Determining the Range of Electron Emission and Absorption in Aluminium and Copper

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Abstract – Various hypotheses were proposed to ascertain exponential fittings explaining the electron emissions from Sr-90 and Am-241 on varying thicknesses of Cu and Al metal sheets, fitted onto a digitally-connected radiation detector. From these models, we ascertained values for the initial activity rates using χ_2 minimisation tests, and the least-squares regression to determine the intersections, which were consistent with expected theoretical values. We determined the hypothesis *n = 3* to be the most valid at the 5% significance level.

I INTRODUCTION

The following experiment aims to determine the maximum electron energies for the emissions of various electrons in decay schemes.

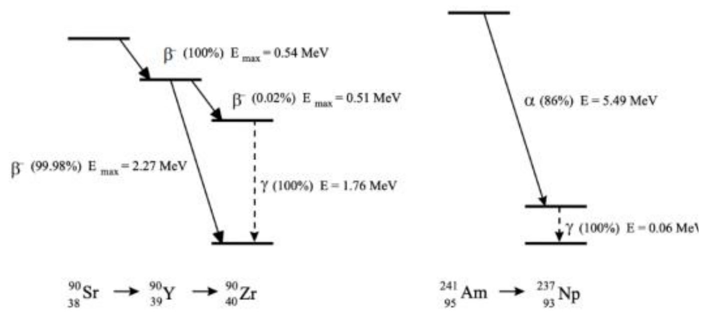

Figure 1: *Left* – the decay scheme for Sr-90. *Right* – the decay scheme for Am-241.

From Figure 1, we are able to ascertain the different possible beta particle emissions for the scenario presented to us, and hence we can determine the maximum electron energies for each decay.

II EXPERIMENTAL SET-UP AND DATA COLLECTION

The energy loss due to inelastic collisions is given below:

$$\overline{\frac{dE}{dx}} = \frac{4\pi e^4 NZ}{mv^2}\ln\left(\frac{1.16E}{I}\right) = 0.306\rho\frac{Z}{A}\beta^{-2}\ln\left(\frac{1.16E}{I}\right)[MeV/cm]$$

The equation for maximum energy is given as follows:

$$R = 0.11\left(\sqrt{1+22.4E_{max}{}^2}-1\right),\quad 0 < E_{max} < 3MeV$$

where R is given by ρd_{max} , along with the calculated constants, as well as the maximum electron energy, E_{max}, which we can rearrange for in the following equation:

$$E_{\max} = \left(\frac{\left(\frac{\rho d_{max}}{0.11}+1\right)^2-1}{22.4}\right)^{0.5}$$

From this, we can also determine the uncertainties in quadrature.

III RESULTS & ANALYSIS

The resultant distributions can be tested against the theoretical distributions using a Chi-squared analysis. We anticipate that both graphs of count rate against metal thicknesses should display an exponential decay relationship in accordance with Beer-Lambert's law, which plateaus towards the background rate. Therefore, we have the intercepts of the three linear gradient fits, as well as their associated uncertainties and the thickness of the material required to fully absorb the different β emissions, from which we can determine the values of R by multiplying by the density of Al and Cu. Then, we can calculate the maximum electron energy.

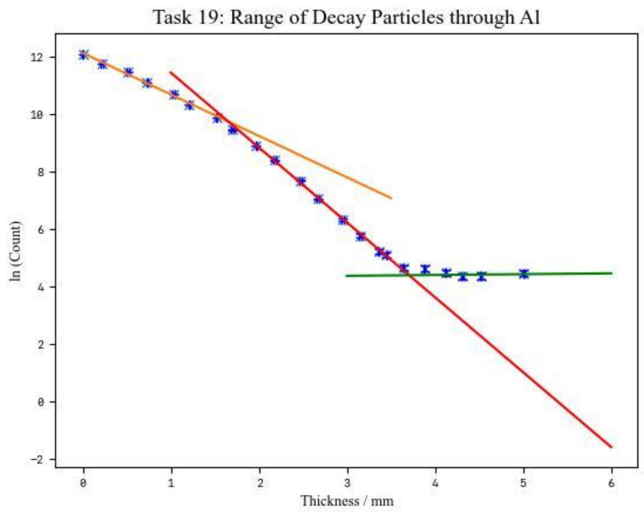

Figure 1: The logarithmic count for the decay particles of Al.

