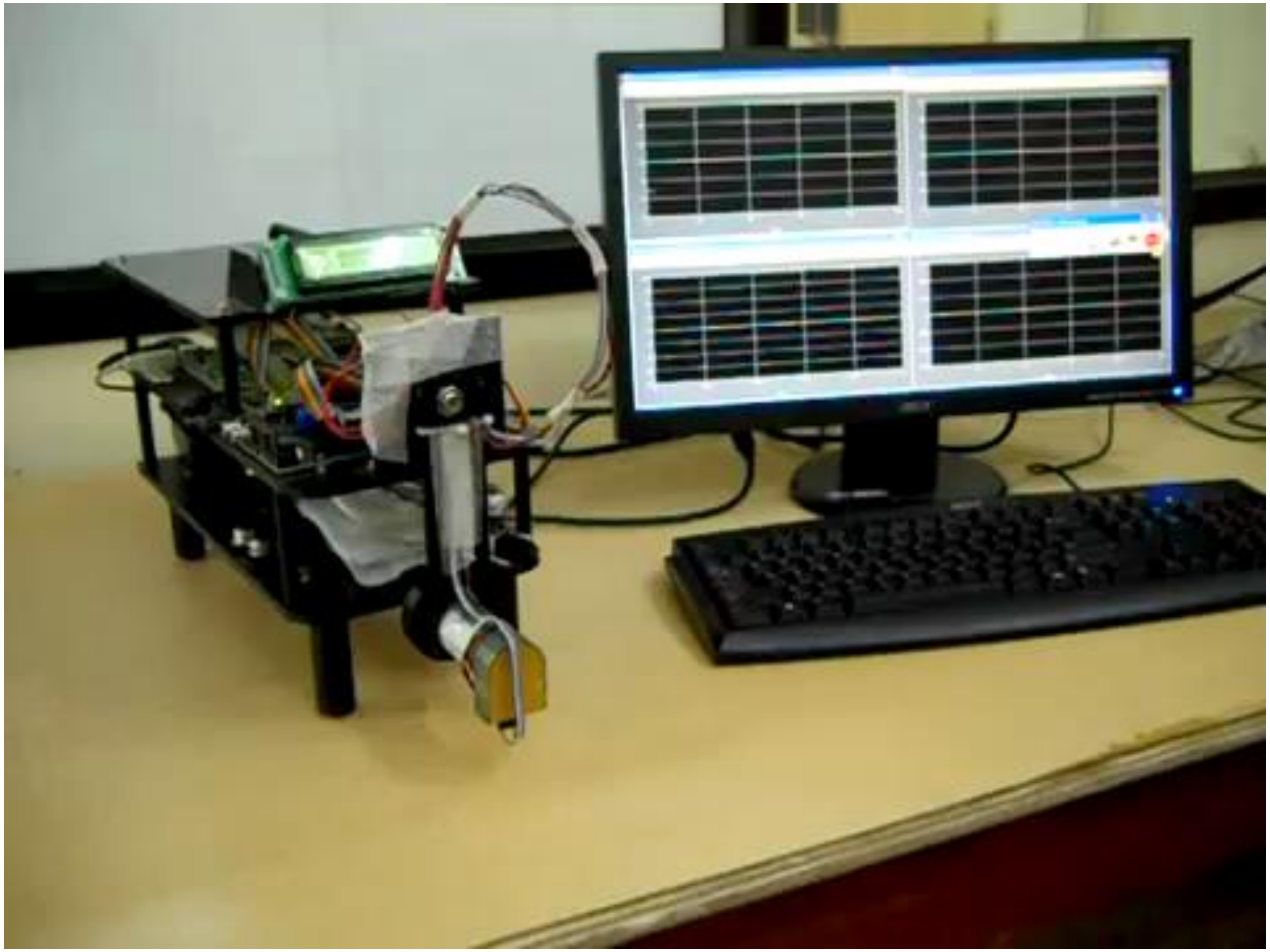
Lecture 19: Rigid body dynamics

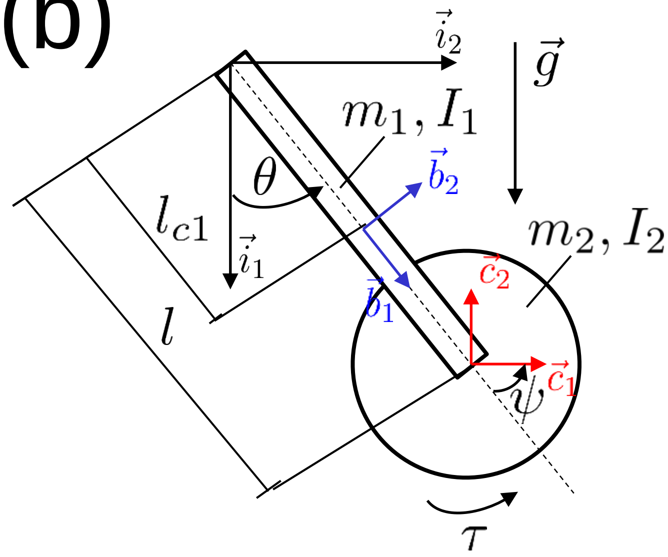
- Block in a pipe example
- Inverted Pendulum example

- Lagrange method of first kind

Gyroscopic pendulum
(Inertia wheel pendulum)



Gyroscopic pendulum (a),(b)



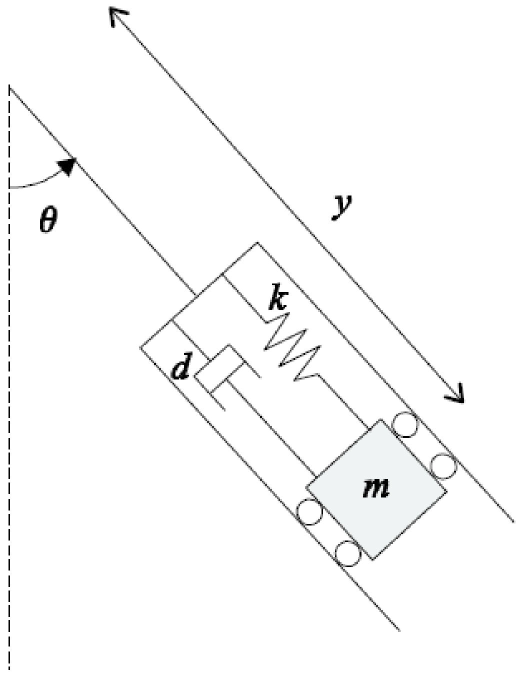


Figure 1: Kloss i rør

Oppgave 3) (15 %)

Figur (1) viser en kloss inne i et rør som svinger om et opphengspunkt. Anta at all masse bortsett fra klossen er neglisjerbar, og at klossens masse er m med massesenter gitt av y som er avtanden mellom massesenteret og opphengspunktet. Videre er fjærkonstanten k og dempekonstanten d. Fjæra er kraftløs når $y = y_0$. Det er ingen friksjon i systemet.

Velg passende generaliserte koordinater $\mathbf{q}$ og bruk Lagranges formulering for å sette opp en matematisk modell.

