Lecture 22: Hydraulic motors, transmission lines

- Hydraulic motors
- Hydraulic transmission lines
- (Electrical transmission lines)

Book: 4.1-4.6, (1.6)

- Info: Ocean Talk «The Polar Regions»
  - 28.03.2019 – 18:00-20:00, EL1
  - https://www.facebook.com/events/263677944559897/

Systems using hydraulics to produce motion

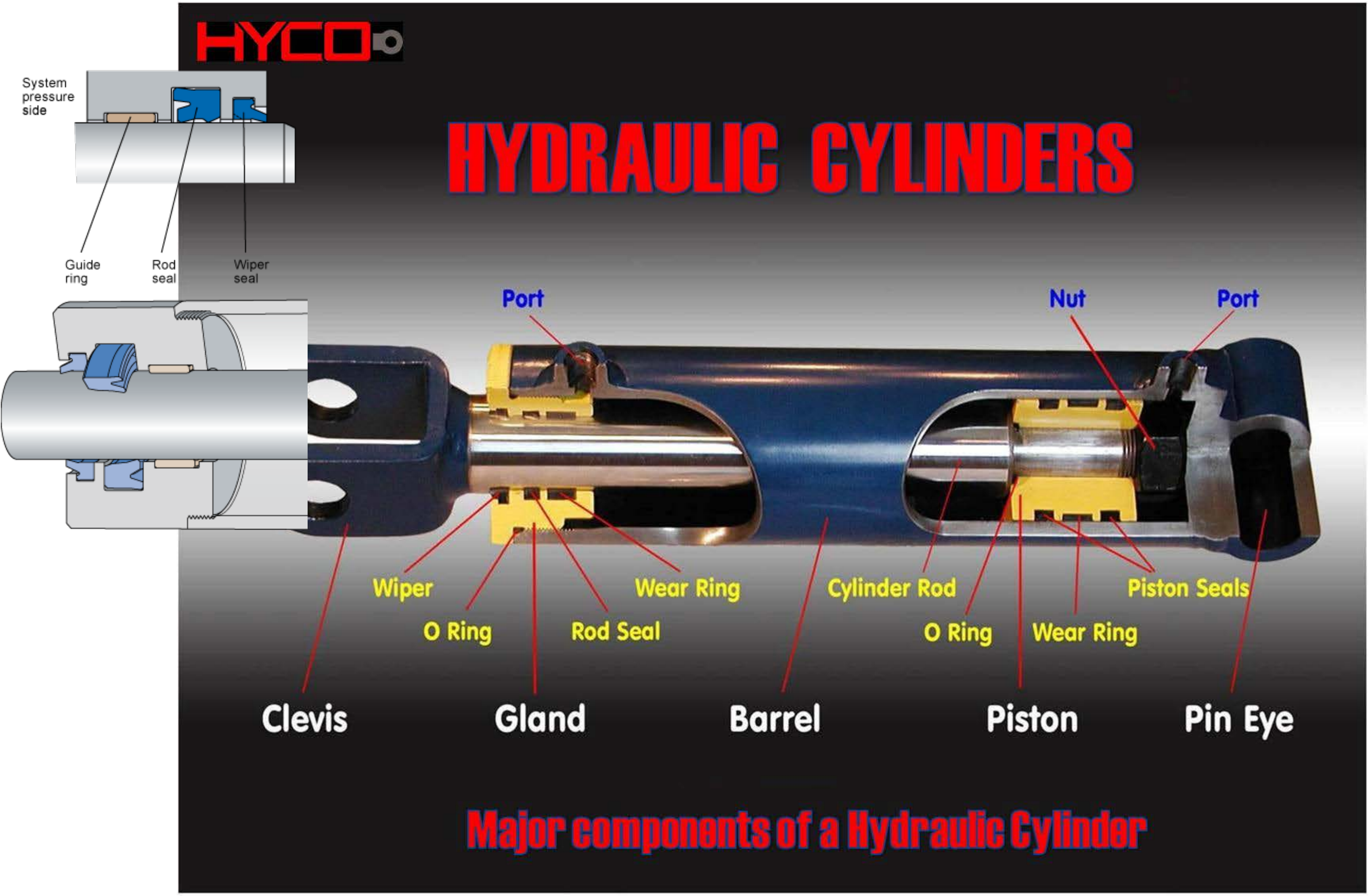
- Excavators

- Robots, cranes, etc.

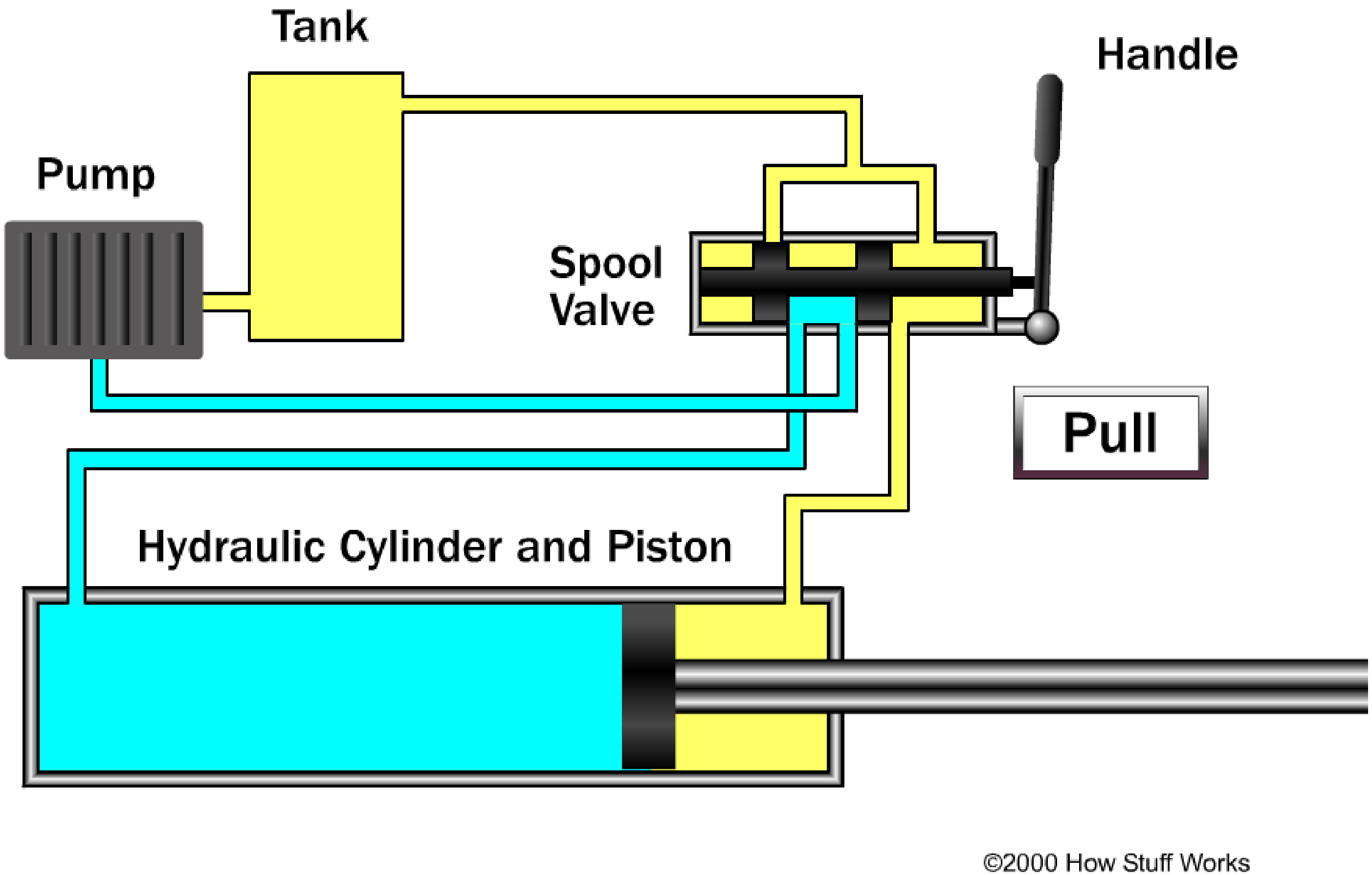
- To control motion of these systems, we need models of the hydraulic actuators

Hydraulic cylinder

For information about seals etc.:
Skf.com



Hydraulic system

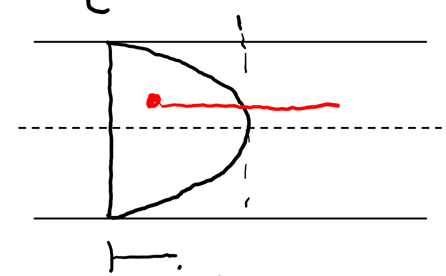


Anna Konda – The fire fighting snake robot

Moody chart

- Circular pipe
- Darcy-Weisbach factor with Reynolds number and relative roughness

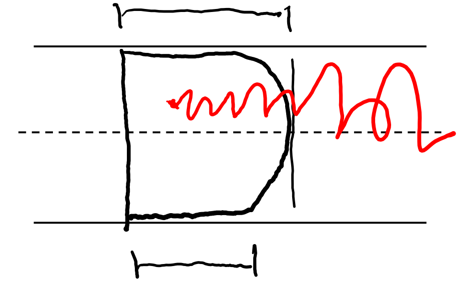
$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} \approx \frac{Dv}{\nu} = \frac{Dv\rho}{\eta}$$



