Lecture 22: Hydraulic motors, transmission lines

- Hydraulic motors
- Hydraulic transmission lines
- (Electrical transmission lines)

Book: 4.1-4.6, (1.6)

- Info: Ocean Talk «The Polar Regions»
  - 28.03.2019 – 18:00-20:00, EL1
  - https://www.facebook.com/events/263677944559897/

Systems using hydraulics to produce motion

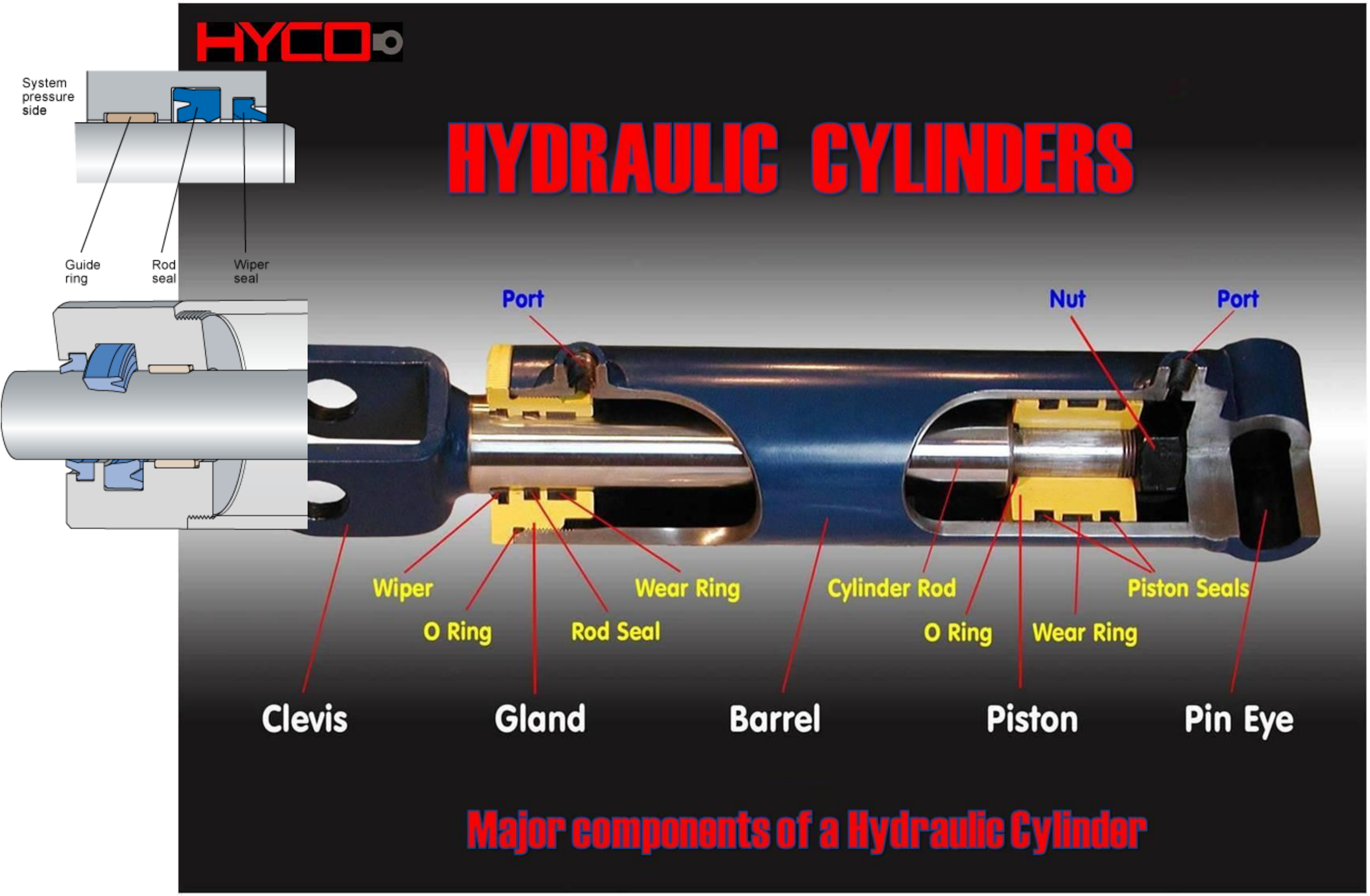
- Excavators

- Robots, cranes, etc.

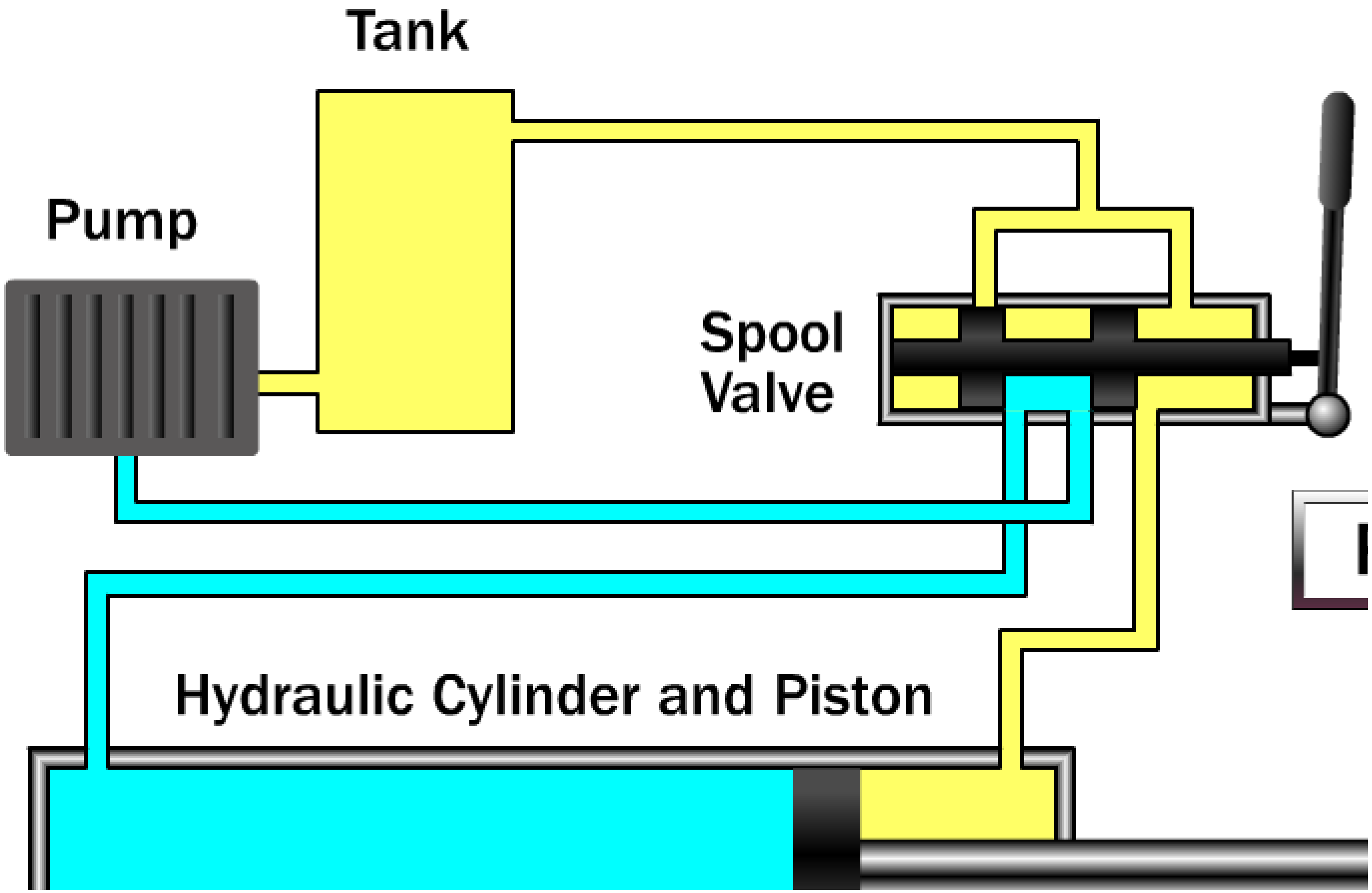
- To control motion of these systems, we need models of the hydraulic actuators

Hydraulic cylinder

For information about seals etc.:
Skf.com



Hydraulic system



Anna Konda – The fire fighting snake robot

Moody chart

- Circular pipe
- Darcy-Weisbach factor with Reynolds number and relative roughness

