1ª FREQUÊNCIA 2018

1. a) $W = \alpha A_1 + \beta A_2 \Leftrightarrow$

$\Leftrightarrow (5,2,0) = \alpha(4,0,2) + \beta(2,4,-4)$

$\Leftrightarrow (5,2,0) = (4\alpha, 0, 2\alpha) + (2\beta, 4\beta, -4\beta)$

$\Leftrightarrow \begin{cases} 4\alpha + 2\beta = 5 \\ 4\beta = 2 \\ 2\alpha - 4\beta = 0 \end{cases} \Leftrightarrow \begin{cases} 4\alpha + 1 = 5 \\ \beta = 1/2 \\ 2\alpha = 2 \end{cases} \Leftrightarrow \begin{cases} \alpha = 1 \\ \beta = \frac{1}{2} \\ \alpha = 1 \Leftrightarrow 1 = 1 \text{ P.V.} \end{cases}$

R.: W é combinação linear das colunas de A

$W = (4,0,2) + \frac{1}{2}(2,4,-4)$

b) $4AA^T + BC$

$4A = 4\begin{bmatrix} 4 & 2 \\ 0 & 4 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 0 & 16 \\ 8 & -16 \end{bmatrix}$ | $A^T = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 4 & -4 \end{bmatrix}$

$BC = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & -3 \\ 1 & -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0+0+2 & 0+2+(-4) & 0+(-3)+6 \\ 4+0+2 & 2+2+(-4) & 0+(-3)+6 \\ -2+0+1 & -1+0+(-2) & 0+0+3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & 3 \\ 6 & 0 & 3 \\ -1 & -3 & 3 \end{bmatrix}$

$4AA^T = \begin{bmatrix} 16 & 8 \\ 0 & 16 \\ 8 & -16 \end{bmatrix}\begin{bmatrix} 4 & 0 & 2 \\ 2 & 4 & -4 \end{bmatrix} = \begin{bmatrix} 16\times4+8\times2 & 0+8\times4 & 8\times4-16\times2 \\ 0+16\times2 & 0+16\times4 & 0+(16)\times(-4) \\ 8\times4+(-16)\times2 & 0+(-16)\times4 & 8\times2+(-16)\times(-4) \end{bmatrix}$

$= \begin{bmatrix} 80 & 32 & 0 \\ 32 & 64 & -64 \\ 64 & +64 & 80 \end{bmatrix}$

Digitalizada com CamScanner

$$4AA^T + BC = \begin{bmatrix} 80 & 32 & 0 \\ 32 & 64 & -64 \\ 64 & 64 & 80 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 3 \\ 6 & 0 & 3 \\ -1 & -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 82 & 30 & 3 \\ 38 & 64 & -61 \\ 63 & 61 & 83 \end{bmatrix}$$

c) $D = [A \mid z] = \left[\begin{array}{cc|c} 4 & 2 & z_1 \\ 0 & 4 & z_2 \\ 2 & -4 & z_3 \end{array}\right]$

$$D^T = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 4 & -4 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$D^T D = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 4 & -4 \\ z_1 & z_2 & z_3 \end{bmatrix} \left[\begin{array}{cc|c} 4 & 2 & z_1 \\ 0 & 4 & z_2 \\ 2 & -4 & z_3 \end{array}\right]$$

$$= \begin{bmatrix} 20 & 0 & 4z_1 + 2z_3 \\ 0 & 36 & 2z_1 + 4z_2 + (-4z_3) \\ 4z_1 + 2z_3 & 2z_1 + 4z_2 - 4z_3 & (z_1^2 + z_2^2 + z_3^2) \end{bmatrix}$$

- para D^TD ser matriz diagonal:

$$\begin{cases} 4z_1 + 2z_3 = 0 \\ 2z_1 + 4z_2 - 4z_3 = 0 \end{cases} \Leftrightarrow \begin{cases} z_1 = -\frac{z_3}{2} \\ -z_3 + 4z_2 - 4z_3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{—} \\ -5z_3 = -4z_2 \end{cases} \Leftrightarrow \begin{cases} z_1 = -\frac{1}{2} z_3 \\ z_2 = \frac{5}{4} z_3 \end{cases}$$

$z\left(-\frac{1}{2}z_3 \,;\, \frac{5}{4}z_3 \,;\, z_3\right)$ seja $z = 1 \Rightarrow z = \left(-\frac{1}{2} ; \frac{5}{4} ; 1\right)$

d) $D = \begin{bmatrix} 4 & 2 & -1/2 \\ 0 & 4 & 5/4 \\ 2 & -4 & 1 \end{bmatrix}$ $\Rightarrow$ para ser conj. ortonormal: $\| \, \|$ de todos os vetores $= 1$

seja: $x(4, 0, 2)$

$y(2, 4, -4)$

$z = \left(-\frac{1}{2} ; \frac{5}{4} ; 1\right)$

Digitalizada com CamScanner

$$\|x\| = \sqrt{4^2+0^2+2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\|y\| = \sqrt{2^2+4^2+(-4)^2} = \sqrt{4+32} = \sqrt{36} = 6$$

$$\|z\| = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{5}{4}\right)^2+1^2} = \sqrt{\frac{1}{4}+\frac{25}{16}+1}$$

$$= \sqrt{\frac{4+25+16}{16}}$$

$$= \sqrt{\frac{45}{16}} = \frac{\sqrt{45}}{4}$$

$$\begin{array}{c|c} 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

$2\sqrt{5}$

$\left\{\frac{x}{\|x\|}; \frac{y}{\|y\|}; \frac{z}{\|z\|}\right\}$ é um conj. ORTONORMAL de vetores

