Lecture 22: Hydraulic motors, transmission lines

- Hydraulic motors
- Hydraulic transmission lines
- (Electrical transmission lines)

Book: 4.1-4.6, (1.6)

- Info: Ocean Talk «The Polar Regions»
  - 28.03.2019 – 18:00-20:00, EL1
  - https://www.facebook.com/events/263677944559897/

Systems using hydraulics to produce motion

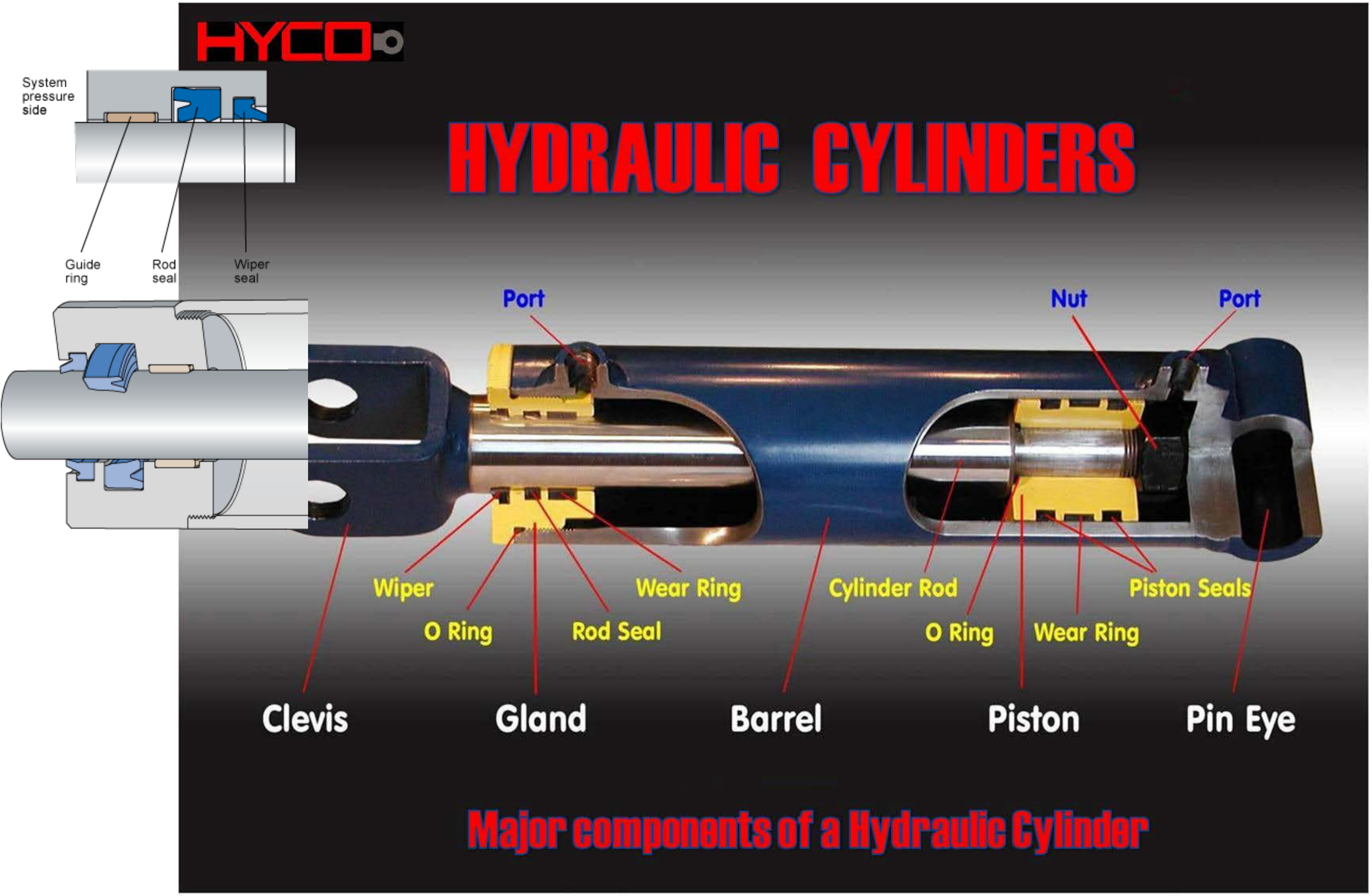
- Excavators

- Robots, cranes, etc.

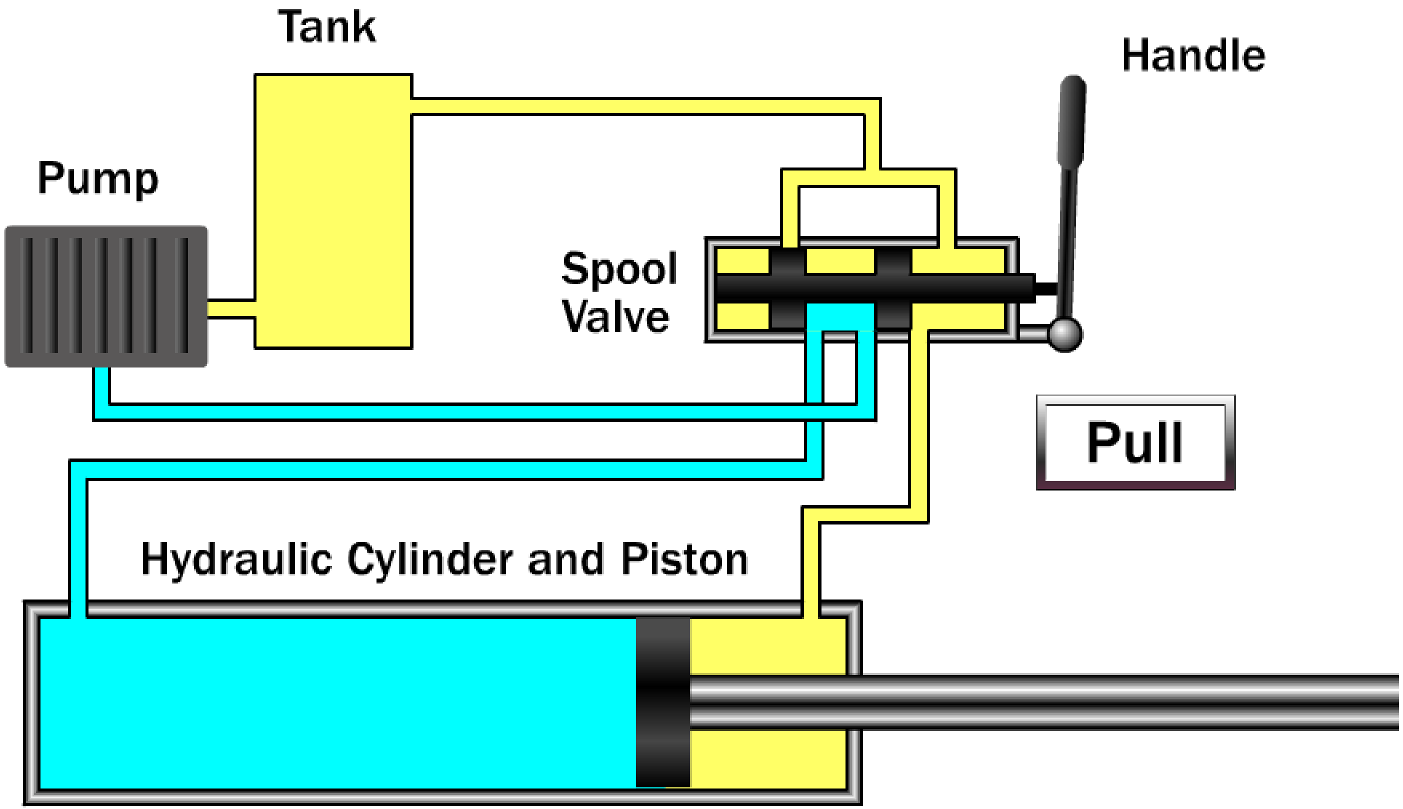
- To control motion of these systems, we need models of the hydraulic actuators