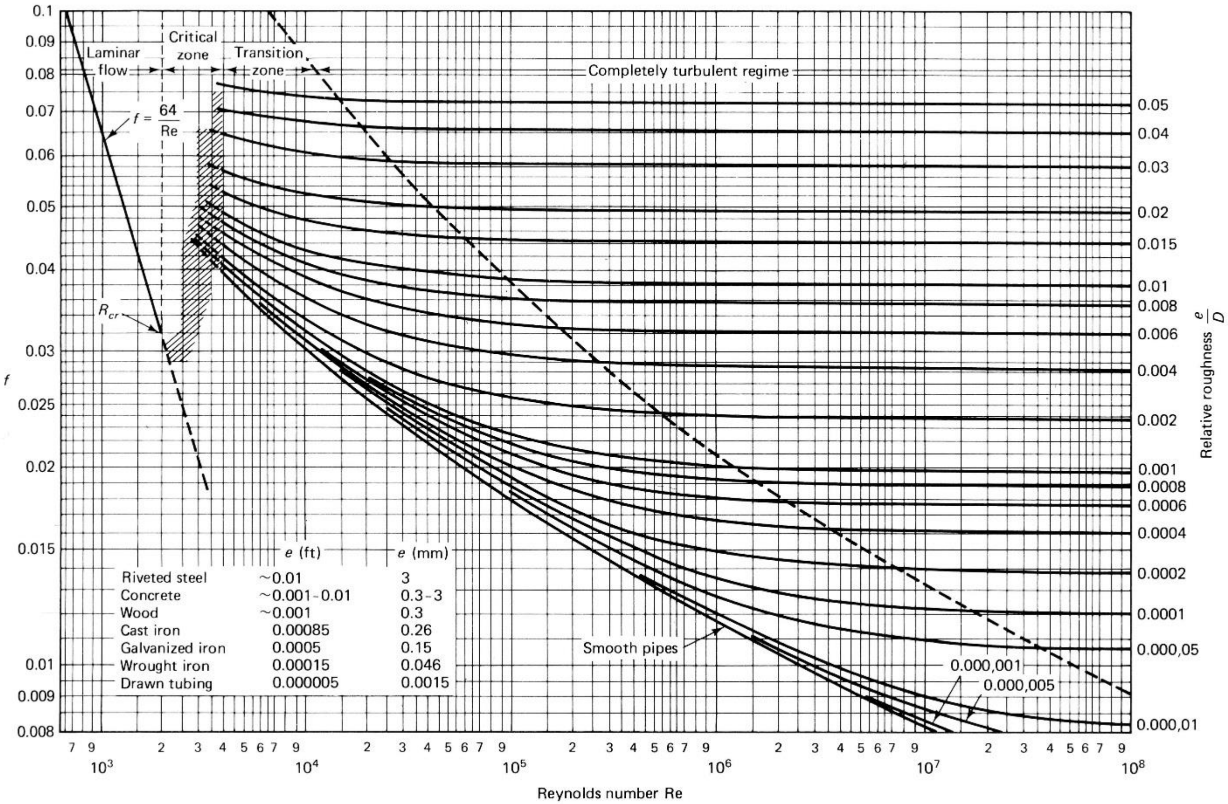
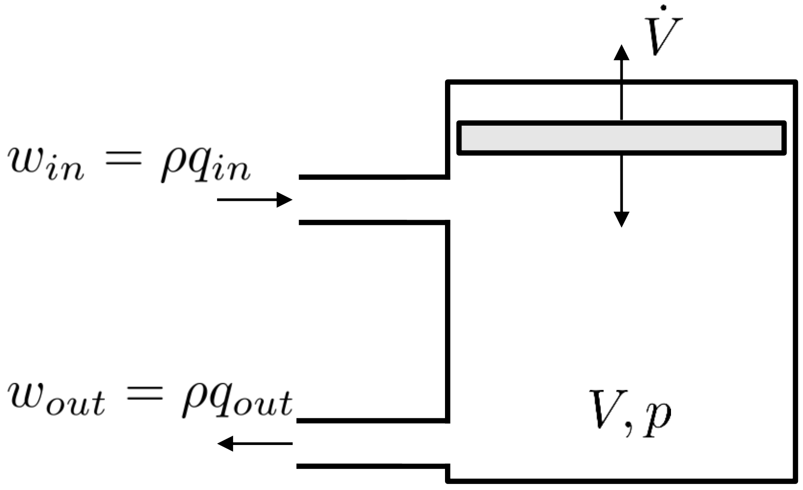


