Image Processing - Assignment 1

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Contents

f =	5	0	2	3	4
	3	2	0	5	6
	4	6	1	1	4

1 Spatial Filtering

1.1 Task 1: Theory

a)

Sampling is the process of converting a continuos-time signal to a discrete-time signal, usually by measuring the continuos-time signal at specific points in time and extending this measurement over a set time step.

b)

Quantization is the process of constraining a signal from a larger to a smaller set of values, like mapping colours to the standard RGB range of 256 integer values.

c)

A high contrast image histogram would look similar to a dirac delta function, with most values grouped together around the same intensity.

d)

$$
\begin{aligned}
n_{\text{pixel}} &= 3 * 5 = 15 \\
L = 7 i_0 &= 2 \\
i_1 &= 2 \\
i_2 &= 2 \\
i_3 &= 2 \\
i_4 &= 3 \\
i_5 &= 2 \\
i_6 &= 2 \\
i_7 &= 0
\end{aligned}
$$

Then using eq. (1) on section 1.1 gives section 1.1.

$$
\left[\begin{array}{c|cccccccc}
n & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
f_n & \frac{2}{15} & \frac{2}{15} & \frac{2}{15} & \frac{2}{15} & \frac{3}{15} & \frac{2}{15} & \frac{2}{15} & \frac{0}{15} \\
F_n & \frac{2}{15} & \frac{4}{15} & \frac{6}{15} & \frac{8}{15} & \frac{11}{15} & \frac{13}{15} & \frac{15}{15} & \frac{15}{15} \\
E_q & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 6
\end{array}\right]
$$

6	0	2	3	4
3	2	0	5	6
4	6	1	1	4

$$g_{i,j} = floor((L-1) * \sum_{n=0}^{f_{i,j}} \frac{i_n}{n_{\text{pixel}}}) = floor((L-1) * F_n) \tag{1}$$

e)

The log transform will increase the output intensity for low input intensities, and flatten for high input intensities. This will make it easier to see the lower ranges of the input intensities. If we then apply this transform to

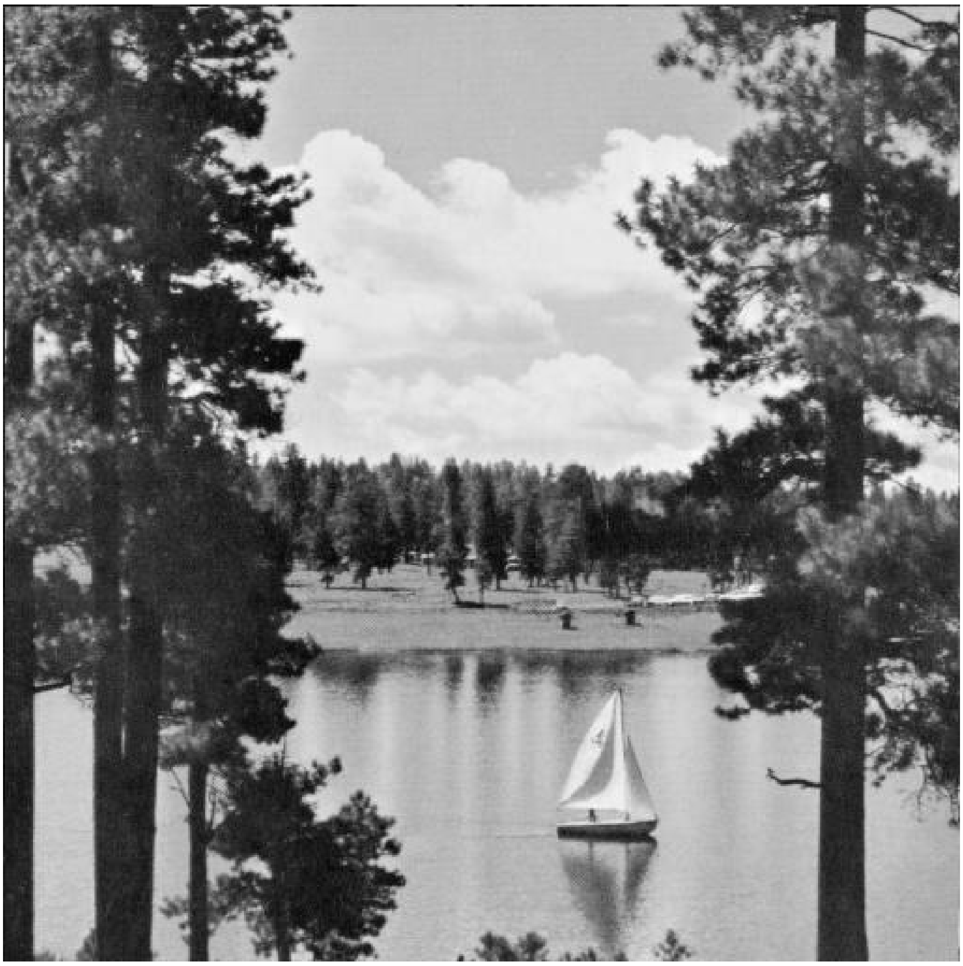

Figure 1: The grey scale version of the lake image

1.2 Task 2: Programming

a)

See the result from fig. 1.

b)

Se the result from fig. 2.

c)

See the results from fig. 3 and fig. 4.

Figure 2: The inverse of the grey scale version of the lake image

Figure 3: Convolution of the lake image with the kernel h_a.

Figure 4: Convolution of the lake image with the kernel h_b.

2 Neural Networks

2.1 Task 3: Theory

a)

The binary operation XOR, or exclusive or, cannot be represented by a single-layer neural network. If we look at the two binary input values as the two digits of a binary number, we get the range $[0, 3]$. Then applying the XOR function for this range, we get $[0, 1, 1, 0]$, which obviously cannot be represented by a linear function.

b)

A hyperparameter is a parameter which is set before, and not changed by, the learning process. Batch size and learning rate are examples of hyperparameters.

c)

The softmax function is a smooth approximation of the arg max function. The values being applied to the are shifted into the range $[0, 1]$, so that they can be interpreted as probabilities.

d)

$$C(y_n, \hat{y}_n) = (y_n - \hat{y}_n)^2, \hat{y}_n = 1 \tag{2}$$

Using eq. (2), we calculate:

$$\begin{aligned}
w_1' &= \frac{\partial C}{\partial w_1} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_1}\frac{\partial c_1}{\partial a_1}\frac{\partial a_1}{\partial w_1} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_1 = -2 * (y_n - 1) \\
w_2' &= \frac{\partial C}{\partial w_2} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_1}\frac{\partial c_1}{\partial a_2}\frac{\partial a_2}{\partial w_2} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_2 = 2 * (y_n - 1) \\
w_3' &= \frac{\partial C}{\partial w_3} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_2}\frac{\partial c_2}{\partial a_3}\frac{\partial a_3}{\partial w_3} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_3 = -2 * (y_n - 1) \\
w_4' &= \frac{\partial C}{\partial w_4} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_2}\frac{\partial c_2}{\partial a_4}\frac{\partial a_4}{\partial w_4} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_4 = -4 * (y_n - 1) \\
b_1' &= \frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_1}\frac{\partial c_1}{\partial b_1} = 2 * (y_n - \hat{y}_n) * 1 * 1 = 2 * (y_n - 1) \\
b_2' &= \frac{\partial C}{\partial b_2} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_2}\frac{\partial c_2}{\partial b_2} = 2 * (y_n - \hat{y}_n) * 1 * 1 = 2 * (y_n - 1)
\end{aligned}$$

e)

Calculating the current value for y:

$$\begin{array}{llll} a_1 = 1, & a_2 = 0, & a_3 = 1, & a_4 = -4 \\ c_1 = 2, & c_2 = -4, & y_n = max(c_1, c_2) = 2 & \end{array}$$

$$\theta_{t+1} = w_t - \alpha \frac{\partial C}{\partial \theta_t} \tag{3}$$

Using eq. (3) and $\alpha = 0.1$, we get:

$$\begin{aligned} w_{1,t+1} &= w_{1,t} - \alpha \frac{\partial C}{\partial w_1} = -1 - 0.1 * 2 * (2 - 1) = -1.2 \\ w_{3,t+1} &= w_{3,t} - \alpha \frac{\partial C}{\partial w_3} = -1 - 0.1 * (-2 * (2 - 1)) = -0.8 \\ b_{1,t+1} &= b_{1,t} - \alpha \frac{\partial C}{\partial b_1} = 1 - 0.1 * 2 * (2 - 1) = 0.8 \end{aligned}$$