Hydraulic cylinder

For information about seals etc.:
Skf.com



Hydraulic system

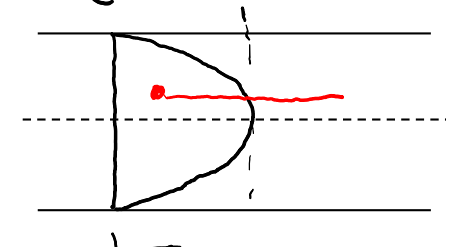


Anna Konda – The fire fighting snake robot

Moody chart

- Circular pipe
- Darcy-Weisbach factor with Reynolds number and relative roughness

$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} \approx \frac{Dv}{\nu} = \frac{Dv\rho}{\eta}$$

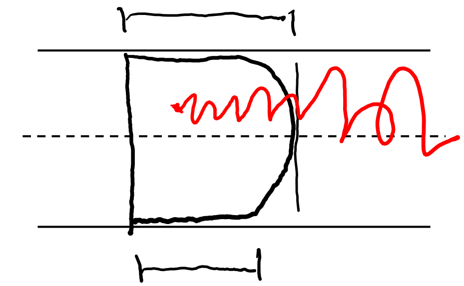


$V_m \approx 1/2\, V_{max}$

laminar flow

$Re \lesssim 2100$

Vmax



$V_m \approx 0.8\, V_{max}$

turbulent

$Re \gtrsim 4000$

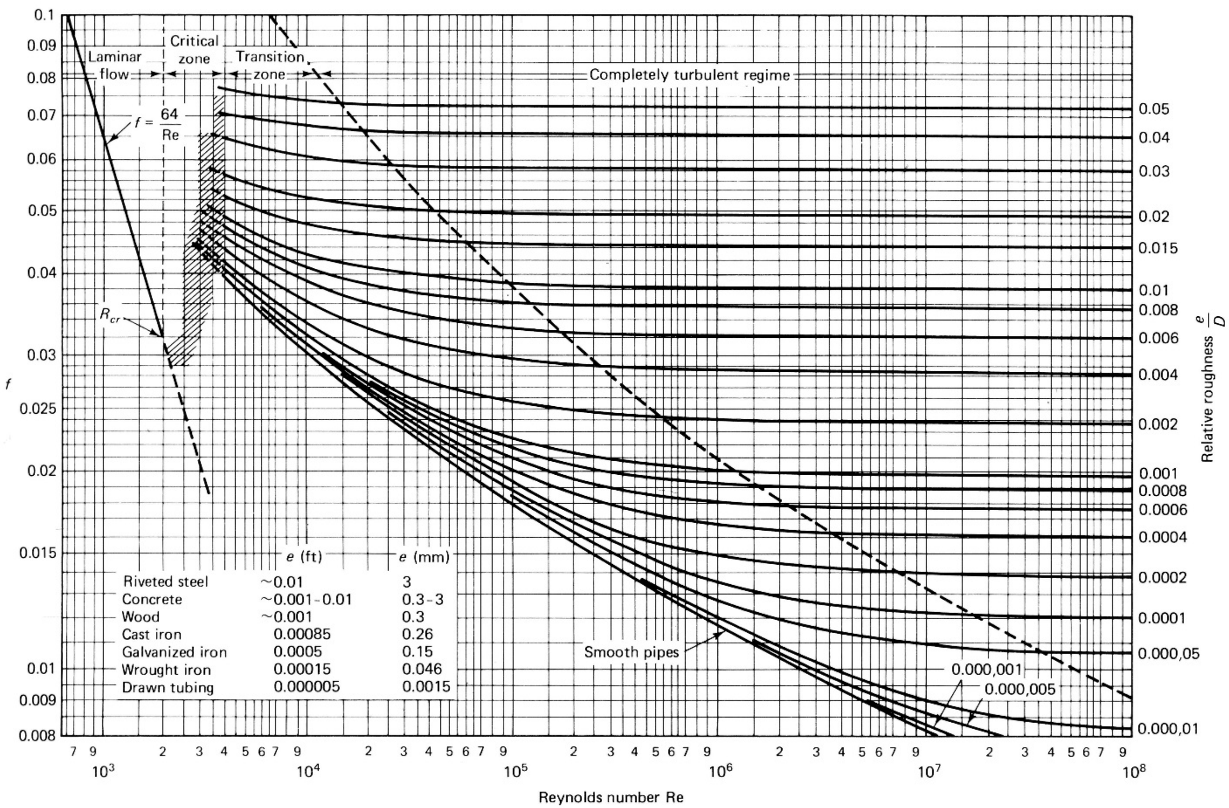
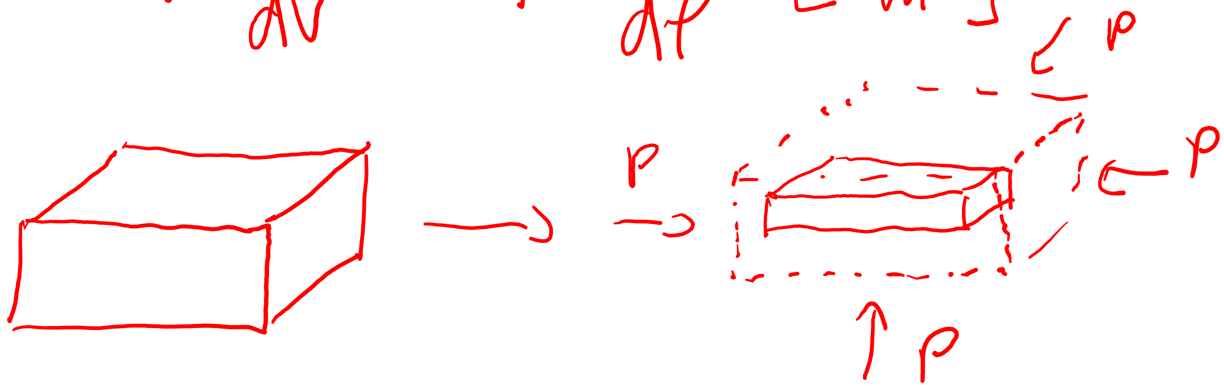


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

Bulk modulus

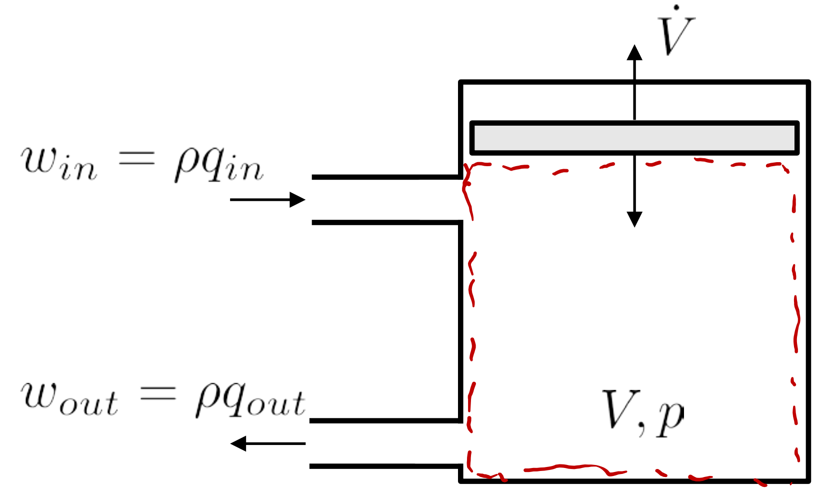
$$\beta = -V\frac{dp}{dV} = \rho\frac{dp}{d\rho} \quad \left[\frac{N}{m^2}\right]$$



example : oil : 7000 bar

water : 22000 bar

Motor models



Mass balance:

$$\frac{dm}{dt} = w_{in} - w_{out}$$

$$\frac{d}{dt}(\rho V) = \rho q_{in} - \rho q_{out}$$

$$\dot{\rho} V + \rho \dot{V} = \rho q_{in} - \rho q_{out}$$

$$\mid \dot{\rho} = \frac{\rho}{\beta}\dot{p}$$

$$\frac{V}{\beta}\dot{p} + \dot{V} = q_{in} - q_{out}$$

Hydraulic cylinder

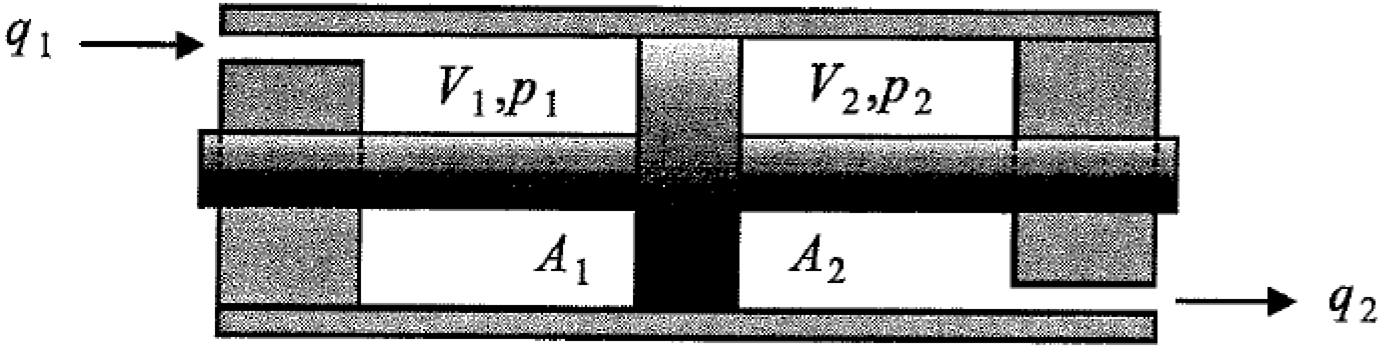


Figure 4.9: Symmetric hydraulic cylinder

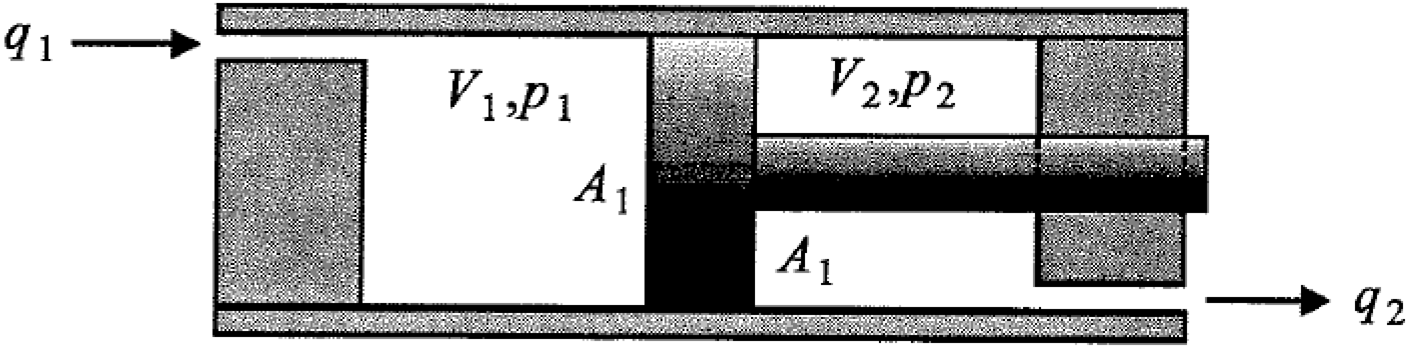


Figure 4.10: Single-rod hydraulic piston

Rotational hydraulic motor I

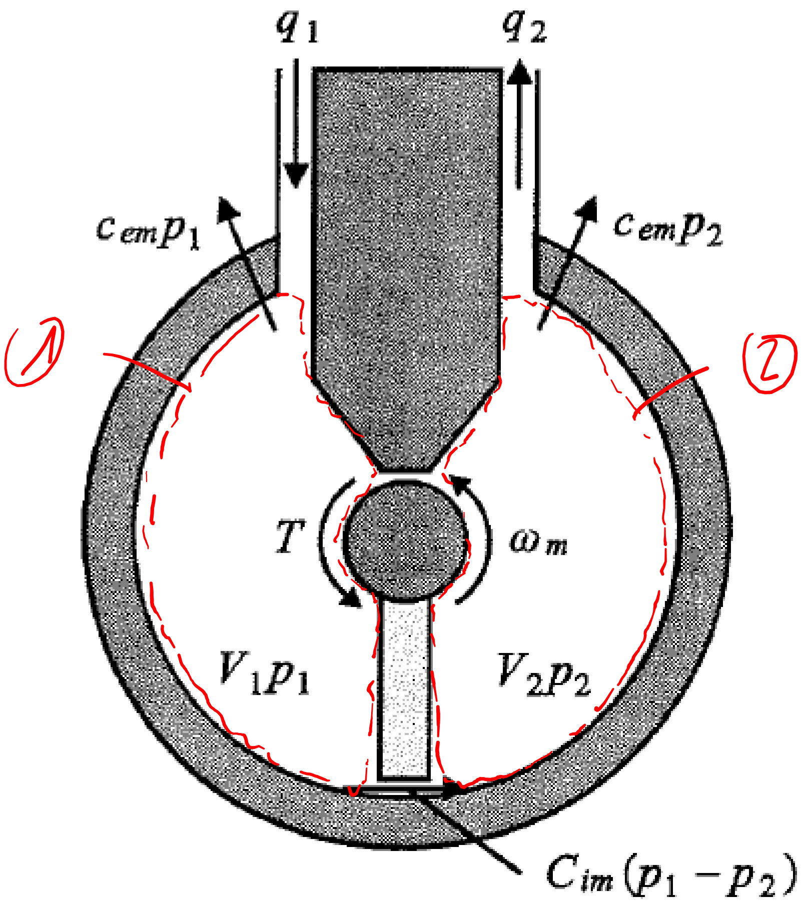


Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

Rotational hydraulic motor II

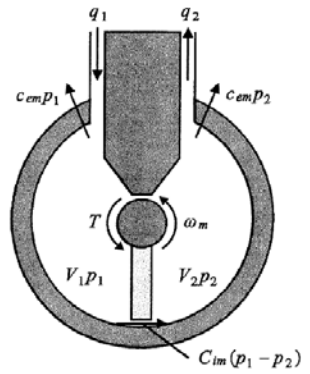


Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

mass balance: $\frac{d}{dt}m = q_1\rho - C_{em}p_1\rho - C_{in}(p_1-p_2)\rho$

① $\frac{V_1}{\beta}\dot{p}_1 + \dot{V}_1 = q_1 - C_{em}p_1 - C_{im}(p_1-p_2)$

② $\frac{V_2}{\beta}\dot{p}_2 + \dot{V}_2 = -q_2 - C_{em}p_2 - C_{im}(p_2-p_1)$

$$\dot{V}_1 = -\dot{V}_2 = D_m\omega_m$$

displacement

momentum equation

③ $J\dot{\omega}_m = T_m - B\omega_m - T_L$

friction

load

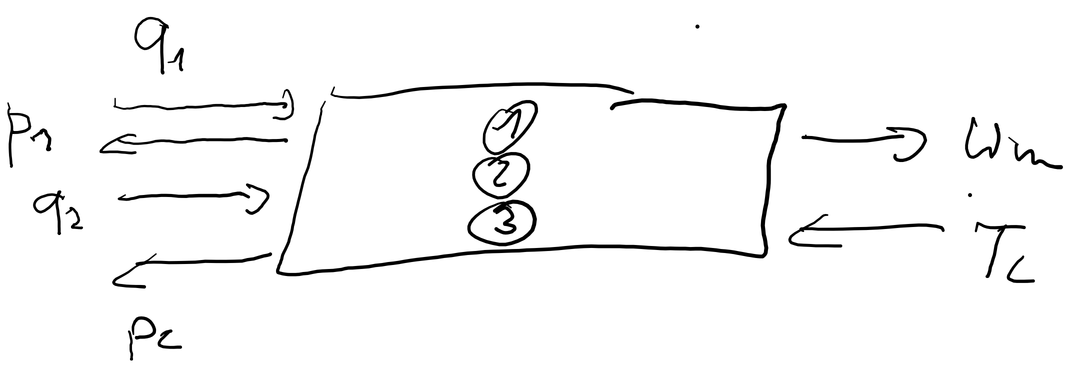
Rotational hydraulic motor III

Assume: loseless: $\text{Power}_{in} = \text{Power}_{out}$

$$\left[\frac{Nm}{J}\right] \quad T_m \omega_m = p_1 \dot{V}_1 + p_2 \dot{V}_2 \quad \left[\frac{Nm}{J}\right]$$

