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Detecting a random variable

- Now we have $H_1 \; : \; x(n) = s(n) + w(n) \;\; n = 0, \ldots, N-1$

  where s is a random variable with density $p(s) = N(A, \sigma_s^2)$

- This leads to the distributions $p(x/H_0) = N(0, \sigma^2)$ and

  $p(x/H_1) = N(A, \sigma_x^2)$ where $\sigma_x^2 = \sigma_s^2 + \sigma^2$

- Deriving the test for the sufficient statistics we get

$$z = T(\mathbf{x}) = \sigma_s^2 \bar{x}_{sp} + 2A\sigma^2 \bar{x}_{sm} \lessgtr \sigma^2 A^2 + \sigma_s^2 \sigma_x^2 [log(\frac{\sigma_x^2}{\sigma^2}) + \frac{2}{N} log(\lambda)] \quad (1)$$

  where $\bar{x}_{sp} = \sum_n x^2(n)/N$ (power estimate) and $\bar{x}_{sm} = \sum_n x(n)/N$ (sample mean)

- z does not have a simple density, thus P_{FA} and P_M are not easily derived

- For the case $A = 0$ we have a power/energy detector; i.e. $z = \bar{x}_{sp}$

Detecting a deterministic sequence

- The hypothesis densities are $p(x(n)/H_1) = N(s(n), \sigma^2)$ and $p(x(n)/H_0) = N(0, \sigma^2)$

- Deriving $LLRT(\mathbf{x})$ we end up with

$$z = T(\mathbf{x}) = \sum_n x(n)s(n) \lesseqgtr 2\sigma^2 log(\lambda) + E_s = \eta \tag{2}$$

where $E_s = \sum_n s^2(n)$

- This detector is called a correlator and/or a matched filter

- We showed that $p(z/H_0) = N(0, \sigma^2 E_s)$ and $p(z/H_1) = N(E_s, \sigma^2 E_s)$

- Thus the false alarm is given by

$$P_{FA} = \int_\eta^\infty p(z/H_0)dz = \int_\eta^\infty N(0, \sigma^2 E_s)dz = Q(\frac{\eta}{\sqrt{E_s}\sigma}) \tag{3}$$

Generalized LLRT

- The value of the constant A in the H_1 hypothesis is not known

- We measure $\mathbf{x} = [x(n),\ n = 0, \ldots, N-1]$ but we do not know the mean A of $p(x/H_1) = N(A, \sigma^2)$

- Thus we need to find an estimate $\hat{A} = \sum_n x(n)/N$.

- Problem is that we do not know if we have the case H_1 (estimate is good) or H_0 (estimate is wrong)

- The estimator gives $H_1 : \hat{A} = A + q(n)$ or $H_0 : \hat{A} = q(n)$ where $p(q) = N(0, \sigma^2/N)$

- If we know the sign of A we can can set up a treshold η based on $P(x/H_0) = P(q/H_0) = \eta << 1$.

- Another option is to use the absolute value $|x(n)|$, however $p(|x|)$ is not Gaussian.