Computational Methods Assignment 3 - Random Numbers

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1 Question 1

The random number generator used for this was found in *Numerical Recipes* page 342, which is a *struct* called *Ran*. This was selected because it is not based on any kind of linear congruential generator, and has a period of $3.138 \times 10^{57} / approx 2^{191} / approx 10^{57}$ which is neither too small to produce feasibly "psuedorandom" numbers nor too large so to be "over-engineered". It combines a 64bit-XORshift method ($x_1 = 17$, $x_2 = 31$, $x_3 = 8$), and 64-bit Multiplut-with-carry method with base $b = 32$ and multiplier $a = 4294957665$. This combination of unrelated methods essentially makes the "randomness" higher, since they do not share a state throughout the generation.

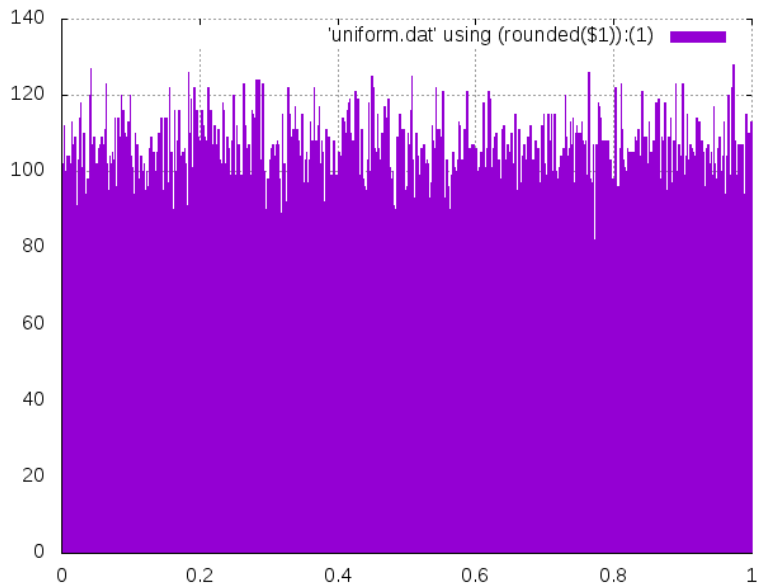

Figure 1: Histogram of uniform deviate x between $0.0 \leq x < 1.0$ plotted frequency bins, from Ran random number generator with seed 42.

Figure 1 shows the distribution of the double precision floating point variable uniform deviate portrayed in a histogram.

2 Question 2

To obtain the correct transformation $y(x)$ on uniform deviate $x \in [0, 1]$ with probability distribution $\text{pdf}(x) = U(x)$ (where U is the uniform distribution) such that the corresponding values follow a probability distribution function $\text{pdf}(y) = \frac{1}{2}\sin(x)$, the cummulative distribution function needs to be calculated and inverted. For this specific case this can be derived through:

$$\begin{aligned}
\mathrm{pdf}(y)|\mathrm{d}y| &= \frac{1}{2}\sin(y)|\mathrm{d}y| = \mathrm{pdf}(x)|\mathrm{d}x| = U(x)|\mathrm{d}x| \\
\mathrm{pdf}(y) &= \frac{1}{2}\sin(y) = \frac{|\mathrm{d}x|}{|\mathrm{d}y|} \implies \mathrm{cdf}(y) = \int_0^y \mathrm{d}y' \frac{1}{2}\sin(y') = \left[-\frac{1}{2}\cos(y')\right]_0^y = \frac{1}{2}(1-\cos(y)) \\
&= \int_0^y \frac{|\mathrm{d}x|}{|\mathrm{y}y'|}\mathrm{d}y' = x(y) \\
&\implies y(x) = \cos^{-1}(1-2x)
\end{aligned}$$

which means that $0 \leq y \leq \pi$. This transformation is applie to 10^5 samples of a uniform deviate x and the resulting histogram is plotted in Figure 2.