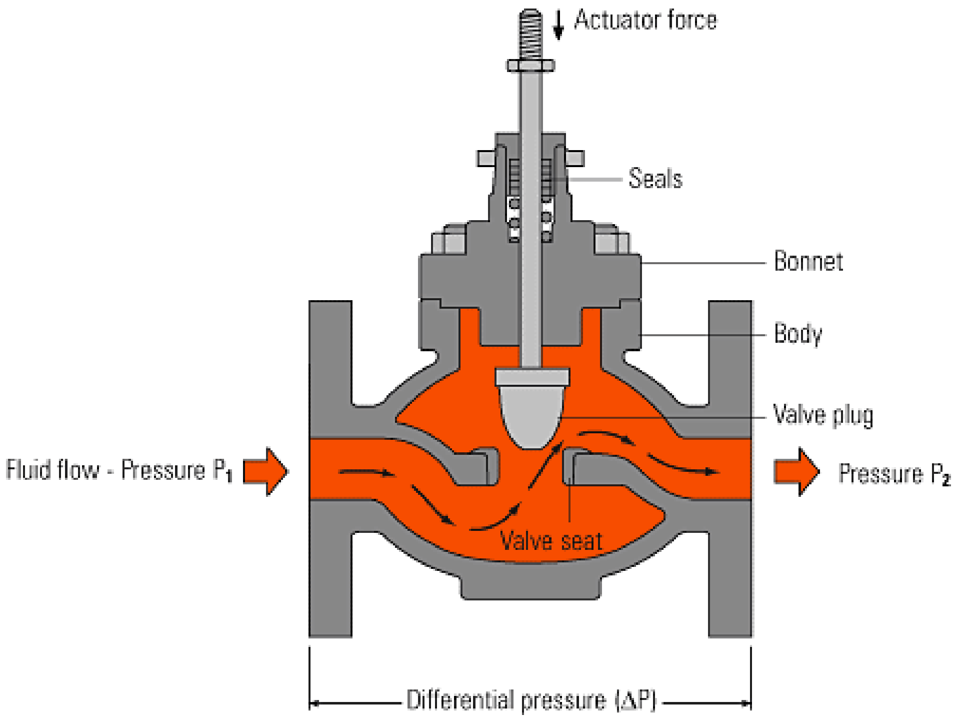
$$= D_m \omega_m (p_1 - p_2)$$

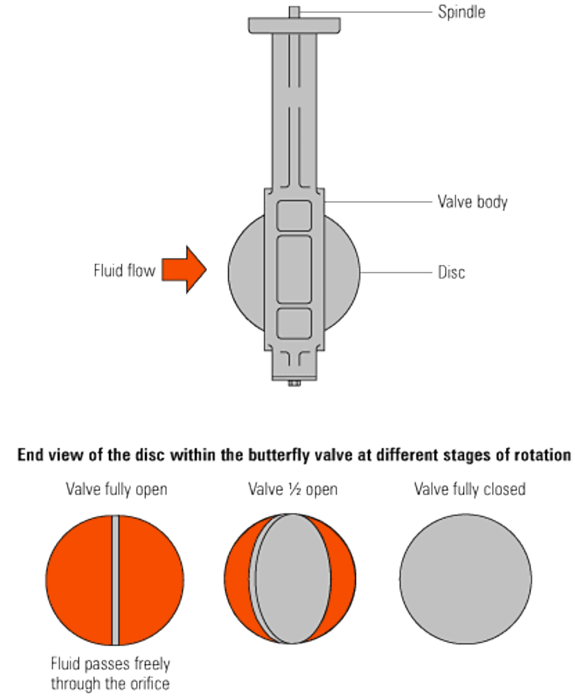
$$T_m = D_m (p_1 - p_2)$$

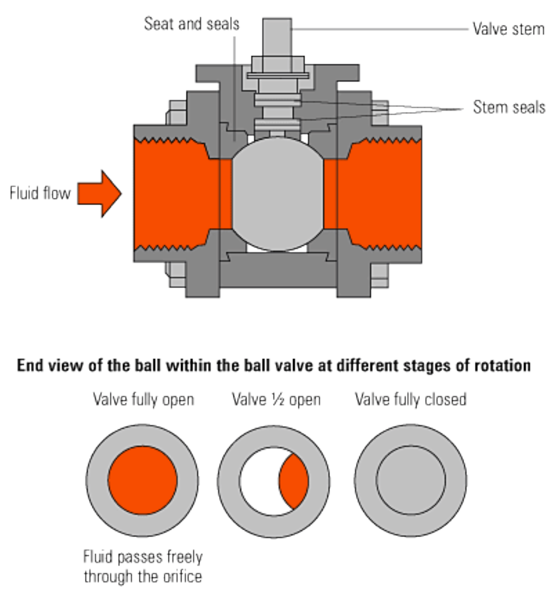


Valves

- Device that regulates flow
- Many different types of valves exist
  - Globe valve, ball valve, butterfly valve, ...





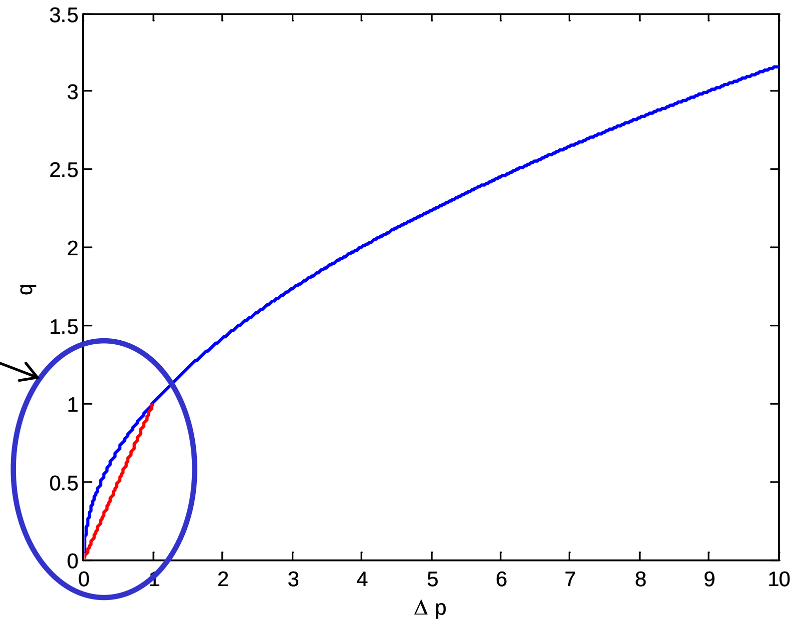


Valve models (book 4.2)

- Flow through a restriction is generally turbulent

$$q = C_d A \sqrt{\frac{2}{\rho} \Delta p}$$

Problematic!

