Norwegian University of Science and Technology
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Exercise 4
Quadratic Programming

Problem 1 (50 %) Quadratic Programming

a Under which conditions is a QP-problem convex? Why is convexity an important property?

b Go through and understand the proof of Theorem 16.2. If you were to formulate the lemma with the condition $Z^\top GZ \geq 0$ instead of $Z^\top GZ > 0$, how would you change the wording of the theorem? How would the proof change?

c Based on Example 16.4 in the textbook, show how Algorithm 16.3 finds the solution if the starting point is $x = [2, 0]^\top$ and we assume that only the constraint $-x_1 + 2x_2 + 2 \geq 0$ is active. This means $\mathcal{W}_0 = \{3\}$.

d Define the dual problem for the QP-problem in Example 16.4. Hint: See Section 12.9 and Example 12.10 and 12.12. Note that in this section, $f(x)$ is the primal objective and $q(x)$ is the dual objective.

e Explain how the dual optimization problem can be used to give an over-estimate of $q(\bar{x}) - q(x^*)$, when x^* is not known. ($q(x)$ is the objective function in a QP.) Hint: See Theorem 12.11 (and note that in this theorem, $f(x)$ is the primal objective and $q(x)$ is the dual objective).

Problem 2 (50 %) Production Planning and Quadratic Programming

Two reactors, R_I and R_{II}, produce two products A and B. To make 1000 kg of A, 2 hours of R_I and 1 hour of R_{II} are required. To make 1000 kg of B, 1 hour of R_I and 3 hours of R_{II} are required. The order of R_I and R_{II} does not matter. R_I and R_{II} are available for 8 and 15 hours, respectively. We want to maximize the profit from selling the two products.

The profit now depends on the production rate:

- the profit from A is $3 - 0.4x_1$ per tonne produced,
- the profit from B is $2 - 0.2x_2$ per tonne produced,

where x_1 is the production of product A and x_2 is the production of product B (both in number of tonnes).

a Formulate this as a quadratic program.

b Make a contour plot and sketch the constraints.

c Find the production of A and B that maximizes the total profit. Do this using the MATLAB files posted on Blackboard; modify the file qp_prodplan.m so that it solves the problem formulated in a) (define G, c, A, and b and run the file). Is the solution found at a point of intersection between the constraints? Are all constraints active? Mark the iterations on the plot made in b), as well as the iteration number.

d The solution is calculated by an active-set method. Explain how this method works based on the sequence of iterations from c).

e Compare the solution in c) with Problem 2 c) in exercise 3 and comment.