TTT4275 Summary for January 21th
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The vector CRLB

- In most problems we need to estimate more than one parameter, i.e. $\Theta = [\theta_1, \theta_2, \ldots, \theta_d]$ based on N observations $x = [x(0), x(1), .., x(N-1)]$

- We then define the Fisher Information matrix :

$$I(\Theta)_{ij} = E\{\frac{-1}{\frac{\delta^2 log[p(x;\Theta)])}{\delta\theta_i\delta\theta_j}}\} \tag{1}$$

- Any estimator must then fulfill the CRLB :

$$var(\hat{\theta})_{ii} \geq I^{-1}(\Theta)_{ii}$$

- Equality is achieved for an efficient (MVU) estimator which also will fulfill

$$\nabla_{\Theta} log[p(x;\Theta)] = I(\Theta)[\hat{\Theta} - \Theta] \tag{2}$$

where the covariance matrix of the estimator is given by

$$C(\hat{\Theta}) = I^{-1}(\Theta)$$

- Example 7 in the compendium gives a good introduction to the vector case

The linear model for a problem and the resulting LSE estimator

- Many problems have a complex form such that good estimators are difficult to find.

- However, for some problems the observations are approximately linear in the unknown parameters. Thus we can write the following :

$$x = H\Theta + w \tag{3}$$

  where w is the model error and H is a known (observation) matrix

- We use the Least Square Error (LSE) criterium to find an estimator

$$LSE(\Theta) = (x - H\Theta)^T(x - H\Theta) \tag{4}$$

- Setting $\nabla LSE(\Theta) = 0$ we find

$$\hat{\Theta} = (H^T H)^{-1} H^T x \tag{5}$$

- The remaining question is how good this estimator is?