# Computational Methods Assignment 6 - Monte Carlo Integration

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## 1 Question 1 - Discussion of the Method

In this program, the *Mcint* class takes a specific, potentially multivalued function of $N$ variables to integrate between upper and lower limits, each individually specified by $N$-dimensional vectors. To integrate the function between the limits using importance sampling the user must supply the function to be integrated, the probability distribution to weight the samples, and an extra function supplied to either (a) generate the distribution of random numbers from a uniform deviate or (b) a function which produces numbers distributed according to the user's desired probability distribution. In this method, the function $f(\vec{x})$ (or $f(\vec{x})/p(\vec{x})$ if importance sampling) is sampled over volume $V$ at $M_{\text{init}}$ values, to calculate the average $V\langle f\rangle$ (or $\langle f/p\rangle$ for importance sampling). At each step, additional amount of samples are taken equal to the number currently included in the average, $M$. The old evaluation of the integral is multiplied by $M$ and then the further $M$ samples are added to it, to then divide by $2M$. As a measure of error, the standard error is used as a metric to determine the point to stop iterating the method. At each step, as well as calculating $\langle f\rangle$ (or $\langle f/p\rangle$), the value of $\langle f^2\rangle$ (or $\langle f^2/p^2\rangle$) is also calculated to obtain the standard error according to:

$$\sigma = \sqrt{\frac{\langle f^2\rangle - \langle f\rangle^2}{M}}$$

This means that the error, as expected, should decrease as $M^{-1/2}$ as the number of samples $M$ is increased. This means that by doubling the samples it is expected that the error should decrease by a factor of $\sqrt{2}$ each time.

## 2 Question 2 - Uniform vs Importance sampling

The error function was supplied as the integrand and firstly the evaluation of erf(2) was performed using the uniform sampling of points in an integral, later to be calculated using a distribution $\text{pdf}(y) = 0.98 - 0.48y$ with a correspond cummulative distribution of:

$$\text{cdf}(y) = -0.24^y + 0.98y = x \implies y = \text{cdf}^{-1}(x) = \frac{0.98 - \sqrt{0.98^2 - 0.96x}}{0.48}$$

This was passed to the generalised integrator class along with the probability distribution function.