Waves Session 2

Wednesday, November 2, 2022   11:58 AM

Welcome to Martin's Lab Book. The work in the following section was completed on: **Friday, February 17th, 2023** **9am – 12pm** in a synchronous manner becoming of a lab workbook.
Aims
• Finish Part II on Thermal Waves. • Plots all of the provided datasets for Part II and analyse them. • Complete a second *back-of-the-envelope* calculation for the two additional datasets, in order to obtain their respective values of D (with error). • Repeat this analysis for other the rest of the datasets. • Plot D as a function of period discussing our plots. Superimpose expected D. • Complete the Fourier Analysis with n = 3 to create a truncated dataset for τ = 4 min. • Perform the Bessel Analysis of the data. • Perhaps, start Task III on Electrical Waves.

2.5 Data Analysis

Task 2.3: First 'back of the envelope' estimate of *D* using the Fundamental mode approximation.

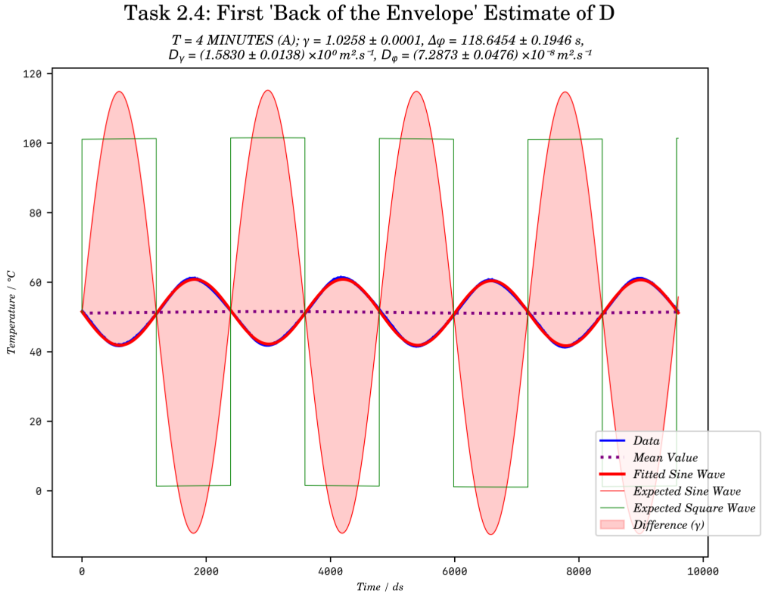
[17/02/2023 - 19/02/2023, Corrections: 26/02/2023]

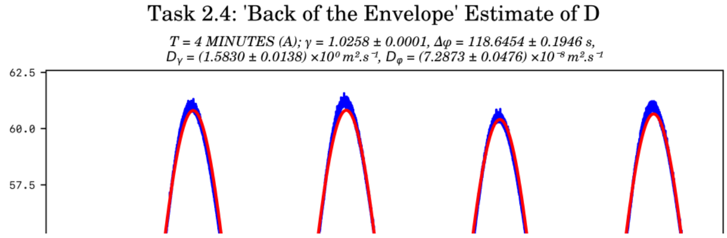
a. Focus specifically on the 4 min data. On the same plot, superimpose the expected ideal square wave at the outer wall of the PTFE cylinder. Can you see the amplitude attenuation and the phase difference between the two waveforms?

b. Assume that the fundamental frequency is the only harmonic present in the temperature measured inside the PTFE and calculate:
I. The Transmission Factor for the fundamental frequency
II. The Phase Lag for the fundamental frequency
III. Show clearly in your plot these two measured quantities
IV. Using Eqs. (2.3) and (2.4), estimate *D* (with error)
NOTE: To calculate $\Delta\varphi$ in Eq. (1.6) use the "2-argument arctangent" function (atan2, available in Excel and Python) as otherwise you might get unphysical phase lags.

c. Look up the expected value of *D* for PTFE (remember to make a note of your sources) and compare with your first estimate for *D*. Are they compatible?

Values: 4min_A





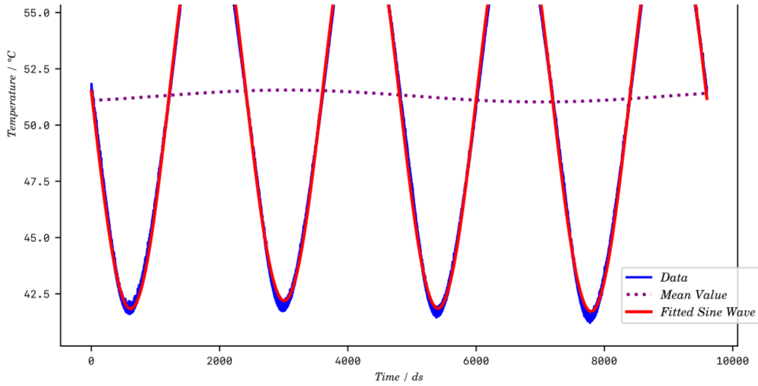


Figure 4(a): The dataset A for τ = *4 mins*. There is very little significant increases or decreases in the system's temperature range, as it stays relatively stable, which is quite interesting to note, as opposed to most of the other graphs, where there is a temperature transient.

$$\Delta r = 20.57 - 2.50 = (18.07 \pm 0.06)\, mm$$

$$\gamma = \frac{A_{inner}}{A_{outer}} = \frac{9.361 \pm 0.004}{50.000 \pm 0.000} = 0.1872 \pm 0.0001$$