Block in a pipe I

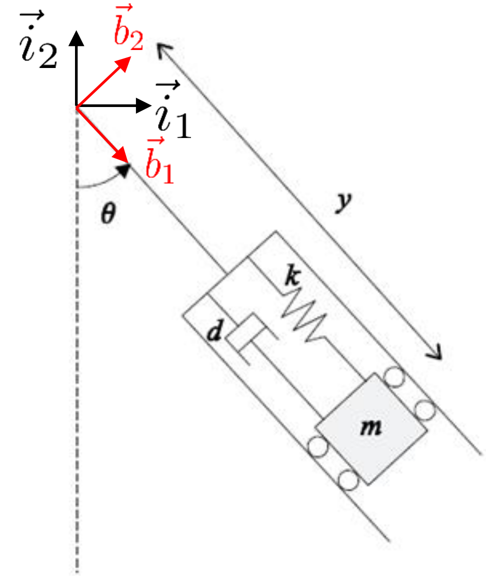


Figure 1: Kloss i rør

Block in a pipe II

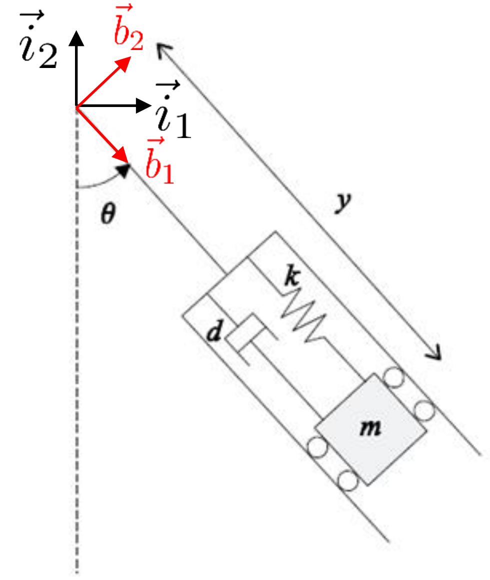


Figure 1: Kloss i rør

Block in a pipe III $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i$

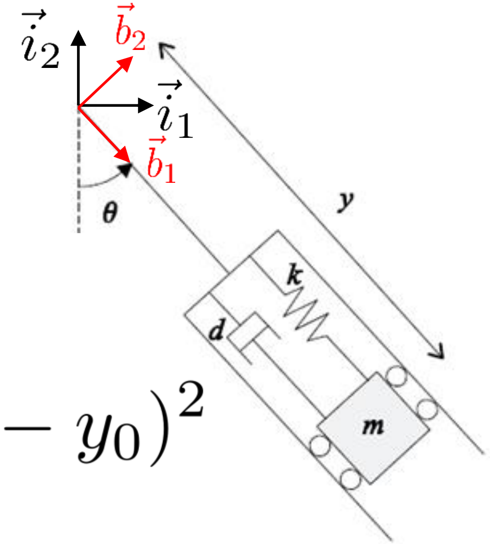
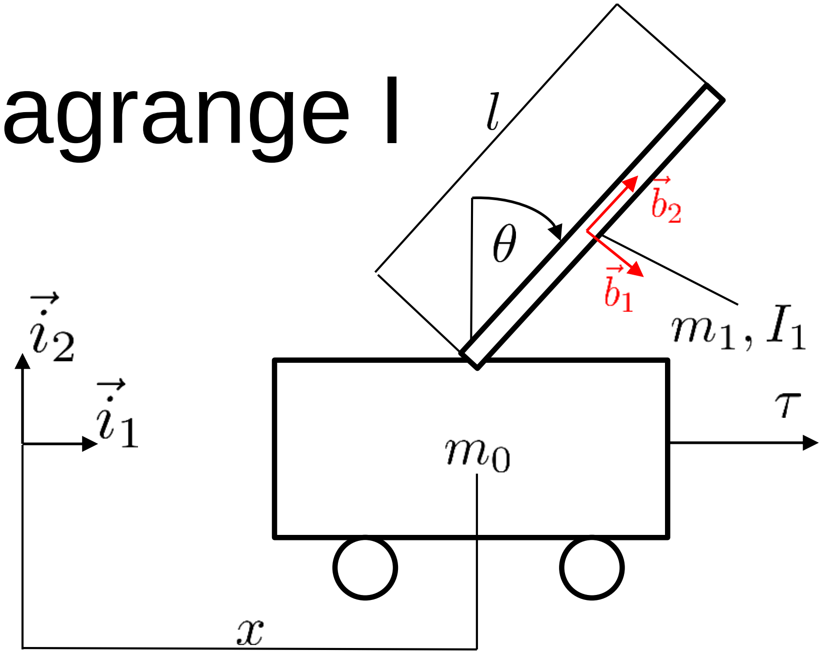


