NTNU – Trondheim
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Science and Technology

Assignment 10

TTK4130 Modeling and Simulation

Problem 1 (Tank with liquid, mass balance. 20 %)

A tank with area A is filled with an incompressible liquid with constant density ρ and level h. The liquid volume is then $V = Ah$, and the mass of the liquid in the tank is $m = V\rho$.

Liquid enters the tank through a pipe with mass flow $w_i = \rho A_i v_i$, where A_i is the pipe cross section, and v_i is the velocity over this cross section, which is assumed constant. Moreover, liquid leaves the tank through a second pipe with mass flow $w_u = \rho A_u v_u$, where A_u is the cross section of this pipe, and v_u is the velocity over its cross section, which is also assumed constant.

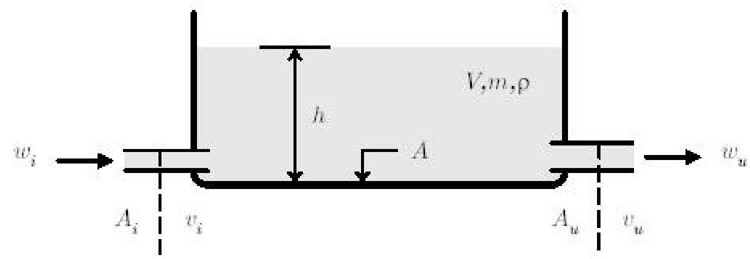

Figure 1: Tank with liquid

Use a mass balance for the tank to set up a differential equation for the level h.

Hint: Read sections 10.2, 10.4 and 11.1 in the book.

Problem 2 (Compressor, momentum balance, Bernoulli's equation. 15 %)

A compressor takes in air with pressure p_0 and velocity $v_0 = 0$ from the surroundings. The air flows through a duct into the compressor. For control purposes, it would be beneficial to have a measurement of the mass flow into the compressor. However, this measurement is not available.

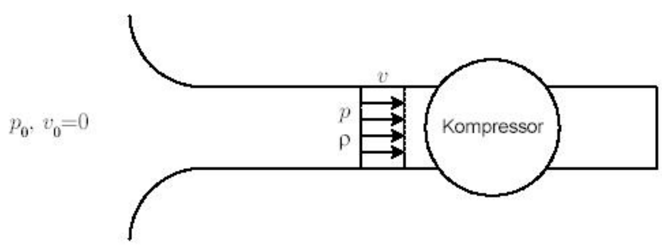

Figure 2: Compressor

Instead, there is a pressure measurement in the duct, giving a measurement p.

How can the mass flow w and velocity v be found from this measurement?

Assume that the density ρ in the duct is constant and known, that there is no friction, and that the velocity is uniform over the cross-section where the pressure transmitter is located.

Hint: Read section 11.2 in the book.

Problem 3 (Stirred tank, mass and energy balance. 30 %)

Figure 3 shows a stirred tank that cools an inlet stream. The tank is cooled by a "jacket" that contains a fluid of presumably lower temperature than the tank. The inlet stream to the tank has density ρ, temperature T_1 and mass flow rate w_1. Moreover, the outlet stream from the tank is given by

$$w_2 = Cu\sqrt{h}\,, \tag{1}$$

where C is a constant, u is the valve opening and h is the level of the liquid in the tank. Furthermore, assume that the outflow is controlled such that the level h does not exceed the height of the jacket.

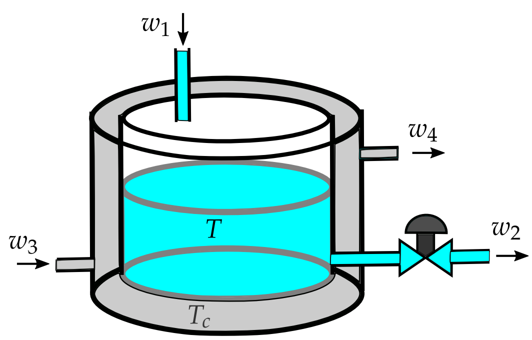

Figure 3: Tank with cooling jacket.

The inlet and outlet mass flow rates for the jacket are matched such that the jacket is always filled with fluid. In symbols, $w_3 = w_4$. Moreover, the cooling fluid has density ρ_c, and the inlet stream to the jacket has temperature T_3. Since the tank is stirred, we assume homogeneous conditions, i.e that the temperature T is the same everywhere in the tank. Similarly, we assume that the temperature T_c is the same everywhere in the jacket.