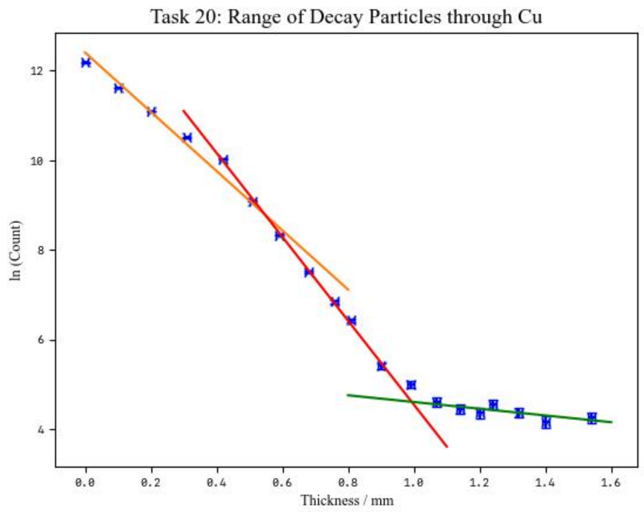

Figure 2: A logarithmic count of the decay particle of Cu.

Copper (Cu)
R_{min} = (8.96 ± 0.03) × (0.55 ± 0.09) mm = (0.490 ± 0.081) g.cm^{-2}
R_{max} = (8.96 ± 0.03) × (0.99 ± 0.08) mm = (0.890 ± 0.072) g.cm^{-2}
E_{min} = (1.13 ± 0.15) MeV, E_{max} = (1.91 ± 0.14) MeV

Aluminium (Al) R_{min} = (2.70 ± 0.03) × (1.65 ± 0.17) mm = (0.165 ± 0.017) g.cm^{-2} R_{max} = (2.70 ± 0.03) × (4.40 ± 0.66) mm = (0.371 ± 0.066) g.cm^{-2} E_{min}= (1.05 ± 0.15) MeV, E_{max} = (2.12 ± 0.35) MeV

Table 1: the resultant maximum electron energies for Cu and Al.

From our results in Table 1, as compared to the results of a theoretical energy distribution, we arrive at the following values, with percentage errors of the following:

Through the examination of alternative energy distributions, we can compare the fittings evaluated for the source through various metals. For instance, we can perform an inverse square distribution on the count rate, an exponential distribution, a linear fit, as well as an inverse square distribution with an exponential, like the Beer-Lambert's Law. This results in the following results for the minimised Chi-square calculations, including for the number of lines needed for a linear fit of the gradient.

Chi-Squared Minimisation Algorithm

A series of nested for-loops combined with a Chi-squared calculator from NumPy were utilised to determine the lowest p-value solution for determining the best fit.

Results of Chi-Squared Tests

We found the idealised value of the gradient polyfit to be for n=3 of the linear graph. For the different distributions, the minimise Chi-squared values are listed below in the following table:

Inverse Square Distribution	
Exponential Distribution	
Linear Distribution	
Inverse Square with Exponential Distribution (Beer-Lambert's Law)	

There appears to be the strongest distribution for the Beer-Lambert's Law as expected: the inverse square with exponential distribution. This is also true for the linear distribution for $n = 3$, in which we can identify the three decay profiles of the electrons.

IV CONCLUSION

There is a high degree of probability that the fitting of the Beer-Lambert's law provides an excellent model for the attenuation of beta particles in air, combined with the inverse square law decrease in the count rate, as shown by the Chi-Squared tests.

We can therefore conclude the maximum electron energy, which is in accordance with what we previously calculated for Copper and Aluminium below:

Thus, we have determined to a __% error ___________. Our predictions have been met, and the actual values lie within the uncertainty ranges provided in the *Results* Section. Therefore, we can conclude that we have an excellent model for the determination of count rate variation over distance.

V DISCUSSION

To improve the discourse of the experiment, multiple changes could be made to the experimental setup to optimise the collection of data, its comparison to the simulation on Geant4, as well as the interpretation of the results collected. This could be extended to being performed in a vacuum chamber and measuring the effects of the attenuation of air in this scenario.

The set-up could additionally be automated through the use of stepper motors and Robot testing on Windows in order to streamline data collection, and host multiple repeated trials to verify the data and eliminate random errors.

Similarly by applying different fits to the attenuation in air, we obtain that the basal activity rate to be approximately 5MBq, as expected. Both the results from Figure 1 and Figure 2 appear to be almost perfectly in tune with what we would expect theoretically, and different polynomial fittings could be applied to determine a wider breadth of possible intersections between the points.

Finally, we could compare this data for different metals of varying densities such as Iron and Steel, in order to further corroborate the findings we have made for this practical on maximum electron energies, and to verify Beer-Lambert's law.

VI REFERENCES

Appendix I: Uncertainty Propagation

Throughout this practical, uncertainties were calculated in quadrature through the use of the Python uncertainties package to eliminate the

Appendix II: Chi-Squared Testing

A Chi-squared minimisation algorithm was utilised in order to find the fitting with the most probable likelihood of being a good fit.

References

martin-he543, GitHub

All the data, as well as figures, uncertainty propagation and Python code can be found on the following GitHub repository, which was needed for the calculation of all the following findings.