Department of Electronic Systems

Examination paper for TTT4275 Estimation, Detection and Classification

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Problem 1 Estimation (4+4+4+4+3=19)

Consider a sensor network consisiting of N sensors measuring an environmental parameter A. The sensors all send their data to a fusion center, where the estimate of A is computed. We assume that the measurement from sensor n is

$$x[n] = A + w[n]$$

where $w[n] \sim \mathcal{N}(0, \sigma_n^2)$, that is, the noise has zero mean, but different variance for each sensor. We assume σ_n is known for all n.

1a) Write the estimation problem as a linear model.

1b) Write down the Cramer-Rao bound for the estimation problem (Hint: For a general linear problem with colored noise, $\mathbf{x} = H\Theta + \mathbf{w}$ we have)

$$\nabla_{\hat{\Theta}} \log p(\mathbf{x}; \Theta) = H^T \Sigma^{-1} H \left((H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1} \mathbf{x} - \Theta \right)$$

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1c) Find a closed form expression for the estimator.

1d) Write down the likelihood function for the problem, and use this to obtain the MLE.

1e) Assume now that A is a random variable,

$$A \sim \mathcal{N}(0, \sigma_A^2).$$

Explain why a you should use a Bayesian estimator in this case, and explain the difference between the Bayesian Mean square error estimator (B_{mse}) and the Maximum a Posteriori (MAP) estimator.

Problem 2 Detection (4+4+3+5+3 = 19)

Consider the following binary hypothesis testing problem

$$H_0 : x[n] \sim N(0,1), n = 0, \ldots, N-1$$
$$H_1 : x[n] \sim N(1,1)$$

2a) For the case of $N = 1$, design an NP detector (decision rule and threshold) that ensures that the probability of false alarm does not exceed $P_{FA} = 0.1$.

2b) Find the probability of detection P_D of the detector developed in a).

2c) Assuming that someone tells you that the occurrence probability of hypothesis H_0 is $\pi_0 = 0.2$. Find the test that will yield the minimum probability of error P_e.

2d) What is the probability of error in Problem 2c)?

2e) Assume that you get access to two samples instead of one, i.e., $N = 2$. How would you modify your NP detector? Will the new detector result in an increased or decreased value of P_D?

Problem 3 Classification (2+3+3+5+4 = 17)

3a) Give the Bayes Decision Rule (BDR) for a C-class problem.

Use Bayes Rule (BR) to reformulate BDR using class priors $P(\omega_i)$ and class densities $p(x/\omega_i)$.

3b) Assume a parametric form of the densities; i.e. $p(x/\theta_i) = p(x/\omega_i)$.

Explain the principle for Maximum Likelihood (ML) based estimation of θ_i from a training set $X = [x_1, \ldots, x_N]$

3c) Given a scalar observation x (1-dimensional) and assume Gaussian densities $p(x/\mu_i) = N(\mu_i, \sigma_i^2)$.

Derive the expression for the ML-estimate of the mean μ_i.

3d) Give the decision rule and the discriminant formula for a linear discriminant classifier for C classes.

Sketch a linear discriminant classifier using sigmoids and binary targets.

Derive the gradient upgrade expression for MSE-based training

3e) Explain shortly the principle of clustering.

What is meant by hierarchical clustering?