$$\frac{x}{\|x\|} = \left(\frac{4}{2\sqrt{5}}; \frac{0}{2\sqrt{5}}; \frac{2}{2\sqrt{5}}\right) = \left(2\sqrt{5}; 0; \frac{1}{\sqrt{5}}\right)$$

$$\frac{y}{\|y\|} = \left(\frac{2}{6}; \frac{4}{6}; -\frac{4}{6}\right) = \left(\frac{1}{3}; \frac{2}{3}; -\frac{2}{3}\right)$$

$$\frac{z}{\|z\|} = \left(\frac{-1/2}{\sqrt{45}/4}; \frac{5/4}{\sqrt{45}/4}; \frac{1}{\sqrt{45}/4}\right) = \left(\frac{-4}{2\sqrt{45}}; \frac{5}{\sqrt{45}}; \frac{4}{\sqrt{45}}\right) =$$

$$= \left(-2\sqrt{45}; \frac{5}{\sqrt{45}}; \frac{4}{\sqrt{45}}\right)$$

$\rightarrow \left\{\left(2\sqrt{5}, 0, \frac{1}{\sqrt{5}}\right); \left(1/3; 2/3; -2/3\right); \left(-2\sqrt{45}; \frac{5}{\sqrt{45}}; \frac{4}{\sqrt{45}}\right)\right\}$ é um conjunto ortonormal de vetores

e)

$$BC = \begin{bmatrix} 2 & -2 & 3 \\ 6 & 0 & 3 \\ -1 & -3 & 3 \end{bmatrix} \xrightarrow[L_3 = L_3 + \frac{1}{2}L_1]{L_2 = L_2 - 3L_1} \begin{bmatrix} 2 & -2 & 3 \\ 0 & 6 & -6 \\ 0 & -4 & \frac{9}{2} \end{bmatrix} \xrightarrow[L_3 = L_3 + \frac{4}{6}L_2]{}$$

$$\rightarrow \begin{bmatrix} 2 & -2 & 3 \\ 0 & +6 & -6 \\ 0 & 0 & 1/2 \end{bmatrix}$$

car(BC) = 3

car (BC) → é o nº de pivots ≠ s 0, logo há 3, s eles:

2 ; 6 ; ½

$$\frac{9}{2} + \frac{4}{6} \times (-6)$$

$$\frac{9}{2} + \left(-\frac{24}{6}\right)$$

$$= \frac{9}{2} - 4$$

$$C^T = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & -2 \\ 0 & -3 & 3 \end{bmatrix} \qquad B^T = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

Digitalizada com CamScanner

$$\mathcal{N}(C^T B^T) = \{0,0,0\}$$

$C^T B^T x = 0$ ← é sempre possível e determinado pois o vetor $(0,0,0)$ é sempre solução do sist homogéneo

f)

$$B = I_3 \Leftrightarrow \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{l_2/l_1} \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[l_3 = l_3 + \frac{1}{2} l_1]{l_2 = l_2} \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 2 & 0 & \frac{1}{2} & 1 \end{array}\right] \xrightarrow{l_3 = l_3 - \frac{1}{2} l_2} \left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \end{array}\right]$$

$\frac{1/2}{1/1} = \frac{1}{2}$

$2 - \frac{1}{2} 2 = 2 - 2 = 0$

R.: como o último pivot resultante da eliminação de Gauss é igual a 0, B não admite inversa

2. a)

$$\begin{cases} x + 2y + 3z = 1 \\ -3x - 5y + (\alpha^2 - a) z = 0 \\ 2x + 3y - 5\alpha z = \alpha \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 3 \\ -3 & -5 & (\alpha^2 - a) \\ 2 & 3 & -5\alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix}$$

b)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ -3 & -5 & (\alpha^2 - a) & 0 \\ 2 & 3 & -5\alpha & \alpha \end{array}\right] \xrightarrow[l_3 = l_3 + 2 l_1]{l_2 = l_2 + 3 l_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & \alpha^2 & 3 \\ 0 & -1 & -5\alpha - 6 & \alpha - 2 \end{array}\right]$$

$$\xrightarrow{l_3 = l_3 - l_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & \alpha^2 & 3 \\ 0 & 0 & \alpha^2 - 5\alpha - 6 & \alpha + 1 \end{array}\right]$$

CA

$+\alpha^2 - 5\alpha - 6 = 0 \Leftrightarrow$

$\Leftrightarrow \alpha = \frac{+5 \pm \sqrt{25 - 4 \times (+1) \times (-6)}}{-2}$

$\Leftrightarrow \alpha = \frac{5 \pm 7}{-2} \Leftrightarrow \alpha = +6 \lor \alpha = -1$

Digitalizada com CamScanner

→ se $\alpha = 6 \vee \alpha = -1$:

$car(A) = 2 < n = 4$ → sist. possível indeterminado

→ se $\alpha \in \mathbb{R} \setminus \{-1; 6\}$:

$car(A) = 3 < \underset{4}{n}$ → sist. possível indeterminado

c) $\boxed{\alpha = -1}$

$$\begin{cases} x + 2y + 3z = 1 \\ y + z = 3 \\ 0 = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2(3-z) + 3z = 1 \\ y = 3 - z \\ 0 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + 6 - 2z + 3z = 1 \\ y = 3 - z \\ 0 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 1 - 6 + 2z - 3z \\ y = 3 - z \\ 0 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -5 - z \\ y = 3 - z \\ 0 = 0 \end{cases} \qquad CS = \{(-5-z, 3-z, z), z \in \mathbb{R}\}$$

$$= \{z(-6, 2, 1), z \in \mathbb{R}\}$$

3. M → 4 linhas, 5 colunas > ⊕ incógnitas q equações

$Mx = b$

→ $Car(M) = 3$:

→ $car(M) = 4$: