Tarea 6:
Lenguajes de Programación

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Considera la siguiente gramatica:

$$e \ := \ x \mid n \mid true \mid false \mid \neg e \mid e+e \mid if\ e\ then\ e\ else\ e \mid let\ x\ =\ e\ in\ e \mid e\ <\ e \mid \lambda x.e \mid e\ e$$

extendida con la expresión:

$$|letcc(k.e)|continue(e_1\ e_2)$$

y con el valor:

$$cont(P)$$

Donde P es una pila de control.

1.Escribe todos los marcos de la operacion:

- Tomamos a x, n, true, false y cont(P) como valores entonces no tienen marco.
- letcc no es un valor pero no necesita marco.
- $\neg e$

$$\frac{}{not(-)\ marco}$$

- $e_1 + e_2$

$$\frac{}{suma(-,e_1)\ marco} \qquad \frac{}{suma(v_1,-)\ marco}$$

- $if\ e\ then\ e_1\ else\ e_2$

$$\frac{}{if(-,e_1,e_2)\ marco}$$

- $let\ x\ =\ e\ in\ e_2$

$$\frac{}{let(-,x.e_2)\ marco}$$

- $e_1\ <\ e_2$

$$\frac{}{menor(-,e_2)\ marco} \qquad \frac{}{menor(v_1,-)\ marco}$$

- $e_1\ e_2$

$$\frac{}{app(-,e_2)\ marco} \qquad \frac{}{app(v_1,-)\ marco}$$

- $continue(e_1\ e_2)$

$$\frac{}{continue(-,e_2)\ marco} \qquad \frac{}{continue(v_1,-)\ marco}$$

2.Describe todas las transiciones de la máquina K.

- Valores

$$\frac{}{P \succ v \to_{\mathcal{K}} P \prec v}$$

- $\neg e$

$$\frac{}{P \succ not(e) \to_{\mathcal{K}} not(-); P \succ e}$$

$$\frac{}{not(-); P \prec v}$$

- $e_1 + e_2$

$$\frac{}{P \succ suma(e_1,e_2) \to_{\mathcal{K}} suma(-,e_2); P \succ e_1}$$

$$\frac{}{suma(-,e_2); P \prec v \to_{\mathcal{K}} suma(v,-); P \succ e_2}$$

- $if\ e\ then\ e_1\ else\ e_2$

$$\overline{P \succ if(e, e_1, e_2) \to_{\mathcal{K}} if(-, e_1, e_2); P \succ e}$$

$$\overline{if(-, e_1, e_2); P \prec true \to_{\mathcal{K}} P \succ e_1}$$

$$\overline{if(-, e_1, e_2); P \prec false \to_{\mathcal{K}} P \succ e_2}$$

- $let\ x\ =\ e\ in\ e_2$

$$\overline{P \succ let(e_1, x.e_2) \to_{\mathcal{K}} let(-, x.e_2); P \succ e_1}$$

$$\overline{let(-, x.e_2); P \prec v \to_{\mathcal{K}} P \succ e[x := v]}$$

- $e_1\ <\ e_2$

$$\overline{P \succ menor(e_1, e_2) \to_{\mathcal{K}} menor(-, e_2); P \succ e_1}$$

$$\overline{menor(-, e_2); P \prec v \to_{\mathcal{K}} menor(v, -); P \succ e_2}$$

- $e_1\ e_2$

$$\overline{P \succ app(e_1, e_2) \to_{\mathcal{K}} if(-, e_2); P \succ e_1}$$

$$\overline{app(-, e_2); P \prec v \to_{\mathcal{K}} app(v, -); P \succ e_2}$$

$$\overline{app(lam(T, x.e), -); P \prec v \to_{\mathcal{K}} P \succ e[x := v]}$$

- $Letcc[T](k.e)$

$$\overline{P \succ letcc[T](k.e) \to_{\mathcal{K}} P \succ e[k := cont(p)]}$$

- $continue(e_1\ e_2)$

$$\overline{P \succ continue(e_1, e_2) \to_{\mathcal{K}} continue(-, e_2); P \succ e_1}$$

$$\overline{continue(-, e_2); P \prec v_1 \to_{\mathcal{K}} continue(v_1, -); P \succ e_2}$$

$$\overline{continue(cont(P'), -); P \prec v_2 \to_{\mathcal{K}} P' \prec v_2}$$

3.Escribe cinco programas y ejecutalos en la máquina K. Cada programa debe usar al menos cuatro expresiones del lenguaje y ademas hacer uso de los operadores *letcc* y *continue*. Debes haber utilizado todas las expresiones del lenguaje entre todos los programas que escribiste.

