Project: Spectrum Sensing in OFDM Cognitive Radios

This project deals with spectrum sensing in OFDM based wireless communication systems. You will design a Neyman-Pearson (NP) detector to determine the spectrum availability.

Background and problem formulation

In wireless communication systems, spectrum is a scarce resource and is regulated by legislation. In practice, service providers (or operators) pay a substantial amount of money to the government in their respective country for licensing of the radio spectrum. Consequently, some part of your monthly mobile subscription cost will finance the spectrum license, while other parts will pay for cost related to infrastructure, maintenance, etc. Paying customers demand a certain quality-of-service (QoS) on their information transfer, and will quickly become dissatisfied if the experienced data rate is below the rate specified in the subscription. From a system point-of-view, a paying customer is considered as the primary user (PU) of the system, and any interference appearing on its communication channel should be kept at a minimum to ensure the promised QoS.

Traditionally, the wireless communication standards, e.g., 3G and 4G, have used a fixed spectrum allocation policy. That is, a slice of the spectrum is allocated over a long time period to be used only by the primary users (or licensees) of the system. However, it has been shown that this approach results in very uneven spectrum utilization that varies with time, frequency, and spatial location. This observation was the main driver behind so-called *cognitive radio* introduced to exploit underutilized spectral resources in an attempt to match the constantly growing demands for high data rates.

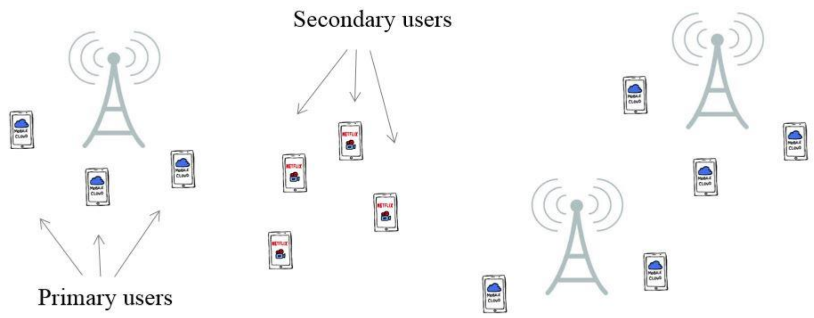

Figure 1. *Network of cognitive radios, wherein secondary users sense the frequency spectrum for spectrum opportunities and exploit them in an agile manner.*

A cognitive radio system, depicted in Fig. 1, allows co-existence of PUs and secondary users (SUs), where SUs try to use the spectrum in an opportunistic manner whenever PUs are idle. It is important that priority to access the spectrum is given to PUs. For this purpose, SUs need to sense the spectrum and *detect* when it can be used without causing interference to the PUs. The introduction of cognitive radio will inevitably lead to increased interference levels, since there is always a risk that an SU decides to transmit at the same time as a PU and, as a consequence, degrade the PU's QoS. The problem of detecting the availability of spectrum is referred to as *spectrum sensing* and can be formulated as a binary hypothesis testing problem. The requirements are that the SUs should be able to detect very weak PU signals and minimize the probability of transmitting at the same time as the PU and, of course, not be overly pessimistic in their strategy so that they refrain from exploiting unused spectrum. This is exactly the type of problem solved by the NP detector, which we have studied extensively in the course.

The detection problem under consideration is given by

$$H_0: x(n) = w(n), \;\; n = 0, 1, \dots, N-1$$

$$H_1: x(n) = s(n) + w(n), \;\; n = 0, 1, \dots, N-1,$$

where $s(n)$ is the sequence (waveform) of the PU and $w(n)$ additive white Gaussian noise.

A popular modulation method to encode digital information and construct sequence $s(n)$, that will be transmitted over the wireless channel, is orthogonal frequency-division multiplexing (OFDM). OFDM is a form of multicarrier modulation employed in today's 4th generation mobile system, wherein each information symbol $S(k)$, $k = 0, 1, \dots, N-1$, is allocated to one out of N carrier frequencies. That is, we construct an *N*-point discrete spectrum wherein the *k*th spectral component is multiplied by $S(k)$. As we learned in TTT4120, the unique time-domain signal $s(n)$, corresponding to the sampled spectrum, is obtained by the inverse discrete Fourier transform (IDFT), whose fast implementation, the FFT, is one of the main reasons for the successful adoption of OFDM.[1] Thus, the PU's time-domain signal is given by

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S(k) e^{\frac{j2\pi nk}{N}}, n = 0, 1, \dots, N-1.$$

We notice that $s(n)$ is a complex-valued quantity while in the lectures we were mostly concerned with real-valued signals. In addition, it cannot be generally assumed that a SU has knowledge of data symbols $S(k)$ of the PU, which means $s(n)$ must be treated as a random sequence with known statistical properties.

In this project, your task will be to develop suitable models that will subsequently be used for spectrum sensing, so that the SU can detect when the spectrum is idle (and not cause interfere to a PU transmission). You will also derive performance guarantees for your detector, e.g., the resulting probability of false alarm and probability of detection. You will work with seven data sets, namely,

- T1_data_Sk_Gaussian.mat: This data set contains 1024 realizations of Gaussian PU data symbols $S(k)$.
- T1_data_Sk_BPSK.mat: This data set contains 1024 realizations of BPSK PU data symbols $S(k)$.
- T3_data_x_H0.mat: This data set contains 1024 realizations of signal observed at the SU $x(n)$ when PU is absent.
- T3_data_x_H1.mat: This data set contains 1024 realizations of signal observed at the SU $x(n)$ when PU is present.
- T3_data_sigma_w.mat: This data set contains 1024 realizations of noise at SU $w(n)$.
- T3_data_sigma_s.mat: This data set contains 1024 realizations of PU signal $s(n)$.
- T8_numerical_experiment.mat: This data set contains 100 realizations of signal observed at the SU $x(n)$ to determine spectrum vacancy. The sample size is $N = 256$, noise variance $\sigma_w^2 = 1$ and signal power $\sigma_s^2 = 5$.

