TTT4275 Summary from February 8th
Spring 2019

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Different detection cases

- The LR treshold λ is mostly dependent on choice of detection method

- The LR is dependent on the problem case

- In this course we will assume gaussian noise $p(w) = N(0, \sigma^2)$, i.e.

$$H_0 \; : \; x(n) = w(n) \quad n = 0, \ldots, N-1$$

- We will investigate the following cases for H_1 :

  - Constant in noise $x(n) = A + w(n)$

  - Random signal in noise $x(n) = s(n) + w(n)$ where $p(s) = N/A, \sigma_s^2)$

  - Deterministic sequence in noise $x(n) = s(n) + w(n)$

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Detection of constant in gaussian noise

- Defining the log lilklihood ratio test

$$LL((\mathbf{x})) = log[p(\mathbf{x}/H_1) - log[p(\mathbf{x}/H_0) \lessgtr log(\lambda)$$

- Using the independence assumption $p(\mathbf{x}) = \prod_{n=0}^{N-1} p(x(n))$ we get

$$\begin{aligned} LL(\mathbf{x})) &= \frac{NA}{\sigma^2}z - \frac{NA^2}{2\sigma^2} \lessgtr log(\lambda) \Rightarrow \\ z &\lessgtr \frac{A\sigma^2}{N}log(\lambda) + \frac{A}{2} = \eta \end{aligned} \tag{1}$$

  where $z = T(x) = \frac{1}{N}\sum x(n)$ is the sample mean

- Note that eq. 1 is an equivalent test for the LRT. In general the term $z = T(x)$ is called a sufficient statistic

- The false alarm is then given by

$$P_{FA} = \int_{\eta}^{\infty} p(z/H_0)dz = \int_{\eta}^{\infty} N(0, \sigma^2/N)dz = Q(\frac{\eta\sqrt{N}}{\sigma}) \tag{2}$$