- $p_1 \rightleftharpoons$

```
e = letcc(k_{1}.8<continue(k_{1},letcc(k_{2}.continue(k_{2},3)+4)))
```

Renombraremos algunas expresiones, por el tamaño que tienen, de la siguiente forma:

$e' = 8 < continue(cont(\text{square}, letcc(k_2.continue(k_2, 3) + 4))$
$e'' = letcc(k_2.continue(k_2, 3) + 4)$
$P' = continue(cont(\Box, -); lt(8, -)$

$$\Box \succ e[k := cont(\Box)] \to_{\mathcal{K}}$$

$$\Box \succ 8 < continue(cont(\Box), e'') \to_{\mathcal{K}}$$

$$lt(-, e') \succ 8 \to_{\mathcal{K}}$$

$$lt(-, e') \prec 8 \to_{\mathcal{K}}$$

$$lt(8, -) \succ e' \to_{\mathcal{K}}$$

$$continue(-, e''); lt(8, -) \succ cont(\Box) \rightarrow_{\mathcal{K}}$$

$$continue(-, e''); lt(8, -) \prec cont(\Box) \rightarrow_{\mathcal{K}}$$

$$continue(cont(\Box), -); lt(8, -) \succ e'' \rightarrow_{\mathcal{K}}$$

$$\mathcal{P}' \succ continue(cont(\mathcal{P}'), 3) + 4 \rightarrow_{\mathcal{K}}$$

$$suma(-, 4); \mathcal{P}' \succ continue(cont(\mathcal{P}'), 3) \rightarrow_{\mathcal{K}}$$

$$continue(-, 3); suma(-, 4); \mathcal{P}' \succ cont(\mathcal{P}') \rightarrow_{\mathcal{K}}$$

$$continue(-, 3); suma(-, 4); \mathcal{P}' \prec cont(\mathcal{P}') \rightarrow_{\mathcal{K}}$$

$$continue(cont(e'), -); suma(-, 4); \mathcal{P}' \succ 3 \rightarrow_{\mathcal{K}}$$

$$continue(cont(e'), -); suma(-, 4); \mathcal{P}' \prec 3 \rightarrow_{\mathcal{K}}$$