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

Bulk modulus

Motor models



Hydraulic cylinder

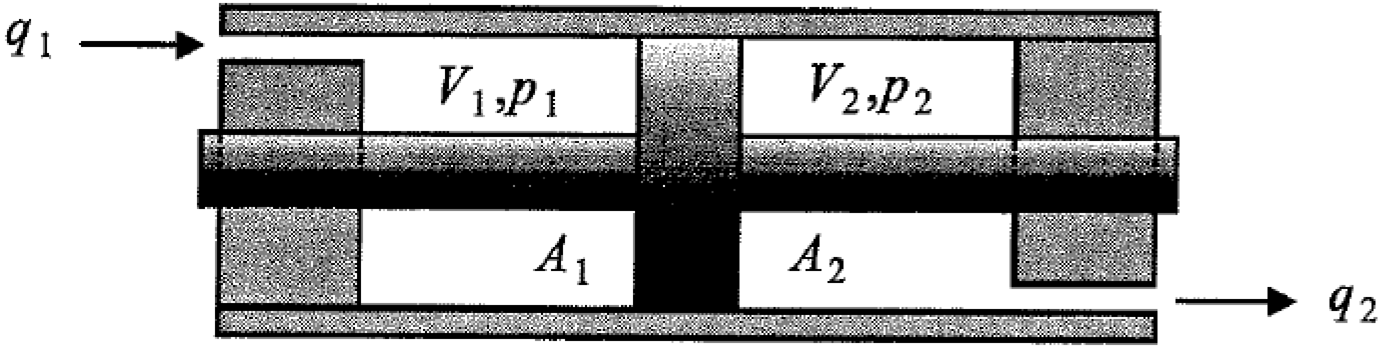


Figure 4.9: Symmetric hydraulic cylinder

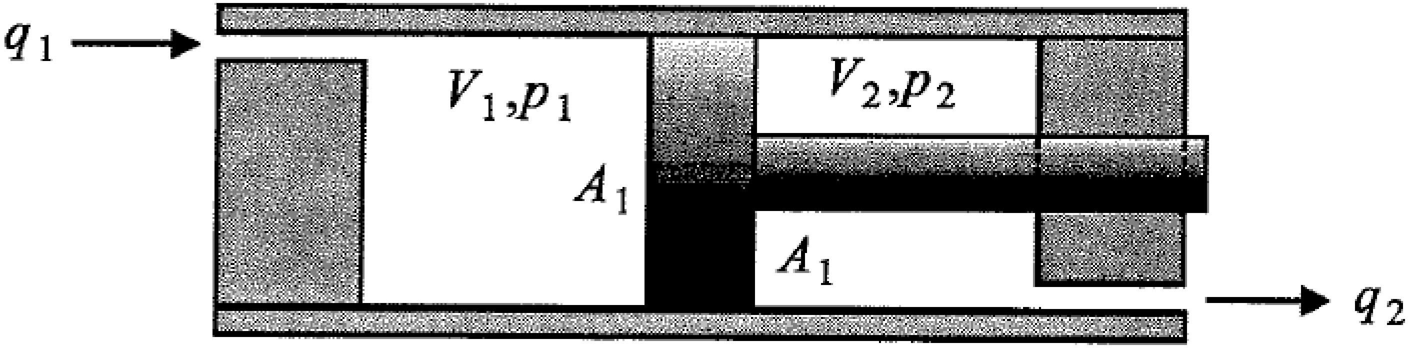


Figure 4.10: Single-rod hydraulic piston

Rotational hydraulic motor I

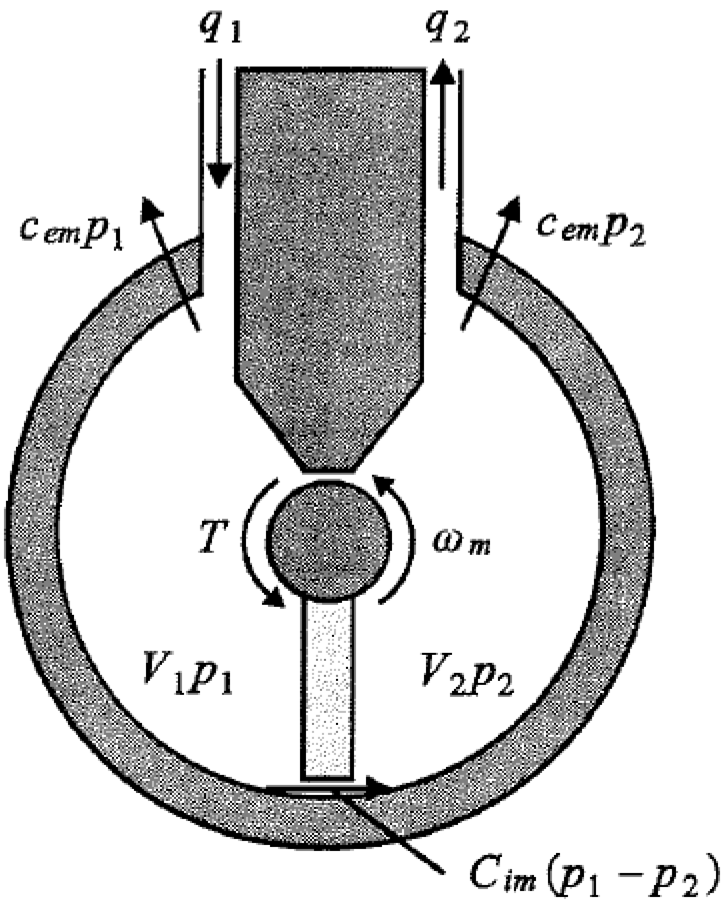


Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

Rotational hydraulic motor II

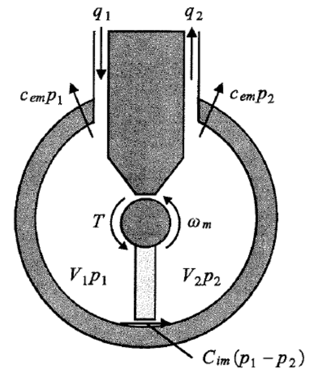
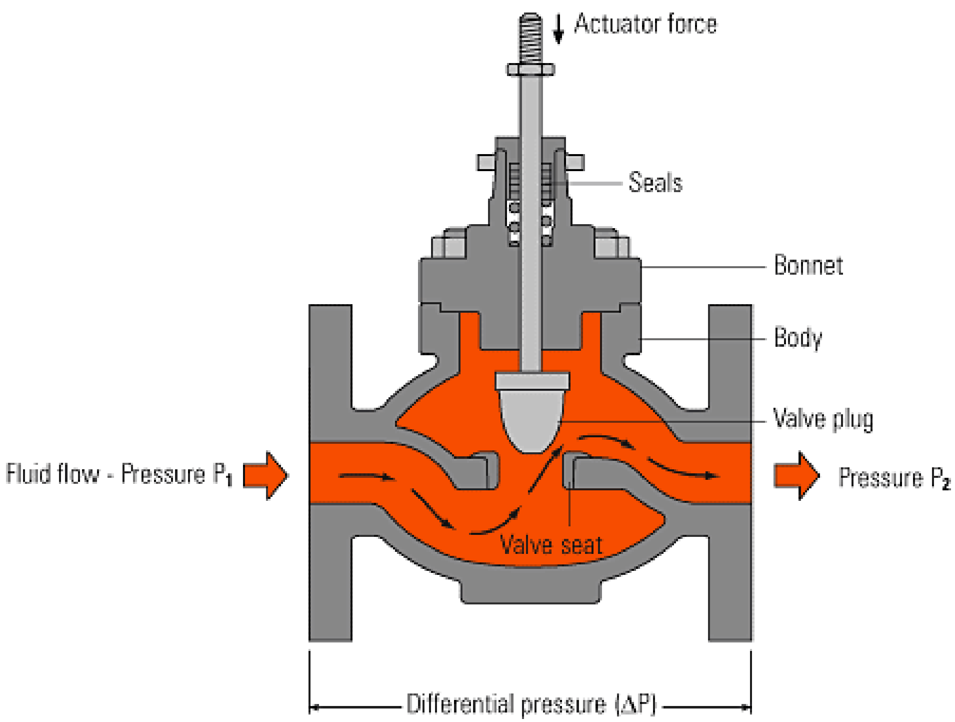


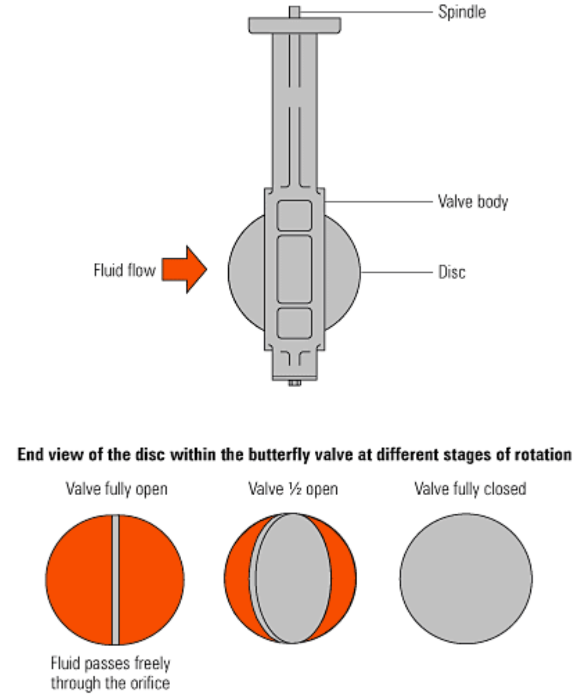
Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

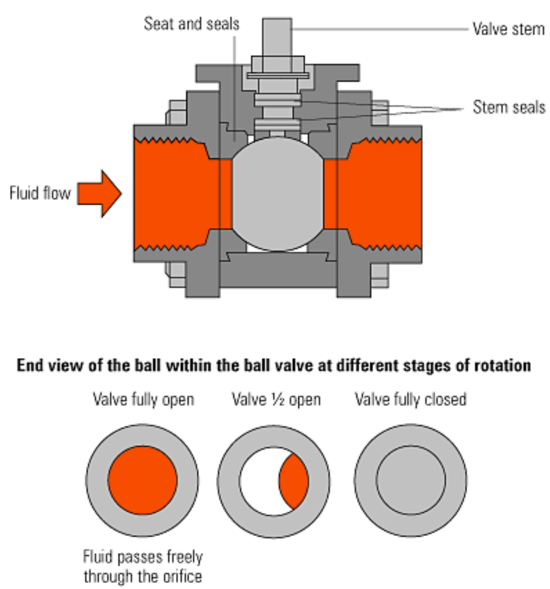
Rotational hydraulic motor III

Valves

- Device that regulates flow
- Many different types of valves exist
  - Globe valve, ball valve, butterfly valve, ...





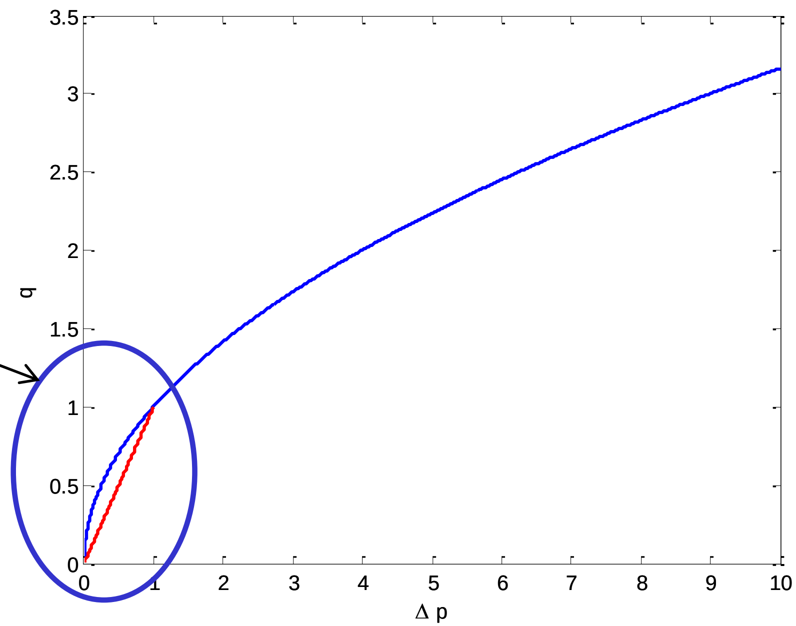


Valve models (book 4.2)

- Flow through a restriction is generally turbulent

$$q = C_d A \sqrt{\frac{2}{\rho}\Delta p}$$

Problematic!