Figure 1: Kloss i rør

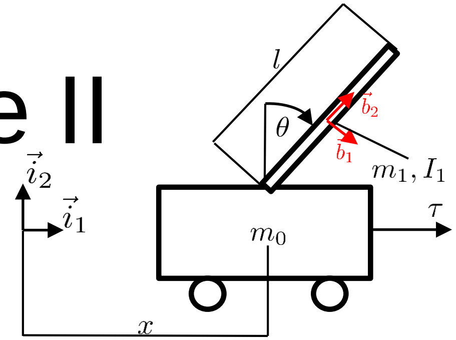
$$\mathcal{L} = \mathbf{T} - \mathbf{U}$$

$$= \frac{1}{2}m(\dot{y}^2 + y^2\dot{\theta}^2) + mgy\cos\theta - \frac{1}{2}k(y - y_0)^2$$

Inverted Pendulum – Lagrange I



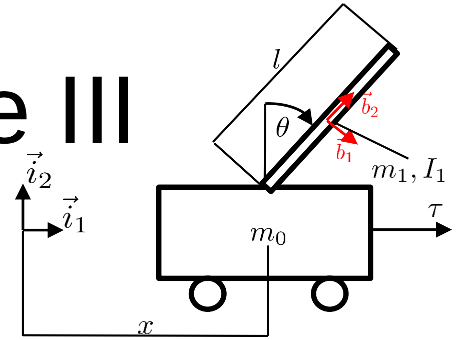
Inverted Pendulum – Lagrange II



$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i$$

$$\mathcal{L} = \tfrac{1}{2}\dot{x}(m_0 + m_1) + m_1 \tfrac{l}{2}\dot{\theta}\dot{x}\cos\theta + \tfrac{l^2}{8}m_1\dot{\theta}^2 + \tfrac{1}{2}I_1\dot{\theta}^2 - \tfrac{1}{2}m_1 g l \cos\theta$$