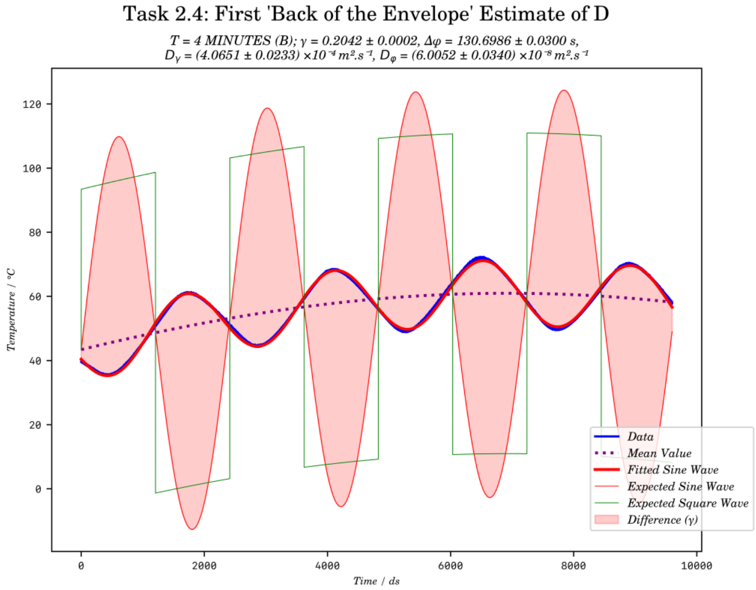
$$\Delta\varphi = Phase_{inner} - Phase_{outer} = (1213.546 \pm 1.946) - (1200.000 \pm 0.000) = (13.55 \pm 0.19)\, ds$$

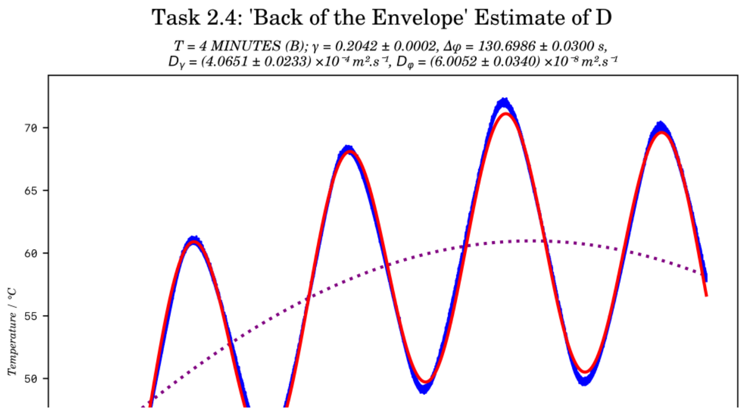
; $\omega 1 = \frac{\pi}{2}$

$$D_\gamma = \frac{\omega \Delta r^2}{2ln(\gamma)^2} = \frac{\left(\frac{\pi}{2}\right)\left(18.07 \times 10^{-3}\right)^2}{2ln(0.1872)^2} = (9.136 \pm 0.052) \times 10^{-5} m^2 \cdot s^{-1}$$

$$D_{\Delta\varphi} = \frac{\omega \Delta r^2}{2\Delta\varphi^2} = \frac{\left(\frac{\pi}{2}\right)\left(18.07 \times 10^{-3}\right)^2}{2(1.355)^2} = (1.398 \pm 0.040) \times 10^{-4} m^2 \cdot s^{-1}$$

Discarded Values: 4min_B





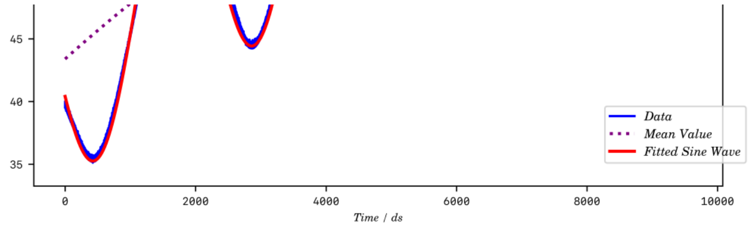


Figure 4(b): The dataset for τ = *4 mins*. As can be seen, as we increase the time elapsed, the system slowly edges upwards, increasing in temperature. The system gains thermal energy. **DISCARDED.** The data with period of 4 minutes, fitted by a sinusoidal, and square wave fitting. The amplitude attenuation occurs as time progresses, reducing ever-so-slightly in temperature range. As a result, our square wave, as well as our sinusoidal fit becomes increasingly worse with a larger error factor without a dampening envelope function (such as an exponential decay). Despite this, the phase difference is not substantial.

$$\Delta r = 20.57 - 2.50 = (18.07 \pm 0.06)\,mm$$

$$\gamma = \frac{A_{inner}}{A_{outer}} = \frac{10.211 \pm 0.008}{50.000 \pm 0.000} = 0.2042 \pm 0.0002$$

$$\Delta\varphi = Phase_{inner} - Phase_{outer} = 1200.000 - 1093.012 = (106.99 \pm 0.30)\,ds \quad ; \quad \omega 1 = \frac{\pi}{2}$$

$$D_\gamma = \frac{\omega \Delta r^2}{2ln(\gamma)^2} = \frac{\left(\frac{\pi}{2}\right)\left(18.07 \times 10^{-3}\right)^2}{(2ln(0.2042))^2} = (1.0163 \pm 0.0058) \times 10^{-4} m^2 s^{-1}$$

$$D_{\Delta\varphi} = \frac{\omega \Delta r^2}{2\Delta\varphi^2} = \frac{\left(\frac{\pi}{2}\right)\left(18.07 \times 10^{-3}\right)^2}{2(10.6986)^2} = (2.2405 \pm 0.0178) \times 10^{-6} m^2 s^{-1}$$

These values are incompatible, but at higher temperatures, it might alter the value of PTFE. The actual value that we're supposed to get according to a demonstrator is

PTFE (Polytetrafluorethylene) at 25 °C	$0.124 mm^2 s^{-1} = \left(1.24 \times 10^{-7}\right) m^2 s^{-1}$

We wish to examine how valid our experimental data is in comparison to the literature value. There is a large difference between what our two thermal diffusivity constants were calculated to be, and this significant uncertainty in our experiment was expected on account of several assumptions which we made during the calculations: we only considered the fundamental frequency, whilst the cylinder is approximated to be infinitely long whilst it is very clearly finite in its length. Therefore, we wish to perform numerous improvements to this experiment.

