Norwegian University of Science and Technology
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Exercise 8
Unconstrained Optimization

Problem 1 (25 %) Second-Order Necessary Conditions

a Formulate Theorem 2.3 as stated in the textbook.

b Go through the proof so that you understand it. Briefly explain the proof.

c Compare Theorem 2.3 and Theorem 2.4. Why does Theorem 2.3 not give sufficient conditions for a strict local minimum?

Problem 2 (40 %) The Newton Direction

Consider the model function m_k based on the second-order Taylor approximation (see equation (2.14) in the textbook):

$$m_k(p) := f_k + p^\top \nabla f_k + \frac{1}{2} p^\top \nabla^2 f_k p \approx f(x_k + p) \tag{1}$$

a Derive the Newton direction

$$p_k^N = -(\nabla^2 f_k)^{-1} \nabla f_k \tag{2}$$

using the model function m_k.

b Assume that $\nabla^2 f_k$ is not positive definite. In this case, is the Newton-direction p_k^N a descent direction? Is it even defined? Explain.

c Given an unconstrained minimization problem with objective function

$$f(x) = \frac{1}{2} x^\top G x + x^\top c \tag{3}$$

with $G = G^\top > 0$ and $x \in \mathbb{R}^n$. Show that an iteration algorithm based on the Newton direction (i.e., $x_{k+1} = x_k + p_k^N$) always converges to the optimum in *one step*.

d Show that

$$f(x) = \frac{1}{2} x^\top G x + x^\top c, \qquad x \in X, \quad X = \{x \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leq 1\} \tag{4}$$

where

$$G = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \qquad c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{5}$$

is a convex function.

Problem 3 (35 %) The Rosenbrock Function

Solve problem 2.1 in the textbook.