TTT4275 Summary for January 25th

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How good is the LSE estimator for the linear model problem

- The linear model for a problem is : $x = H\Theta + w$
  where w is the deviation from linearity

- The LSE estimator is given by

$$\hat{\Theta} = (H^T H)^{-1} H^T x \tag{1}$$

- The quality of the estimator can only be evaluated if we can calculate CRLB; i.e if we know $p(x, \Theta)$

- In the first case we assumed that the deviation can be approximated by independent (white) Gaussian noise; i.e. $p(w) = N(0, \sigma^2 I)$

- We then showed that the LSE-estimator fulfilled the requirement :

$$\nabla_{\Theta} log[p(x; \Theta)] = I(\Theta)[\hat{\Theta} - \Theta] \tag{2}$$

  with the corresponding CRLB equality :

  $Cov(\hat{\Theta}) = I^{-1}(\Theta) = \sigma^2 (H^T H)^{-1}$

The linear model for a problem and the resulting LSE estimator

- The white noise assumption is often wrong. A more general approximation is to assume colored (correlated) Gaussian noise, i.e $\Sigma \neq \sigma^2 I$

- We solved the problem by filtering/whitening the noise and observation, i.e.

$$x' = Sx = SH\Theta + Sw = H'\Theta + w' \quad \text{where} \quad \Sigma^{-1} = S^T S \tag{3}$$

- We showed that $p(w') = N(0, I)$, i.e. white noise with unit power.

- Thus we ended up with the following efficient MVU estimator

$$\hat{\Theta} = (H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1} x \tag{4}$$

$$Cov(\hat{\Theta}) = I^{-1}(\Theta) = (H^T \Sigma^{-1} H)^{-1} \tag{5}$$