Task 2.4: Physical meaning and key features of all datasets.

[17/02/2023 - 19/02/2023, Corrections: 26/02/2023]

a. Plot all datasets on separate axes. Do not forget to label the axes and the period represented on each figure.
b. Explain why none of the measured temperature waves are square waves.
c. Identify the datasets which are affected by strong temperature transients and discard them from the analysis.
d. For the remaining datasets, decide which ones look approximately sinusoidal. In each case, explain your reasoning. Can you identify any pattern for increasing or decreasing period?
e. In which cases could the fundamental mode approximation be sufficient? In which cases would we benefit the most from performing the Fourier analysis of the measured wave?

Task 2.5: More 'back of the envelope' analysis – comparing different values for *D*

[17/02/2023 - 19/02/2023, Corrections: 26/02/2023]

a. Now that you have done 4 min, you will look at the datasets for τ = 1 min and τ = 8 min.
   I. Repeat the 'back of the envelope' calculation for these two additional datasets and obtain their respective values of D (with error).
b. Time permitting, repeat the analysis for τ = 2 min and τ = 6 min.
c. Plot your values of D as a function of period and discuss your plot. Superimpose the expected value of D as a constant.
d. Looking at the measured wave + square wave plot for each investigated period, can you identify a pattern in the overall amplitude attenuation? Is this what you expected?

Recall: to obtain the Fourier series of your dataset you need to calculate the constant coefficients a_n and b_n. The numerical integration methods discussed in Task 1.3 might be useful in this process!

Task 2.4a

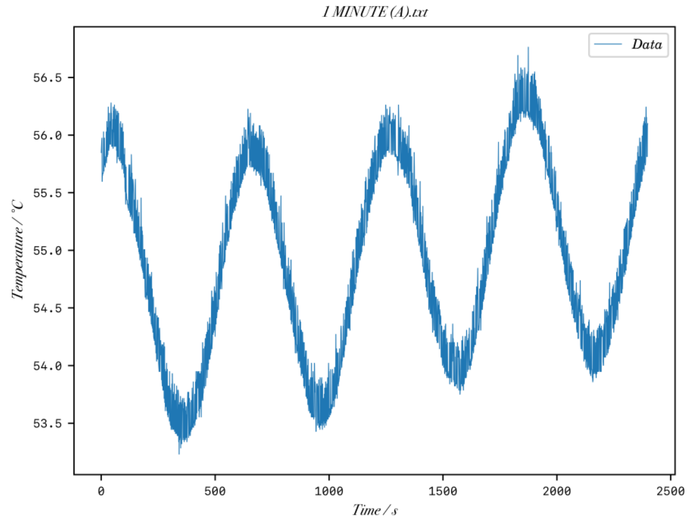
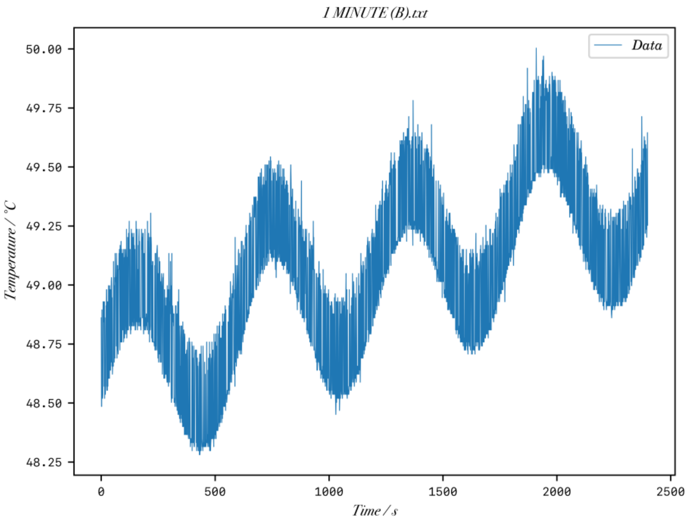


Figure 4(b): The dataset for τ = *4 mins*.



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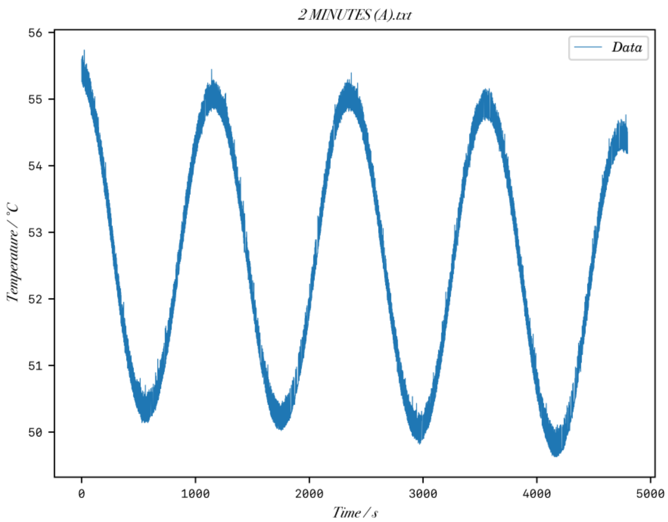

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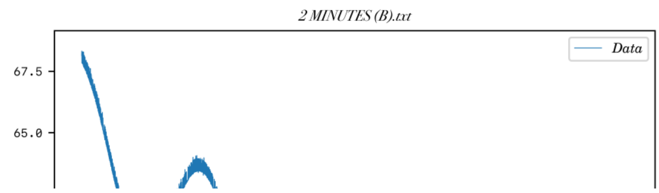

