$P = P_m$$

$$i_a \cdot e_a = T \cdot \omega_m = K_T \cdot i_a \cdot \omega_m$$

$$\Rightarrow \quad e_a = K_E \, \omega_m \qquad K_E = K_T$$

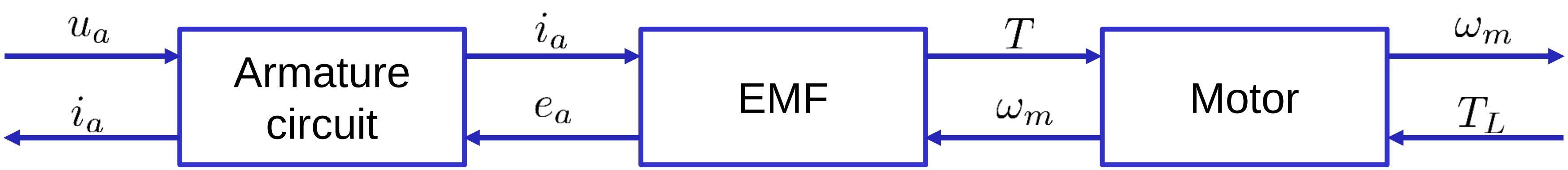
Network modeling of DC-motor:



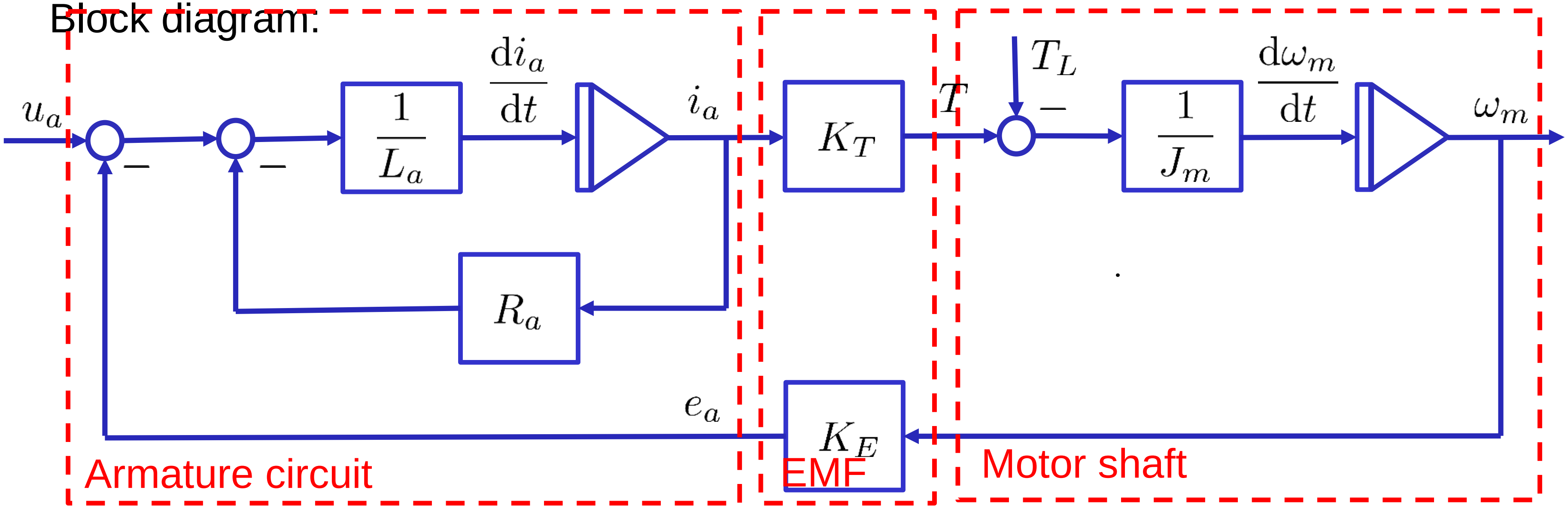
$$L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} = -R_a i_a - e_a + u_a \qquad T = K_T i_a \qquad J_m \frac{\mathrm{d}\omega_m}{\mathrm{d}t} = T - T_L$$

$$e_a = K_E \omega_m$$

Signal flow modeling of DC-motor:



Block diagram:



Dymola Demo: Motor Drive

- File -> Demos -> Motor Drive
- Modelica.Electrical.Machines

Passivity

- A system with input u and output y is passive if

$$\int_0^t y(\tau)u(\tau)d\tau \geq -E_0$$

for all $t \geq 0$, for all input trajectories.

