Frequência 2017/2018 (18/12/17)

1. a)

método: Regra de Laplace

$$|D| = (-2)(-1)^{1+2}\begin{vmatrix} 2 & -3 & 0 \\ 1 & -1 & 1 \\ -2 & 0 & 3 \end{vmatrix} + (-1)(-1)^{1+4}\begin{vmatrix} 2 & 0 & -3 \\ 1 & 0 & -1 \\ -2 & 2 & 0 \end{vmatrix}$$

$$= 2\left(2\times(-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 0 & 3 \end{vmatrix} + (-3)(-1)^{1+2}\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix}\right) + 1\left(2(-1)^{1+1}\begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} + (-3)(-1)^{1+3}\begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix}\right)$$

$$= 2[2\times(-3) + 3(3+2)] + [2(2) - 3(2)]$$

$$= 2(-6+15) + [4-6]$$

$$= 18 - 2 = 16$$

b)

$$cof(D)_{2,3} = (-1)^{2+3}\begin{vmatrix} 0 & -2 & -1 \\ 1 & 0 & 1 \\ -2 & 2 & 3 \end{vmatrix} =$$

$$= -\left(1(-1)^{2+1}\begin{vmatrix} -2 & -1 \\ 2 & 3 \end{vmatrix} + (-2)(-1)^{3+1}\begin{vmatrix} -2 & -1 \\ 0 & 1 \end{vmatrix}\right) =$$

$$= -\left(-(-6+2) - 2(-2)\right) = -[4+4] = -8$$

$$\operatorname{adj}(D)_{4,2} = \operatorname{cof}(D)_{2,4} =$$

$$= (-1)^{2+4} \begin{vmatrix} 0 & -2 & 0 \\ 1 & 0 & -1 \\ -2 & 2 & 0 \end{vmatrix} = 1 \left((-2)(-1)^{1+2} \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} \right) =$$

$$= 1 \left(-2 \, (2) \right) = -4$$

c)

$$D^{-1} = \frac{1}{|D|} \times \operatorname{adj} D = \frac{[\operatorname{cof}(D)]^T}{|D|}$$

$$D^{-1}_{1,2} = \frac{\operatorname{adj}_{1,2}}{16} = \frac{-4}{16} = -\frac{1}{4}$$

$$D^{-1}_{2,2} = \frac{\operatorname{adj}_{2,2}}{16} = \frac{2}{16} = \frac{1}{8}$$

$$D^{-1}_{2,3} = \frac{\operatorname{cof}_{3,2}}{16} = \frac{(-1)^{3+2} \begin{vmatrix} 0 & -2 & -1 \\ 1 & 0 & 1 \\ -2 & 2 & 3 \end{vmatrix}}{16} = \frac{-8}{16} = -\frac{1}{2}$$

$$D^{-1}_{2,4} = \frac{\operatorname{cof}_{4,2}}{16} = \frac{(-1)^{4+2} \begin{vmatrix} 0 & -2 & 0 \\ 1 & 0 & -1 \\ -2 & 2 & 0 \end{vmatrix}}{16} = -\frac{4}{16} = -\frac{1}{4}$$

$$D^{-1} = \begin{bmatrix} x & -\frac{1}{4} & x & x \\ x & \frac{1}{8} & x & x \\ x & -\frac{1}{2} & x & x \\ x & -\frac{1}{4} & x & x \end{bmatrix}$$

d)

$$Dx = b \iff \begin{bmatrix} 0 & -2 & 0 & -1 \\ -2 & 0 & 3 & 0 \\ 1 & 0 & -1 & 1 \\ -2 & 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ -2 \end{bmatrix}$$

$$x_3 = \frac{|D[x_3]|}{|D|} =$$

$$= \frac{\begin{vmatrix} 0 & -2 & 0 & -1 \\ +2 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 \\ -2 & 2 & -2 & 3 \end{vmatrix}}{16} =$$

$$= \frac{(-2)(-1)^{1+2}\begin{vmatrix} +2 & 1 & 0 \\ 1 & 3 & 1 \\ -2 & -2 & 3 \end{vmatrix} + (-1)(-1)^{1+4}\begin{vmatrix} +2 & 0 & 1 \\ 1 & 0 & 3 \\ -2 & 2 & -2 \end{vmatrix}}{16}$$

$$= \frac{2 \times \left[+2(-1)^{1+1}\begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} + 1(-1)^{1+2}\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix}\right] + 1 \times \left[+2(-1)^{1+1}\begin{vmatrix} 0 & 3 \\ 2 & -2 \end{vmatrix} + 1(-1)^{1+3}\begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix}\right]}{16}$$