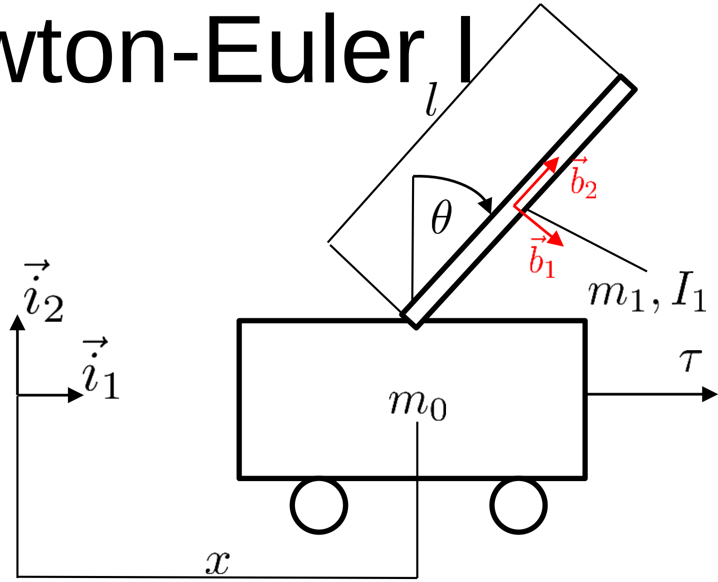
Inverted Pendulum – Lagrange III



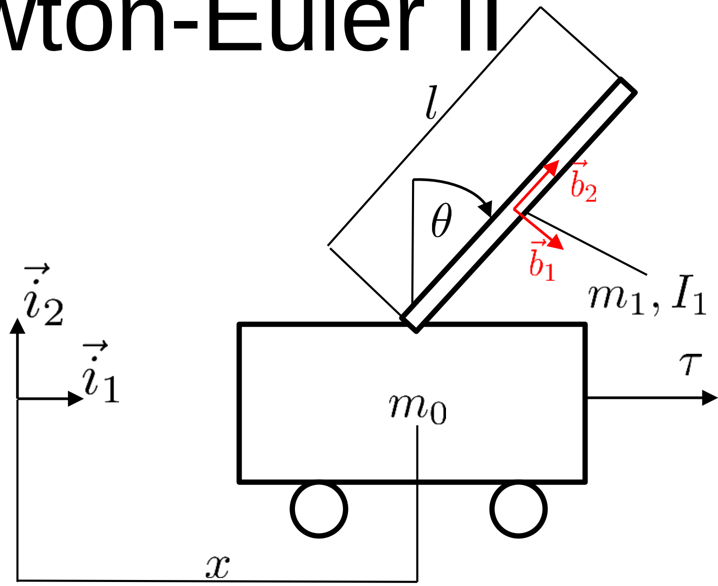
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i$$

$$\mathcal{L} = \tfrac{1}{2}\dot{x}(m_0 + m_1) + m_1 \tfrac{l}{2}\dot{\theta}\dot{x}\cos\theta + \tfrac{l^2}{8}m_1\dot{\theta}^2 + \tfrac{1}{2}I_1\dot{\theta}^2 - \tfrac{1}{2}m_1 g l\cos\theta$$