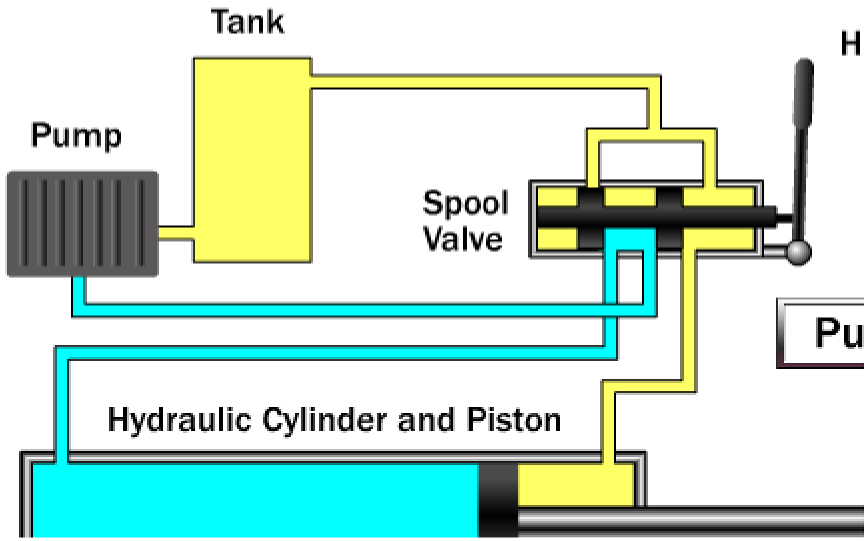


- Solution: Regularize by assuming laminar flow for small *Δp*

$$q = C_l \Delta p$$

- Book: Make transition smooth

Four-way valve



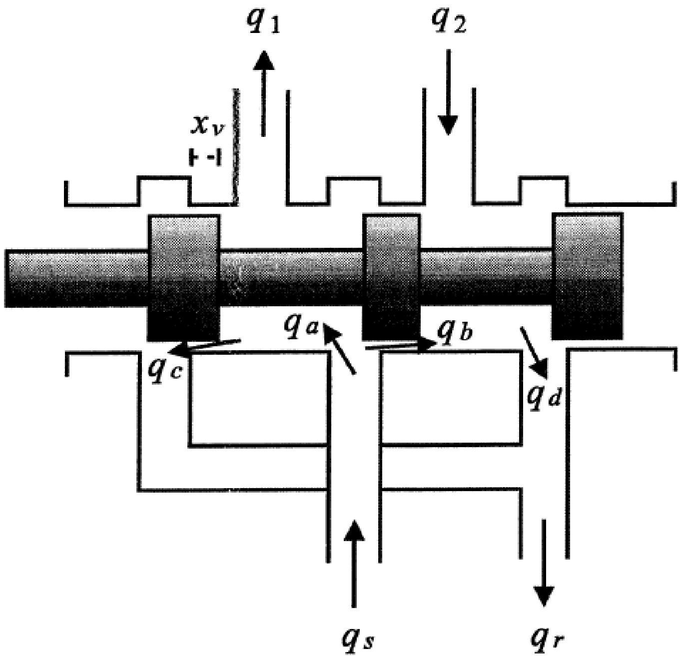


Figure 4.2: A matched and symmetric four-way valve.

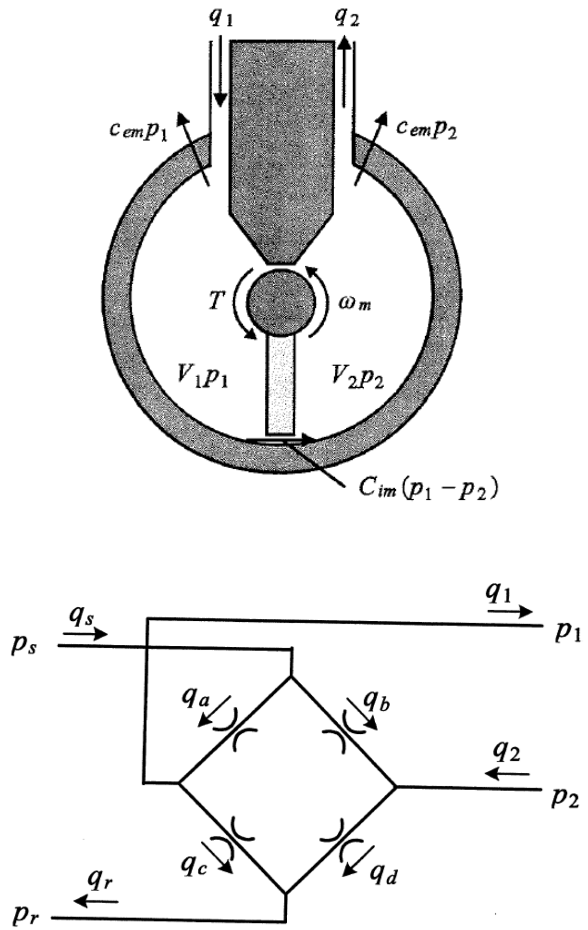


Figure 4.1: Four-way valve

Modeling of four-way valve

- Define load pressure

$$p_L = p_1 - p_2$$

- Define load flow

$$q_L = \frac{q_1 + q_2}{2}$$

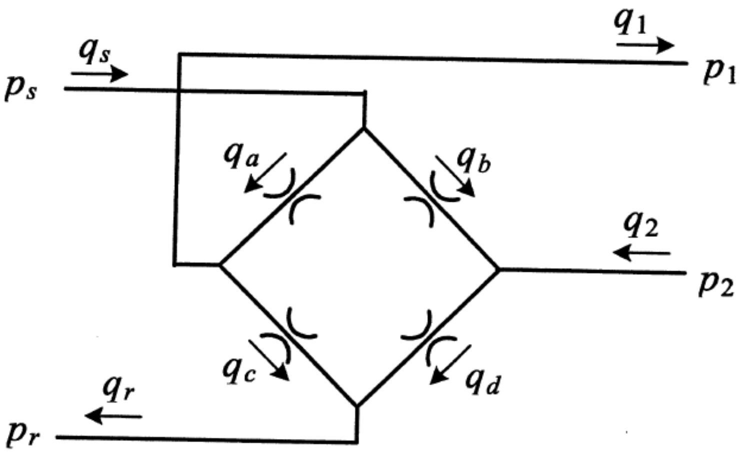


Figure 4.1: Four-way valve

- Symmetric load assumption (motor)

$$q_1 = q_2$$

- Symmetric valve and symmetric load

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \mathrm{sign}(x_v) p_L\right)}$$

Characteristic of four-way valve

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \mathrm{sign}(x_v) p_L\right)}$$