$V_m \approx 1/2\, V_{max}$

laminar flow

$Re \lesssim 2100$



$V_m \approx 0.8\, V_{max}$

turbulent

$Re \gtrsim 4000$

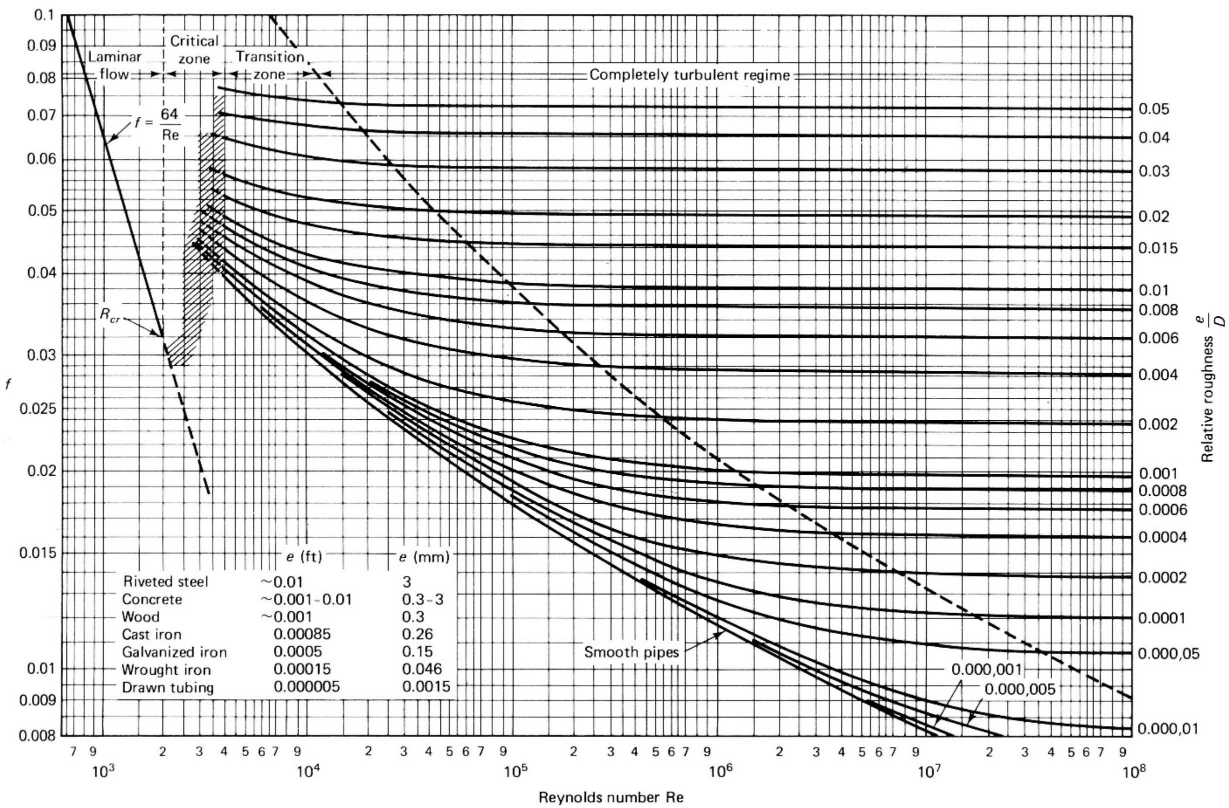
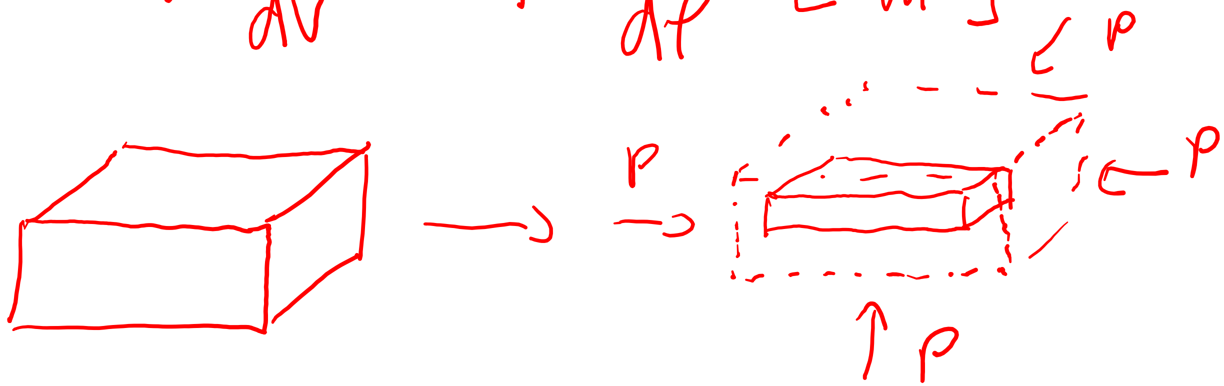


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

Bulk modulus

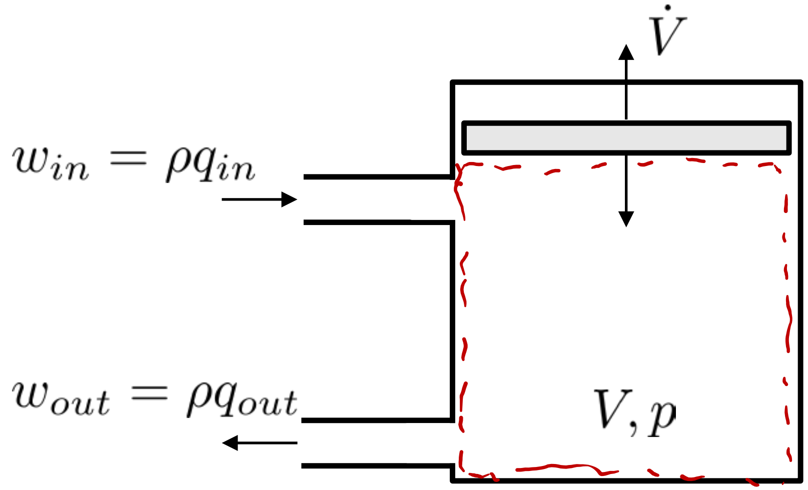
$$\beta = -V \frac{dp}{dV} = \rho \frac{dp}{d\rho} \quad \left[\frac{N}{m^2}\right]$$



example: oil: 7000 bar

water: 22000 bar

Motor models



Mass balance:

$$\frac{dm}{dt} = w_{in} - w_{out}$$

$$\frac{d}{dt}(\rho V) = \rho q_{in} - \rho q_{out}$$

$$\dot{\rho} V + \rho \dot{V} = \rho q_{in} - \rho q_{out}$$

$$\dot{\rho} = \frac{\rho}{\beta} \dot{p}$$

$$\frac{V}{\beta} \dot{p} + \dot{V} = q_{in} - q_{out}$$

Hydraulic cylinder

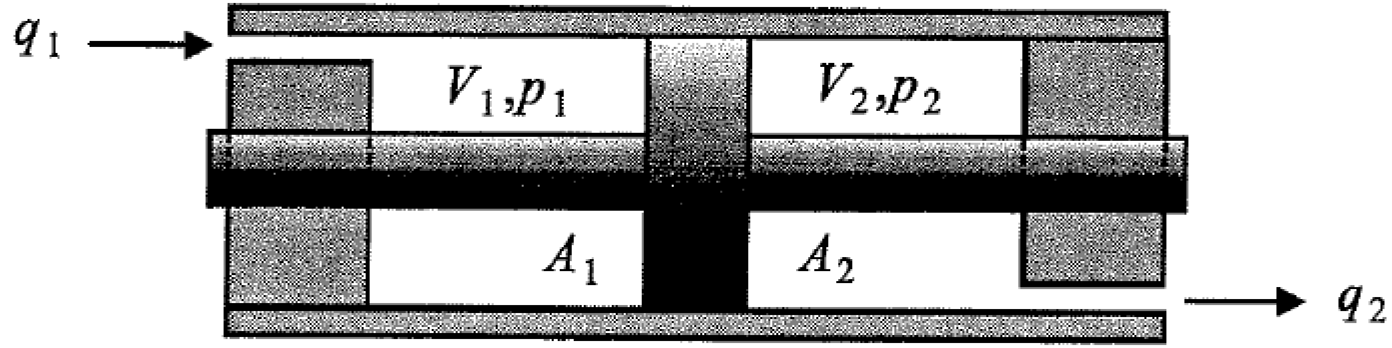


Figure 4.9: Symmetric hydraulic cylinder

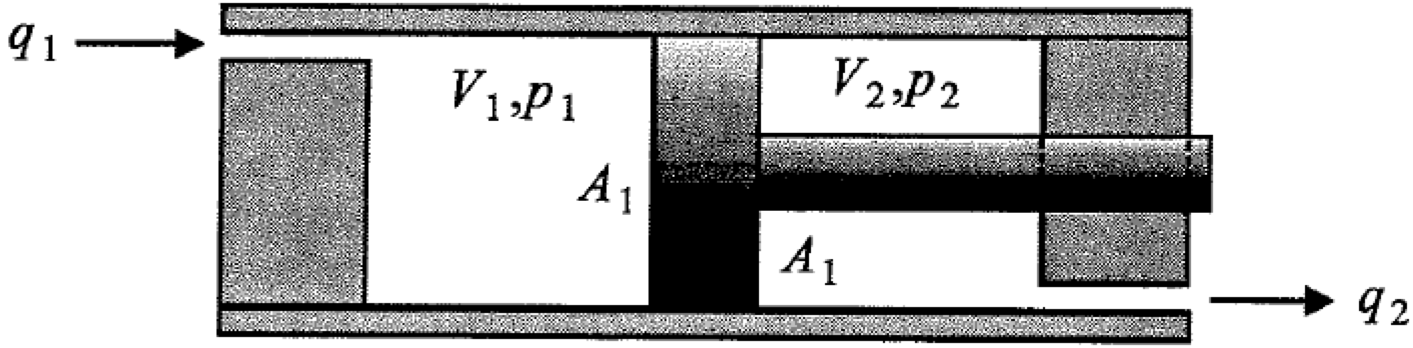


Figure 4.10: Single-rod hydraulic piston

Rotational hydraulic motor I

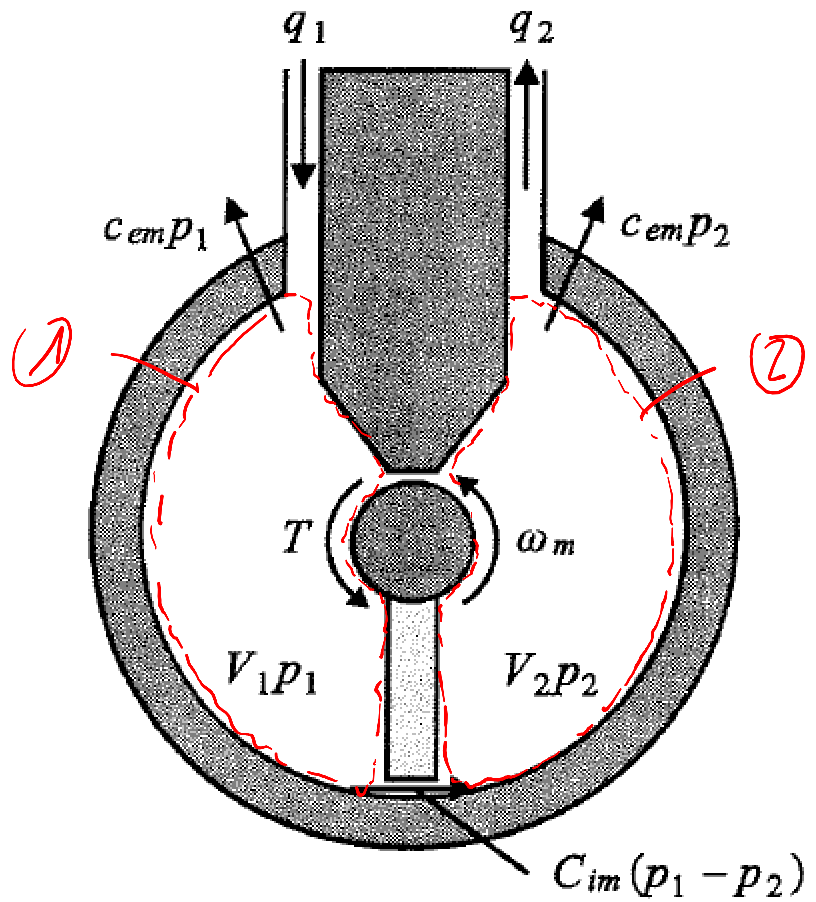


Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

Rotational hydraulic motor II

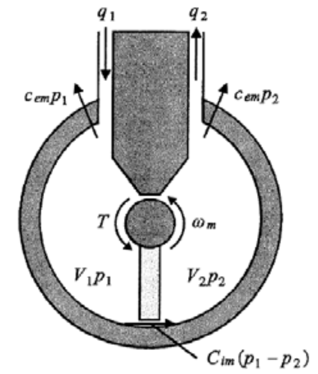


Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

mass balance: $\frac{d}{dt} m = q_1 \rho - C_{em} p_1 \rho - C_{in}(p_1 - p_2)\rho$

① $\frac{V_1}{\beta} \dot{p}_1 + \dot{V}_1 = q_1 - C_{em} p_1 - C_{im}(p_1 - p_2)$

② $\frac{V_2}{\beta} \dot{p}_2 + \dot{V}_2 = -q_2 - C_{em} p_2 - C_{im}(p_2 - p_1)$

$$\dot{V}_1 = -\dot{V}_2 = D_m \omega_m$$

displacement

momentum equation

③ $J \dot{\omega}_m = T_m - B \omega_m - T_L$

friction

load

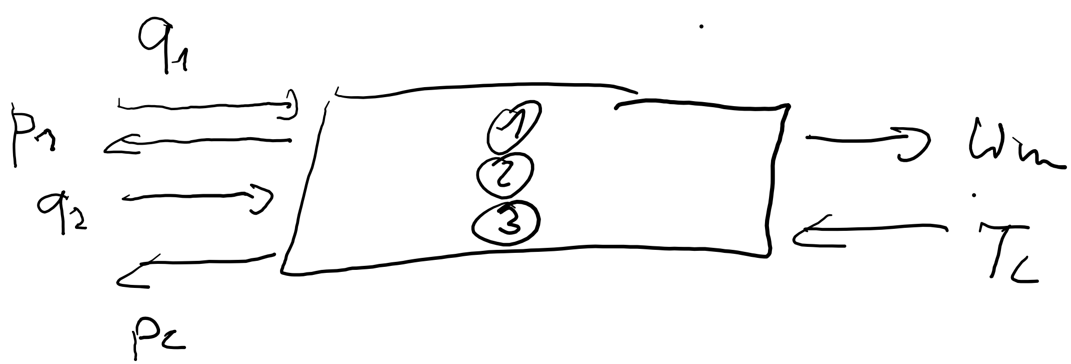
Rotational hydraulic motor III

Assume: loseless : $\text{Power}_{in} = \text{Power}_{out}$

$$\left[\frac{Nm}{J}\right] \quad T_m \omega_m = p_1 \dot{V}_1 + p_2 \dot{V}_2 \quad \left[\frac{Nm}{J}\right]$$

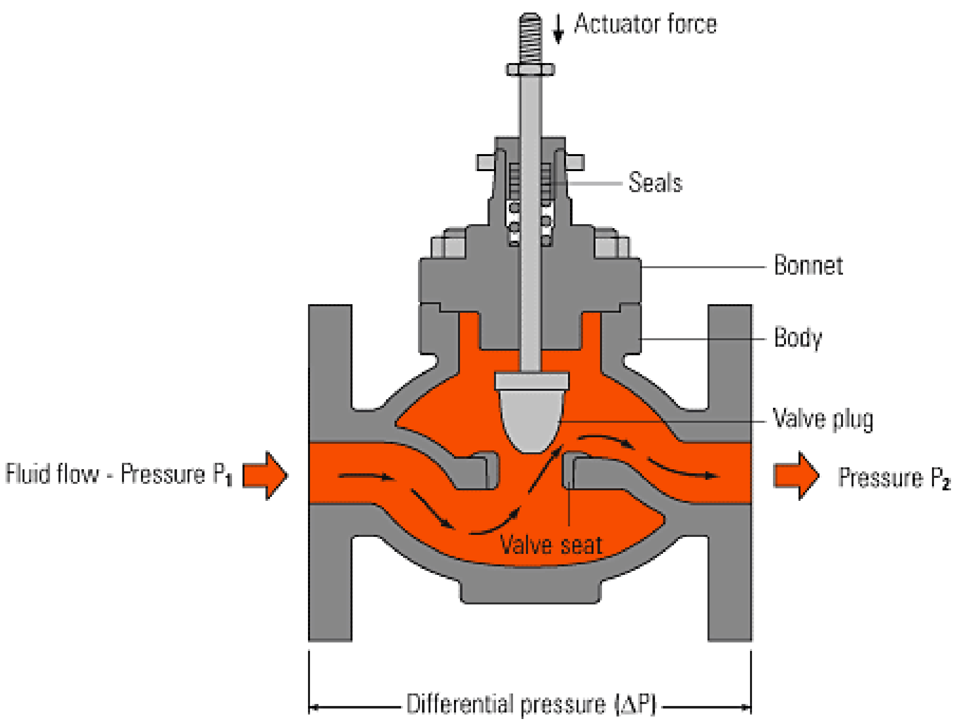
$$= D_m \omega_m (p_1 - p_2)$$

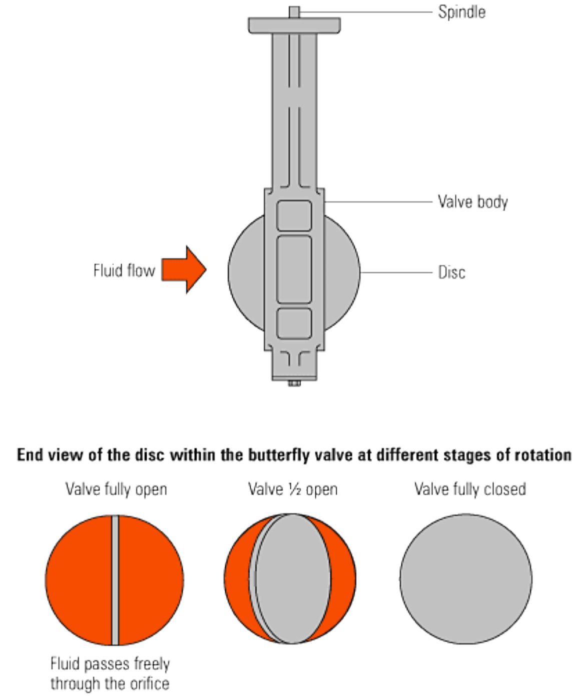
$$T_m = D_m (p_1 - p_2)$$

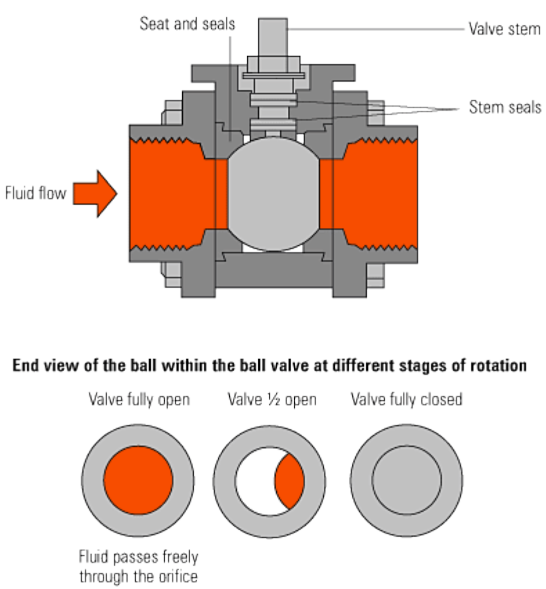


Valves

- Device that regulates flow
- Many different types of valves exist
  - Globe valve, ball valve, butterfly valve, ...





