Appendix :

1. The normal density for a vector $\vec{x}$ of dimension N is given by

$$p(\vec{\mathbf{x}}/\theta) = (2\pi)^{-N/2}|\Sigma|^{-1/2} \exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu})\right]$$

where the parameters are $\theta = \{\vec{\mu}, \Sigma\}$. In the BLUE explanation on the next page the symbol $\mathbf{C_x}$ is used instead of Σ for the covariance matrix.

2. The Cramer Rao Lower Bound (CRLB) is a lower bound for the variance of an estimator $\hat{\theta}$ based on N observations $\vec{x} = [x(0), x(1), \ldots, x(N-1)]$ The distribution $p(\vec{x}/\theta)$ must be known in order to find the bound. The CRLB is only valid if the following condition applies :

$$E\left[\frac{\delta log[p(\vec{x}/\theta)]}{\delta\theta}\right] = 0$$

Then the CRLB is given by

$$var[\hat{\theta}] \geq CRLB = \frac{-1}{E\left[\frac{\delta^2 log[p(\vec{x}/\theta)]}{\delta^2\theta}\right]}$$

3. The Minimum Variance Unbiased (MVU) estimator fulfills

$$\begin{aligned} E[\hat{\theta}] &= \theta \\ var[\hat{\theta}] &= CRLB \end{aligned}$$

Further, the following applies :

$$\frac{\delta log[p(\vec{x}/\theta)]}{\delta\theta} = I(\theta)(\hat{\theta}-\theta)$$

where

$$I(\theta) = \frac{1}{var[\hat{\theta}]}$$

4. The Best Linear Unbiased Estimator (BLUE) is given by the following where $\vec{s}$ and the covariance matrix $\mathbf{C_x}$ are assumed known:

$$
\begin{aligned}
\hat{\theta} &= \vec{a}^T \vec{x} = \sum_n a_n x(n) \\
E[\hat{\theta}] &= \theta \\
E[\vec{x}] &= \vec{s}\theta \\
E[\hat{\theta}] &= E[\vec{a}^T \vec{x}] = \vec{a}^T \vec{s}\theta \;\Rightarrow\; \vec{a}^T \vec{s} = 1 \\
var[\hat{\theta}] &= \vec{a}^T \mathbf{C_x} \vec{a} \\
\vec{a} &= \frac{\mathbf{C_x^{-1}} \vec{s}}{\vec{s}^T \mathbf{C_x^{-1}} \vec{s}}
\end{aligned}
$$

5. The Likelihood Ratio Test (LRT) for binary detection is given by

$$LR = \frac{p(\vec{x}/H_1)}{p(\vec{x}/H_0)} \lessgtr \lambda$$

where $\vec{x} = [x(0), \ldots, x(N-1)]$

6. The Bayes Rule is given by

$$P(\omega/x) = \frac{p(x/\omega)P(\omega)}{p(x)}$$