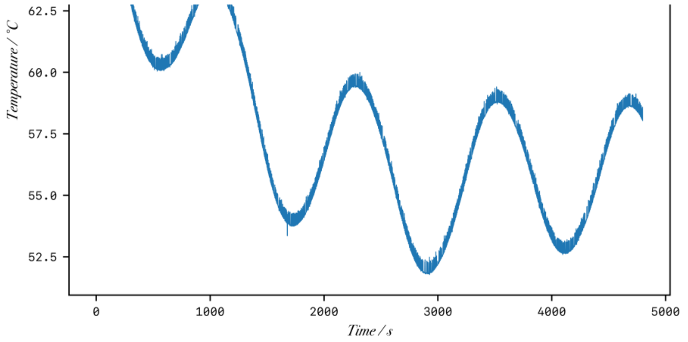

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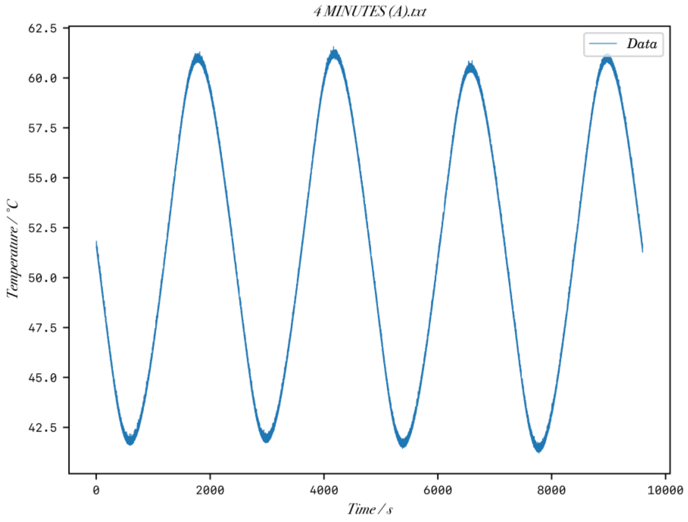

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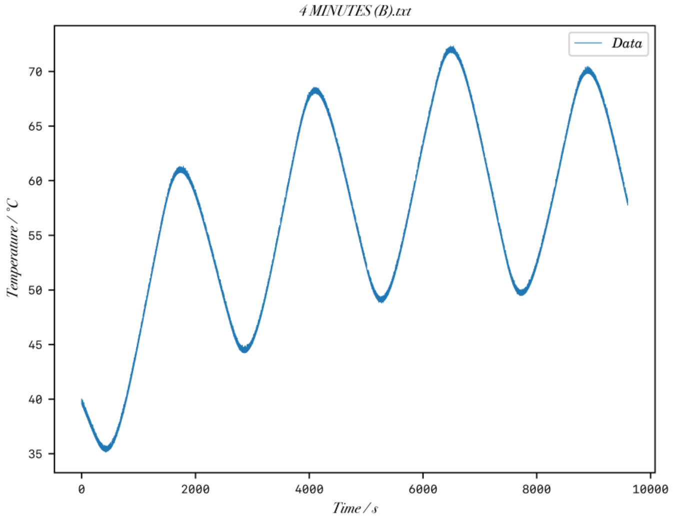

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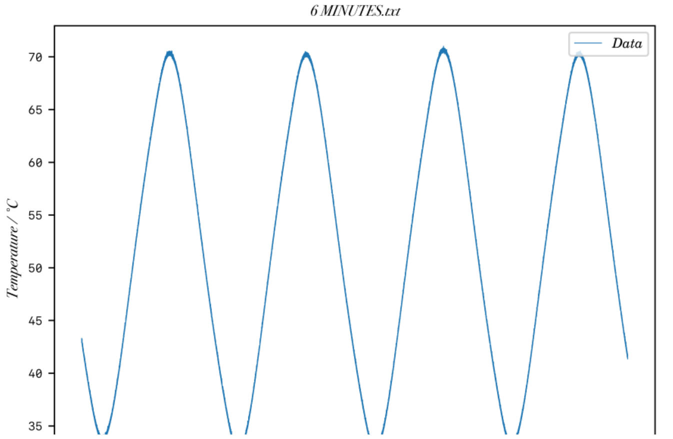

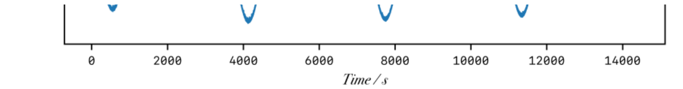

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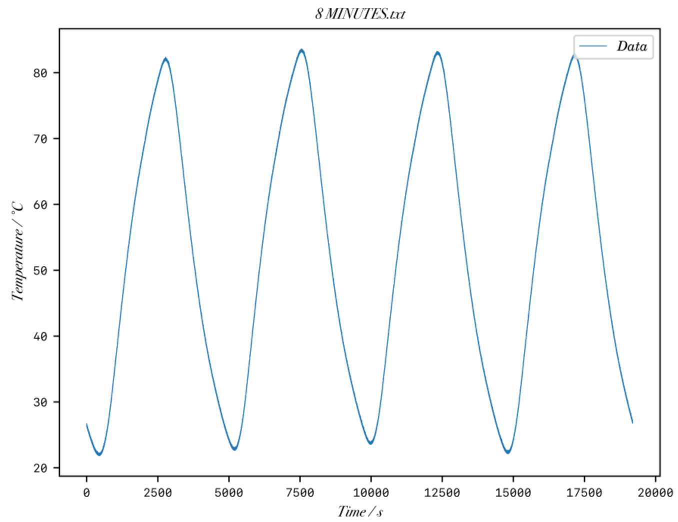

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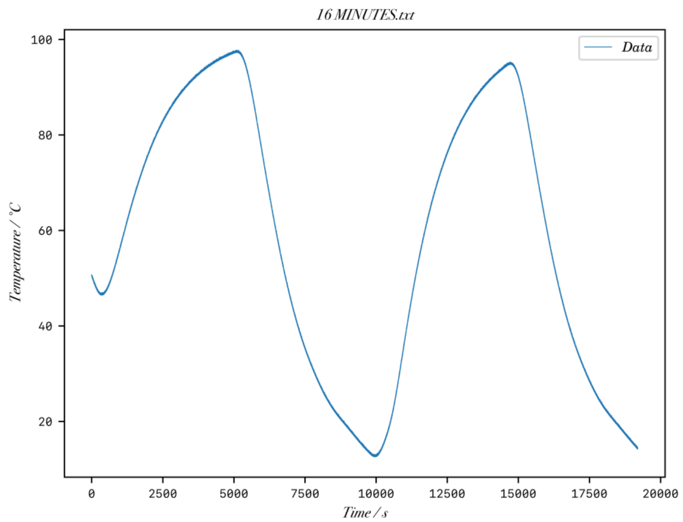

Figure 4(b): The dataset for τ = *4 mins*.

