



### Problem 1 (30 %) The Mean Value Theorem

- a If you follow the same procedure as in Example A.2 (p. 629), but with a different  $p$ , you should find an  $\alpha$  between zero and one. The mean value theorem is discussed in most calculus books; there is also a good Wikipedia page about the theorem.
- b Lipschitz continuity is described on page 624. Informally, a function is Lipschitz continuous if its derivative is bounded by a real number (the real number is called the Lipschitz constant, if it exists).

### Problem 2 (25 %) LP and KKT-conditions (Exam August 2000)

This is both covered in class and in the textbook. Remember that we use  $\lambda$  as a multiplier for  $Ax = b$  and  $s$  as a multiplier for  $x \geq 0$ . Make sure you understand how the Lagrangean  $\mathcal{L}(x, \lambda)$  is differentiated (see the note on Blackboard). Try your best to understand the derivation, and do not just copy it from your notes or the textbook.

### Problem 3 (45 %) Linear Programming

- a Make sure you formulate this as a minimization problem, even though the objective is to *maximize* profit. Read the problem description carefully to determine which constraints are equalities and which are inequalities (we have both in this problem).
- b Check how many variables ( $n$ ) and constraints ( $m$ ) there are in the problem — this determines the size of the basis. Then, you can find out how many possible bases (“bases”) there are. Remember that basic variables are nonzero, and that nonbasic variables are zero. Equation (13.18) in the textbook is a good place to start when finding basic feasible points. Note that a basic feasible point is defined by the value of all variables, not just the basic ones (that is, state the value of  $x$ , not just  $x_B$ ).
- c Make sure you remember/understand why it is sufficient to check the KKT conditions to determine the solution of an LP. You will need to solve a matrix equation to determine the value of  $s$  at different basic feasible points. Note that all KKT conditions must be checked. You can verify by calculating the value of  $c^\top x$  at all the basic feasible points.
- d Note that the dual problem (13.7) in the textbook is the dual problem for an LP in standard form!

- e This is shown in the textbook, but show more details of the derivation. Make sure you understand how the variables are transposed, and why this is OK.
- f This is a question about sensitivity, which is described in Chapter 13 for LPs. Use MATLAB to check your answer (read the documentation for the function *linprog*).