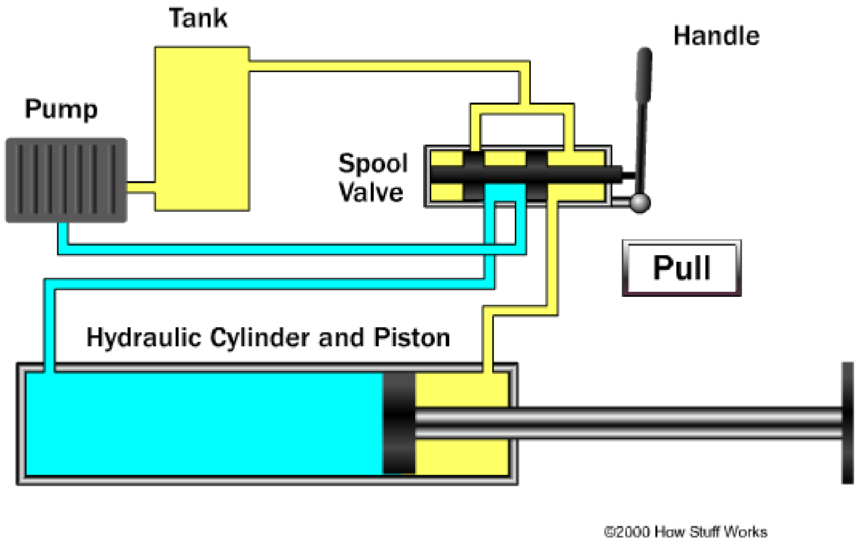


- Solution: Regularize by assuming laminar flow for small *Δp*

$$q = C_l \Delta p$$

- Book: Make transition smooth

Four-way valve



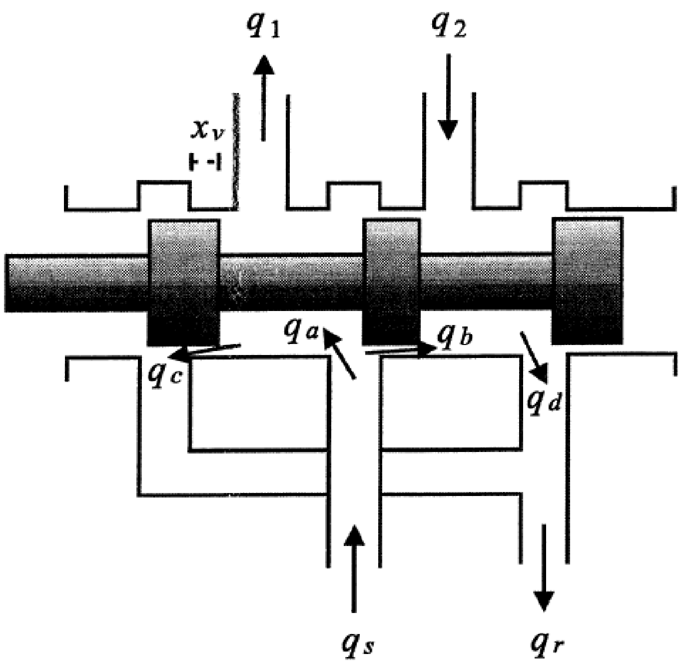
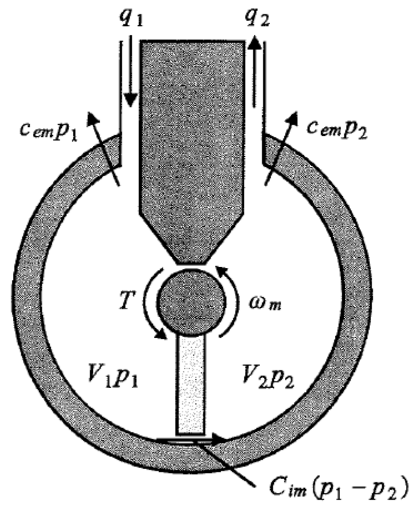


Figure 4.2: A matched and symmetric four-way valve.



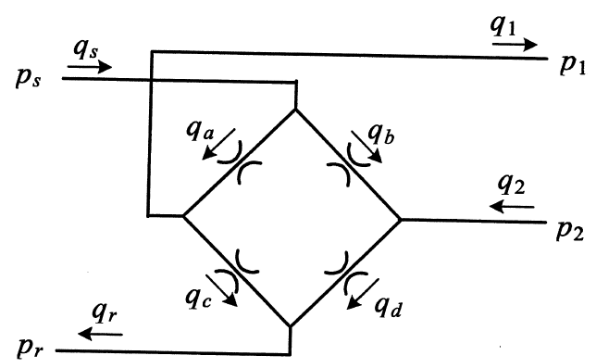


Figure 4.1: Four-way valve

Modeling of four-way valve

- Define load pressure

$$p_L = p_1 - p_2$$

- Define load flow

$$q_L = \frac{q_1 + q_2}{2}$$

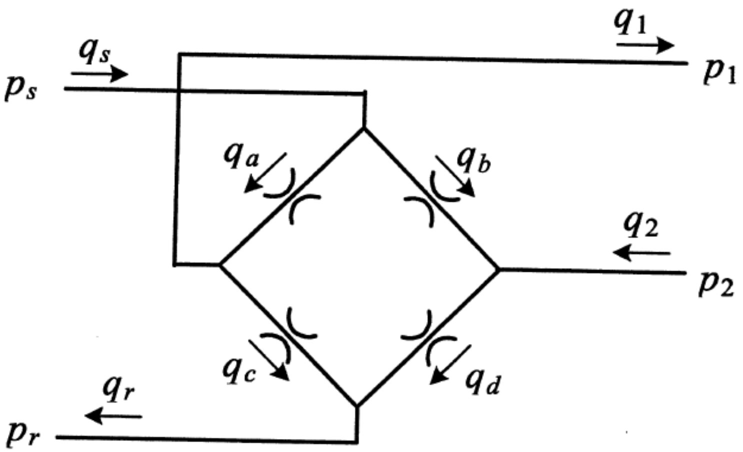


Figure 4.1: Four-way valve

- Symmetric load assumption (motor)

$$q_1 = q_2$$

- Symmetric valve and symmetric load

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \text{sign}(x_v) p_L\right)}$$

Characteristic of four-way valve

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \mathrm{sign}(x_v) p_L\right)}$$

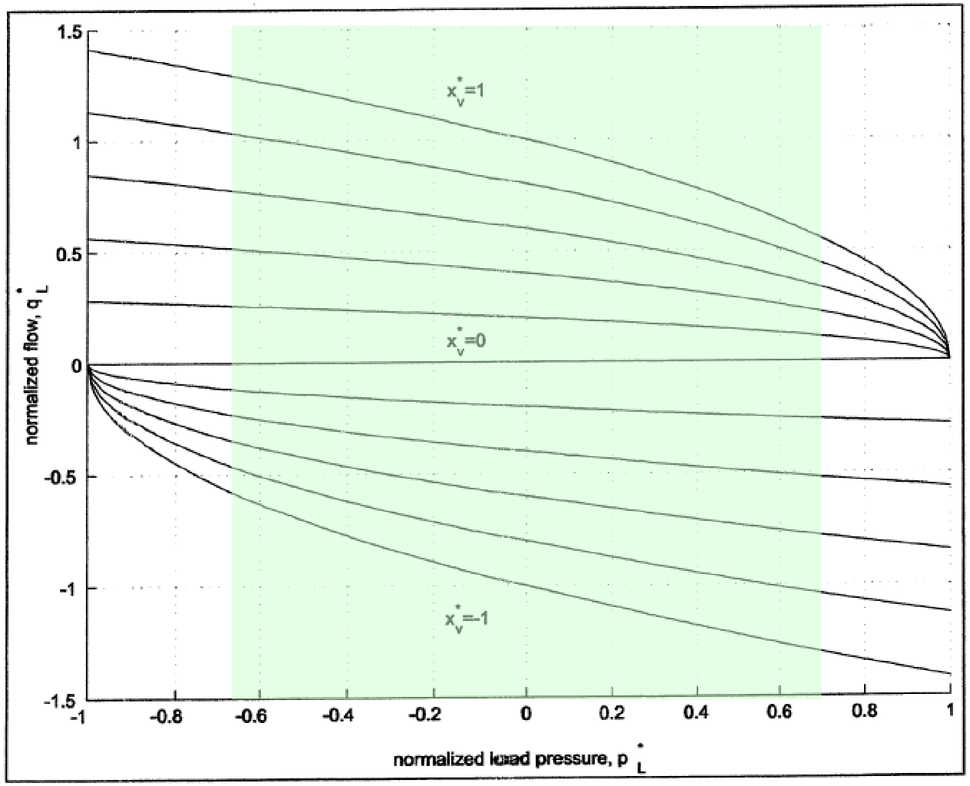


Figure 4.3: Valve characteristic

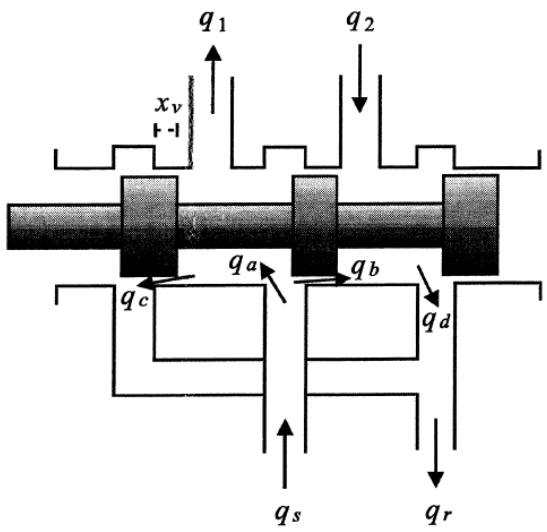
Linearized model:

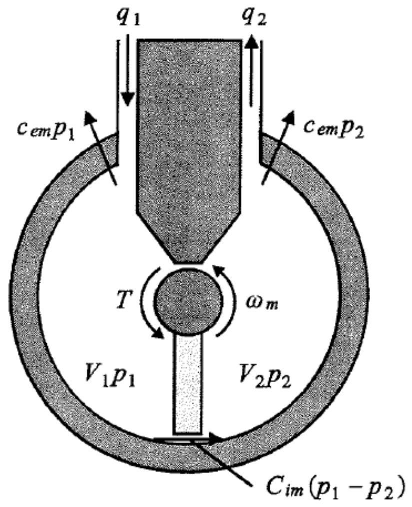
$$|p_L| \leq \frac{2}{3} p_s : \quad \boxed{q_L = K_q x_v - K_c p_L}$$

Gain uncertainty:

$$0.58 K_{q0} \leq K_q \leq 1.29 K_{q0}$$

Transfer function valve+motor

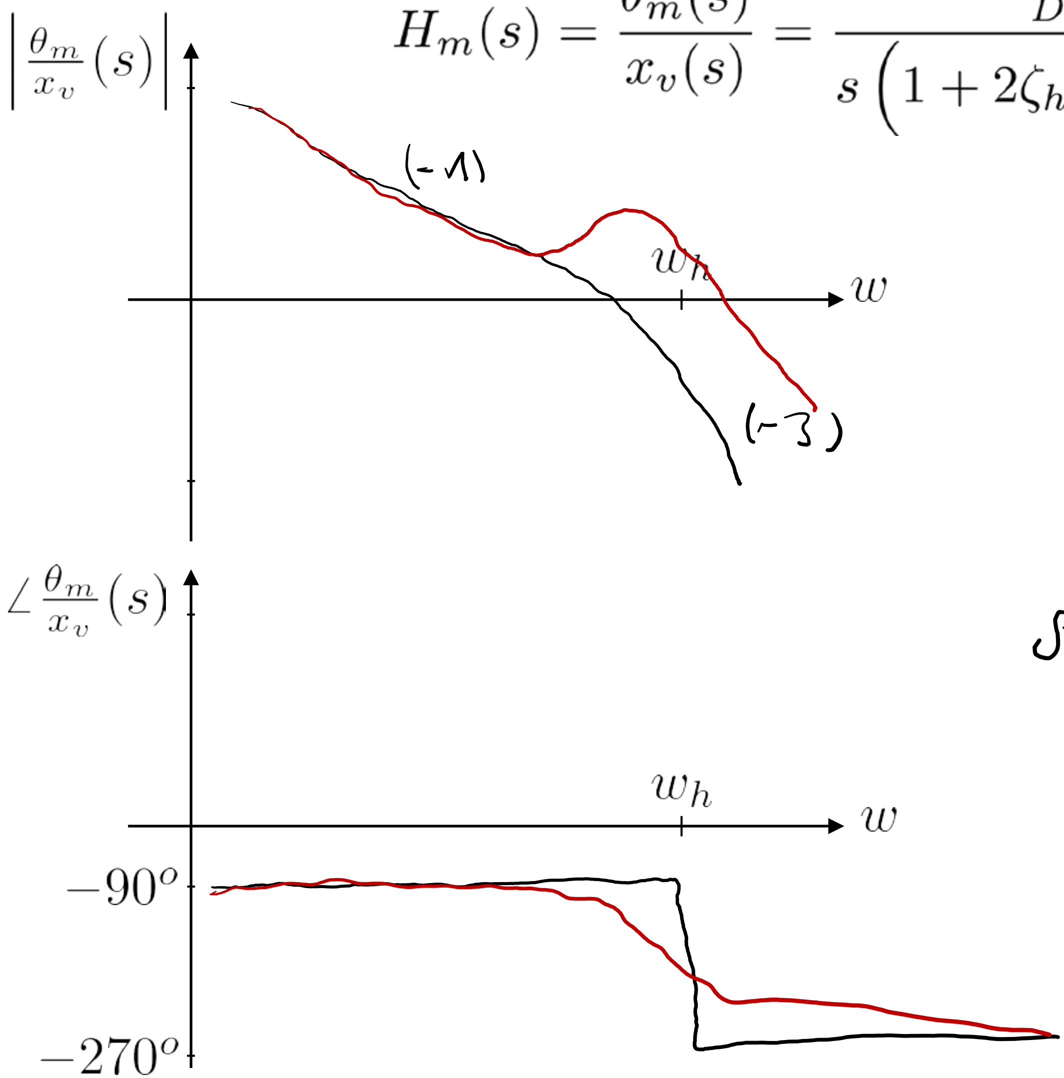




$$\theta_m(s) = \frac{\frac{K_q}{D_m}x_v(s) - \frac{K_{ce}}{D_m^2}\left(1+\frac{s}{\omega_t}\right)T_L(s)}{s\left(1+2\zeta_h\frac{s}{\omega_h}+\frac{s^2}{\omega_h^2}\right)}$$

Transfer function spool to shaft

$$H_m(s) = \frac{\theta_m(s)}{x_v(s)} = \frac{\underbrace{\frac{K_q}{D_m}}_{K_v}}{s\left(1 + 2\zeta_h \frac{s}{\omega_h} + \frac{s^2}{\omega_h^2}\right)}$$



P - controll

$w_h = w_{180}$ bandwidth of system

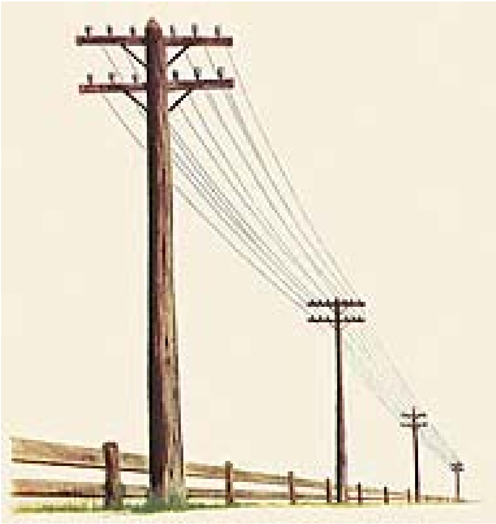
rule thumb $w_c \approx 0.1 w_h$

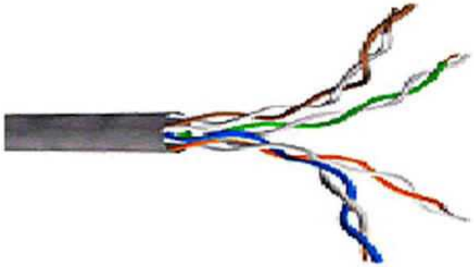
stable if:

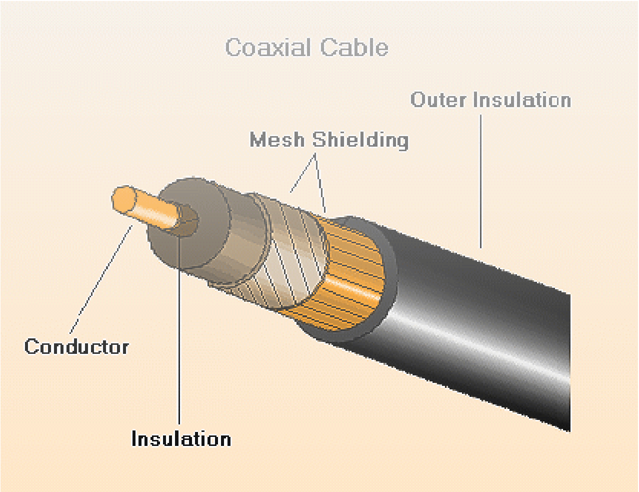
$$K_v \leq 2 \zeta_h w_h$$

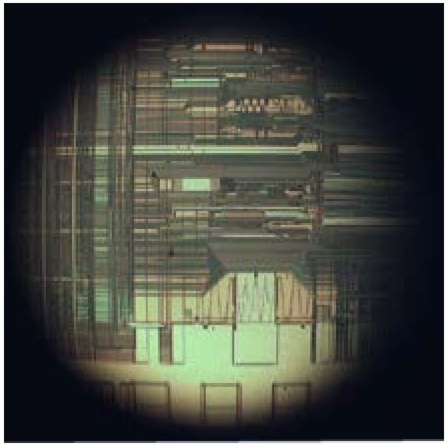
$$\Rightarrow K_p = 2 \frac{D_m}{K_q} \zeta_h w_h$$

Electrical transmission lines



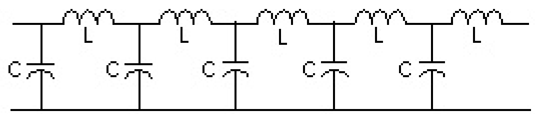




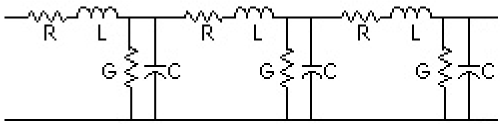


Telegrapher's equation (Wave equation)

- Lossless:



Lossy:



- Model (Ch. 1.6):

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

- Laplace:

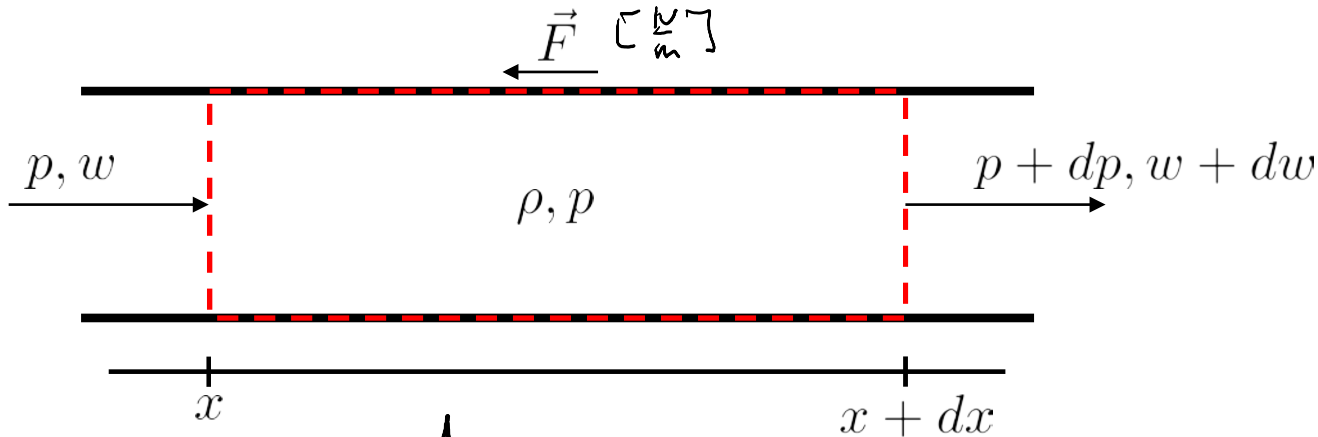
$$\frac{\partial u(x,s)}{\partial x} = -X(s)i(x,s)$$
$$\frac{\partial i(x,s)}{\partial x} = -Y(s)u(x,s)$$

Series impedance:
$$X(s) = R + Ls$$
Parallel admittance:
$$Y(s) = G + Cs$$

Characteristic impedance:
$$Z_c(s) = \sqrt{\frac{X(s)}{Y(s)}}$$

Example: Transmission line I

$$w = \iint_A \rho_v \, dA = \rho q$$



mass balance: $$\frac{d}{dt} m = w_1 - w_2$$

$$A \, dx \frac{d\rho}{dt} = w - (w + dw) = -dw$$

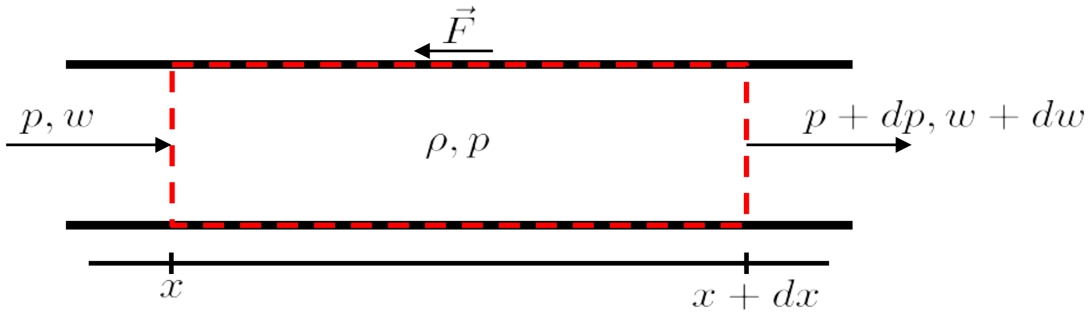
$$\frac{\partial \rho}{\partial t} = -\frac{1}{A} \frac{\partial w}{\partial x}$$ [pde] [differential balance]

bulk modulus

$$dp = \frac{\beta}{\rho} d\rho$$

$$\frac{\partial p}{\partial t} = -\frac{\beta}{\rho A} \frac{\partial w}{\partial x}$$

Example: Transmission line II



momentum balance:

$$\frac{d}{dt} \underbrace{\iiint_{V_c} \rho v \, dV}_{= w\,dx} = A p - A(p + dp) - F\,dx + \iint_{A_1} \rho v^2 \, dA - \iint_{A_2} \rho (v + dv)^2 \, dA$$

$$\longrightarrow \quad \frac{\partial w}{\partial t} = -A \frac{\partial p}{\partial x} - F + \frac{\partial}{\partial x} \left(\rho \iint_{A_1} v^2 \, dA - \iint_{A_2} (v + dv)^2 \, dA \right.$$

Linearize around $v = 0$ and $\rho = \rho_0$

$$\boxed{\begin{aligned} \frac{\partial p}{\partial t} &= -\frac{B}{A} \frac{\partial q}{\partial x} \\ \frac{\partial q}{\partial t} &= -\frac{A}{\rho_0} \frac{\partial p}{\partial x} - \frac{F}{\rho_0} \end{aligned}}$$

Same equations for electrical and fluid/hydraulical transmission lines

Electrical transmission lines:

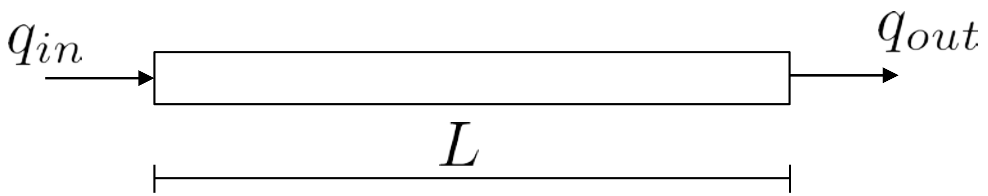
$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

Fluid transmission lines:

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\beta}{A}\frac{\partial q(x,t)}{\partial x}$$
$$\frac{\partial q(x,t)}{\partial t} = -\frac{A}{\rho}\frac{\partial p(x,t)}{\partial x} - \frac{F\left[q(x,t)\right]}{\rho}$$