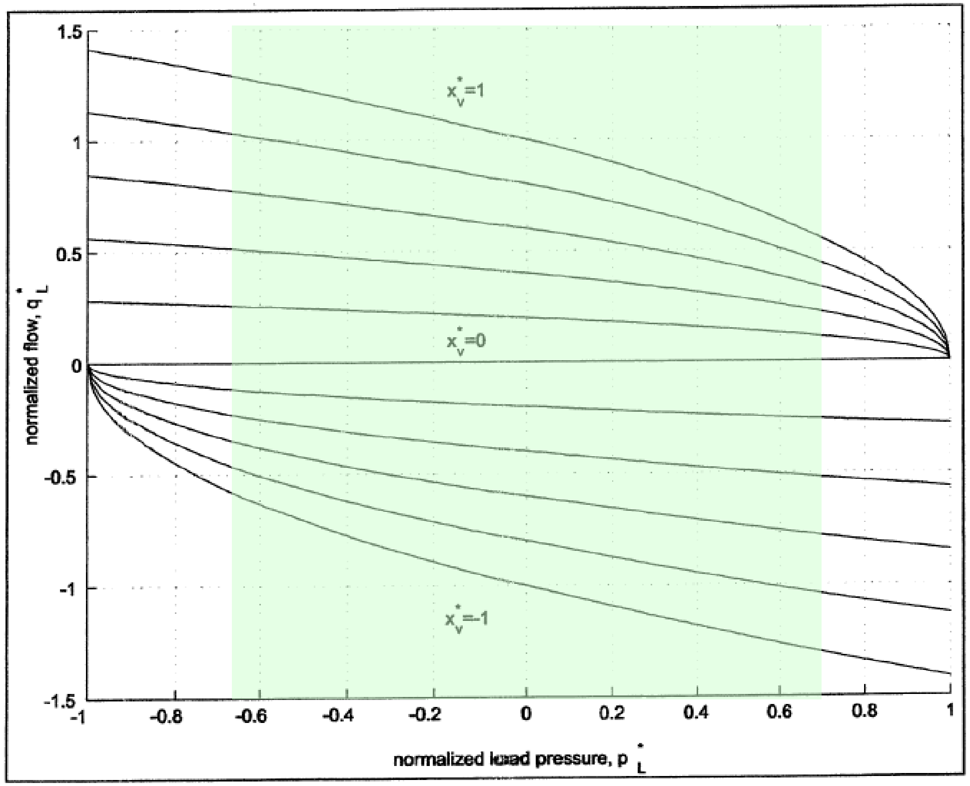


Figure 4.3: Valve characteristic

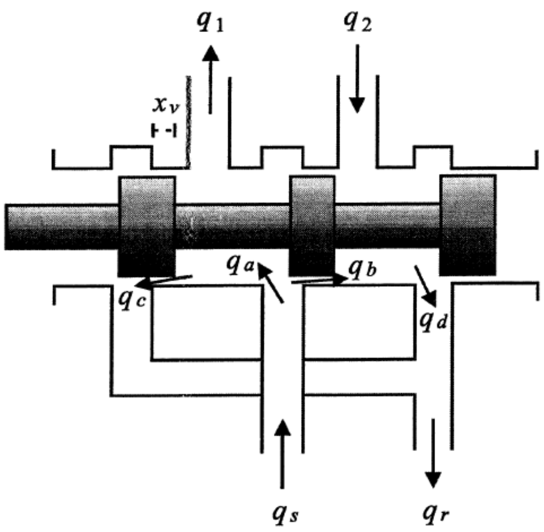
Linearized model:

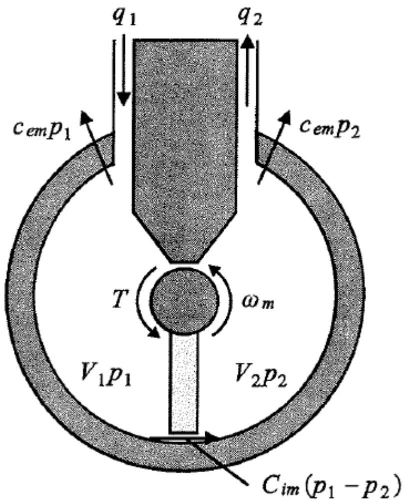
$$|p_L| \leq \frac{2}{3} p_s : \quad \boxed{q_L = K_q x_v - K_c p_L}$$

Gain uncertainty:

$$0.58 K_{q0} \leq K_q \leq 1.29 K_{q0}$$

Transfer function valve+motor

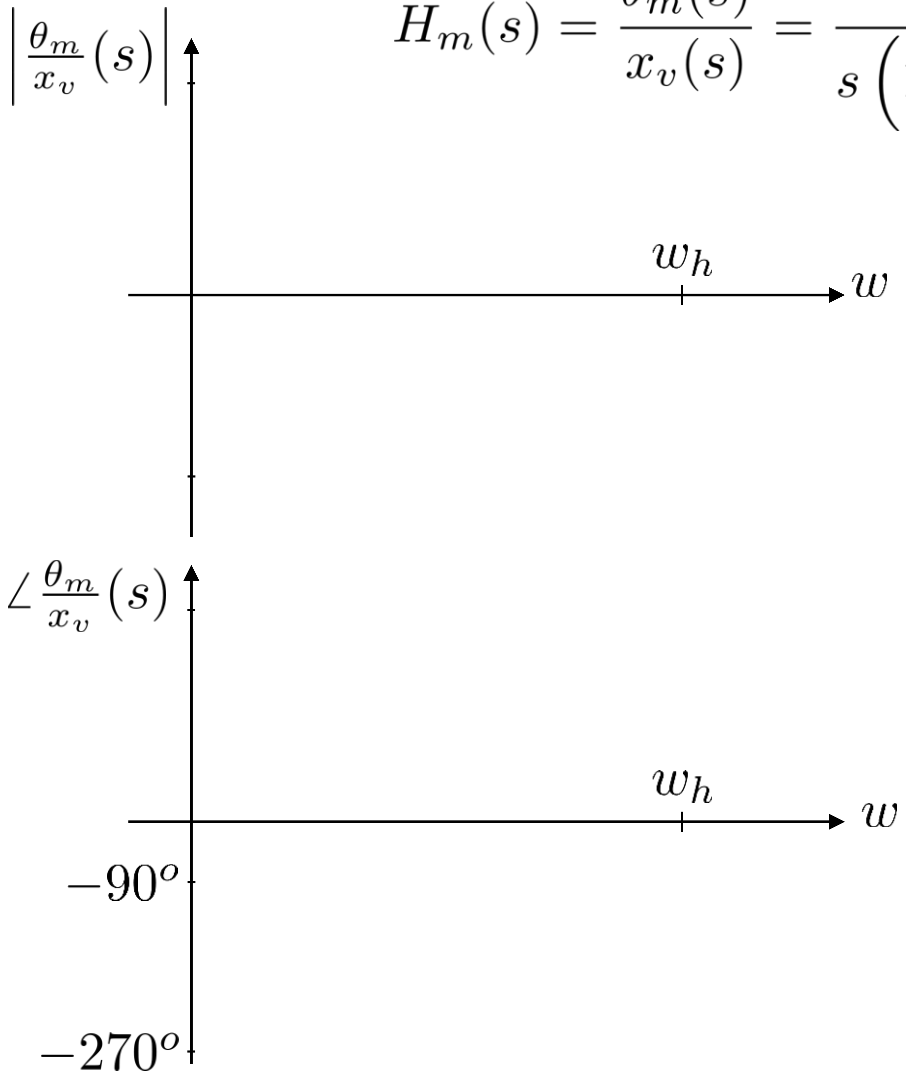




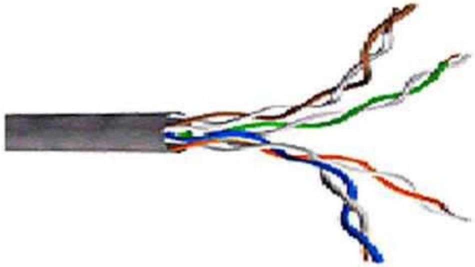
$$\theta_m(s) = \frac{\frac{K_q}{D_m}x_v(s) - \frac{K_{ce}}{D_m^2}\left(1+\frac{s}{\omega_t}\right)T_L(s)}{s\left(1+2\zeta_h\frac{s}{\omega_h}+\frac{s^2}{\omega_h^2}\right)}$$

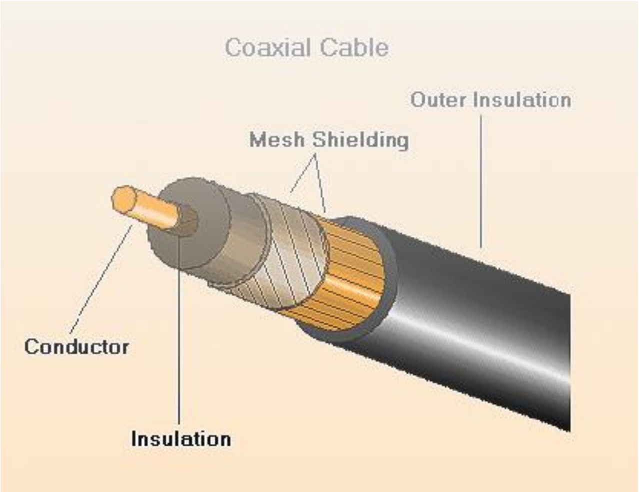
Transfer function spool to shaft

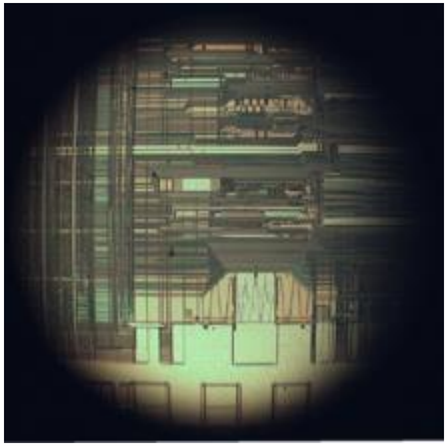
$$H_m(s) = \frac{\theta_m(s)}{x_v(s)} = \frac{\frac{K_q}{D_m}}{s\left(1 + 2\zeta_h \frac{s}{\omega_h} + \frac{s^2}{\omega_h^2}\right)}$$



Electrical transmission lines

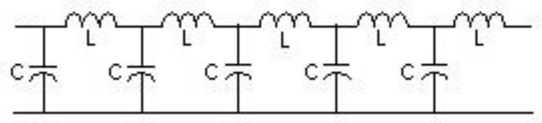




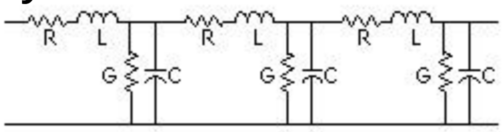


Telegrapher's equation (Wave equation)