- If the product yu has power as unit, then if
  - $\int_0^t y(\tau)u(\tau)d\tau \geq 0$: Energy is absorbed within the system, nothing delivered to the outside
  - $\int_0^t y(\tau)u(\tau)d\tau \geq -E_0$: Some energy can be delivered to the outside, limited (typically) by the initial energy in the system.
  - $\int_0^t y(\tau)u(\tau)d\tau \rightarrow -\infty$: There is an inexhaustible energy source in the system. Not passive!

Storage function

- We can proof passivity via the storage function
- Consider the system:

$$\dot{x} = f(x, u)$$
$$y = h(x)$$

- Assume we have a storage function $V(x) \geq 0$ and a dissipation function $g(x) \geq 0$
- Such that the time derivative for all control inputs u is:

$$\dot{V} = \frac{\partial V}{\partial x} f(x, u) = u^T y - g(x)$$

→ System with input u and output y is passiv

Positive real transfer functions

$$\underbrace{\xrightarrow{u} \boxed{H(s)} \xrightarrow{y}}_{\text{Passive}} \quad \Leftrightarrow \quad H(s) \text{ is positive real}$$

Definition: The transfer function $H(s)$ (rational or irrational) is positive real if

1. $H(s)$ analytic in $\mathrm{Re}[s] > 0$.
2. $H(s)$ is real for all positive and real s.
3. $\mathrm{Re}[H(s)] \geq 0$ for all $\mathrm{Re}[s] > 0$.

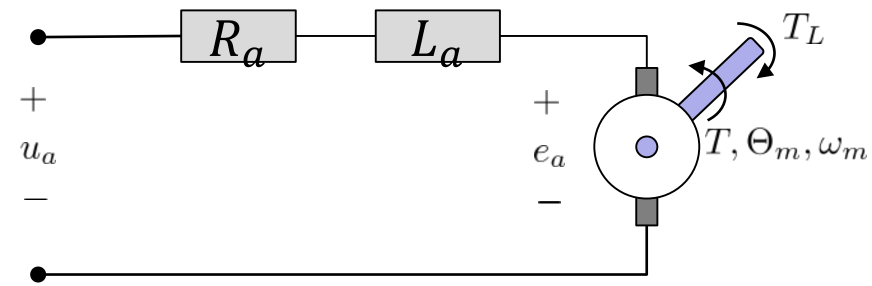
Check rational TFs for PRness

Theorem: A rational, proper transfer function $H(s)$ is positive real (and hence passive) if and only if

1. $H(s)$ has no poles in $\mathrm{Re}[s] > 0$.
2. $\mathrm{Re}[\mathrm{H}(j\omega)] \geq 0$ for all $\omega \in [-\infty, \infty]$ such that $j\omega$ is not a pole of $H(s)$.
3. If $j\omega_0$ is a pole of $H(s)$, then it is a simple pole, and the residual in $s = j\omega_0$ is real and greater than zero, that is,

$$\operatorname*{Res}_{s=j\omega_0} H(s) = \lim_{s \to j\omega_0} (s - j\omega_0) H(j\omega) > 0.$$

Storage function DC motor



$$V = \frac{1}{2} L_a i_a^2 + \frac{1}{2} J_m \omega_m^2 \geq 0$$

$$\dot{V} = i_a L_a \frac{di_a}{dt} + \omega_m J_m \cdot \frac{d\omega_m}{dt}$$

$$= i_a (u_a - R_a i_a - K_E \omega_m) + \omega_m (K_T i_a - T_L)$$

$$= i_a u_a - \omega_m T_L - R_a i_a^2$$

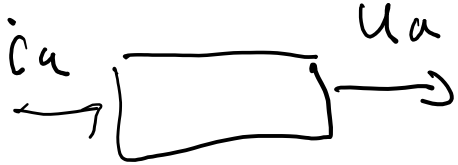
→ passive if load with ω_m and out T_L is passive

Example 42 (load = «friction»)

Assume: load is "friction" with linear friction law $T_L = D\omega_m$

$$\dot{V} = i_a \cdot u_a - \underbrace{D\omega_m^2 - R_a i_a^2}_{\text{dissipation term}}$$

$\rightarrow$ lose



passive

general: passive load $\rightarrow$ motor + load: passive

Transfer function of current controlled DC motor

$$\frac{i_a}{u_a}(s) = \frac{J_m}{K_T K_E} \frac{s}{1+T_m s+T_m T_a s^2}$$

$$T_m = \frac{J_m R_a}{K_E K_T} \qquad T_a = \frac{L_a}{R_a}$$

$$\frac{i_a}{u_a}(s) \approx \frac{J_m}{K_T K_E} \frac{s}{(1+T_m s)(1+T_a s)}$$

Passivity current controlled DC motor

$$\frac{i_a}{u_a}(s) \approx \frac{J_m}{K_T K_E}\frac{s}{(1+T_m s)(1+T_a s)}$$