TTT4275 Summary from February 11th Spring 2019

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The Likelihood Ratio Test (LRT) - 1

- Given hypothesis distributions $p(x, H_i) = p(x, \theta_i)$ $i = 0, 1$ the socalled **"most powerful"** test is the LRT

$$L(x) = \frac{p(x/H_1)}{p(x/H_0)} \lessgtr \lambda \tag{1}$$

- The ratio (i.e distributions) is problem dependent while the threshold λ is mainly dependent of the choice of detection method.

- The simplest method is the maximum likelihood test, i.e. $\lambda = 1$

- The most general method is called the Bayes risk where

$$\lambda = \frac{P_0 C_{10}}{P_1 C_{01}} \tag{2}$$

  where P_i $i = 0, 1$ is the hypothesis priors while C_{01} and C_{10} are respectively the costs of miss and false alarm. These two costs are usually chosen by the developer.

The Likelihood Ratio Test (LRT) - 2

- In for instance medical applications the misses usually are more costly than false alarms.

- In other applications the two types of errors are equally costly, i.e $C_{01} = C_{10}$. Using the Bayes law $p(x/H_i)P_i = P(H_i/x)p(x)$ we then can reformulate the LRT to
$$L(x) = \frac{P(H_1/x)}{P(H_0/x)} \lessgtr 1 \tag{3}$$

- This is called the Maximum A Posteriori (MAP) detector and corresponds to the detector with minimum number of errors.

- In some applications there is a required maximum value on one of the error types.

- The Neyman-Pearson (NP) detector assumes a fixed P_{FA}. From this the threshold and thus the P_M is found.

- Instead we can start with a fixed P_M and derive the threshold and P_{FA}