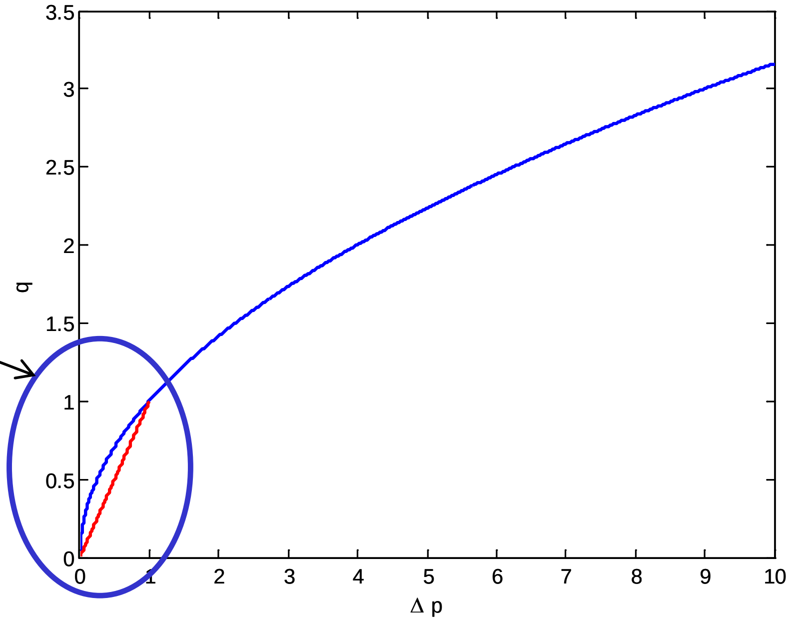


Valve models (book 4.2)

- Flow through a restriction is generally turbulent

$$q = C_d A \sqrt{\frac{2}{\rho} \Delta p}$$

Problematic!

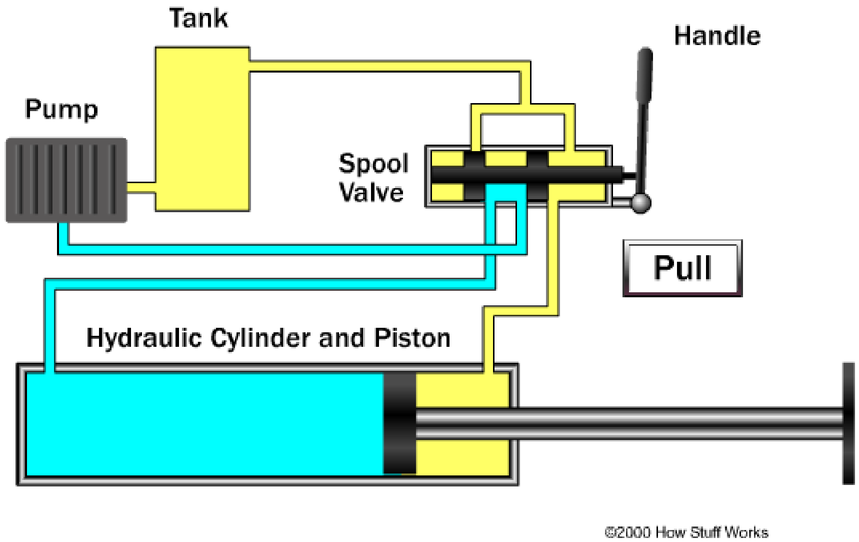


- Solution: Regularize by assuming laminar flow for small *Δp*

$$q = C_l \Delta p$$

- Book: Make transition smooth

Four-way valve



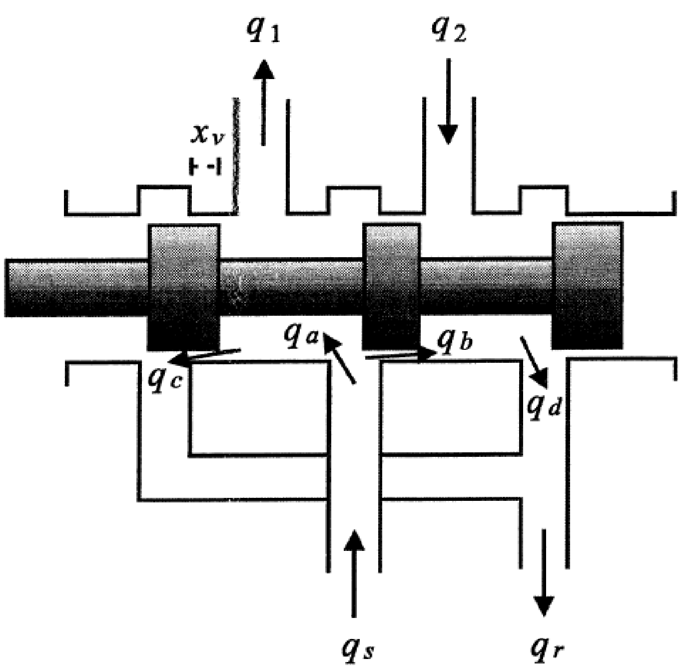
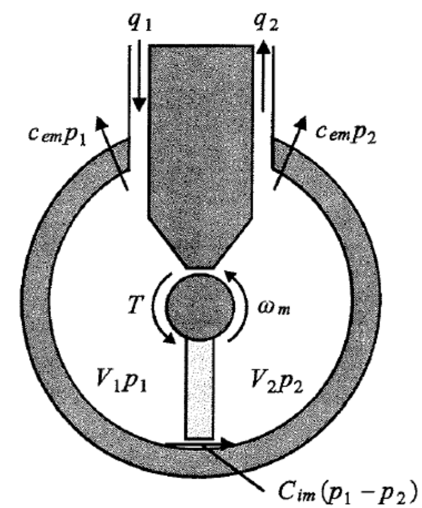


Figure 4.2: A matched and symmetric four-way valve.



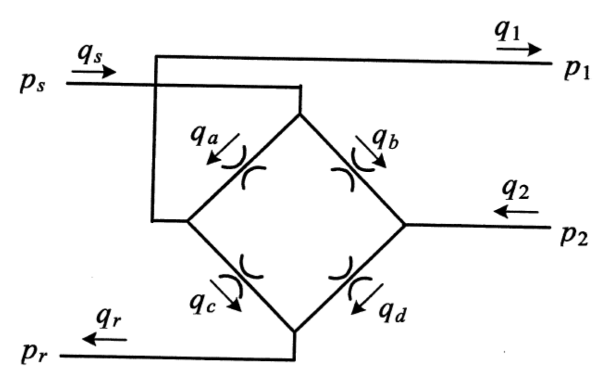


Figure 4.1: Four-way valve

Modeling of four-way valve

- Define load pressure

$$p_L = p_1 - p_2$$

- Define load flow

$$q_L = \frac{q_1 + q_2}{2}$$

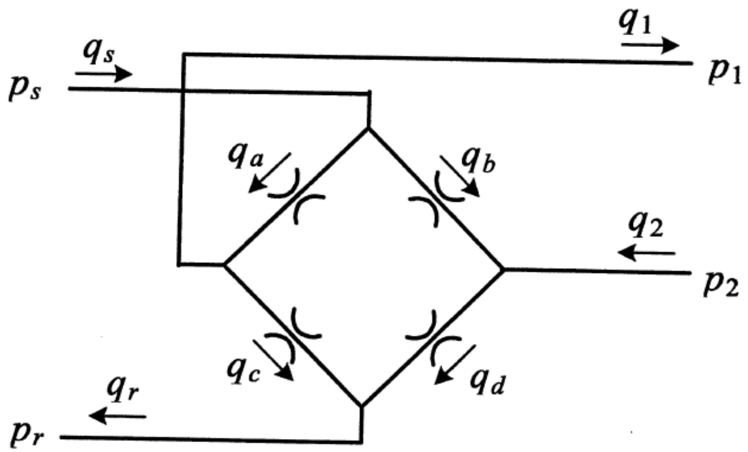


Figure 4.1: Four-way valve

- Symmetric load assumption (motor)

$$q_1 = q_2$$

- Symmetric valve and symmetric load

$$\boxed{q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \operatorname{sign}(x_v) p_L\right)}}$$

Characteristic of four-way valve

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \operatorname{sign}(x_v) p_L\right)}$$

