Part 2: Thermal Waves Experiment

Thursday, 12 January 2023 09:26

Thursday 12th of January

The aim of today's session is to get started on the data analysis for part 2 as we anticipate this will be the most challenging section of this part.

Task 2.3a

As instructed the task, we started by analyzing the data with a 4 minute period. We were given two data sets corresponding to two different runs. We have plotted of these data sets on the same axes and obtained the plot that can be seen in the summary table below.

It can clearly be seen that the plot that data set A has a high transient, which will lead to high uncertainties in the values for D calculated and should therefore be discarded. As such, data set B will be used to the remainder of the task.

The raw data obtained from data set B was plotted and the x-axis was adjusted to display time in seconds. The theoretical square wave for the outer part of the cylinder was then plotted on top of this. Furthermore, the fundamental frequency as calculated in Part 1 was then also plotted, which allowed us to make the calculations for Task 2.3b.

Task 2.3b

We can calculate the diffusivity coefficient as indicated in Task 2.3b. We measured the outer diameter of the cylinder using Duratool caliper to be to be 20.51 ± 0.01mm with the uncertainty given by the measuring tool used. This gives us: r_{outer} = 10.26 ± 0.01mm. We were given r_{inner} to be 2.50 ± 0.05mm. The values for γ and $\Delta\phi$ were estimated using Figure 2.2. Therefore:

$$\Delta r = r_{OUTER} - r_{INNER} = (10.26 \pm 0.01) - (2.50 \pm 0.05) = 7.76 \pm 0.06 \text{ mm}$$

$$\gamma = \frac{A_{INNER}}{A_{OUTER}} \simeq \frac{10}{65}$$

$$\Delta\phi = \text{PHASE}_{INNER} - \text{PHASE}_{OUTER} \sim \frac{1}{2}\text{PERIOD} \sim \pi \text{ rad}$$

$$W_n = \frac{2\pi n}{T} = \frac{2\pi(1)}{4} = \frac{\pi}{2}$$

As such, using the equations for diffusivity given in the lab script, we can calculate two different values for the thermal diffusivity constant as follows:

$$\text{USING TRANSMISSION FACTOR}: D_{TF} = \frac{W_n \Delta r^2}{2(\ln(\gamma_n))^2} \simeq 1.350 \pm 0.021 \times 10^{-5} \text{ m}^2\text{s}^{-1}$$

$$\text{USING PHASE LAG}: D_{PL} = \frac{W_n \Delta r^2}{2(\Delta\phi)^2} \simeq 4.990 \pm 0.070 \times 10^{-6} \text{ m}^2\text{s}^{-1}$$

In order to check the validity of these values we compared them with the literature value which we found to be 0.124×10^{-6} ms^{-1} [1]. We can see that the diffusion constant from the transamination factor is off by 2 orders of magnitude while the constant calculated from the phase lag is off by 1 order of magnitude. This large deviation from the literature value as all as the large difference between the two diffusivity coefficients indicates significant uncertainty in the experiment. This is expected due to the numerous assumptions made, such as:

- The cylinder is approximate to have infinite length, while its real length is very much finite
- Only the fundamental frequency was considered

As such, the aim of the remainder of the experiment and the upcoming tasks is to progresivelly improve this method and obtain more accurate results.

Source [1]: https://www.engineersedge.com/heat_transfer/thermal_diffusivity_table_13953.htm

Monday 16th of January

The aim of today's session is to continue with part 2, specifically from Task 2.4.

Task 2.4

2.4a

Below are all of the plots from the data sets given.

Data = Raw Temperature Data (τ=1)	Data fm ~敽 = 2 Raw Temperature Data (τ=2)	Raw Temperature Data (τ=4)
Data fm = 6 Raw Temperature Data (τ=6)	асп опт мог 07 Raw Temperature Data (τ=8)	04t ｢ ｜ = 16 mm Raw Temperature Data (τ=16)

2.4b

A square shape would require instant heating and cooling. You cannot heat the cylinder from 0°C to 100°C instantly, it takes time for the temperature to change, which is shown by the gradual raising and lowering in temperature, giving the sinusoidal shape as opposed to a square shape.

2.4c

Discarded Data (affected by strong temperature transients):

- τ=1 min -- data set b
- τ=2 min -- data set b
- τ=4 min -- data set b

2.4d

The short period (τ= 1min, 2 min, 4min) plots look the most sinusoidal. The graphs seems to get less and less sinusoidal and essentially get closer to what looks like a triangle function.

Task 2.5

2.5a + b

The value for r_{inner} and r_{outer} are the same as in Task 2.3. This time we decided to use a bigger uncertainty for the measured outer radius, to correspond to the uncertainty of the given value for the inner radius. This gives us r_{outer} = 10.26 ± 0.05mm and hence Δr = 7.71 ± 0.1mmThe values for γ and $\Delta\phi$ were estimated using the figure in the summary table below. The calculation were made as shown below and the final results can be seen in the summary table:

$\tau = 1$ min

$\Delta r = 7.76 \pm 0.10$ mm

$\gamma = \frac{A_{INNER}}{A_{OUTER}} \approx \frac{(56-53.3)/2}{200/\pi} \approx 2.12 \times 10^{-2}$

$\Delta\phi = PHASE_{INNER} - PHASE_{OUTER} = \frac{75.3-65 \pm 3}{60} 2\pi = 1.08 \pm 0.31$ rad

$w_1 = 2\pi$

$\tau = 2$ min

$\Delta r = 7.76 \pm 0.10$ mm

$\gamma = \frac{(55.5-50)/2}{200/\pi} \approx 4.32 \times 10^{-2}$

$\Delta\phi = \frac{150.31-115 \pm 9}{120} 2\pi = 1.85 \pm 0.47$ rad

$w_1 = \pi$

$\tau = 8$ MIN

$\Delta r = 7.76 \pm 0.10$ mm

$\gamma = \frac{(83.5-22.5)/2}{...} \approx 0.479$

$\tau = 6$ min

$\Delta r = 7.76 \pm 0.10$ mm

$\gamma = \frac{(70.5-32.5)/2}{200/\pi} \approx 0.298$

$\Delta\phi = \frac{231 \pm 6 - 90}{360} 2\pi = 2.46 \pm 0.16$ rad

$\theta = 200/\pi$

$\Delta \emptyset = \frac{277 \pm 7 - 120}{480} \pi = 2.06 \pm 0.09$ rad

$W_1 = \pi/3$

$W_1 = \frac{\pi}{4}$

2.5d

When comparing the theoretical square wave with the measured wave, a clear difference can be seen between the amplitude of the theoretical graph and the measured graph.

Summary Table

Below is a summary table of the graphs and respective values for different time periods:

Time Period (τ)	Plot	Phase Lag (rad)	Amplitude Transmission Factor (unitless)	D_{TF} (m^2s^{-1})	D_{PL} (m^2s^{-1})
1min		1.08±0.31	$2.12*10^{-2}$	$1.27±0.24*10^{-6}$	$7.0±0.9*10^{-6}$
2min		1.85±0.47	$4.32*10^{-2}$	$1.92±0.25*10^{-5}$	$9.6±2.1*10^{-6}$
6min	해 = 6 min	2.46±0.16	$2.98*10^{-1}$	$1.29±0.007*10^{-4}$	$3.1±0.4*10^{-5}$
8min	acn	2.06±0.09	$4.79*10^{-1}$	$3.49±0.01*10^{-4}$	$4.5±0.4*10^{-5}$