- Current and flow “same” variables, as is voltage and pressure
- In both cases, we can define line impedance, characteristic impedance, propagation operator, etc.
- Solution to equations have same structure/form: waves propagating back and forth

When do we need these equations?



if L is "small" $\frac{V}{\beta}\dot{p} + \dot{V} = q_{in} - q_{out}$

if L is "large" → transmission line

$c = \sqrt{\beta/\rho}$ "sonic velocity" [1 bar = $10^5 \frac{N}{m^2}$]

Example: hydraulic oils: $c = \sqrt{\frac{7000\,\text{bar}}{870\,\text{kg/m}^3}} \approx 1000\,\text{m/s}$

$c = \frac{L}{T}$

L	1m	10m	~~100~~ 500m
T	1ms	10ms	0.5ms

Drilling fluid: $L = 10\,\text{km}$ $c = \sqrt{\frac{15000\,\text{bar}}{1600\,\text{kg/m}^3}} \approx 970\,\text{m/s}$

$T \approx 10\,\text{s}$

Laplace transformation

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\beta}{A}\frac{\partial q(x,t)}{\partial x}$$
$$\frac{\partial q(x,t)}{\partial t} = -\frac{A}{\rho}\frac{\partial p(x,t)}{\partial x} - \frac{F\left[q(x,t)\right]}{\rho}$$

$$Z_0 = \frac{\rho c}{A}$$

$$\frac{\partial q(x,s)}{\partial x} = -\frac{s}{c Z_0} p(x,s)$$

$$\frac{\partial p(x,s)}{\partial x} = -\frac{Z_0 s}{c} q(x,s) - \frac{Z_0 F[q(x,s)]}{c\rho}$$

$$= -\frac{Z_0 \Gamma^2(s)}{LTs} q(x,s)$$

$\Gamma(s)$: propagation operator

Friction - Examples

- linear friction : $F = \rho B q$ [Hagen-Poiseuille]

$$\Gamma(s) = T s \sqrt{\frac{s+B}{s}} \qquad Z_c = Z_0 \sqrt{\frac{s+B}{s}}$$

characteristic impendence

- no friction (special case) $F = 0$

$$\frac{Z_0 \Gamma^2(s)}{L T s} = \frac{Z_0 s}{C}$$

$\rightarrow \Gamma(s) = T \cdot s$

Wave variables

$$\frac{\partial}{\partial x}\begin{pmatrix} q(x,s) \\ p(x,s) \end{pmatrix} = \begin{pmatrix} 0 & -\frac{Ts}{L Z_0} \\ -\frac{Z_0 \Gamma^2(s)}{C T s} & 0 \end{pmatrix}\begin{pmatrix} q(x,s) \\ p(x,s) \end{pmatrix}$$

Wave variables:

$$a(x,s) = p(x,s) + Z_c\, q(x,s)$$

$$b(x,s) = p(x,s) - Z_c\, q(x,s)$$

$$Z_c(s) = Z_0 \frac{\Gamma(s)}{Ts}$$

$$\frac{\partial a(x,s)}{\partial x} = -\frac{\Gamma(s)}{L}\, a(x,s)$$

$$\frac{\partial b(x,s)}{\partial x} = \frac{\Gamma(s)}{c}\, b(x,s)$$

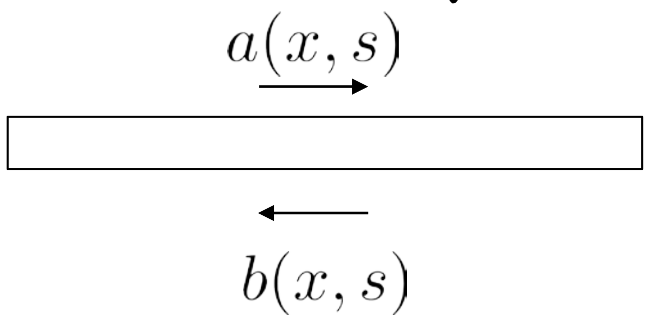
Solution: Wave variables

$$a(x,s) = \exp\left(-\Gamma \frac{x}{L}\right) a(0,s)$$

$$b(x,s) = \exp\left(-\Gamma \frac{L-x}{L}\right) b(L,s)$$

$a(0,s) = a_1(s)$

$b(0,s) = b_1(s)$



$a(L,s) = a_2(s)$

$b(L,s) = b_2(s)$

lossless: $\Gamma = Ts$

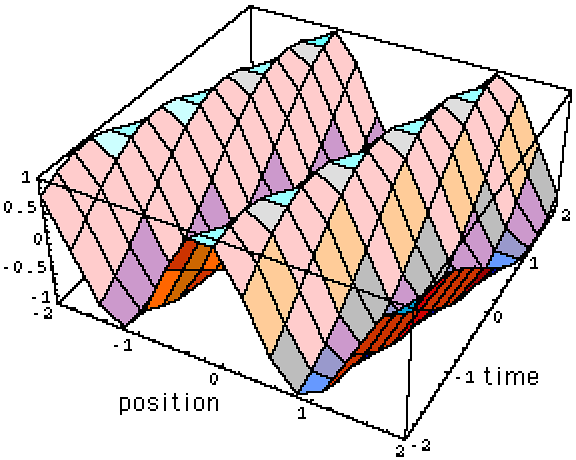
$\rightarrow$ only time delay e^{Ts}

Solution: Waves

- Solution: $$u_{out}(s) = e^{-\Gamma(s)} u_{in}(s)$$

- Propagation operator $\Gamma(s) = L\sqrt{X(s)Y(s)}$
  - Attenuation factor $\alpha = Re[\Gamma(j\omega)]$: How much is wave reduced
  - Phase factor: $\beta = Im[\Gamma(j\omega)]$: How long does it take

- Lossless (*R* = *G* = 0): $\Gamma(s) = Ts$
  - Attenuation factor: 0
  - Phase factor: Pure time-delay



When should we care?

- Solution lossless case: Time delay

$$e^{-Ts}$$

- Rule-of-thumb from control theory: We can ignore time-delay for frequencies much less than 1/*T*

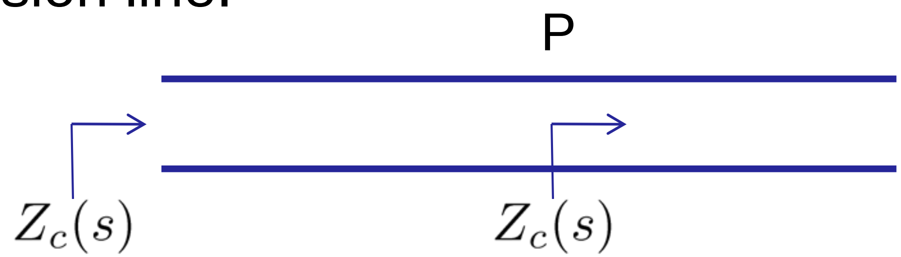
$$\omega \leq \frac{1}{T} \Rightarrow 2\pi\frac{c}{\lambda} \leq \frac{c}{L} \Rightarrow L \leq \frac{\lambda}{2\pi}$$

- Rule-of-thumb for transmission lines: When L is larger than one tenth of wavelength, treat as transmission line

- Power lines, f = 50Hz: λ = 6000km
- Personal computers, f = 10GHz: λ = 1.5cm

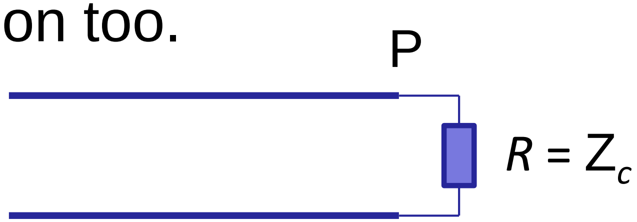
Impedance matching

- Suppose we have an imaginary joint at P in a very long transmission line.



The wave goes through the joint without reflection because there is actually no joint (just imagined).

- Now, let us terminate a resistance of value Z_c *at the same position of this* imaginary joint. The wave will go through without reflection too.



This is called a **matched load.**

http://cktse.eie.polyu.edu.hk/eie403/Transmissionline.pdf

Homework

- Read 4.5 (Hydraulic transmission lines)