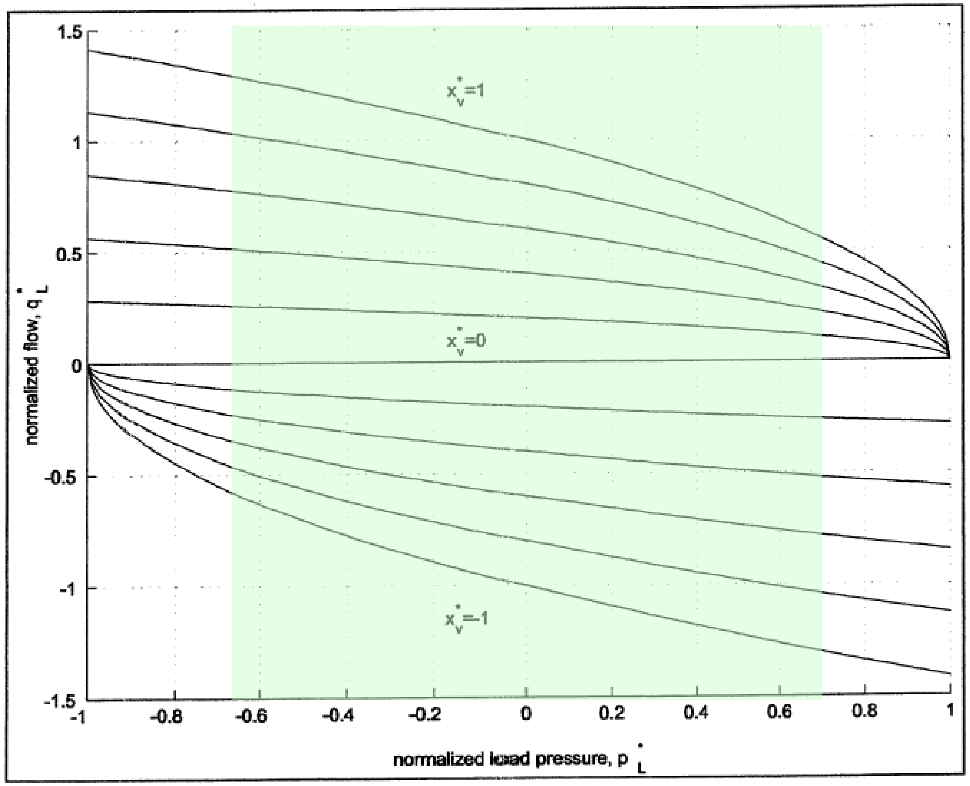


Figure 4.3: Valve characteristic

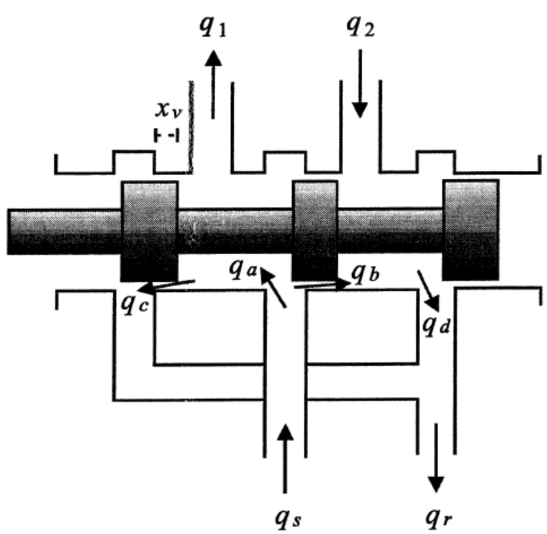
Linearized model:

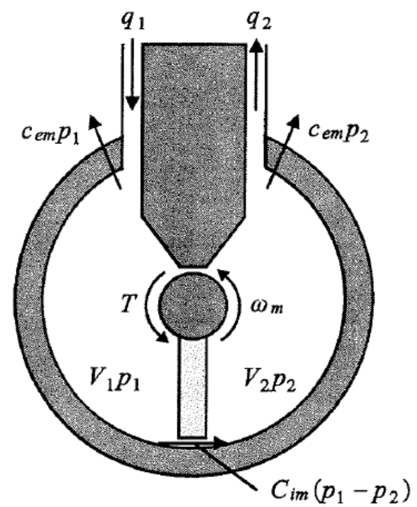
$$|p_L| \leq \frac{2}{3} p_s : \quad \boxed{q_L = K_q x_v - K_c p_L}$$

Gain uncertainty:

$$0.58 K_{q0} \leq K_q \leq 1.29 K_{q0}$$

Transfer function valve+motor

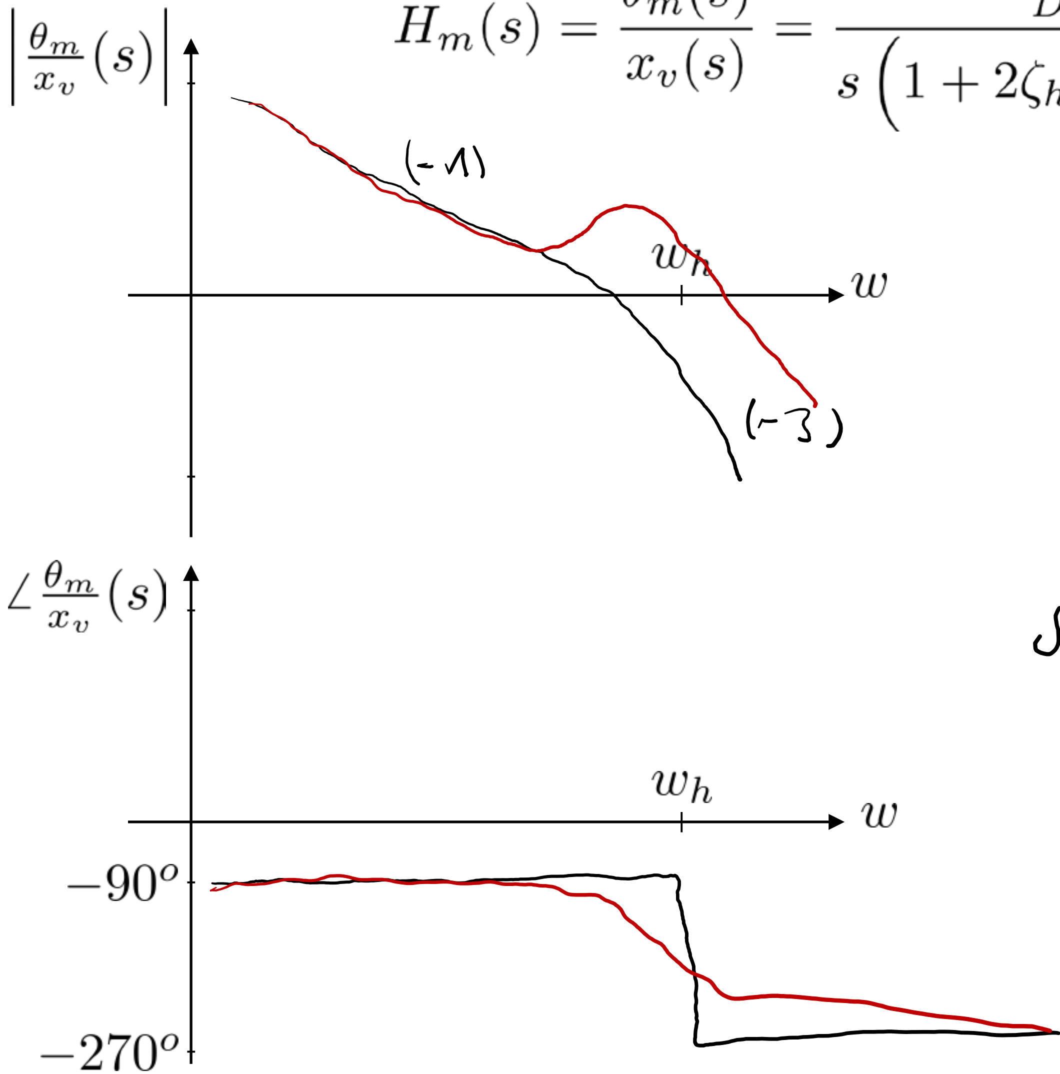




$$\theta_m(s) = \frac{\frac{K_q}{D_m}x_v(s) - \frac{K_{ce}}{D_m^2}\left(1+\frac{s}{\omega_t}\right)T_L(s)}{s\left(1+2\zeta_h\frac{s}{\omega_h}+\frac{s^2}{\omega_h^2}\right)}$$

Transfer function spool to shaft

$$H_m(s) = \frac{\theta_m(s)}{x_v(s)} = \frac{\overbrace{\frac{K_q}{D_m}}^{K_v}}{s\left(1 + 2\zeta_h \frac{s}{\omega_h} + \frac{s^2}{\omega_h^2}\right)}$$



P - controll

$w_h = w_{180}$ bandwidth of system

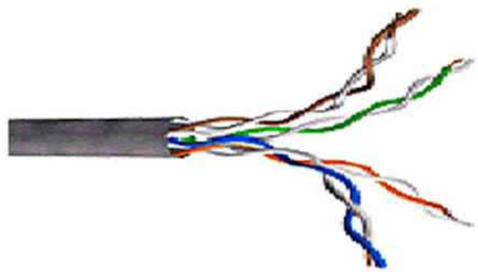
rule thumb $w_c \approx 0.1 w_h$

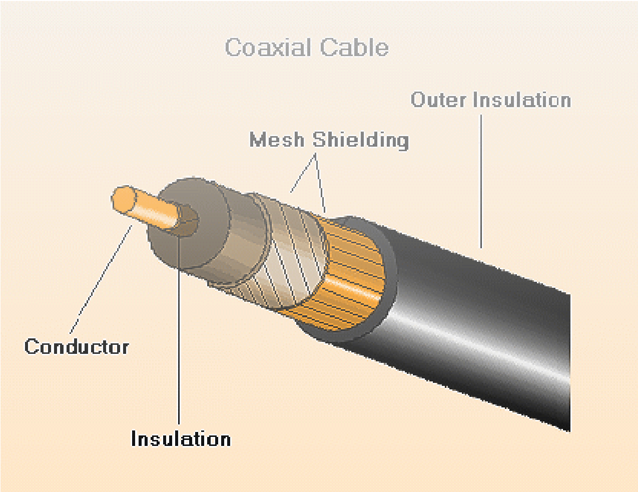
stable if:

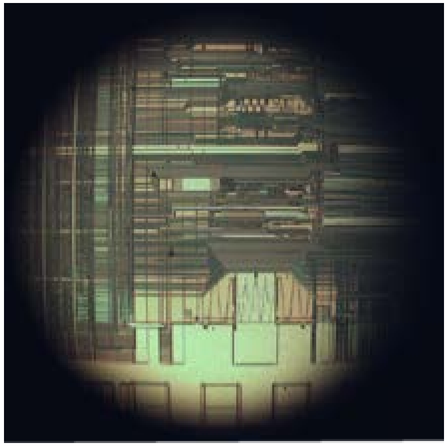
$K_v \leq 2 \zeta_h w_h$

$\Rightarrow K_p = 2 \frac{D_m}{K_q} \zeta_h w_h$

Electrical transmission lines

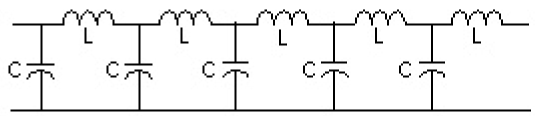




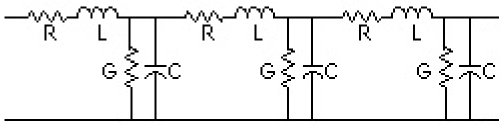


Telegrapher's equation (Wave equation)

- Lossless:



Lossy:



- Model (Ch. 1.6):

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

- Laplace:

$$\frac{\partial u(x,s)}{\partial x} = -X(s)i(x,s)$$
$$\frac{\partial i(x,s)}{\partial x} = -Y(s)u(x,s)$$

Series impedance:
$$X(s) = R + Ls$$
Parallel admittance:
$$Y(s) = G + Cs$$

Characteristic impedance:
$$Z_c(s) = \sqrt{\frac{X(s)}{Y(s)}}$$

Same equations for electrical and fluid/hydraulic transmission lines

Electrical transmission lines:

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

Fluid transmission lines:

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\beta}{A}\frac{\partial q(x,t)}{\partial x}$$
$$\frac{\partial q(x,t)}{\partial t} = -\frac{A}{\rho}\frac{\partial p(x,t)}{\partial x} - \frac{F\left[q(x,t)\right]}{\rho}$$

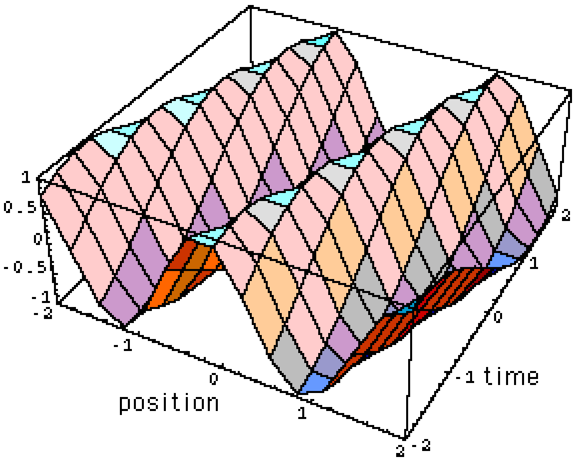
- Current and flow “same” variables, as is voltage and pressure
- In both cases, we can define line impedance, characteristic impedance, propagation operator, etc.
- Solution to equations have same structure/form: waves propagating back and forth

Solution: Waves

- Solution: $$u_{out}(s) = e^{-\Gamma(s)} u_{in}(s)$$

- Propagation operator $\Gamma(s) = L\sqrt{X(s)Y(s)}$
  - Attenuation factor $\alpha = Re[\Gamma(j\omega)]$: How much is wave reduced
  - Phase factor: $\beta = Im[\Gamma(j\omega)]$: How long does it take

- Lossless (*R* = *G* = 0): $\Gamma(s) = Ts$
  - Attenuation factor: 0
  - Phase factor: Pure time-delay



When should we care?

- Solution lossless case: Time delay

$$e^{-Ts}$$

- Rule-of-thumb from control theory: We can ignore time-delay for frequencies much less than 1/*T*

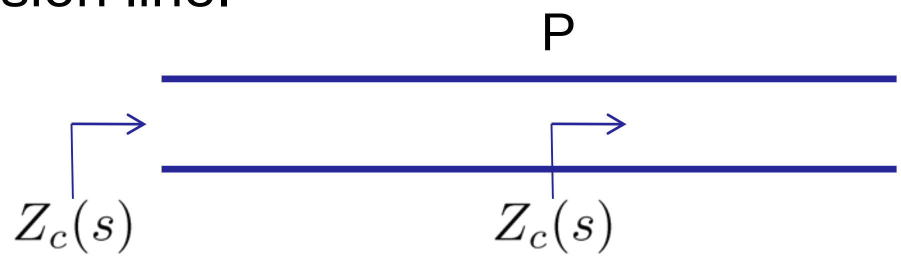
$$\omega \leq \frac{1}{T} \;\Rightarrow\; 2\pi\frac{c}{\lambda} \leq \frac{c}{L} \;\Rightarrow\; L \leq \frac{\lambda}{2\pi}$$

- Rule-of-thumb for transmission lines: When L is larger than one tenth of wavelength, treat as transmission line

- Power lines, f = 50Hz: λ = 6000km
- Personal computers, f = 10GHz: λ = 1.5cm

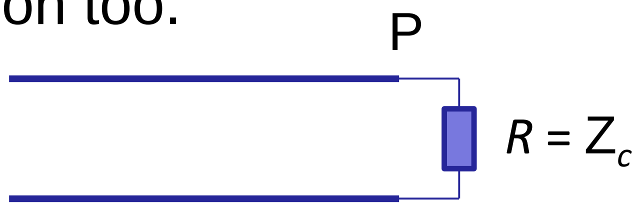
Impedance matching

- Suppose we have an imaginary joint at P in a very long transmission line.



The wave goes through the joint without reflection because there is actually no joint (just imagined).

- Now, let us terminate a resistance of value Z_c *at the same position of this* imaginary joint. The wave will go through without reflection too.



This is called a **matched load.**

http://cktse.eie.polyu.edu.hk/eie403/Transmissionline.pdf

Lecture 23: Process modeling & balance laws

- Process modeling, structure and methodolgy
- Balance laws
  - Closure relations

Book: 10.4, 11.1-11.4

Process equations

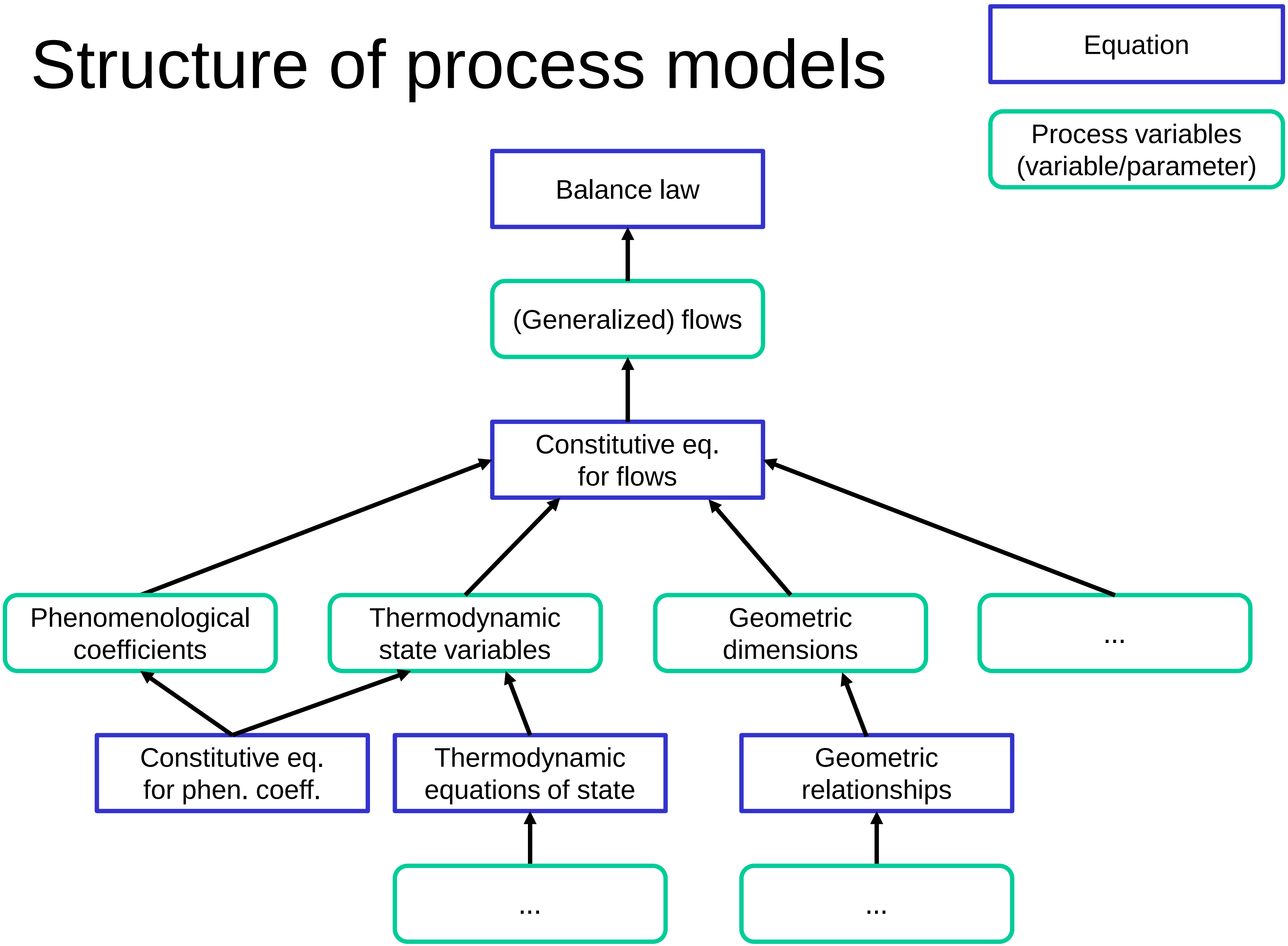
- Balance laws
  - Mass
  - Momentum
  - Energy
  - ...