- Lossless:



Lossy:



- Model (Ch. 1.6):

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

- Laplace:

$$\frac{\partial u(x,s)}{\partial x} = -X(s)i(x,s)$$
$$\frac{\partial i(x,s)}{\partial x} = -Y(s)u(x,s)$$

Series impedance:
$$X(s) = R + Ls$$
Parallel admittance:
$$Y(s) = G + Cs$$

Characteristic impedance:
$$Z_c(s) = \sqrt{\frac{X(s)}{Y(s)}}$$

Same equations for electrical and fluid/hydraulic transmission lines

Electrical transmission lines:

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

Fluid transmission lines:

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\beta}{A}\frac{\partial q(x,t)}{\partial x}$$
$$\frac{\partial q(x,t)}{\partial t} = -\frac{A}{\rho}\frac{\partial p(x,t)}{\partial x} - \frac{F\left[q(x,t)\right]}{\rho}$$

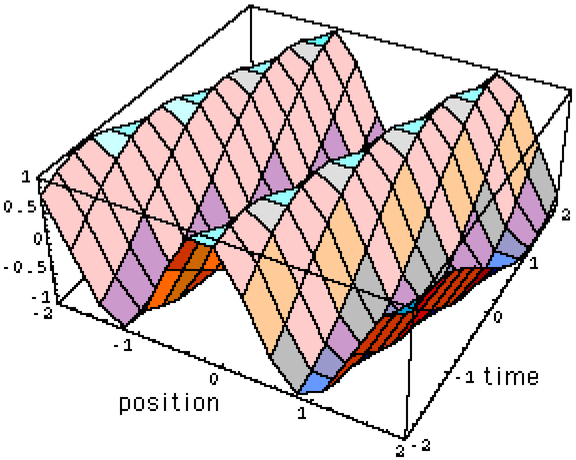
- Current and flow “same” variables, as is voltage and pressure
- In both cases, we can define line impedance, characteristic impedance, propagation operator, etc.
- Solution to equations have same structure/form: waves propagating back and forth

Solution: Waves

- Solution: $$u_{out}(s) = e^{-\Gamma(s)} u_{in}(s)$$

- Propagation operator $\Gamma(s) = L\sqrt{X(s)Y(s)}$
  - Attenuation factor $\alpha = Re[\Gamma(j\omega)]$: How much is wave reduced
  - Phase factor: $\beta = Im[\Gamma(j\omega)]$: How long does it take

- Lossless (*R* = *G* = 0): $\Gamma(s) = Ts$
  - Attenuation factor: 0
  - Phase factor: Pure time-delay



When should we care?

- Solution lossless case: Time delay

$$e^{-Ts}$$

- Rule-of-thumb from control theory: We can ignore time-delay for frequencies much less than 1/*T*

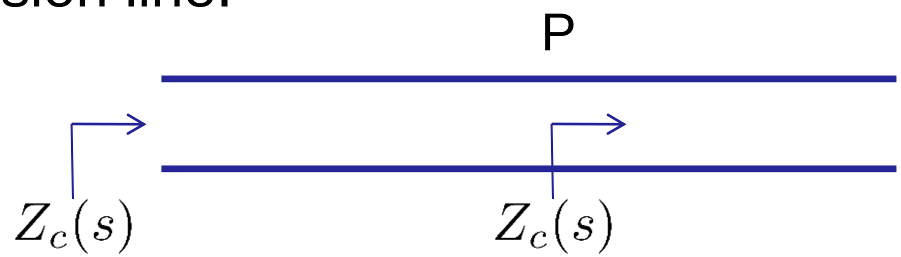
$$\omega \leq \frac{1}{T} \;\Rightarrow\; 2\pi\frac{c}{\lambda} \leq \frac{c}{L} \;\Rightarrow\; L \leq \frac{\lambda}{2\pi}$$

- Rule-of-thumb for transmission lines: When L is larger than one tenth of wavelength, treat as transmission line

- Power lines, f = 50Hz: λ = 6000km
- Personal computers, f = 10GHz: λ = 1.5cm

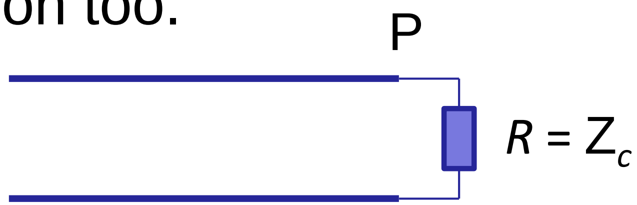
Impedance matching

- Suppose we have an imaginary joint at P in a very long transmission line.



The wave goes through the joint without reflection because there is actually no joint (just imagined).

- Now, let us terminate a resistance of value Z_c *at the same position of this* imaginary joint. The wave will go through without reflection too.



This is called a **matched load.**

http://cktse.eie.polyu.edu.hk/eie403/Transmissionline.pdf

Lecture 23: Process modeling & balance laws

- Process modeling, structure and methodolgy
- Balance laws
  - Closure relations

Book: 10.4, 11.1-11.4

Process equations

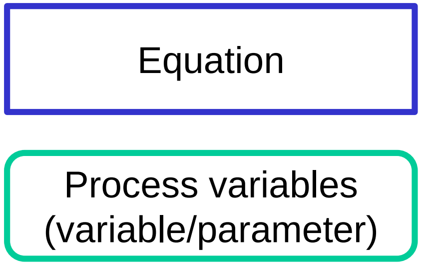
- Balance laws
  - Mass
  - Momentum
  - Energy
  - ...

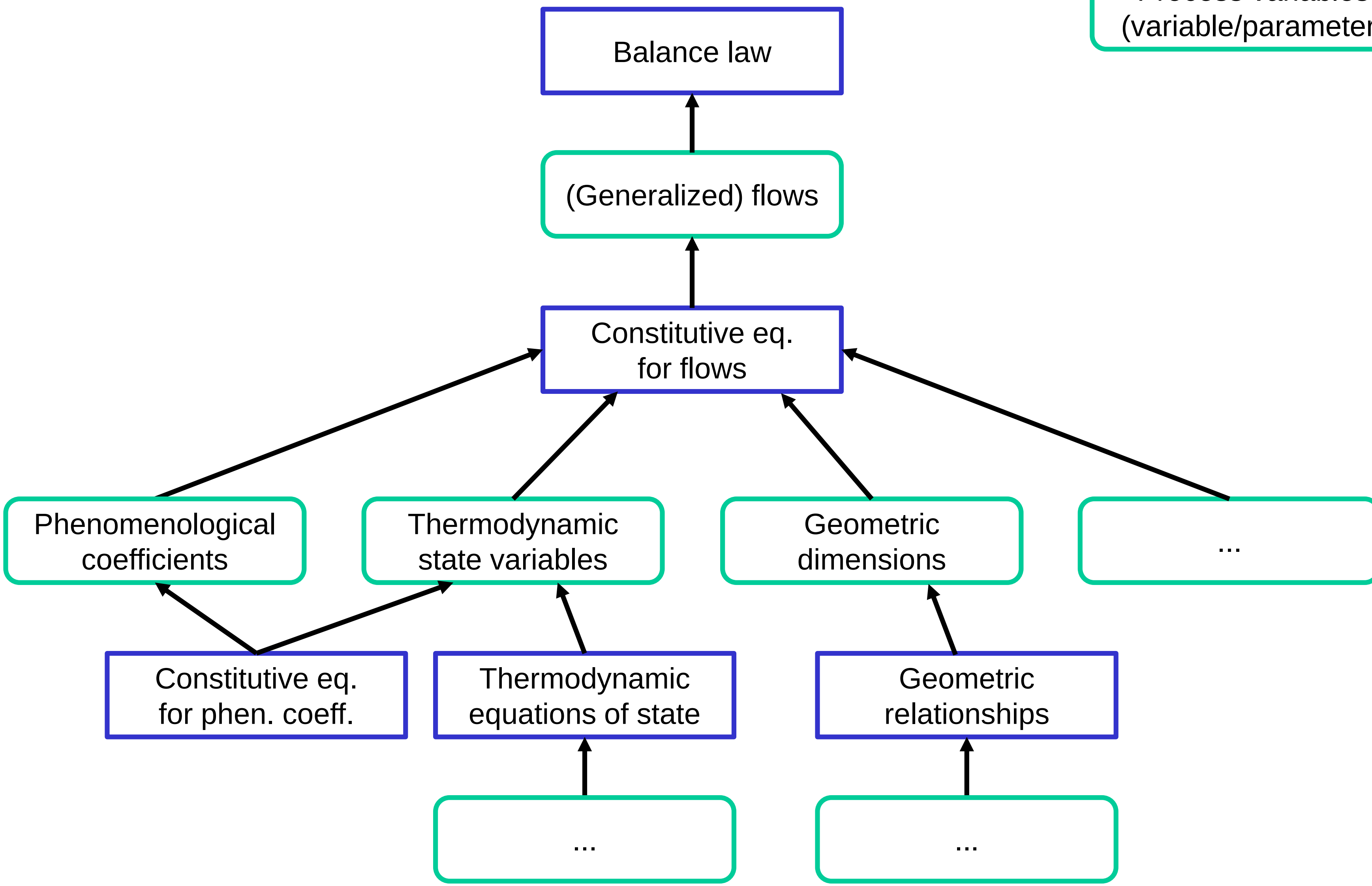
- Constitutive equations
  - For (generalized) flows
  - Thermodynamic equations of state (e.g. ideal gas law)
  - Phenomenological relationships (e.g. between friction force and flow in a pipe)
  - ...