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[1]*Another reason is the simple demodulation of symbols, at the receiver, in presence of a frequency-selective channel. However, this feature is something that will be dwelled upon in other courses, e.g., TTT4130 Digital Communication.*

Project tasks

1. *Model building:* To develop a suitable detector, we need a model that explains how the observed data $x(n)$ is generated. In other words, we seek the PDFs, $p_0(\boldsymbol{x})$ and $p_1(\boldsymbol{x})$, associated with each hypothesis. First, we will verify that the complex-valued time-domain OFDM signal sequence $s(n) = s_R(n) + js_I(n)$ is independent and identically distributed (*i.i.d*) and accurately modeled with a complex Gaussian distribution. That is, through the supplied data sets, you should illustrate that the PDF of a single sample of $s(n)$ can be modeled as a complex normal variable whose probability density function (PDF) is given by

$$p(s) = p(s_R)p(s_I) = \frac{1}{\pi\sigma_s^2} e^{-\frac{1}{\sigma_s^2}|s-\mu_s|^2}$$

where $s_R(n) \sim N\left(0, \frac{\sigma_s^2}{2}\right)$, $s_I(n) \sim N\left(0, \frac{\sigma_s^2}{2}\right)$, $\mu_s = E\{s(n)\} = E\{s_R(n)\} + jE\{s_I(n)\}$. For this purpose, plot histograms of $s_R(n)$ and $s_I(n)$, compute estimates of $E\{s_R(n)s_I(n)\}$ and $E\{s(n)\} = E\{s_R(n)\} + jE\{s_I(n)\}$. Two sets of data for the PU data symbols $S(k)$ are provided. The dataset "T1_data_Sk_Gaussian.mat" contains samples from standard normal Gaussian distribution and dataset "T1_data_Sk_BPSK.mat" contains binary phase shift keying (BPSK) symbols with $P(S(k) = 1) = P(S(k) = -1) = 0.5$. Comment on your observations.

2. *One-sample-detector:* Let us assume that a single sample is observed, i.e., $N = 1$. Let $w(0)$ be a zero-mean complex Gaussian random variable with variance σ_w^2, *cf.* PDF for $s(n)$. Given that you have only knowledge of σ_s^2 and σ_w^2, derive the NP detector and verify that it is given by

$$|x(0)|^2 = x_R^2(0) + x_I^2(0) > \lambda'$$

3. *Performance of the one-sample-detector:* Verify from datasets "T3_data_x_H0.mat" and "T3_data_x_H1.mat" that

$2x_R^2(0)/\sigma_w^2 + 2x_I^2(0)/\sigma_w^2$ (under H_0) and

$2x_R^2(0)/(\sigma_w^2 + \sigma_s^2) + 2x_I^2(0)/(\sigma_w^2 + \sigma_s^2)$ (under H_1)

are both accurately modeled as chi-square random variable with two-degrees of freedom. Simply form the histogram of $2|x(0)|^2/\sigma_s^2$ and $2|x(0)|^2/(\sigma_s^2 + \sigma_w^2)$ and compare with the true PDFs given by function e.g., by using Matlab function 'pdf'. Use a suitable estimator for σ_w^2 and σ_s^2. Use the data set "T3_data_sigma_w.mat" and "T3_data_sigma_s.mat" to estimate σ_w^2 and σ_s^2, respectively. Compute the probability of detection P_D and probability of false alarm P_{FA} for the one-sample-detector.

4. *NP detector with data set of K samples:* Based on the knowledge acquired above, compute the NP detector for the case with *K* samples and the associate threshold λ' that maximizes the probability of detection P_D while ensuring that the probability of false alarm P_{FA} is less than a given limit.

5. *Performance of a general NP detector:* Compute the distribution of the test statistic obtained in Task 4 under hypothesis H_0 and H_1. Plot the receiver operating characteristics (ROCs). You may use the estimates of σ_w^2 and σ_s^2 obtained in Task 3.

6. *Approximate performance of a general NP detector:* Use the central limit theorem to approximate the test statistic as Gaussian random variable and compute the PDF of the test

statistic. Plot P_D and P_{FA} as a function of the threshold and compare it with the exact P_D and P_{FA} obtained in Task 4.

Note: The central limit theorem states that for large n the random variable $X = \sum_{i=1}^{n} X_i$, where X_i is a random variable with any distribution but with finite mean μ_i and variance σ_i^2 , can be approximated as a Gaussian random variable with mean $\mu = \sum_{i=1}^{n} \mu_i$ and variance $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$. You may refer any introductory book on probability and random variables for a more rigorous statement, cf. chapter 7, pg. 278, *Probability, Random Variables, and Stochastic Processes*, by A. Papoulis and S. U. Pillai.

7. *Complexity of detector:* Using the approximation in Task 6, find an expression compute the number of samples required to attain a given P_D and P_{FA}.

8. *Numerical experiments in PU detection:* Take the dataset named "T8_numerical_experiment.mat" and apply your NP detector to decide whether a PU is present or not. This data set contains 100 realizations of signal observed at the SU $x(n)$ to determine spectrum vacancy. The sample size is $N = 256$, noise variance $\sigma_w^2 = 1$ and signal power $\sigma_s^2 = 5$. Chose $P_{FA} = 0.1$ and $P_{FA} = 0.01$. Tabulate the decision made by SU at different time instants. How confident are you in your result?