- Constitutive equations
  - For (generalized) flows
  - Thermodynamic equations of state (e.g. ideal gas law)
  - Phenomenological relationships (e.g. between friction force and flow in a pipe)
  - ...

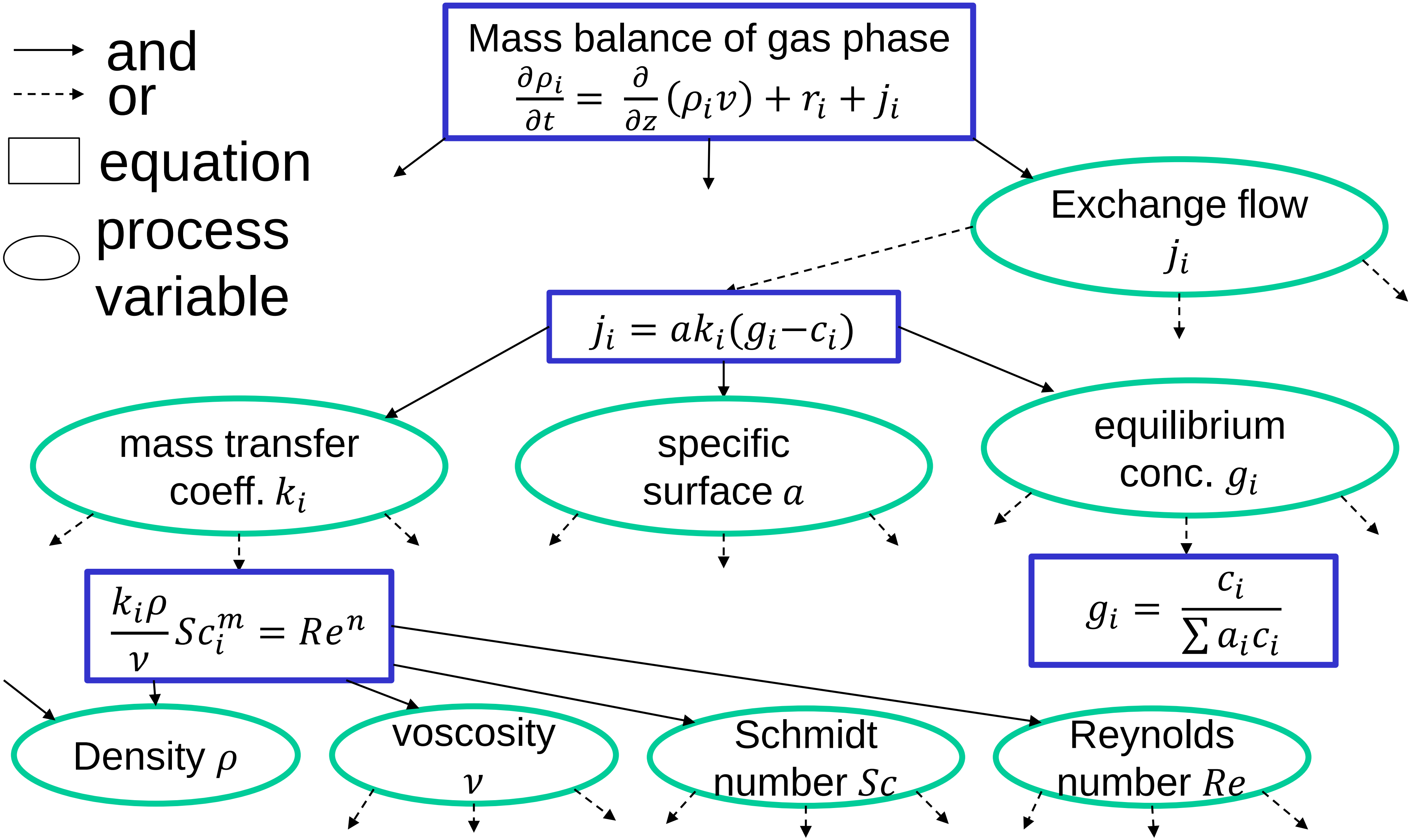
- Constraints
  - Geometric relationships
  - Equilibrium conditions
  - ...

Also called «closure relations» as they «close» the balance laws (such that #equations = #variables)

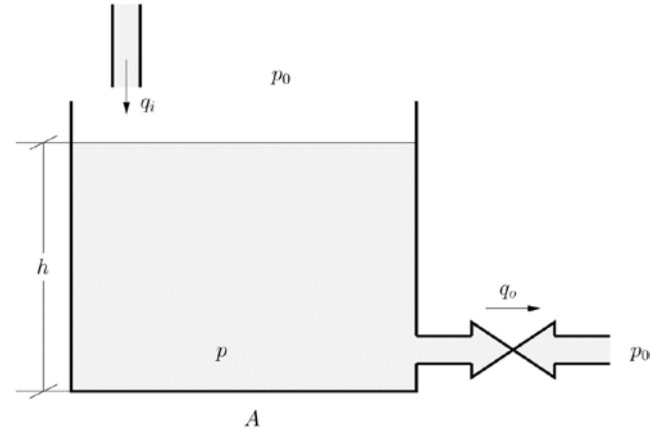
Structure of process models



Example – structure of process models



Example: Tank



- Mass balance: $\frac{\overset{1}{dm}}{dt} = (\overset{2}{q_i} - \overset{3}{q_0})\overset{4}{\rho}$
- Constitutive equation: $q_0 = \overset{5}{C}\sqrt{\overset{6}{p} - \overset{7}{p_0}}$ (2)

  $p = p_0 + \overset{8}{\rho}\overset{9}{g}h$ (3)
- Constraints: $m = \overset{10}{V}\rho$ (4)

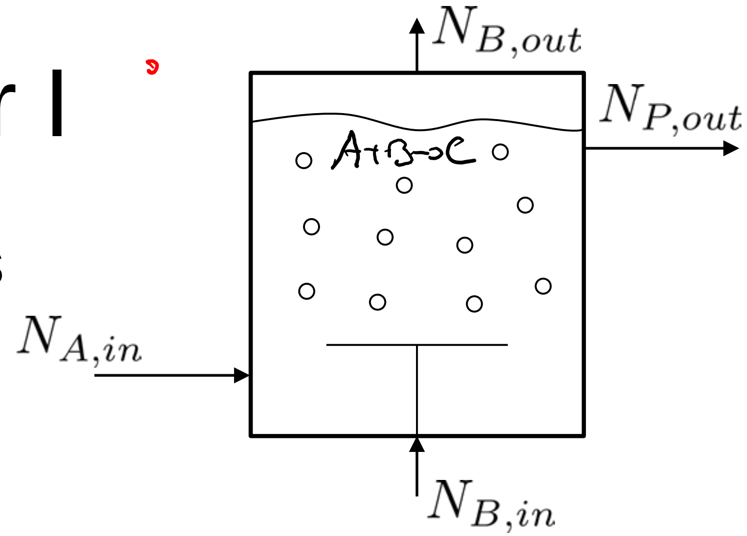
  $V = \overset{11}{A}h$ (5)
- How many variables?
- Need to define parameter and inputs
  - Parameters: C, g, A, ρ
  - Inputs: q_i, p_0

Structural index:

	q_0	p	V	h
(2)	x	x		
(3)		x		x
(4)			x	
(5)				x

→ regular str. index

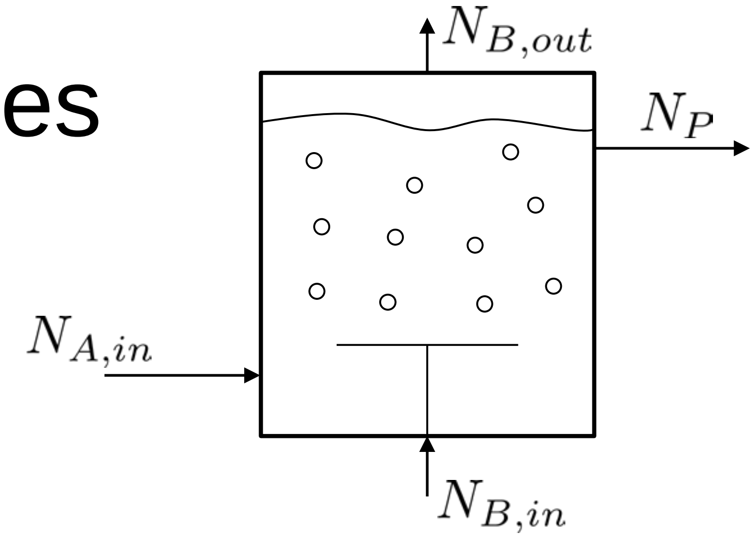
Example: Bubble reactor I



Model reactor as quasi-homogenous

- Assumptions:
  - Ideally mixed
  - Inflows are pure substances
  - Substance A and C are in liquid phase, substance B is gaseous
  - The total surface area of the bubbles depends on the inflow B
    - $S_R = S_R(N_{B,in})$
  - The reaction rate can be calculated based on the concentration of A and the pressure in the reactor
    - $R_0 = R_0(c_{A,liq}, p)$
  - Densities ρ_A and ρ_C and mole masses M_A and M_C are constant and known
  - The gas phase can be described by the ideal gas law
    - $p\, V_{gas} = n_B R_m T$
  - The volume of the reactor is constant and known