$$continue(cont(\Box), -); lt(8, -) \prec 3 \rightarrow_{\mathcal{K}}$$

$$\Box \prec 3$$

- $p_2 \rightleftharpoons$

```
e = (λx.x + 3)(let x=true in if ¬x then 7
  else 2 + (letcc(k.3 + continue(k,(λy.y + y)5)))
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Renombraremos algunas expresiones, por el tamaño que tienen, de la siguiente forma:
$e' = let \quad x = true \quad in \quad if \quad \neg x \quad then \quad 7 \quad else \quad 2 + (letcc(k.3 + continue(k, (\lambda y.y + y)5)))$
$e'' = if \quad \neg x \quad then \quad 7 \quad else \quad 2 + (letcc(k.3 + continue(k, (\lambda y.y + y)5)))$
$e''' = 2 + (letcc(k.3 + continue(k, (\lambda y.y + y)5)))$
$e'''' = letcc(k, (\lambda y.y + y)5))$
$e''''' = continue(cont(\mathcal{P}'), (\lambda y.y + y)5)$
$P'' = continue(cont(P'), -), suma(3, -; P')$

$$\Box \succ e \rightarrow_{\mathcal{K}}$$

$$app(-, e') \succ \lambda x.x + 3 \rightarrow_{\mathcal{K}}$$

$$app(-, e') \prec \lambda x.x + 3 \rightarrow_{\mathcal{K}}$$

$$app(\lambda x.x + 3, -) \succ e' \rightarrow_{\mathcal{K}}$$

$$let(-, x.e''); app(\lambda x.x + 3, -) \succ true \rightarrow_{\mathcal{K}}$$

$$let(-, x.e''); app(\lambda x.x + 3, -) \prec true \rightarrow_{\mathcal{K}}$$

$$app(\lambda x.x + 3, -) \succ if \quad \neg true \quad then \quad 7 \quad else \quad e''' \rightarrow_{\mathcal{K}}$$

$$if(-, 7, e'''); app(\lambda x.x + 3, -) \succ \neg true \rightarrow_{\mathcal{K}}$$

$$not(-); if(-, 7, e'''); app(\lambda x.x + 3, -) \succ true \rightarrow_{\mathcal{K}}$$

$$not(-); if(-, 7, e'''); app(\lambda x.x + 3, -) \prec true \rightarrow_{\mathcal{K}}$$

$$if(-, 7, e'''); app(\lambda x.x + 3, -) \succ false \rightarrow_{\mathcal{K}}$$

$$if(-, 7, e'''); app(\lambda x.x + 3, -) \prec false \rightarrow_{\mathcal{K}}$$

$$app(\lambda x.x + 3, -) \succ e''' \rightarrow_{\mathcal{K}}$$

$$suma(-, e''''); app(\lambda x.x + 3, -) \succ 2 \rightarrow_{\mathcal{K}}$$

$$suma(-, e''''); app(\lambda x.x + 3, -) \prec 2 \rightarrow_{\mathcal{K}}$$

$$suma(2, -); app(\lambda x.x + 3) \succ e'''' \rightarrow_{\mathcal{K}}$$

$$P' = suma(2, -); app(\lambda x.x + 3) \succ 3 + continue(cont(P'), (\lambda y.y + y)5) \rightarrow_{\mathcal{K}}$$

$$suma(-, e'''''); P' \succ 3 \rightarrow_{\mathcal{K}}$$

$$suma(-, e'''''); P' \prec 3 \rightarrow_{\mathcal{K}}$$

$$suma(3,-); P' \succ e''''' \rightarrow_{\mathcal{K}}$$
$$continue(-,(\lambda y.y+y)5); suma(3,-); P' \succ cont(P') \rightarrow_{\mathcal{K}}$$
$$continue(-,(\lambda y.y+y)5); suma(3,-); P' \prec cont(P') \rightarrow_{\mathcal{K}}$$
$$continue(cont(P'),-); suma(3,-); P' \succ (\lambda y.y+y)5 \rightarrow_{\mathcal{K}}$$
$$app(-,5); continue(cont(P'),-); suma(3,-); P' \succ \lambda y.y+y \rightarrow_{\mathcal{K}}$$
$$app(-,5); continue(cont(P'),-); suma(3,-); \mathcal{P}\prime \prec \lambda y.y+y \rightarrow_{\mathcal{K}}$$
$$app(\lambda y.y+y,-); continue(cont(P'),-); suma(3,-); P' \succ 5 \rightarrow_{\mathcal{K}}$$
$$app(\lambda y.y+y,-); continue(cont(P'),-); suma(3,-); P' \prec 5 \rightarrow_{\mathcal{K}}$$
$$continue(cont(P'),-); suma(3,-); P' \succ \lambda y.y+y[y:=5] \equiv 5+5 \rightarrow_{\mathcal{K}}$$
$$suma(-,5); P'' \succ 5 \rightarrow_{\mathcal{K}}$$
$$suma(-,5); P'' \prec 5 \rightarrow_{\mathcal{K}}$$
$$suma(5,-); P'' \succ 5 \rightarrow_{\mathcal{K}}$$
$$suma(5,-); P'' \prec 5 \rightarrow_{\mathcal{K}}$$
$$P'' \succ 10 \rightarrow_{\mathcal{K}}$$
$$P'' \prec 10 \rightarrow_{\mathcal{K}}$$
$$suma(2,-); app(\lambda x.x+3,-) \prec 10 \rightarrow_{\mathcal{K}}$$
$$app(\lambda x.x+3,-) \succ 12 \rightarrow_{\mathcal{K}}$$
$$app(\lambda x.x+3,-) \prec 12 \rightarrow_{\mathcal{K}}$$
$$\Box \succ \lambda x.x+3[x:=12] \equiv 12+3 \rightarrow_{\mathcal{K}}$$
$$suma(-,3) \succ 12 \rightarrow_{\mathcal{K}}$$
$$suma(-,3) \prec 12 \rightarrow_{\mathcal{K}}$$
$$suma(12,-) \succ 3 \rightarrow_{\mathcal{K}}$$
$$suma(12,-) \prec 3 \rightarrow_{\mathcal{K}}$$
$$\Box \succ 15 \rightarrow_{\mathcal{K}}$$
$$\Box \prec 15$$

- $p_3 \rightleftharpoons$