$$= \frac{2 \times \left[+2 \times (11) - 5\right] + \left[+2[-6] + (2)\right]}{16}$$

$$= \frac{2 \times [+22 - 5] + (-10)}{16} = \frac{34 - 10}{16} = \frac{24}{16} = \frac{12}{8} = \frac{3}{2}$$

$$\boxed{x_3 = \frac{3}{2}}$$

2. a) $Av = \lambda v \iff$

$$\iff \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ \alpha \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 0 \\ \alpha \\ 1 \end{bmatrix}$$

$$\iff \begin{bmatrix} -\alpha \\ 0 \\ 2 \end{bmatrix} = \lambda \begin{bmatrix} 0 \\ \alpha \\ 1 \end{bmatrix} \iff \begin{cases} -\alpha = 0 \\ 0 = \alpha \\ 2 = \lambda \end{cases} \iff \begin{cases} \alpha = 0 \\ \alpha = 0 \\ \lambda = 2 \end{cases}$$

→ para $\alpha = 0$, v é vetor próprio de A, associado ao valor próprio de $\lambda = 2$

b) $\det(A - \lambda I_3) = 0$

$$\Leftrightarrow \det\left(\begin{bmatrix} 2-\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ -1 & 0 & 2-\lambda \end{bmatrix}\right) = 0$$

$$\Leftrightarrow (2-\lambda)(-1)^{1+1}\begin{vmatrix} -\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} + (-1)(-1)^{1+2}\begin{vmatrix} 1 & 0 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (2-\lambda)\left[(-\lambda)(2-\lambda)\right] + (2-\lambda) = 0$$

$$\Leftrightarrow (2-\lambda)^2(-\lambda) + (2-\lambda) = 0$$

$$\Leftrightarrow (2-\lambda)\left[(2-\lambda)(-\lambda) + 1\right] = 0$$

$$\Leftrightarrow \lambda = 2 \quad \vee \quad -2\lambda + \lambda^2 + 1 = 0$$

$$\Leftrightarrow \lambda = 2 \quad \vee \quad \lambda = \frac{2 \pm \sqrt{4 - 4\times 1\times 1}}{2}$$

$$\Leftrightarrow \lambda = 2 \quad \vee \quad \lambda = 1 \quad \vee \quad \lambda = 1$$

$m.a(2) = 1 \qquad m.a.(1) = 2$

c) $(A - I_3)x = 0 \quad \Leftrightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \xrightarrow[L_3+L_1]{L_2-L_1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[L_3 / L_1]{} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \text{car} = 2$$

$$\Leftrightarrow \begin{cases} x - y = 0 \\ -y + z = 0 \\ 0 = 0 \end{cases} \Leftrightarrow \begin{cases} x = y \\ z = y \\ 0 = 0 \end{cases}$$

$$E(1) = \{(y, y, y),\ y \in \mathbb{R}\} = \{y(1, 1, 1),\ y \in \mathbb{R}\}$$

d.

para A ser diagonalizável ma(1) = mg(1) e
m.a.(2) = m.g.(2)

$\boxed{\lambda = 2}$

m.a.(2) = 1

m.g.(2) = n − car(A−2I)

= 3 − 2 = 1

$(A-2I)x = 0 \iff \begin{bmatrix} 0 & -1 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\iff \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right] \xrightarrow{L_2/L_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{L_3+L_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right] \xrightarrow{L_3-2L_2} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ car(A − 2I) = 2

m.a(2) = m.g(2) = 1, logo A é diagonalizável

→ vetor próprio associado a λ=2 é (0, 0, 1)

$\boxed{\lambda = 1}$

m.a.(1) = 2

m.g.(1) = n − car(A − I_n)

= 3 − 2 = 1

→ m.a.(1) ≠ m.g(1), logo A <u>não é</u> diagonalizável

3. $B = B^T$ ← matriz simétrica

valores próprios → 2 e 6

vetores próprios → (−1, 1); (1, 1)

$B = SDS^{-1}$

$S = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$

$S^{-1} = \dfrac{[\text{cof } S]^T}{\text{Det}(S)}$ $(\text{cof } S)^T = \begin{bmatrix} (-1)^{1+1}(1) & (-1)^{1+2}(1) \\ (-1)^{2+1}(1) & (-1)^{2+2}(-1) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

$$|S| = -1-1 = -2$$

$$S^{-1} = \frac{[\mathrm{COF}\,S]^T}{-2} = \begin{bmatrix} \frac{1}{-2} & -\frac{1}{-2} \\ +\frac{1}{2} & \frac{1}{-2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$B = SDS^{-1}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1/2 & 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} +1+3 & -1+3 \\ -1+3 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = B^T$$

4.