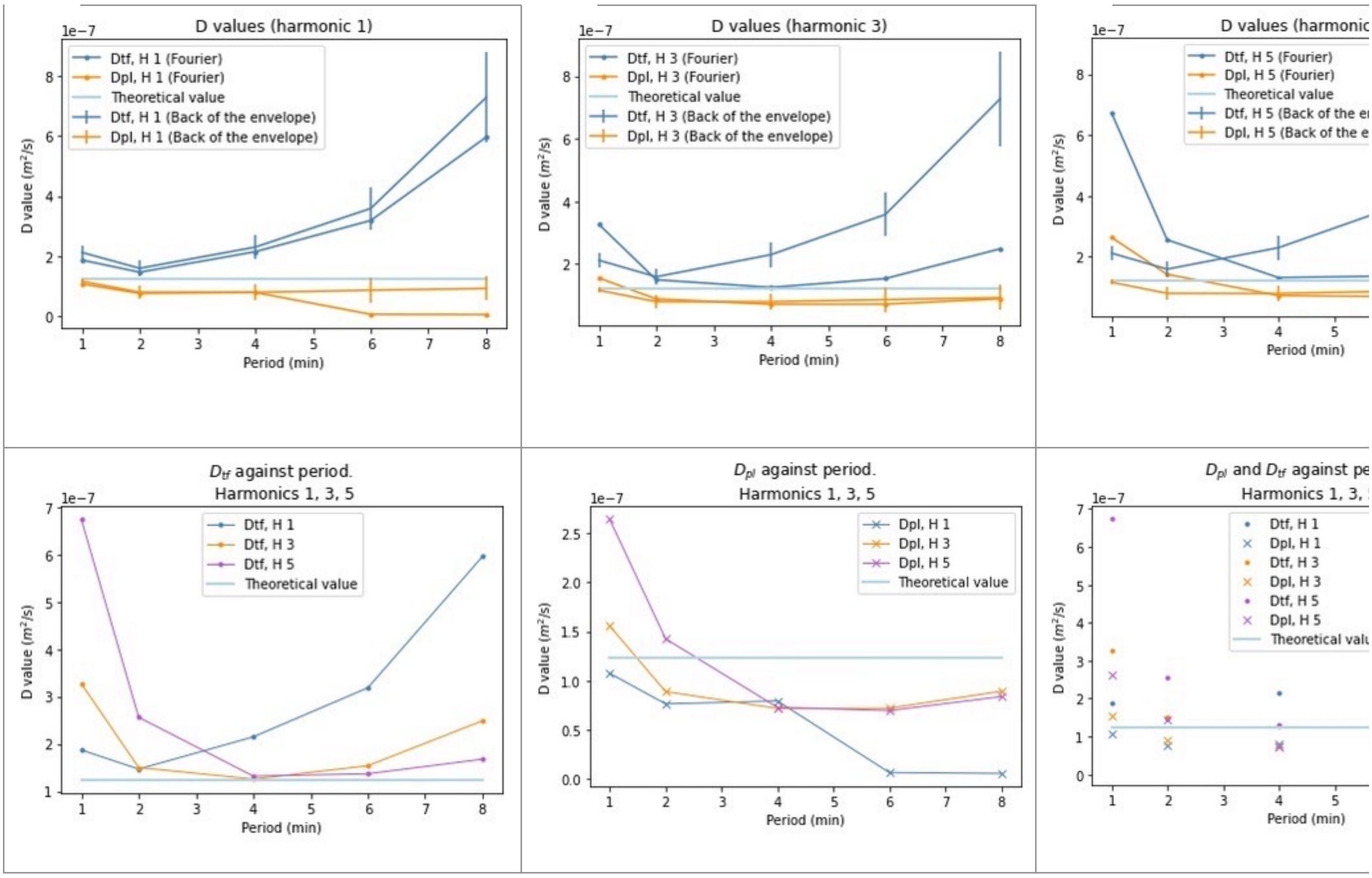
Table 2.5: Table showing all of the values of D calculated using the back of the envelope method

Thursday 19th of January

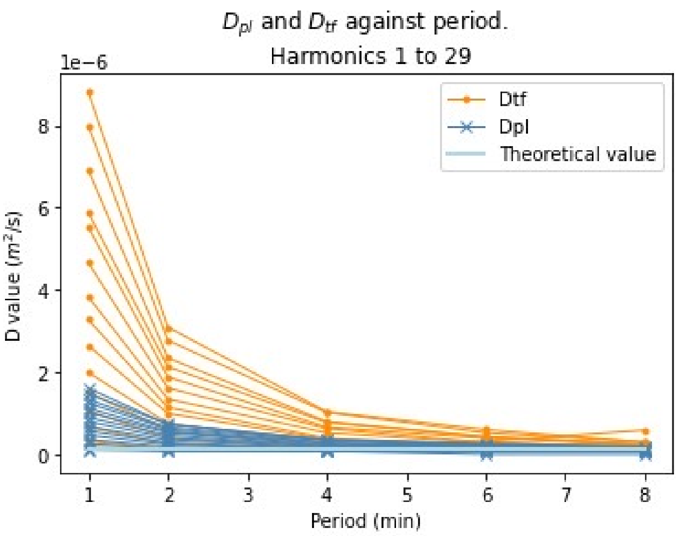
The aim of today's session is to continue working from Task 2, specifically starting with Task 2.6

Task 2.6 and Task 2.7

In order to obtain a better approximation for the diffusivity than the back-of the envelope estimation, we have used Fourier Analysis of the data sets, truncated at n=3. When doing the math, we have noticed that the even harmonics will disappear leaving us with only odd harmonics. The values of the Diffusivity can be seen summarized in the plots below:



As an extension, to see if we would notice a clearer trend, we have decided to extend out analysis to 29 harmonics and plot all of them together.



A clear trend can be seen here, where the D values are decreasing as period increases, and they hence get closer and closer to the theoretical value. We will later use the phase lag and transmission factor values obtained the Fourier analysis to obtain even more accurate values for diffusivity, using Bessel functions. We will declare these as out final diffusivity values.

Task 2.8

The aim of this task is to use a more physically correct and mathematically rigorous method to estimating the diffusivity of the material rather than the plane slab model. This was done through the mathematics of Bessel functions and is more accurate as it takes into account the actual shape of the PTFE used in the experiment when modelling the head condition. The Bessel calculator given to us finds D using the following equation:

$$D = \frac{r_{inner}^2 \omega_n}{\alpha_n^2},$$

where α_n is found through a root-findin algorithm that uses Bessel functions.

The value for the transmission factors and phase lags for harmonics 1,3 and 5 for all of the time periods, found through Fourier Analysis were used to calculate these values of D. The respective graphs and values of D for each time period can be found in the table below:

Time Period (τ)	Plot	D (Transmission Factor)	D (Phase Lag)
1min		Harmonic 1: 1.3427470890311249e-07 Harmonic 3: 2.5416007124912964e-07 Harmonic 5: 5.095793531690722e-07	Harmonic 1: 6.352214062538903e-07 Harmonic 3: 4.540769266655294e-07 Harmonic 5: 7.798688890640681e-07
2min		Harmonic 1: 6.713735445155624e-08 Harmonic 3: 1.2708003562456482e-07 Harmonic 5: 2.547896765845361e-07	Harmonic 1: 7.182465253981977e-07 Harmonic 3: 2.88724280427525e-07 Harmonic 5: 4.4770227837063507e-07
4min	- (4 min	Harmonic 1: 3.356867722577812e-08 Harmonic 3: 6.354001781228241e-08 Harmonic 5: 1.2739483829226806e-07	Harmonic 1: 9.867121955750399e-05 Harmonic 3: 3.7155226302349904e-07 Harmonic 5: 2.3379327064936503e-07
6min		Harmonic 1: 2.237911815051875e-08 Harmonic 3: 4.236001187485494e-08 Harmonic 5: 8.49298921948454e-08	Harmonic 1: 1.6966678459612934e-08 Harmonic 3: 6.233358678453182e-07 Harmonic 5: 3.1189343676143343e-07
8min		Harmonic 1: 1.678433861288906e-08 Harmonic 3: 3.1770008906141204e-08 Harmonic 5: 6.369741914613403e-08	Harmonic 1: 1.6315897541256236e-08 Harmonic 3: 1.896162883560435e-06 Harmonic 5: 6.568399787434341e-07

Table 2.8: Table showing all of the values of D calculated using the Bessel method

It is logical to compute an average of all of these values to obtain a final experimental value for D. As it can be seen in Table 2.8 above, all of the values are relatively close to each other aside from the value of D for the 1st Harmonic for τ=4min, which is 2-3 orders of magnitude off of the rest of the data. While we currently do not know the exact reason for this (will be discussed later with a demonstrator, should time permit), we have decided to exclude this data point from the average, as it would significantly skew the average and decrease the accuracy of the final result. The plot of the values of D calculated experimentally for all of the time periods, as well as the average and theoretical values can be seen in Figure 2.8 below:

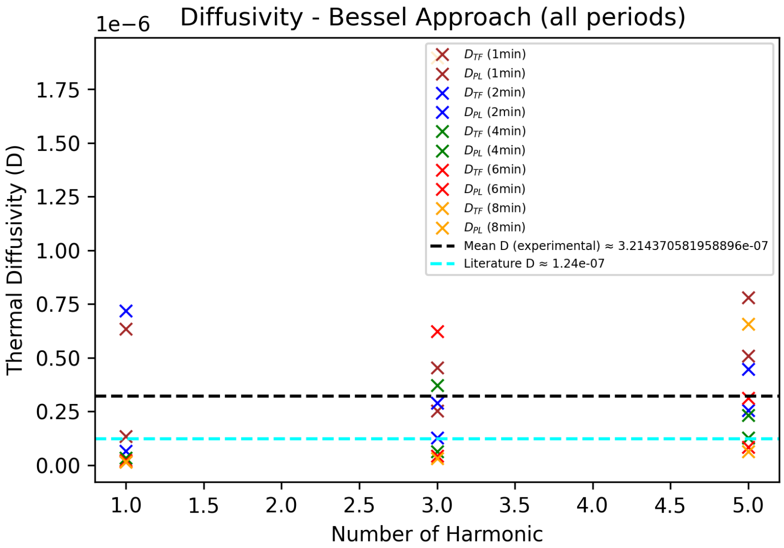


Figure 2.8: Thermal Diffusivity - Bessel Approach (all periods)

As can be seen in Figure 2.8 above, the mean value of D obtain experimentally is on the same order of magnitude as the literature value but it is off by 159%, which indicates that there are significant sources of uncertainty int he experiment of that our method for finding the thermal diffusivity is still quite inaccurate. On the other side, it can be seen in Figure 2.8 that some of the values of D are actually very close to the theoretical value (within 10-20% discrepancy) and hence provide a very good experimental value for thermal diffusivity.