Image Processing - Assignment 1

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Contents

	5	0	2	3	4
I =	3	2	0	5	6
	4	6	1	1	4

1 Spatial Filtering

1.1 Task 1: Theory

a)

Sampling is the process of converting a continuos-time signal to a discrete-time signal, usually by measuring the continuos-time signal at specific points in time and extending this measurement over a set time step.

b)

Quantization is the process of constraining a signal from a larger to a smaller set of values, like mapping colours to the standard RGB range of 256 integer values.

c)

A high contrast image histogram would look similar to a dirac delta function, with most values grouped together around the same intensity.

d)

Couldn't be bothered to do this in Latex, so see fig. 1. The top matrix is the image before equalization, and the bottom is the image after equalization. The columns are the following: intensities, counts of intensities, pdf, cdf, cdf multiplied with scaling, intensity rounding, new intensity count. Then, using the intensity rounding and mapping the old intensities to the new, we get the final, equalized image.

e)

The log transform will increase the output intensity for low input intensities, and flatten for high input intensities. This will make it easier to see the lower ranges of the input intensities. If we then apply this transform to a high variance image, then the lower part of the dynamic range will be enhanced, while the higher part looses

f)

Using zero-padding for this convolution. Convolving section 1.1 and section 1.1. See section 1.1:

```
5 0 2 3 4
3 2 0 5 6
4 6 1 1 4

0 2 2/15 2/15  7*2/15  0 2
1 2 2/15 4/15  7*4/15  1 2
2 2 2/15 6/15  7*6/15  2 2
3 2 2/15 8/15  7*8/15  3 2
4 3 3/15 11/15 7*11/15 5 3
5 2 2/15 13/15 7*13/15 6 2
6 2 2/15 15/15 7*15/15 7 2
7 0 0/15 15/15 7*15/15 7
 15

6 0 2 3 5
3 2 0 6 7
5 7 1 1 5
```

Figure 1: Histogram equalization

K =

0	0	0
1	-4	1
0	1	0

-4 * 5 + 3	5 + 2 + 2	2 - 4 * 2 + 3	2 - 4 * 3 + 4 + 5	3 - 4 * 4 + 6
5 - 4 * 3 + 2 + 4	3 - 4 * 2 + 6	2 + 2 + 5 + 1	3 - 4 * 5 + 6 + 1	4 + 5 - 4 * 6 + 1
3 - 4 * 4 + 6	2 + 4 - 4 * 6 + 1	6 - 4 * 1 + 1	5 + 1 - 4 * 1 + 4	6 + 1 + 4