The cross-sectional area of the tank is A and the volume of the jacket is V_c.

The heat transfer from the tank to the jacket is given by

$$Q = Gh(T - T_c)\,, \tag{2}$$

where G is the heat transfer coefficient (a constant). We assume that the jacket and tank are well insulated from the surroundings, i.e. there are no other heat losses. Furthermore, we assume that both fluids are incompressible, i.e. that their specific internal energy and enthalpy can be assumed equal and proportional to the temperature, with constant of proportionality being c_p and c_{pc} for the two fluids, respectively.

Set up differential equations for the temperatures T in the tank and T_c in the jacket, and the level h in the tank.

Hint: Read sections 11.1 and 11.4 in the book.

Problem 4 (Mixing, reactions, mass balance. 35 %)

An incompressible liquid of substance C enters a perfectly mixed tank (a continuous stirred tank reactor, CSTR) with mass flow w_C and temperature T_C. In the tank, the substance reacts (e.g. due to the presence of a catalyst) to form the substance D with a rate JV, where J is the reaction rate per unit volume, and $V = Ah$ is the volume of the tank. The tank then contains a mixture of C and D, which leaves the tank with mass flow w and temperature T. The mass of substance C in the tank is denoted m_C, and the mass of substance D is denoted m_D.

(a) Assume that the average density ρ is constant, and set up a differential equation for the level of the tank.

Hint: Use the ordinary overall mass balance. Read section 11.1 in the book.

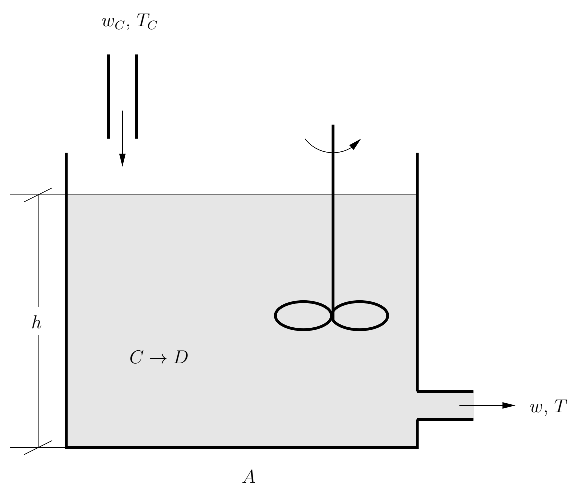

Figure 4: Tank reactor

(b) In a material volume V_m, the following holds:

$$\frac{D}{Dt}\iiint\limits_{V_m} \rho_C \, dV = -\iiint\limits_{V_m} J \, dV.$$

Use this together with an appropriate form of the transport theorem to show that the mass balance for substance C in integral form in a fixed control volume V_f is

$$\frac{d}{dt}\iiint\limits_{V_f} \rho_C \, dV = -\iiint\limits_{V_f} J \, dV - \iint\limits_{\partial V_f} \rho_C \vec{v} \cdot \vec{n} \, dA.$$

Hint: Read section 10.4 in the book.

In this problem, the natural control volume is the volume of the liquid in the tank. Although this volume is not fixed, this can be ignored since the expansion of the volume does not accumulate more substance C. In symbols, $\rho_C \vec{v}_c \cdot \vec{n} = 0$.

Assume from now onwards that J is proportional to the density of substance C: $J = k\frac{m_C}{V}$. Moreover, assume that the outflow of substance C is proportional to the mass ratio of substance C to the total mass in the tank, and the total outflow. In symbols, $w_{C,out} = \frac{m_C}{m_C + m_D} w$.

(c) Set up differential equations for the mass of substance C in the tank ($\frac{d}{dt} m_C = \ldots$).
Hint: Use the mass balance from part b.

(d) What is the mass balance equation in integral form for substance D in a fixed volume?
Use this to write up the mass balance for substance D.
Hint: This task is similar to the ones in part b. and part c.

(e) Verify that the differential equations from part c. and part d. agree with the answer in part a.

(f) Set up a differential equation for the temperature in the tank. Assume that the heat generated by the reaction is proportional to J, with proportionality constant c. Disregard kinetic energy, potential energy and pressure work. Furthermore, assume no "heat flux", i.e. that the tank is well insulated, and that the internal energy is $u = c_p T$.
Hint: The book does not treat energy balances with "internally generated" energy. Therefore, you must derive the energy balance in integral form for this case, as you did for the mass balance.