Task 2.4b
In order to obtain the measured square waves, we would have to have instant heating and cooling, which is impossible given that instantaneous heat, and therefore energy transfer (nothing can travel faster than the speed of light of course, and thus the vibration of atoms can never be instantaneous). Instead, we have that there is a time lag for which the heat is able to increase and decrease, which can be seen by the approximate sinusoidal shape which this gives rise to. Additionally, the accumulation of heat by the system can be seen by the ever-increasing position in the troughs of the waveforms, e.g. **Figure 5**.

Task 2.4c
To summarise, we have chosen to discard the Data Set B graphs of: τ = 1 min, 2 mins, 4 mins, due to their being strongly affected by temperature transients. Additionally, we find sinusoidal behaviour to exhibited most by the graphs with shorter time periods, such as τ = 1 min, 2 mins, 4 mins, 6 mins, with the best approximation probably being 4 mins, as opposed to any longer time periods, where this begins to approximate a *"shark fin"* shape, like a triangular function.

Task 2.4d
The fundamental mode approximation is most likely to be sufficient in the situations with minimal temperature transients, such as with τ = 4 minutes (A), 6 minutes, and possibly 8 minutes, depending on how well the crest of each wave needs to be approximated. Towards the extremities on both sides, i.e. the shorter time durations and longer ones, we find there to be issues concerning shifting equilibrium positions, and pointed triangular function-like *"shark-fin"* shapes, respectively. Evidently, higher order approximations should be used to capture key behaviour for these time periods, such as taking it to n = 3, 5, 7 or beyond *(as only odd harmonics count)*.

Note to self: *as wavelengths are being attenuated, the value of $\gamma < 1$.*
Note to reader: *The order of terms in $\Delta\varphi$ calculations has been swapped where appropriate to ensure that the value obtained is positive, though this doesn't matter as the value is subsequently squared.*

PTFE (Polytetrafluoroethylene) at 25 °C	$0.124 mm^2 s^{-1} = (1.24 \times 10^{-7}) m^2 s^{-1}$

If we make use of the same values of r_{inner} and r_{outer} which we have seen in Task 2.3, which are: r_{outer} = (20.57 ± 0.01) mm, r_{inner} = (2.50 ± 0.05) mm, then we have **Table 1**:

Task 2.5a, 2.5b

<table>
<tr><th>τ / mins</th><th>Calculations</th></tr>
<tr><td>1</td><td>

Values: 1min_A

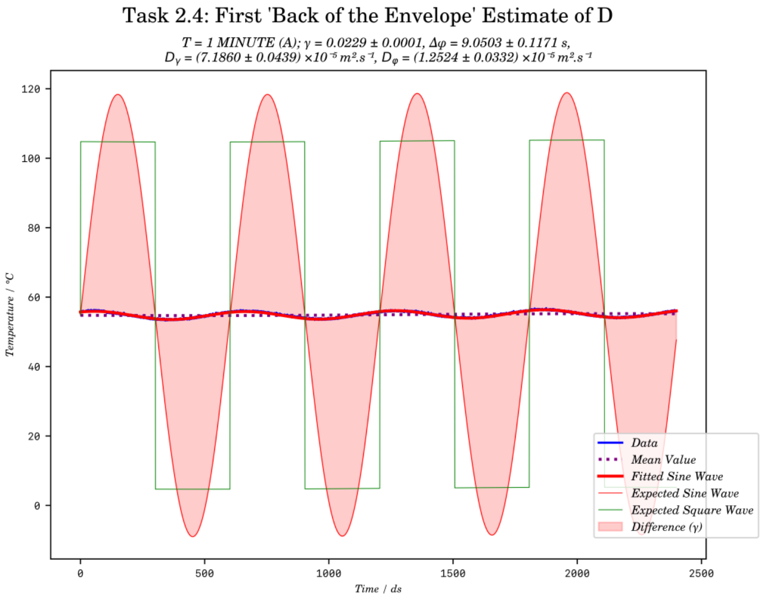

Figure 5: ▶ The dataset A for *τ = 1 min*. As can be seen, as we increase the time elapsed, the system slowly edges upwards, increasing in temperature. The system gains thermal energy.

Discarded Values: 1min_B

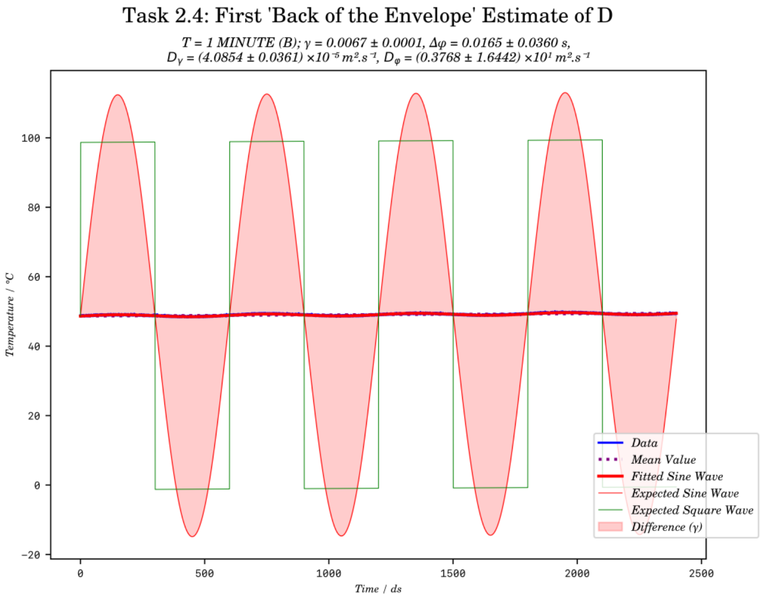

Figure 6: ▶ The dataset B for *τ = 1 min*. As can be seen, as we increase the time elapsed, the system slowly edges upwards, increasing in temperature. The system gains thermal energy. This increase is significant enough for us to class it as strong temperature transients. **DISCARDED**

