Total Variation

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1 Introduction

Reintroducing notation:

$$\mathbf{y} = A\mathbf{f} + \sigma \mathbf{w} \tag{1}$$

In our setting, using $A = Id$, we can write the Total variation problem as

$$\min_{z \in L^2(\Omega)} \frac{1}{2} \int_\Omega ((z-f)^2) dx + \lambda \int_\Omega |\nabla z| dx \tag{2}$$

Effings utleding, $p = (p_x + p_y)$

$$\nabla(\lambda \div p - f) = \nabla(\lambda(\frac{\partial p_x}{\partial x} + \frac{\partial p_y}{\partial y} - f) = (\frac{\partial}{\partial x}(\lambda(\frac{\partial p_x}{\partial x} + \frac{\partial p_y}{\partial y} - f), \frac{\partial}{\partial y}((\frac{\partial p_x}{\partial x} + \frac{\partial p_y}{\partial y} - f)) \tag{3}$$

which in finite difference world is

$$(\frac{(p_x)_{i+1j} - 2(p_x)_{ij} + (p_x)_{i-1j}}{h_x^2} + \frac{(p_y)_{i+1j+1} - (p_y)_{i+1j-1} - (p_y)_{i-1j+1} + (p_y)_{i-1j-1}}{4h_x h_y} - \frac{f_{i+1j} - f_{i-1j}}{2h_x}, \text{same bu} \tag{4}$$