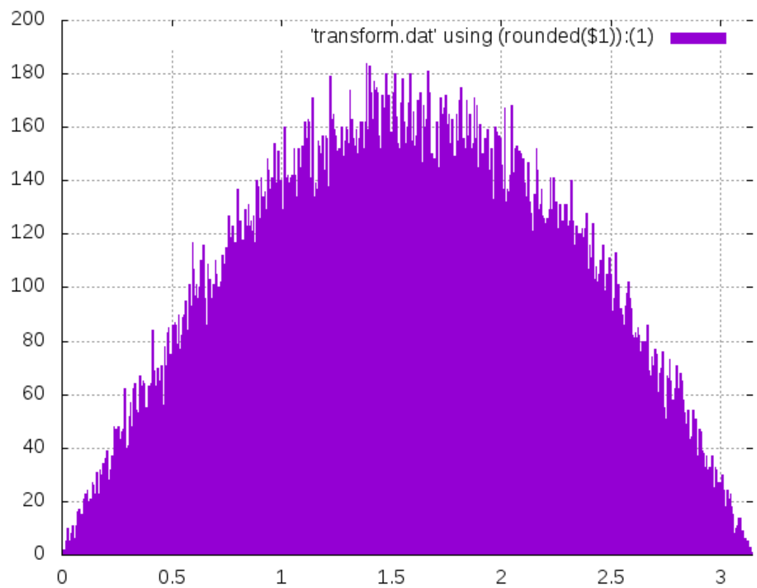

Figure 2: Histogram of transformed random variable $y(x) = \cos^{-1}(1-2x)$ between $0.0 \leq y < \pi$ plotted frequency bins, from a uniform deviate x generated from Ran with seed 42.

3 Question 3

For implementing the rejection method, a good comparison function $c(y)$ to obtain a $\mathrm{pdf}(y) = \frac{2}{\pi}\sin^2(y)$ such that $c(y) \geq \mathrm{pdf}(y)$, $\forall y$, that was chose was $c(y) = \frac{2}{\pi}\sin(y)$ since:

$$c(y) - \mathrm{pdf}(y) = \frac{2}{\pi}\sin(y)(1-\sin(y)) \geq 0 \ \forall y \in [0, \pi]$$

Using this comparison function the rejection method produced this probability distribution function from a uniform deviate x generated by Ran with seed 42. The resulting histogram is shown in Figure 3.

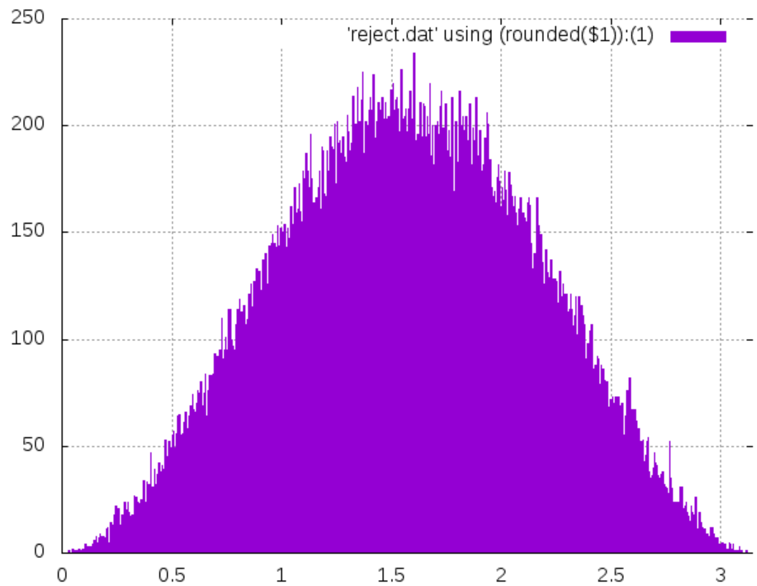

Figure 3: Histogram of random variable y between $0.0 \leq y < \pi$ following probability distribution $\mathrm{pdf}(y) = \frac{2}{\pi}\sin^2(y)$ using the rejection method. The comparison function used was $c(y) = \frac{2}{\pi}\sin(y)$ with samples obtained from a uniform deviate x generated from Ran with seed 42.