- Constraints
  - Geometric relationships
  - Equilibrium conditions
  - ...

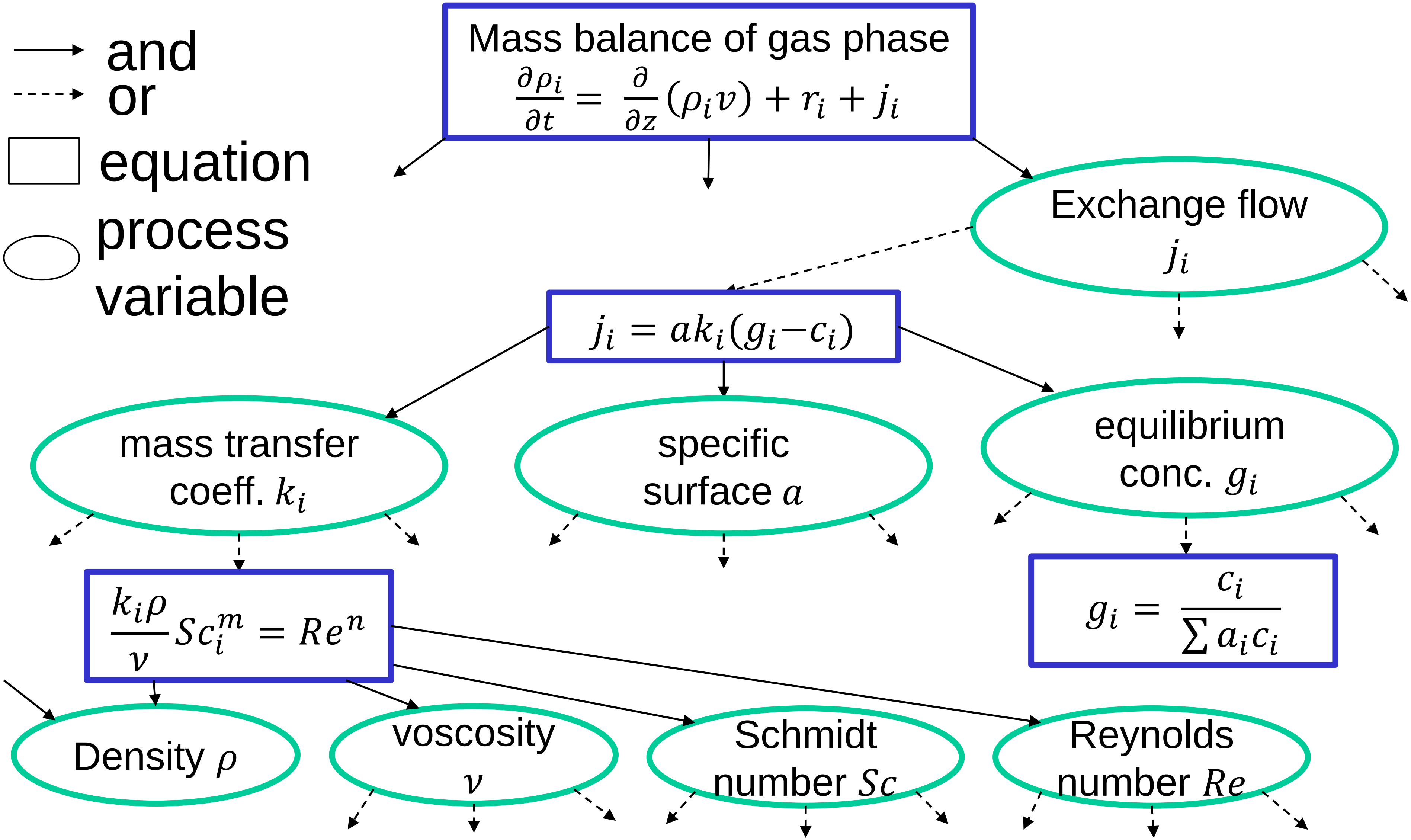
Also called «closure relations» as they «close» the balance laws (such that #equations = #variables)

Structure of process models

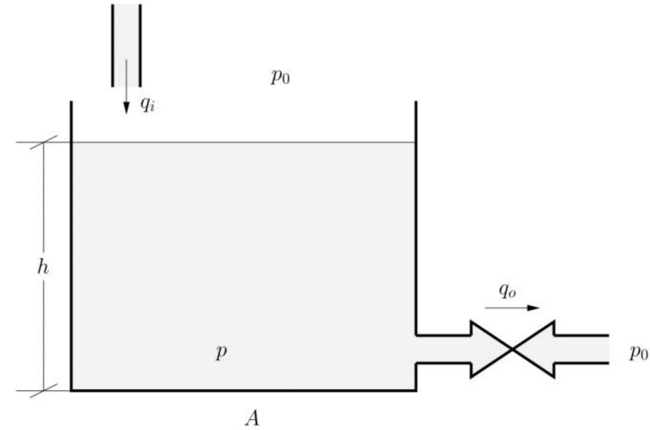




Example – structure of process models



Example: Tank



- Mass balance: $\dfrac{d\overset{1}{m}}{dt} = (\overset{2}{q_i} - \overset{3}{q_0})\overset{4}{\rho}$
- Constitutive equation: $q_0 = \overset{5}{C}\sqrt{\overset{6}{p} - \overset{7}{p_0}}$ (2)

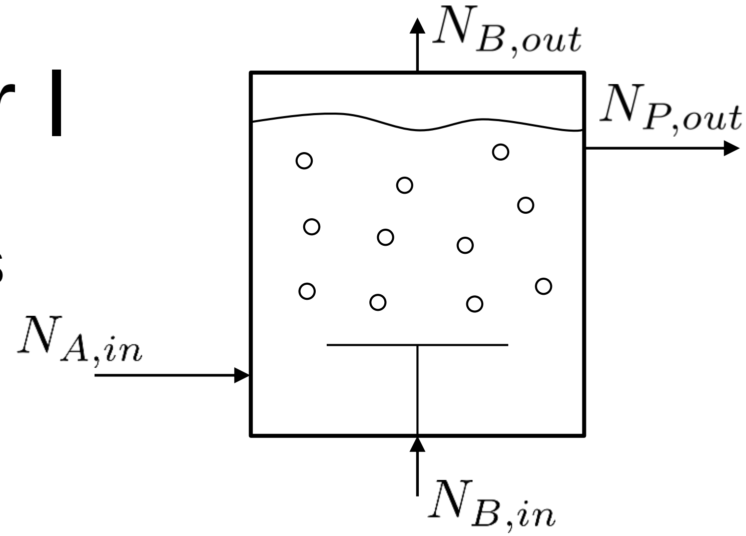
  $p = p_0 + \rho \overset{8}{g} \overset{9}{h}$ (3)
- Constraints: $m = \overset{10}{V}\rho$ (4)

  $V = \overset{11}{A}h$ (5)
- How many variables?
- Need to define parameter and inputs
  - Parameters: C, g, A, ρ
  - Inputs: q_i, p_0

