The Error Function - Latex trial document for Practical Programming 2018

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Introduction

The error function is a particular important function that occurs in many areas of mathematics including probability theory and statistics. It is defined as in equation(1).[1]

$$u(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} \mathrm{d}t \tag{1}$$

and looks like figure 1.

The meaning of the error function in statistics is that for a random normally distributed variable Y which has mean 0 and variance $1/2$ (1) describes the probability of Y falling in the range $[-x, x]$. [1].
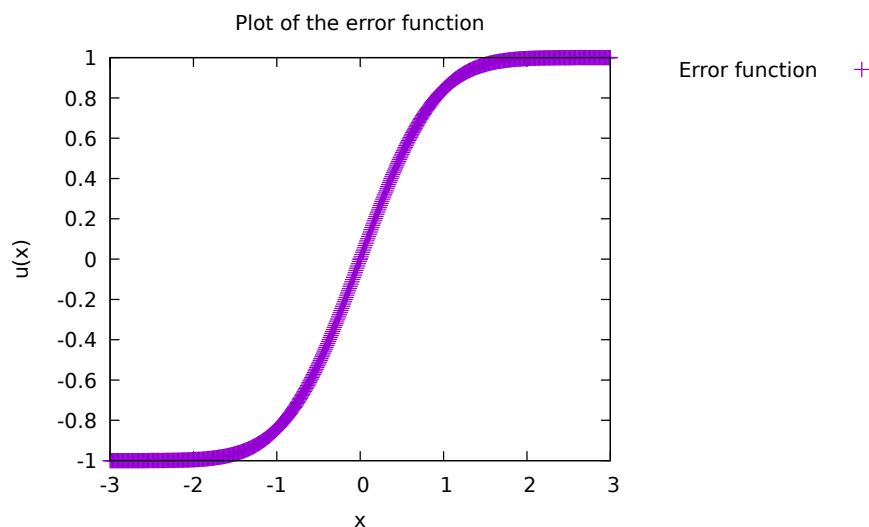

Figure 1: Plot of the Error function as a function of the argument x. Found using the odeiv2 environment of GSL.

Properties

It has some interesting properties such as being an odd function $u(-x) = -u(x)$ and that the complex conjugate of the function is a equal to the function of the complex conjugate of the argument $u(\overline{x}) = \overline{u(x)}$.[1]

Applications

When measurements are descibed by a normal distribution with standard deviation σ and expectation value 0 then $u\left(\frac{a}{\sigma\sqrt{2}}\right)$ is the probsbility that a mea-

surement will lie in the interval $[-a, a]$[1]. One can see that this is coherent with the definition above when $\sigma = 1/2$. An example of such a situation is when detecting scattered particles in Rutherford scattering. The energy of the scattered particles are normally distributed and hence the error function can be used (if σ is known) to determine the properbility to measure energy outside a certain interval around the mean.

Bibliography

[1] Wikipedia contributors, (02.27.2018) Error function, Retrieved 12.03.2018, from `https://en.wikipedia.org/wiki/Error_function`.