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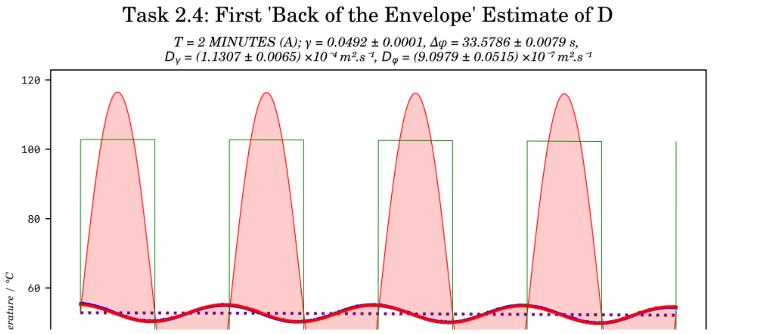
<tr><td>2</td><td>

Values: 2min_A



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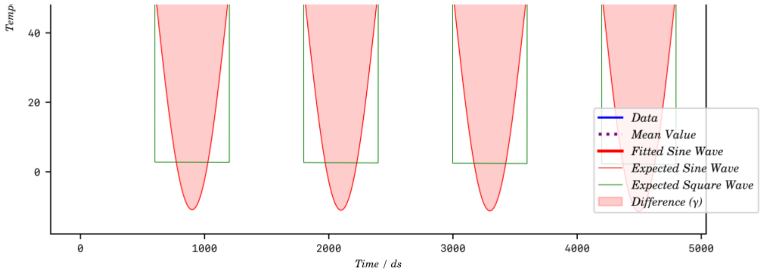


Figure 7: ▶ The dataset A for τ = 2 mins. As can be seen, as we increase the time elapsed, the system slowly edges upwards, increasing in temperature. The system gains thermal energy.

Discarded Values: 2min_B

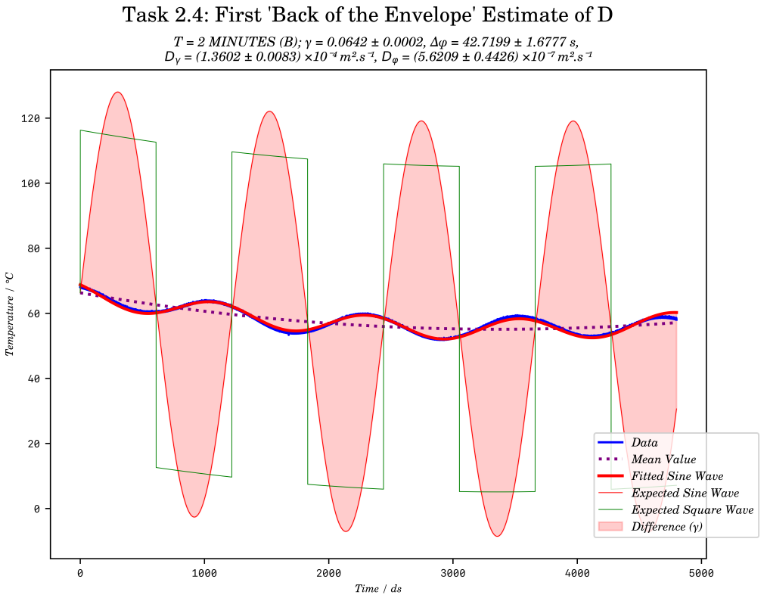


Figure 8: ▶ The dataset B for *τ = 2 mins*. As can be seen, as we decrease the time elapsed, the system slowly edges upwards, increasing in temperature. The system loses thermal energy from a strong initial amount of energy. This decrease is significant to count as affected by a strong temperature transient.
DISCARDED

4

Values: 4min_A

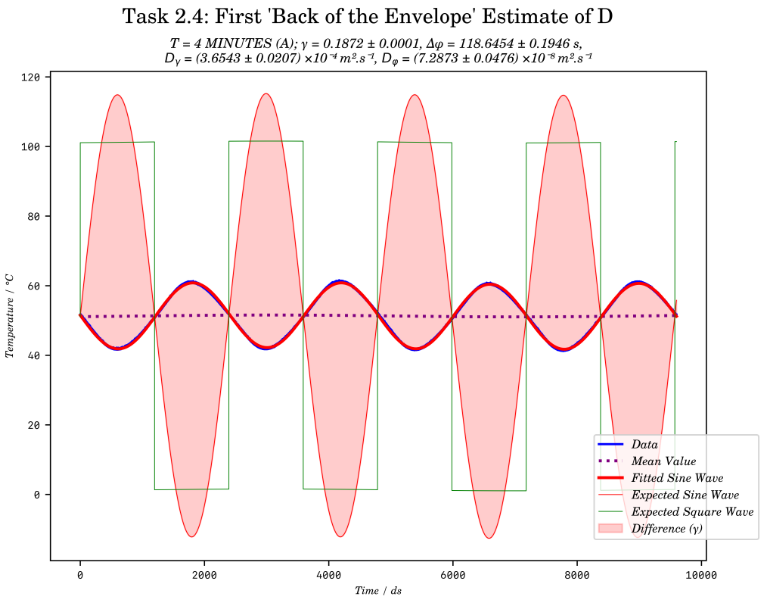
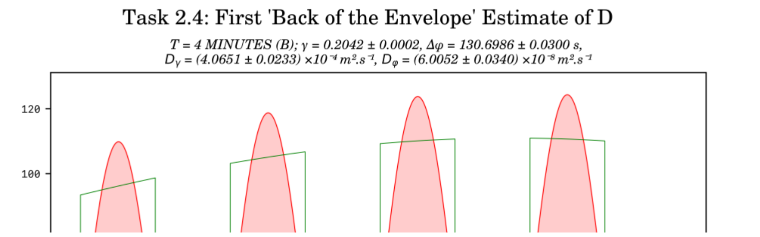


Figure 9: ▶ The dataset A for *τ = 4 mins*. There is no significant increase or decrease in the system's temperature range, as it stays relatively stable, which is quite interesting to note, as opposed to most of the other graphs.

Discarded Values: 4min_B



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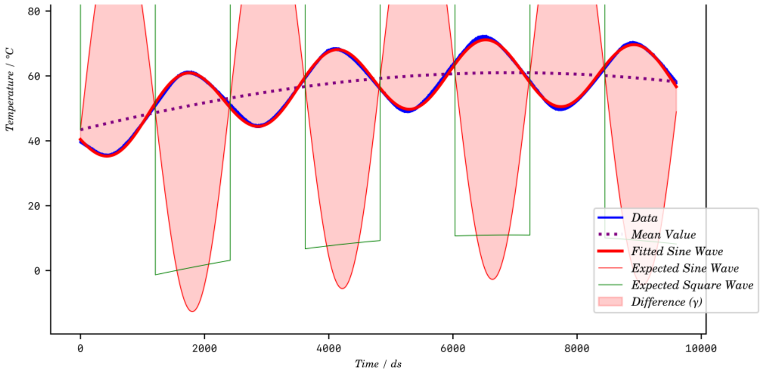

Figure 10: ▶ The dataset for τ = *4 mins*. As can be seen, as we increase the time elapsed, the system slowly edges upwards, increasing in temperature. The system gains thermal energy. **DISCARDED**

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<tr><td>6</td><td>

Values: 6min

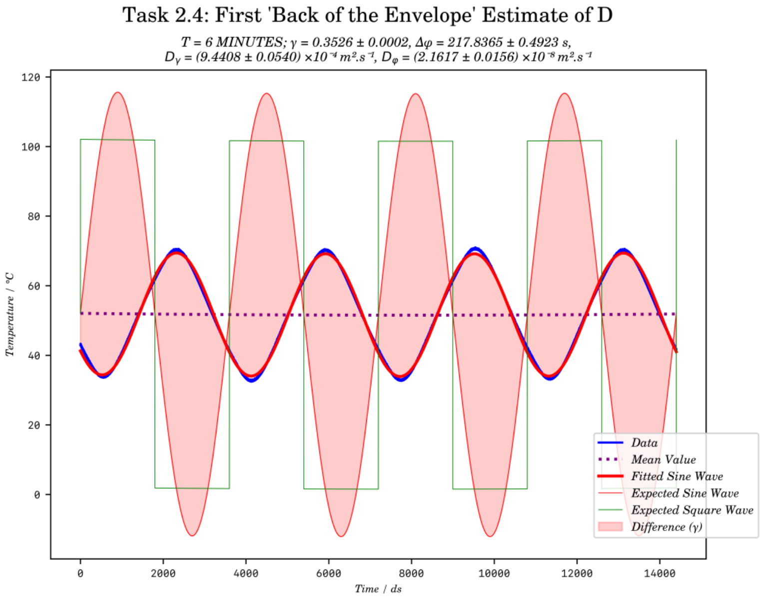

Figure 11: ▶ The dataset for τ = *6 mins*. This graph also stays stable interestingly enough, like.

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<tr><td>8</td><td>

Values: 8min

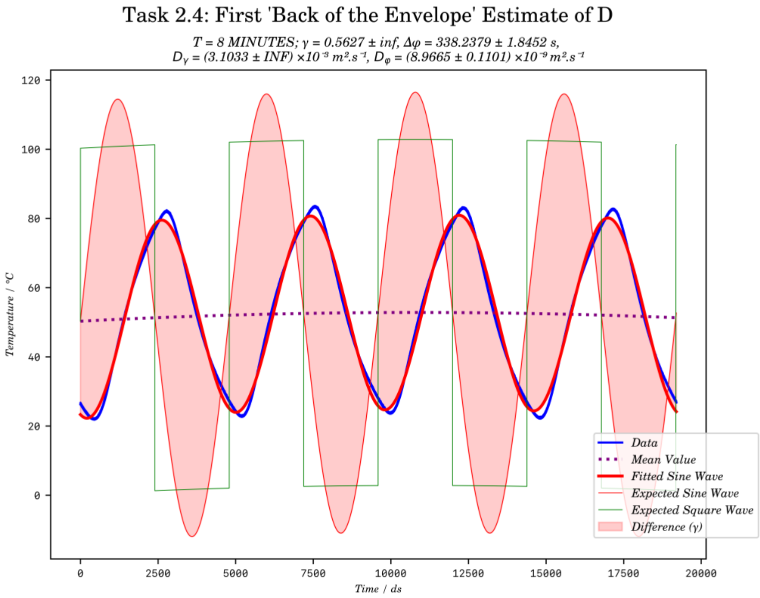

Figure 12: ▶ The dataset for τ = *8 mins*. As can be seen, as we increase the time elapsed, the system very slowly edges upwards, increasing in temperature, before decreasing slightly. Each wave cycle demonstrates the *"shark-fin"* shape.

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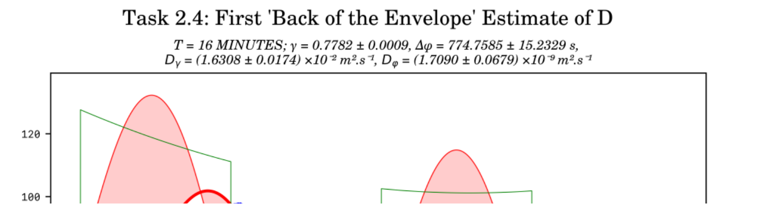
<tr><td>16</td><td>

Values: 16min



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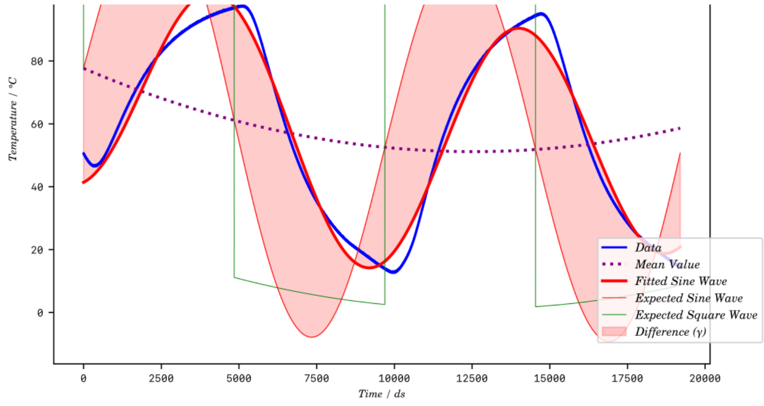

Figure 13: ▶ The dataset for τ = *16 mins*. This is the aforementioned *"shark-fin shape"* very clearly, for each wave cycle.

Example Output Data:

```
> Given: a(bx + c) + d(ex + f) + g, we have:

a :  38.90885963235642 ± 0.04413803226569785
b :  -0.0006480498473992863 ± 3.1599888754987045e-07
c :  -1.9411351274787774 ± 0.003221397068933095
d :  232054.37124694407 ± 1549205.5754588493
e :  -1.2047565378081985e-06 ± 4.021560178888501e-06
f :  -1.55568359924627 ± 0.0504473731264641
g :  232105.5362198636 ± 1549205.5689815972
γ: 0.778177+/-0.000883 unitless
φ₁: 1852.415000+/-152.329000 ds
Δφ: 294.758500+/-15.232900 s
Dᵧ: 0.016308257412+/-0.000173882968 m²·s⁻¹
Dᵩ: 0.000000011807+/-0.000000001222 m²·s⁻¹

>>> END
```

Table 1: Summary of values of Δr, γ, $\Delta\varphi$, ω_1, D_γ, $D_{\Delta\varphi}$.

Task 2.5c

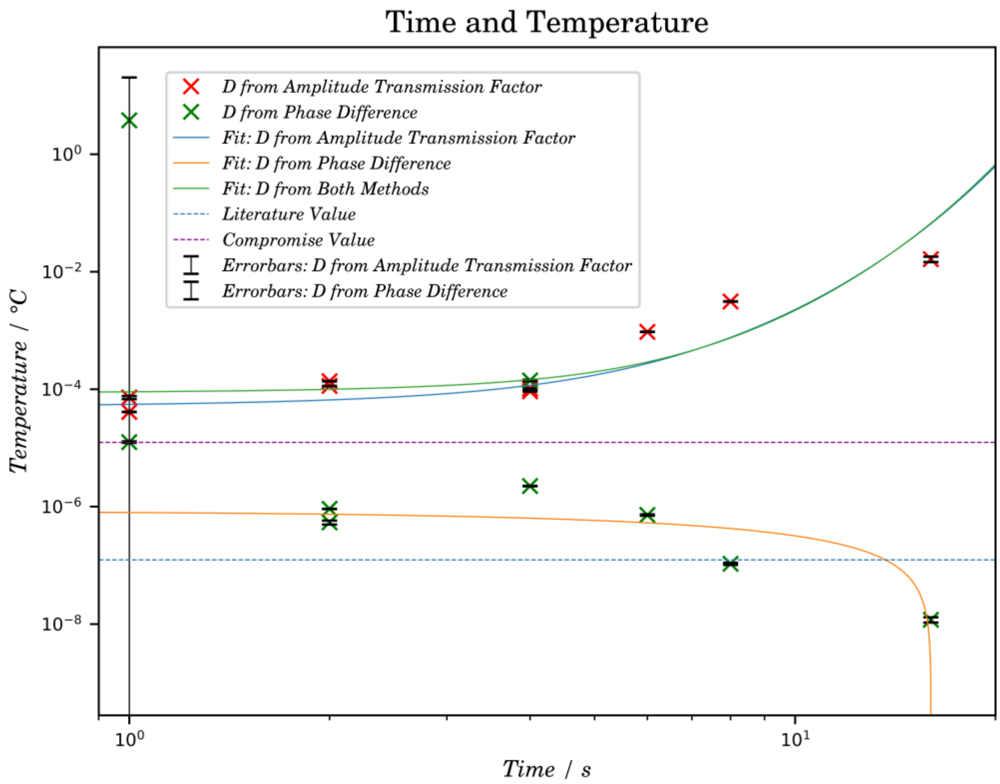

Figure 14: Summary of values of D from both methods, including their associated uncertainties, as well as the possible fits. As can be seen, the value we obtained appears to be a systematic shift from the literature value of $0.124\ mm^2.s^{-1}$.

Output Data
Fit: *D from Amplitude Transmission Factor*
a : 7.86107145564618e-06 ± 1.2721347589963877e-07
b : 0.5640730603980436 ± 0.0024407955947868733
c : 4.1103722553119214e-05 ± 4.45614198682398 66e-07

$$y \cong \left(7.8611 \times 10^{-6}\right) e^{0.5641x} + 4.1104 \times 10^{-5}$$

Fit: *D from Phase Difference*
a : -0.001971662732567049 ± inf
b : 2.6757698445654013e-05 ± inf
c : 0.0019725077371553893 ± inf

$$y \cong \left(1.9717 \times 10^{-3}\right) e^{2.6758 \times 10^{-5}x} + 1.9725 \times 10^{-3}$$

Fit: *D from Both Methods*
a : 6.6251303354888065e-06 ± 1.7103548896418795e-07
b : 0.5761939350613134 ± 0.0027949177819578718
c : 7.816703021961541e-05 ± 8.432397816398805e-07

$$y \cong \left(6.6251 \times 10^{-6}\right) e^{0.5762x} + 7.8167 \times 10^{-5}$$

Task 2.5d