# TTT4275 Summary for January 18th
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# The CRLB 1

**a)** Assume we know (or has estimated) $p(x, \theta)$; i.e. we "know" the problem. Thus point b) and c) applies for any estimator.

**b)** Further assume the 'regularity' condition is fulfilled :

$$E\{\frac{\delta log[p(x;\theta)])}{\delta\theta}\} = 0 \tag{1}$$

**c)** Then the CRLB is given by the right hand side of equation 2 and the inequality applies for any estimator $\widehat{\theta}$ :

$$var(\widehat{\theta}) \geq E\{\frac{-1}{\frac{\delta^2 log[p(x;\theta)])}{\delta^2\theta}}\} = E\{\frac{1}{(\frac{\delta log[p(x;\theta)])}{\delta\theta})^2}\} \tag{2}$$

**d)** If equality is achieved we call the MVU estimator for **efficient** and the following reformulation applies

$$\delta log[p(x;\theta)] = I(\theta)[g(x) - \theta] \tag{3}$$

where $\widehat{\theta} = g(x)$ and $var(\widehat{\theta}) = I^{-1}(\theta)$

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# The CRLB 2

- The problem $x = A + w$ where $p(w) = N(0, \sigma^2)$ results in
  - The joint distribution $p(x; A) = N(A, \sigma^2)$
  - The estimator $\hat{A} = x$ was shown to be efficient with $var(\hat{A}) = CRLB = \sigma^2$
- The problem $x(n) = A + w(n)\ n = 0, \ldots, N-1$ where $p(w) = N(0, \sigma^2)$ results in
  - The joint distribution $p(x; A) = N(A, \sigma^2/N)$
  - The estimator $\hat{A} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)$ (sample mean) was shown to be efficient with $var(\hat{A}) = CRLB = \sigma^2/N$
- The phase problem $x(n) = Asin(2\pi f_0 n + \phi) + w(n)$ where $p(w) = N(0, \sigma^2)$ results in a CRLB which no efficient MVU estimator.