Bubble reactor – Balances

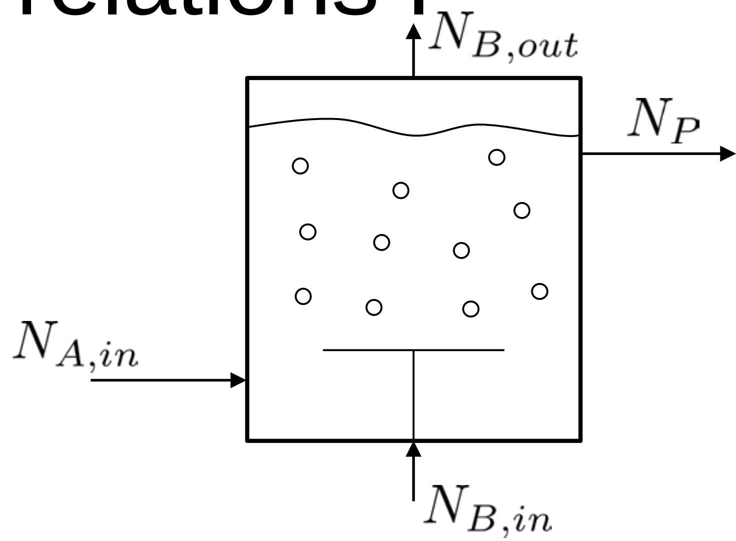


(1) $$\frac{\overset{(1)}{dn}}{dt} = \overset{2}{N_{A,in}} + \overset{3}{N_{B,in}} - \overset{4}{N_{B,out}} - \overset{5}{N_P} + \overset{6}{S_R}\left(\overset{7}{R_A} + \overset{8}{R_B} + \overset{9}{R_C}\right)$$

(2) $$\frac{\overset{10}{dn_A}}{dt} = N_{A,in} - \overset{11}{x_A} N_P + S_R R_A$$

(3) $$\frac{\overset{12}{dn_B}}{dt} = N_{B,in} - N_{B,out} + S_R R_B$$

Bubble reactor – closure relations I



(4) $$S_R = S_R(N_{B,in})$$

(5) $$R_A = -R_0{}^{13}$$

(6) $$R_B = -R_0$$

(7) $$R_C = R_0$$

(8) $$R_0 = R_0(\overset{14}{c_{A,liq}}, \overset{15}{p})$$

(9) $$x_A = \frac{n_A}{n_A + n_C}{}^{16}$$

(10) $$n = n_A + n_B + n_C$$

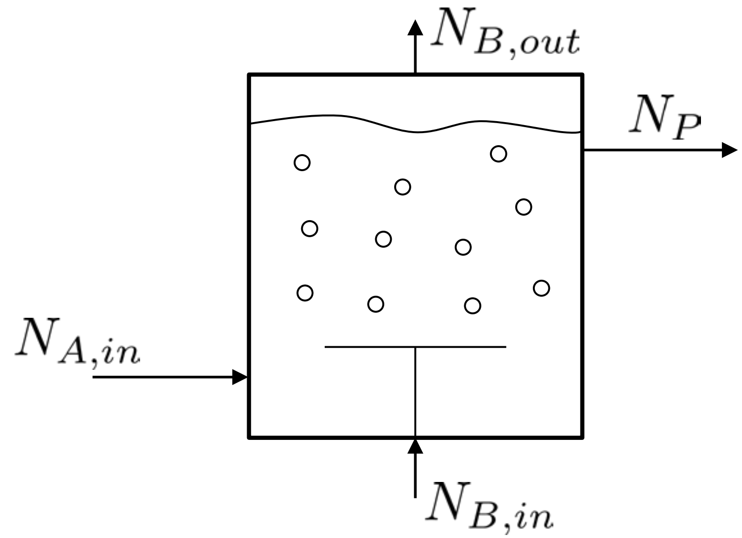
(11) $$c_{A,liq} = \frac{n_A}{V_{liq}{}^{17}}$$

(12) $$V_{liq} = \underbrace{\overset{18}{n_A} M_A \cdot \frac{1}{\rho_A{}^{19}}}_{V_A} + \underbrace{\overset{20}{n_C} M_C \frac{1}{\rho_C{}^{21}}}_{V_C}$$

(13) $$p\overset{22}{V_{gas}} = n_B \overset{23}{R_m} \overset{24}{T}$$

(14) $$\overset{25}{V} = V_{gas} + V_{liq}$$

Bubble reactor – DoF



Equation : #14
Variables: #25

DoF #11

- Variables: $[n; N_{A,in}; N_{B,in}; N_{B,out}; N_P; S_R; R_A; R_B; R_C; n_A; x_A; n_B; R_0; c_{A,liq}; p; n_C; V_{liq}; M_A; \rho_A; M_C; \rho_C; V_{gas}; R_m; T; V]$

parameter: $p = [V, \rho_A, \rho_C, M_A, M_C, R_m]^T$

inputs: $u = [N_{A,in}; N_{B,in}; N_{B,out}; N_P; T]$

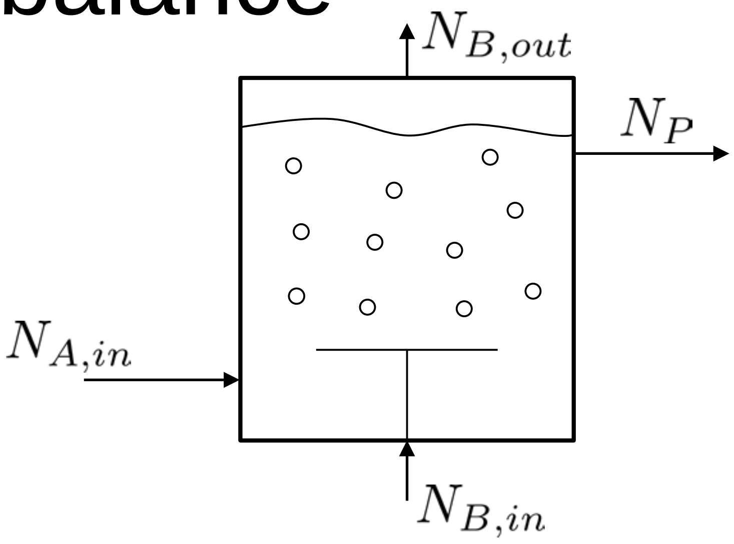
$\rightarrow$ 11 DoF fixed

Bubble reactor – structural index

	S_R	R_A	R_B	R_c	R_0	n_C	x_A	$c_{A,liq}$	p	V_{gas}	V_{liq}
(4)	X										
(5)		X			X						
(6)			X		X						
(7)				X	X						
(8)					X			X	X		
(9)						X	X				
(10)						X					
(11)								X			X
(12)						X					X
(13)									X	X	
(14)										X	X

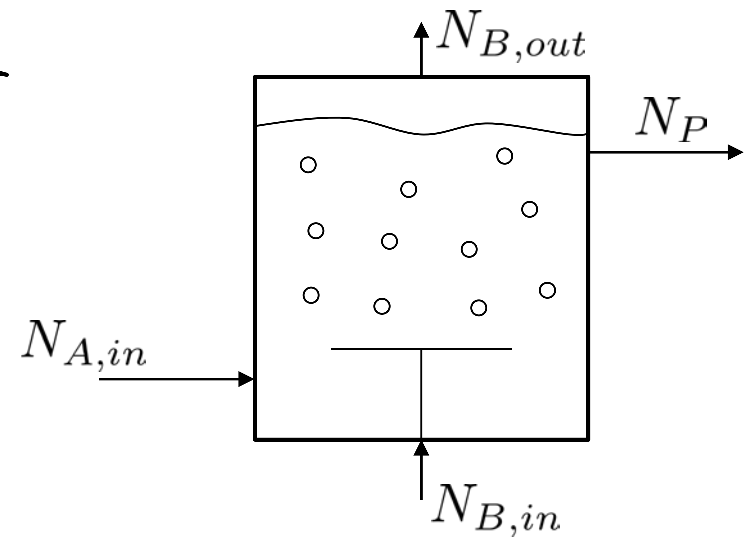
→ structural index : regular

Bubble reactor – energy balance



Bubble reactor – closure relations II

- Assumptions: Adiabat reactor
  - Specific enthalpies of inputs are model inputs
  - Spedific enthalpies of pure substances A, B, C are given by $h_i = h_i(T,p)$



(15) $$\frac{d\overset{26}{U}}{dt} = N_{A,in}\,\overset{27}{h_{A,in}} + N_{B,in}\,\overset{28}{h_{B,in}} - N_{B,out}\cdot \overset{29}{h_B} - N_P\,\overset{30}{h_P}$$

(16) $$h_B = h_B(T,p)$$

(17) $$h_P = x_A \overset{33}{h_A} + \overset{31}{x_C}\,\overset{32}{h_C}$$

(18) $$h_A = h_A(T,p)$$

(19) $$h_C = h_C(T,p)$$

(20) $$x_C = \frac{n_C}{n_C + n_A}$$

(21) $$U = \overset{34}{H} - pV$$

(22) $$H = n_A h_A + n_B h_B + n_C h_C$$

Bubble reactor – structural index II

- Variables: $[n; N_{A,in}; N_{B,in}; N_{B,out}; N_P; S_R; R_A; R_B; R_C; n_A; x_A;$
  $n_B; R_0; c_{A,liq}; p; n_C; V_{liq}; M_A; \rho_A; M_C; \rho_C; V_{gas}; R_m; T; V$
  $U; h_{A,in}; h_{B,in}; h_B; h_P; h_A; x_C; h_C; H]$

#34 variables - #22 equations : #12 DoF

Inputs : $[N_{A,in}; N_{B,in}; N_{B,out}; N_P; h_{A,in}; h_{B,in}]$

Parameter $[V, \rho_A, \rho_C, M_A, M_C, R_m]$

	h_B	h_P	x_C	h_C	h_A	T	H	n_C	p	x_A
(16)	X					X			X	
(17)		X	X	X	X					X
(18)					X	X			X	
(19)				X		X			X	
(20)			X					X		
(21)							X		X	
(22)	X			X	X		X	X		

→ regular :)