TTT4275 Summary for January 25th
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The Best Linear Unbiased Estimator (BLUE) - 1

- The linear model gave an LSE-estimator which was linear in the observations $x = [x(0), \ldots, x(N-1)]$.

  This indicates that we should investigate linear estimators independent of the type of problem!

- Again we assume that we do not know the distribution $p(x;\theta)$

- It turns out that in order to find such an estimator we need to know the following (for a scalar case):

  a) The covariance matrix C_x
  b) The mean $E\{x(n)\} = s_n\theta \ n = 0, \ldots, N-1$ where all s_n are known

- The linear estimator is given by

$$\widehat{\theta} = \sum_n a_n x(n) = a^T x \tag{1}$$

  where a must be found.

BLUE 2

- Forcing the estimator to be unbiased results in the constraint $a^T s = 1$

- We also found the variance : $var(\widehat{\theta}) = a^T C_x a$

- To find the best estimator we need to minimise $var(\widehat{\theta})$, however while fulfilling the constraint. Thus we have to introduce the Lagrangian λ

$$L(\theta, \lambda) = a^T C_x a + \lambda(a^T s - 1) \tag{2}$$

- Minimizing $L(\theta, \lambda)$ results in

$$\widehat{\theta} = \frac{s^T C_x^{-1} x}{s^T C_x^{-1} s} \tag{3}$$

$$var(\widehat{\theta}) = \frac{1}{s^T C_x^{-1} s} \tag{4}$$

- Note that since we do not know $p(x; \theta)$ we do not know how close this variance is to the CRLB!