Structural index:

	q_0	p	V	h
(2)	x	x		
(3)		x		x
(4)			x	
(5)				x

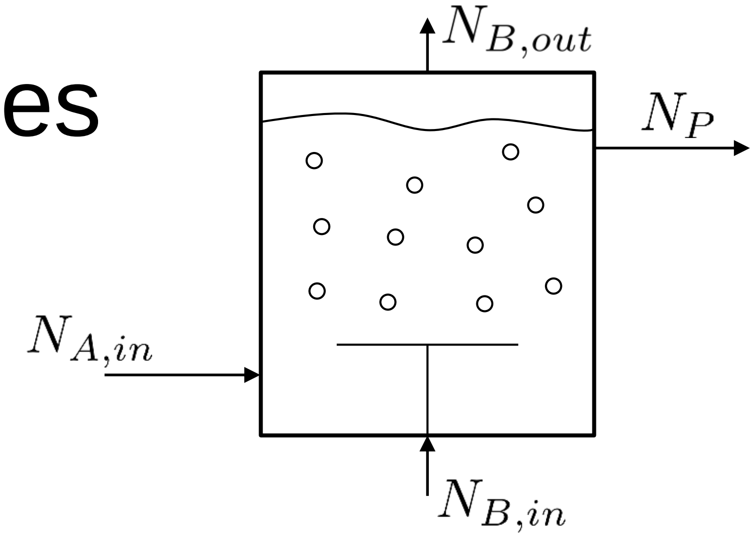
Example: Bubble reactor I



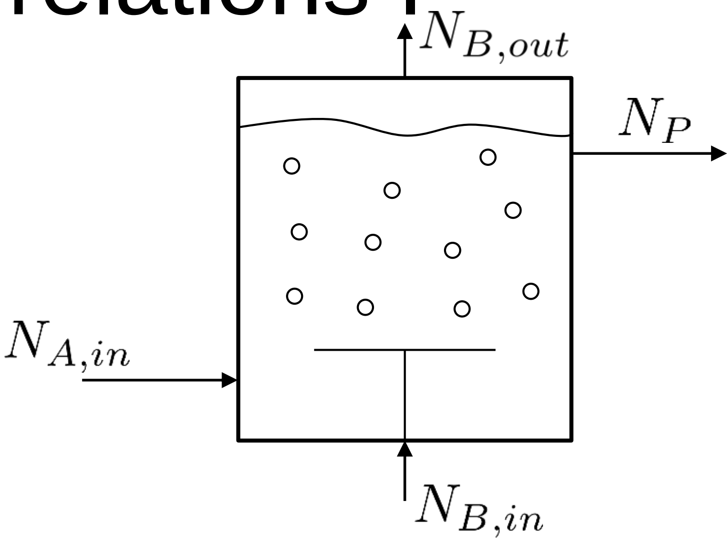
Model reactor as quasi-homogenous

- Assumptions:
  - Ideally mixed
  - Inflows are pure substances
  - Substance A and C are in liquid phase, substance B is gaseous
  - The total surface area of the bubbles depends on the inflow B
    - $S_R = S_R(N_{B,in})$
  - The reaction rate can be calculated based on the concentration of A and the pressure in the reactor
    - $R_0 = R_0(c_{A,liq}, p)$
  - Densities ρ_A and ρ_C and mole masses M_A and M_C are constant and known
  - The gas phase can be described by the ideal gas law
    - $p\, V_{gas} = n_B R_m T$
  - The volume of the reactor is constant and known

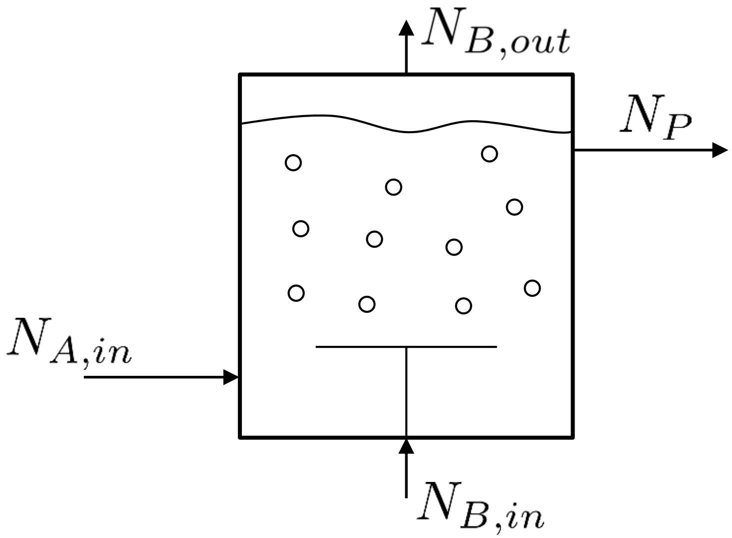
Bubble reactor – Balances



Bubble reactor – closure relations I



Bubble reactor – DoF

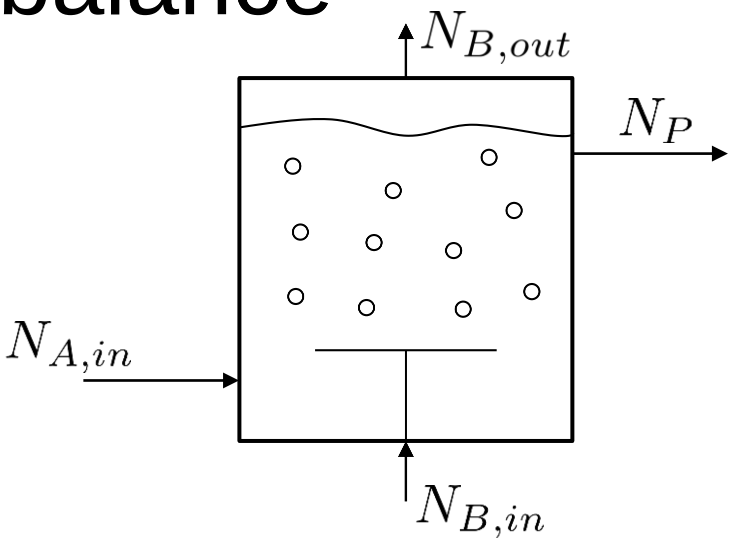


- Variables: $[n; N_{A,in}; N_{B,in}; N_{B,out}; N_P; S_R; R_A; R_B; R_C; n_A; x_A; n_B; R_0; c_{A,liq}; p; n_C; V_{liq}; M_A; \rho_A; M_C; \rho_C; V_{gas}; R_m; T; V]$

Bubble reactor – structural index

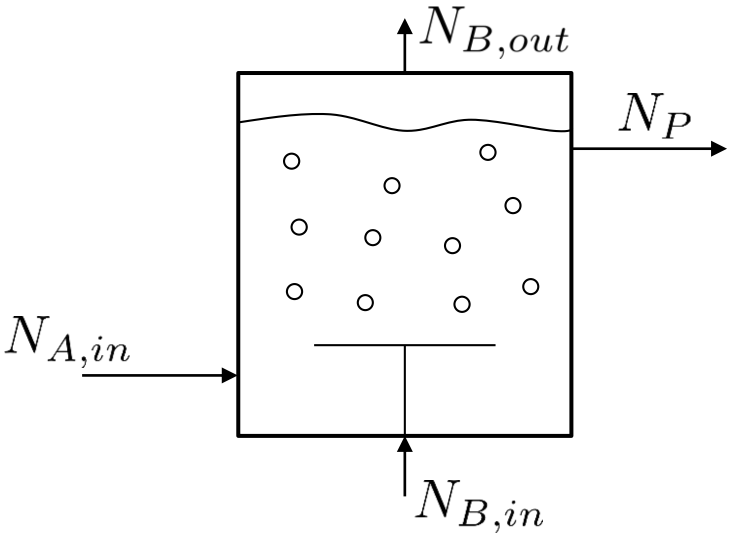
	S_R	R_A	R_B	R_c	R_0	n_C	x_A	$c_{A,liq}$	p	V_{gas}	V_{liq}
(4)											
(5)											
(6)											
(7)											
(8)											
(9)											
(10)											
(11)											
(12)											
(13)											
(14)											

Bubble reactor – energy balance



Bubble reactor – closure relations II

- Assumptions:
  - Specific enthalpies of inputs are model inputs
  - Spedific enthalpies of pure substances A, B, C are given by $h_i = h_i(T, p)$



Bubble reactor – structural index II

- Variables: $[n; N_{A,in}; N_{B,in}; N_{B,out}; N_P; S_R; R_A; R_B; R_C; n_A; x_A;$
  $n_B; R_0; c_{A,liq}; p; n_C; V_{liq}; M_A; \rho_A; M_C; \rho_C; V_{gas}; R_m; T; V$
  $U; h_{A,in}; h_{B,in}; h_B; h_P; h_A; x_C; h_C; H]$

	h_B	h_P	x_C	h_C	h_A	T	H	n_C	p	x_A
(16)										
(17)										
(18)										
(19)										
(20)										
(21)										
(22)										