Frequência 2018/2019 - Normal (10/01/2019)

1. a)

método: Eliminação de Gauss

$$A=\begin{bmatrix} -1 & 3 & 0 & 0 \\ -3 & 5 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 1 & 0 & 2 & 2 \end{bmatrix} \xrightarrow[\substack{L_2-3L_1 \\ L_4+L_1}]{} \begin{bmatrix} -1 & 3 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 3 & 2 & 2 \end{bmatrix}$$

$$\xrightarrow[L_4-\frac{3}{4}L_2]{} \begin{bmatrix} -1 & 3 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \xrightarrow[L_4+2L_3]{} \begin{bmatrix} -1 & 3 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$\det(A) = (-1)\times(-4)\times(-1)\times 6 = -4\times 6 = -24$$

propriedade $\rightarrow$ o determinante de uma matriz triangular superior é o produto dos elementos da diagonal principal

b)

$$x_3 = \frac{|A[x_3]|}{|A|} = \frac{\begin{vmatrix} -1 & 3 & 2 & 0 \\ -3 & 5 & 3 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 2 \end{vmatrix}}{-24}$$

$$= \frac{2\times(-1)^{3+4}\begin{vmatrix} -1 & 3 & 2 \\ -3 & 5 & 3 \\ 1 & 0 & 0 \end{vmatrix}}{-24} = \frac{-2\times\left(1\times(-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix}\right)}{-24}$$

$$= \frac{-2\times -1}{-24} = -\frac{2}{24} = -\frac{1}{12}$$

c)

$$A^{-1} = \frac{[\mathrm{cof}(A)]^T}{|A|}$$

$$\mathrm{cof}\,A = \begin{bmatrix} X & X & (-1)^{1+3}\begin{vmatrix} -3 & 5 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 2 \end{vmatrix} & X \\ X & X & (-1)^{2+3}\begin{vmatrix} -1 & 3 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 2 \end{vmatrix} & X \\ X & X & (-1)^{3+3}\begin{vmatrix} -1 & 3 & 0 \\ -3 & 5 & 0 \\ 1 & 0 & 2 \end{vmatrix} & X \\ X & X & (-1)^{4+3}\begin{vmatrix} -1 & 3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 2 \end{vmatrix} & X \end{bmatrix} =$$

$$=\begin{bmatrix} x & x & \left(2\times(-1)^{1+3}\times\begin{vmatrix}-3 & 5\\ 1 & 0\end{vmatrix}\right) & x \\ x & x & -\left(2\times(-1)^{2+3}\times\begin{vmatrix}-1 & 3\\ 1 & 0\end{vmatrix}\right) & x \\ x & x & \left(1(-1)^{3+1}\begin{vmatrix}3 & 0\\ 5 & 0\end{vmatrix}+2(-1)^{2+3}\begin{vmatrix}-1 & 3\\ -3 & 5\end{vmatrix}\right) & x \\ x & x & -\left(2(-1)^{3+3}\begin{vmatrix}-1 & 3\\ -3 & 5\end{vmatrix}\right) & x \end{bmatrix}$$

$$=\begin{bmatrix} x & x & 10 & x \\ x & x & -6 & x \\ x & x & 8 & x \\ x & x & -8 & x \end{bmatrix}$$

$$S^{-1}=\begin{bmatrix} x & x & x & x \\ x & x & x & x \\ \frac{10}{24} & \frac{6}{24} & -\frac{8}{24} & \frac{8}{24} \\ x & x & x & x \end{bmatrix}$$

2. a) $Bv=\lambda v \Leftrightarrow$

$$\Leftrightarrow \begin{bmatrix} 2 & 3 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \alpha \\ 1 \end{bmatrix}=\lambda\begin{bmatrix} 0 \\ 0 \\ \alpha \\ 1 \end{bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} 0 \\ 0 \\ \alpha+2 \\ 2\alpha-2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \lambda\alpha \\ \lambda \end{bmatrix} \Leftrightarrow \begin{cases} 0=0 \\ 0=0 \\ \alpha+2=\lambda\alpha \\ 2\alpha-2=\lambda \end{cases} \Leftrightarrow \begin{cases} = \\ = \\ \frac{\lambda+2}{2}+2=\lambda\left(\frac{\lambda+2}{2}\right) \\ \alpha=\frac{\lambda+2}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} = \\ = \\ \frac{\lambda+6}{}=\frac{\lambda^2+2\lambda}{} \\ = \end{cases} \Leftrightarrow \begin{cases} = \\ = \\ \lambda=\frac{-1\pm\sqrt{1^2-4\times1\times(-6)}}{2} \\ = \end{cases}$$

$$\Leftrightarrow \begin{cases} = \\ = \\ \lambda=\frac{-1\pm5}{2} \\ = \end{cases} \Leftrightarrow \begin{cases} = \\ = \\ \lambda=-3 \vee \lambda=2 \\ \alpha=\frac{\lambda+2}{2} \end{cases} \Leftrightarrow \begin{cases} 0=0 \\ 0=0 \\ \lambda=-3 \\ \alpha=-\frac{1}{2} \end{cases} \vee \begin{cases} 0=0 \\ 0=0 \\ \lambda=2 \\ \alpha=2 \end{cases}$$