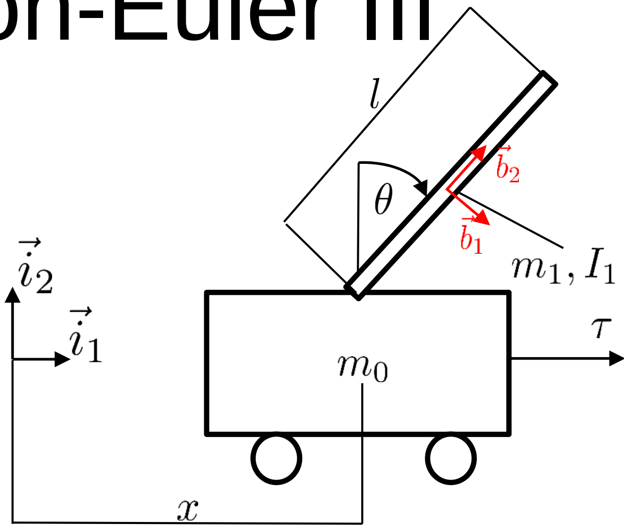
Inverted Pendulum – Newton-Euler I



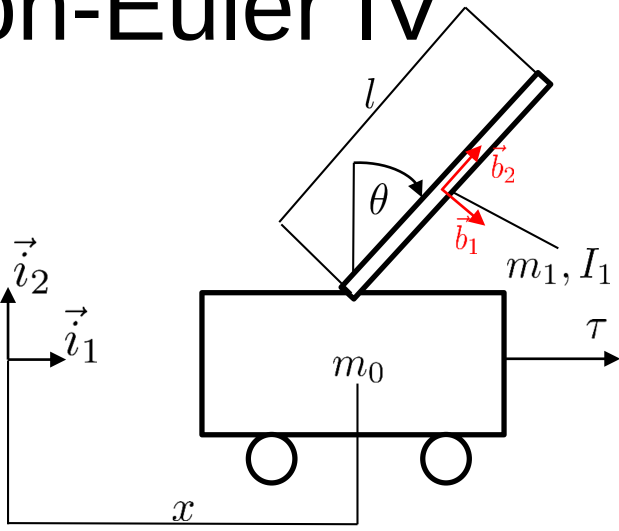
Inverted Pendulum – Newton-Euler II



Inverted Pendulum – Newton-Euler III



Inverted Pendulum – Newton-Euler IV

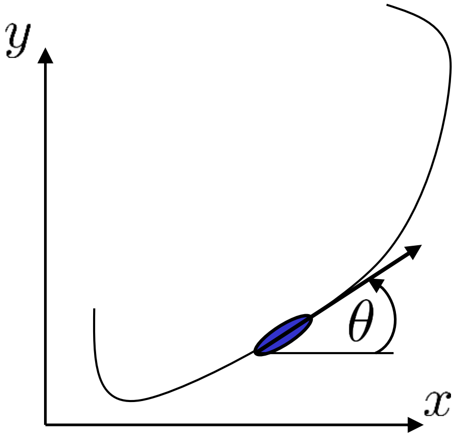


Lagrange's equation of first kind

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} - \sum_{k=1}^{m} \lambda_k \frac{\partial f_k}{\partial q_i} = \tau_i}$$

- Well suited if constraints contain derivatives (and connot be integrated):
  - Non-holonom constraints

Example: Non-holonomic constraint



Revisit d'Alembert's principle

- d'Alembert's principle:

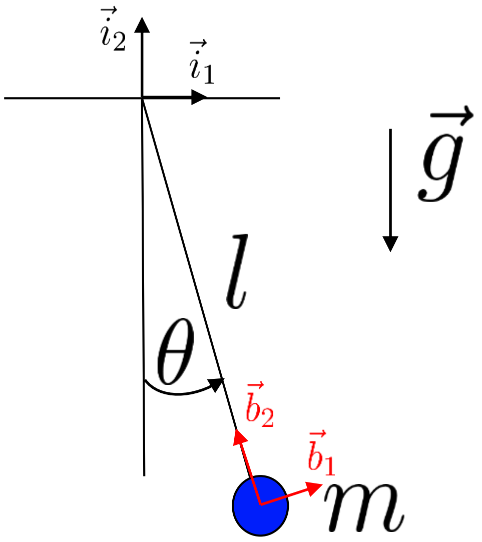
$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} - Q_i\right)\delta q_i = 0 \quad i = 1, \ldots, n$$

- Virtual displacement ($dt = 0$): $f_{ki}\delta q_i = 0$

- Possible to add zero to d'Alembert's principle!

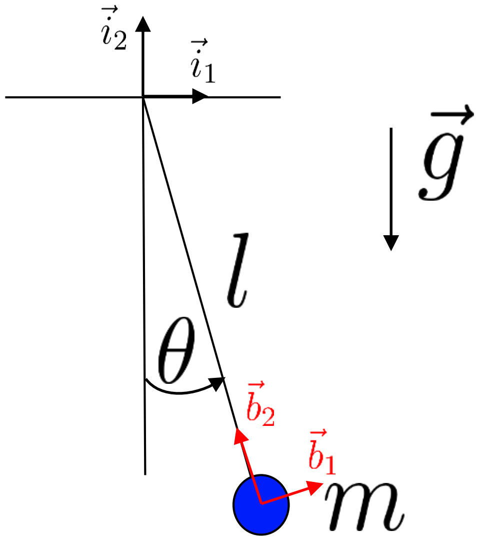
$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} - Q_i - \sum_{k=1}^{m}\lambda_k f_{ki}\right)\delta q_i = 0 \quad i = 1, \ldots, r$$

Example: Pendulum I



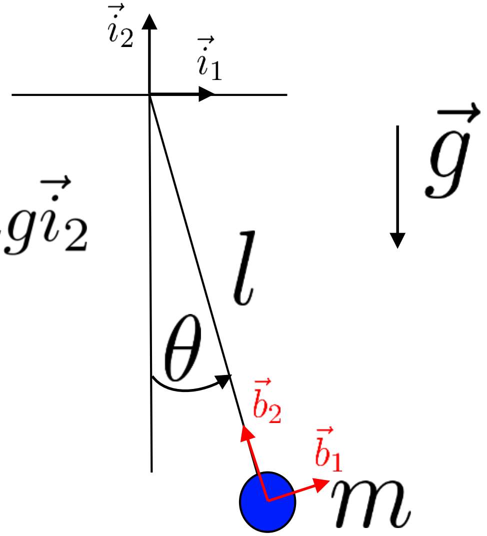
Example: Pendulum II $\mathcal{L} = \frac{1}{2}mr^2\dot{\theta}^2 + mgr\cos\theta$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} - \sum_{k=1}^{m} \lambda_k \frac{\partial f_k}{\partial q_i} = \tau_i$$