```
2 + letcc k in 3 + (continue k 0)
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Renombraremos algunas expresiones, por el tamaño que tienen, de la siguiente forma:
$e' = letcc \quad k \quad in \quad 3 \quad + \quad (continue \quad k \quad 0)$
$e'' = cont(suma(2,-);\Box)$
$P' = suma(-, continue \quad e'' \quad 0); suma(2,-); \Box \succ 3)$
$P'' = suma(3,-); suma(2,-;\Box)$

$$suma(-,e'); \Box \succ 2 \rightarrow_{\mathcal{K}}$$
$$suma(-,e'); \Box \prec 2 \rightarrow_{\mathcal{K}}$$
$$suma(2,-); \Box \succ e' \rightarrow_{\mathcal{K}}$$
$$(3+(continue \quad k0))[k:=e''] \rightarrow_{\mathcal{K}}$$
$$suma(2,-); \Box \succ 3+(continue \quad e'' \quad 0) \rightarrow_{\mathcal{K}}$$
$$P' \succ 3 \rightarrow_{\mathcal{K}}$$
$$P' \prec 3 \rightarrow_{\mathcal{K}}$$

$$P'' \prec continue \quad e'' \quad 0 \rightarrow_{\mathcal{K}}$$
$$continue(-,0); P'' \succ e'' \rightarrow_{\mathcal{K}}$$
$$continue(-,0); P'' \prec e'' \rightarrow_{\mathcal{K}}$$
$$continue(e'',-); P'' \succ 0 \rightarrow_{\mathcal{K}}$$
$$continue(e'',-); P'' \prec 0 \rightarrow_{\mathcal{K}}$$
$$suma(2,-); \square \prec 0 \rightarrow_{\mathcal{K}}$$
$$\square \prec 2$$

- $p_4 \rightleftharpoons$

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e = if ¬true then 2 else (letcc k in 1 + (continue k 2))
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Renombraremos algunas expresiones, por el tamaño que tienen, de la siguiente forma:
$e' = 2$
$e'' = letcc \quad k \quad in \quad 1 + (continue \quad k \quad 2)$

$$\square \succ e \rightarrow_{\mathcal{K}}$$
$$if(-,e',e''; \square \succ \neg true) \rightarrow_{\mathcal{K}}$$
$$not(-); if(-,e',e''); \succ true \rightarrow_{\mathcal{K}}$$
$$not(-); if(-,e',e''); \prec true \rightarrow_{\mathcal{K}}$$
$$if(-,e',e''); \succ false \rightarrow_{\mathcal{K}}$$
$$if(-,e',e''); \prec false \rightarrow_{\mathcal{K}}$$
$$if(false,e',e''); \succ e'' \rightarrow_{\mathcal{K}}$$
$$suma(-, continue(cont(\square),2)); \square \succ 1 \rightarrow_{\mathcal{K}}$$
$$suma(-, continue(cont(\square),2)); \square \prec 1 \rightarrow_{\mathcal{K}}$$
$$suma(1,-); \square continue(cont(\square),2) \rightarrow_{\mathcal{K}}$$
$$continue(-,2); suma(1,-); \square \succ cont(\square) \rightarrow_{\mathcal{K}}$$
$$continue(-,2); suma(1,-); \square \prec cont(\square) \rightarrow_{\mathcal{K}}$$
$$continue(cont(\square),-); suma(1,-); \square \succ 2 \rightarrow_{\mathcal{K}}$$
$$continue(cont(\square),-); suma(1,-); \square \prec 2 \rightarrow_{\mathcal{K}}$$
$$\square \prec 2$$