$\rightarrow \lambda = -3$ é o valor próprio associado ao vetor próprio $v = (0, 0, -\frac{1}{2}, 1)$;

$\rightarrow \lambda = 2$ é valor próprio associado ao vetor próprio $v = (0, 0, 2, 1)$;

b) $\det(B - \lambda I_4) = 0$

$$\Leftrightarrow \det\left(\begin{bmatrix} 2-\lambda & 3 & 0 & 0 \\ 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 1-\lambda & 2 \\ 0 & 0 & 2 & -2-\lambda \end{bmatrix}\right) = 0$$

$$\Leftrightarrow (2-\lambda)(-1)^{1+1} \begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 1-\lambda & 2 \\ 0 & 2 & -2-\lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (2-\lambda)\left((2-\lambda)(-1)^{1+1} \begin{vmatrix} 1-\lambda & 2 \\ 2 & -2-\lambda \end{vmatrix}\right) = 0$$

$$\Leftrightarrow (2-\lambda)(2-\lambda)\left[(1-\lambda)(-2-\lambda) - 4\right] = 0$$

$$\Leftrightarrow (2-\lambda)(2-\lambda)\left[-2-\lambda+2\lambda+\lambda^2-4\right] = 0$$

$$\Leftrightarrow (2-\lambda) = 0 \lor 2-\lambda = 0 \lor \lambda^2+\lambda-6 = 0$$

$$\Leftrightarrow \lambda = 2 \lor \lambda = 2 \lor \lambda = \frac{-1 \pm \sqrt{1-4\times1\times(-6)}}{2}$$

$$\Leftrightarrow \lambda = 2 \lor \lambda = 2 \lor \lambda = 2 \lor \lambda = -3$$

m.a $(2) = 3$

m.a. $(-3) = 1$

c) $E(2) = ?$

$$(A-2I)x=0 \Leftrightarrow \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} 3y=0 \\ 0=0 \\ -z=0 \\ -4t=0 \end{cases} \Leftrightarrow \begin{cases} y=0 \\ 0=0 \\ z=0 \\ t=0 \end{cases} \qquad car = 3$$

$$E(2) = \{(x,0,0,0),\ x \in \mathbb{R}\}$$

$$= \{x(1,0,0,0),\ x \in \mathbb{R}\}$$

d)

B não é diagonalizável pois $ma(2) \neq mg(2)$

3. a)

$$\det(C - \lambda I) = 0$$

$$\Leftrightarrow \det\left(\begin{bmatrix} 1-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix}\right) = 0$$

$$\Leftrightarrow [(1-\lambda)(-2-\lambda) - 4] = 0$$

$$\Leftrightarrow [-2-\lambda+2\lambda+\lambda^2-4] = 0$$

$$\Leftrightarrow \lambda^2 + \lambda - 6 = 0$$

$$\Leftrightarrow \lambda = 2 \ \vee \ \lambda = -3$$

$$(C-2I)x = 0 \Leftrightarrow \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 2 & -4 & 0 \end{array}\right] \xrightarrow[L_2+2L_1]{} \left[\begin{array}{cc|c} -1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \Leftarrow car = 1$$

$$\begin{cases} -x+2y=0 \\ 0=0 \end{cases} \Leftrightarrow \begin{cases} x=2y \\ 0=0 \end{cases} \qquad E(2) = \{(2y, y),\ y \in \mathbb{R}\}$$

$$= \{y(2,1),\ y \in \mathbb{R}\}$$

$(C+3I)x=0 \iff \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 4 & 2 & 0 \\ 2 & 1 & 0 \end{array}\right] \xrightarrow[L_2-\frac{1}{2}L_1]{} \left[\begin{array}{cc|c} 4 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ car = 1

$\begin{cases} 4x+2y=0 \\ 0=0 \end{cases} \iff \begin{cases} x=-\frac{1}{2}y \\ 0=0 \end{cases}$

$E(1)=\left\{ \left(-\frac{1}{2}y, y\right), y\in\mathbb{R} \right\} = \left\{ y\left(-\frac{1}{2}, 1\right), y\in\mathbb{R} \right\}$

b)

C é ortogonalmente diagonalizável pois

m.a.(2) = m.g.(2) = 1 e m.a(-3) = m.g(-3) = 1

$C = 2\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \end{bmatrix}^T + 3\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & 1 \end{bmatrix}^T$

4.