=

-17	9	-3	-1	-7
-1	1	10	-10	-14
-7	-17	3	6	11

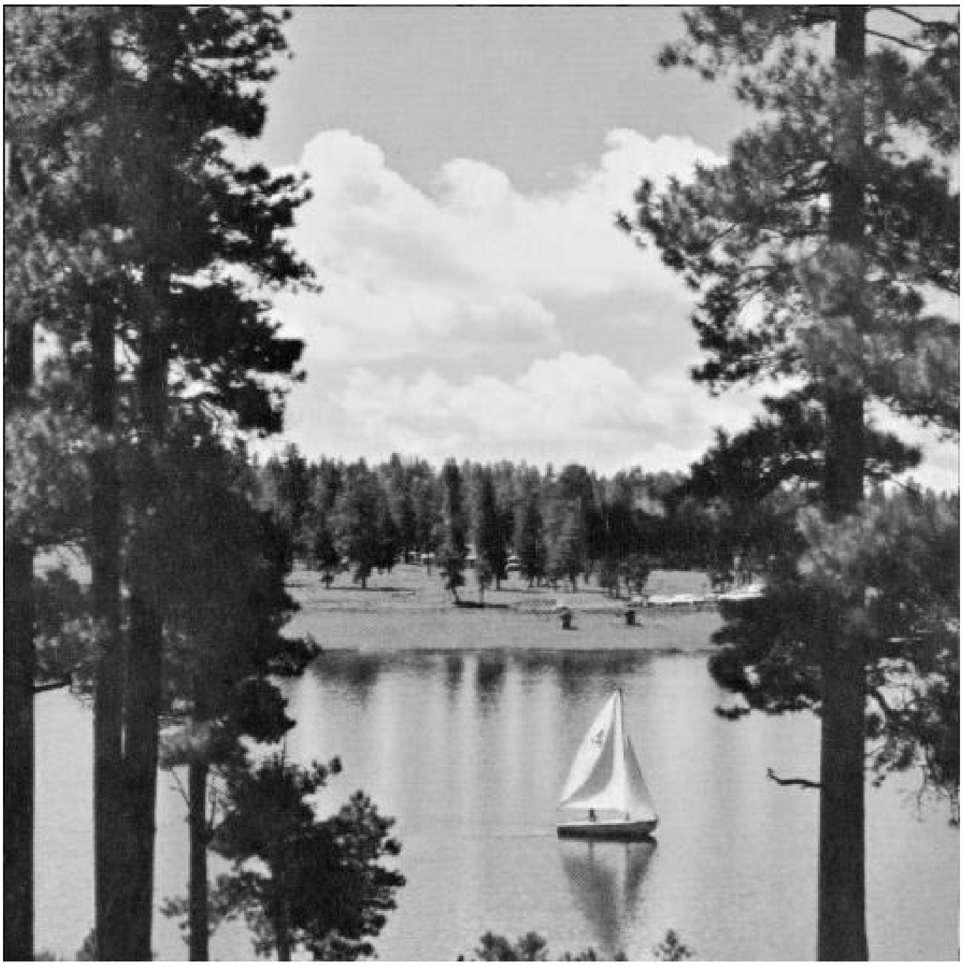

Figure 2: The grey scale version of the lake image

1.2 Task 2: Programming

a)

See the result from fig. 2.

b)

Se the result from fig. 3.

c)

See the results from fig. 4 and fig. 5.

Figure 3: The inverse of the grey scale version of the lake image

Figure 4: Convolution of the lake image with the kernel h_a.

Figure 5: Convolution of the lake image with the kernel h_b.

2 Neural Networks

2.1 Task 3: Theory

a)

The binary operation XOR, or exclusive or, cannot be represented by a single-layer neural network. If we look at the two binary input values as the two digits of a binary number, we get the range $[0, 3]$. Then applying the XOR function for this range, we get $[0, 1, 1, 0]$, which obviously cannot be represented by a linear function.

b)

A hyperparameter is a parameter which is set before, and not changed by, the learning process. Batch size and learning rate are examples of hyperparameters.

c)

The softmax function is a smooth approximation of the arg max function. The values being applied to the are shifted into the range $[0, 1]$, so that they can be interpreted as probabilities, and further as a probability density function, where the sum of the result is 1.

d)

$$C(y_n, \hat{y}_n) = (y_n - \hat{y}_n)^2, \hat{y}_n = 1 \tag{1}$$

Using eq. (1), we calculate:

$$
\begin{aligned}
w_1' &= \frac{\partial C}{\partial w_1} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_1}\frac{\partial c_1}{\partial a_1}\frac{\partial a_1}{\partial w_1} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_1 = -2 * (y_n - 1) \\
w_2' &= \frac{\partial C}{\partial w_2} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_1}\frac{\partial c_1}{\partial a_2}\frac{\partial a_2}{\partial w_2} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_2 = 2 * (y_n - 1) \\
w_3' &= \frac{\partial C}{\partial w_3} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_2}\frac{\partial c_2}{\partial a_3}\frac{\partial a_3}{\partial w_3} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_3 = -2 * (y_n - 1) \\
w_4' &= \frac{\partial C}{\partial w_4} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_2}\frac{\partial c_2}{\partial a_4}\frac{\partial a_4}{\partial w_4} = 2 * (y_n - \hat{y}_n) * 1 * 1 * w_4 = -4 * (y_n - 1) \\
b_1' &= \frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_1}\frac{\partial c_1}{\partial b_1} = 2 * (y_n - \hat{y}_n) * 1 * 1 = 2 * (y_n - 1) \\
b_2' &= \frac{\partial C}{\partial b_2} = \frac{\partial C}{\partial y}\frac{\partial y}{\partial c_2}\frac{\partial c_2}{\partial b_2} = 2 * (y_n - \hat{y}_n) * 1 * 1 = 2 * (y_n - 1)
\end{aligned}
$$

e)

Calculating the current value for y:

$$\begin{aligned} &a_1 = 1, && a_2 = 0, && a_3 = 1, && a_4 = -4 \\ &c_1 = 2, && c_2 = -4, && y_n = max(c_1, c_2) = 2 \end{aligned}$$

$$\theta_{t+1} = w_t - \alpha \frac{\partial C}{\partial \theta_t} \tag{2}$$

Using eq. (2) and $\alpha = 0.1$, we get:

$$\begin{aligned} w_{1,t+1} &= w_{1,t} - \alpha \frac{\partial C}{\partial w_1} = -1 - 0.1 * 2 * (2 - 1) = -1.2 \\ w_{3,t+1} &= w_{3,t} - \alpha \frac{\partial C}{\partial w_3} = -1 - 0.1 * (-2 * (2 - 1)) = -0.8 \\ b_{1,t+1} &= b_{1,t} - \alpha \frac{\partial C}{\partial b_1} = 1 - 0.1 * 2 * (2 - 1) = 0.8 \end{aligned}$$