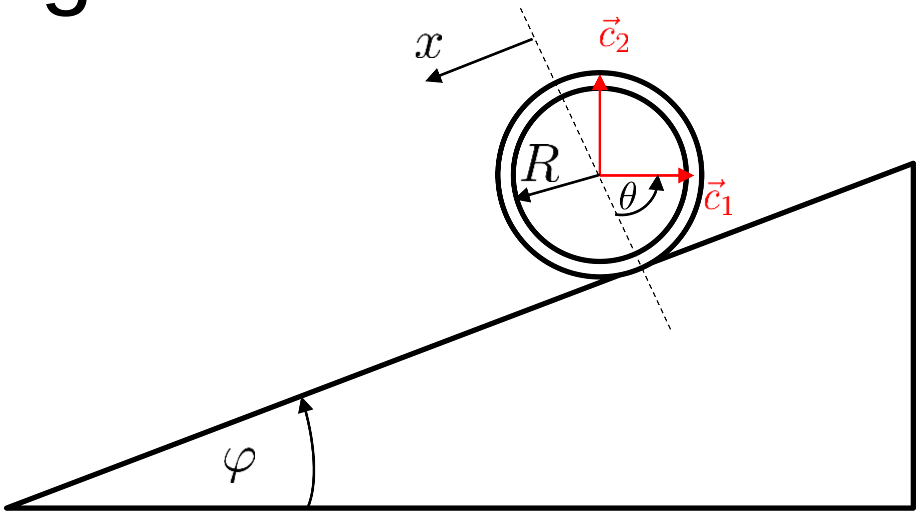


Example: Pendulum – body II

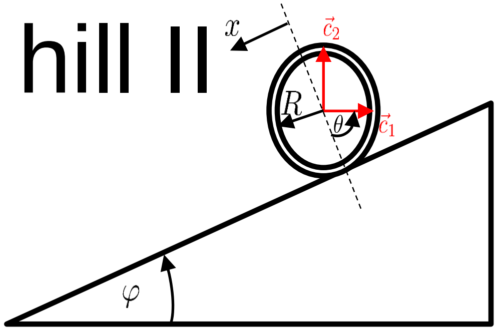
$$m\frac{{}^i d^2}{dt^2}\vec{r} = \delta\vec{b}_2 - mg\vec{i}_2$$



Hollow cylinder rolling down a hill I



Hollow cylinder rolling down a hill II



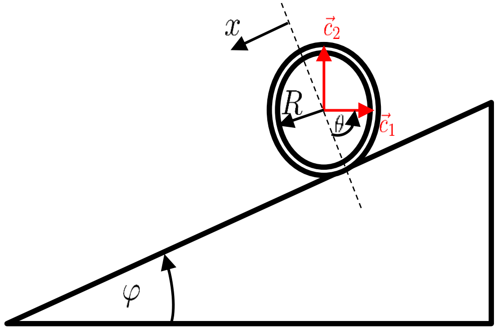
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} - \sum_{k=1}^{m} \lambda_k \frac{\partial f_k}{\partial q_i} = \tau_i$$

$$\mathcal{L} = \tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2} m R^2 \dot{\theta} + m g x \sin\varphi$$

Hollow cylinder rolling down a hill III

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} - \sum_{k=1}^{m} \lambda_k \frac{\partial f_k}{\partial q_i} = \tau_i$$

$$\mathcal{L} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m R^2 \dot{\theta} + mgx \sin\varphi$$



Hollow cylinder rolling down a hill IV (Lagrange second kind)