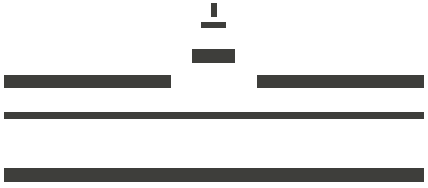


Westfälische
Wilhelms-Universität
Münster

Masterthesis

Efficient Implementation and Optimization of Geometric Multigrid Operations in the Lift Framework

living.knowledge
WWU Münster

Martin Lücke

22. Juni 2018

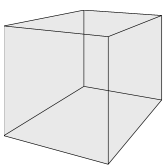
Geometric Multigrid

- Class of Iterative Solvers for Partial Differential Equations (PDE)

Geometric Multigrid

- Class of Iterative Solvers for Partial Differential Equations (PDE)

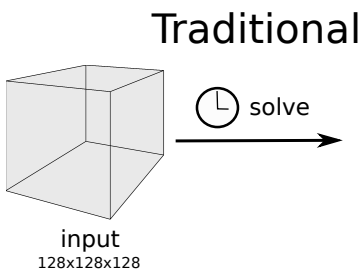
Traditional



input
128x128x128

Geometric Multigrid

- Class of Iterative Solvers for Partial Differential Equations (PDE)



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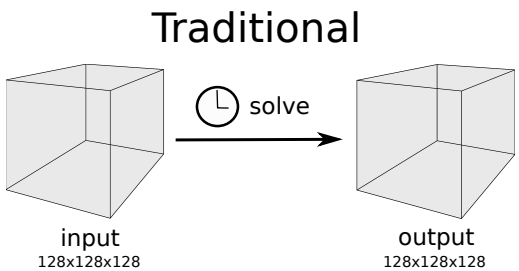
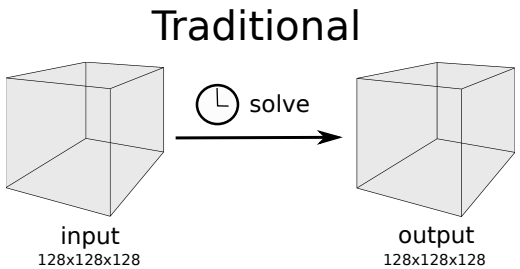


Geometric Multigrid

- Class of Iterative Solvers for Partial Differential Equations (PDE)
- Idea: Correction of the fine grid solution approximation by solving a coarser problem



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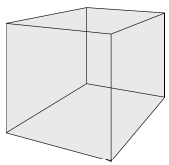

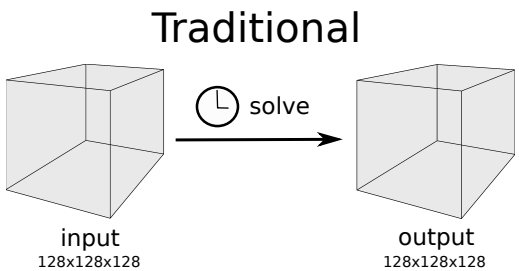

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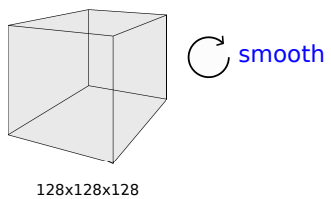

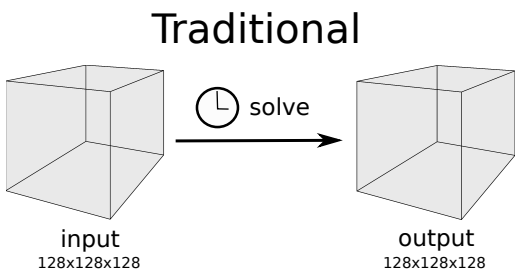

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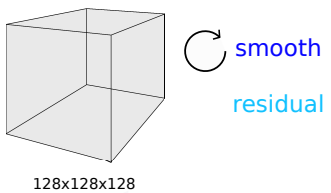

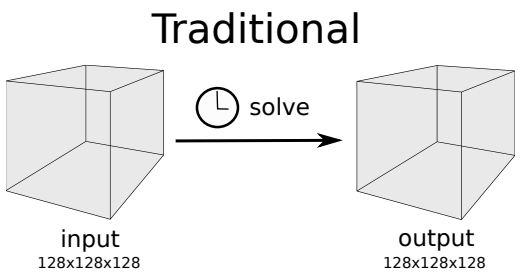

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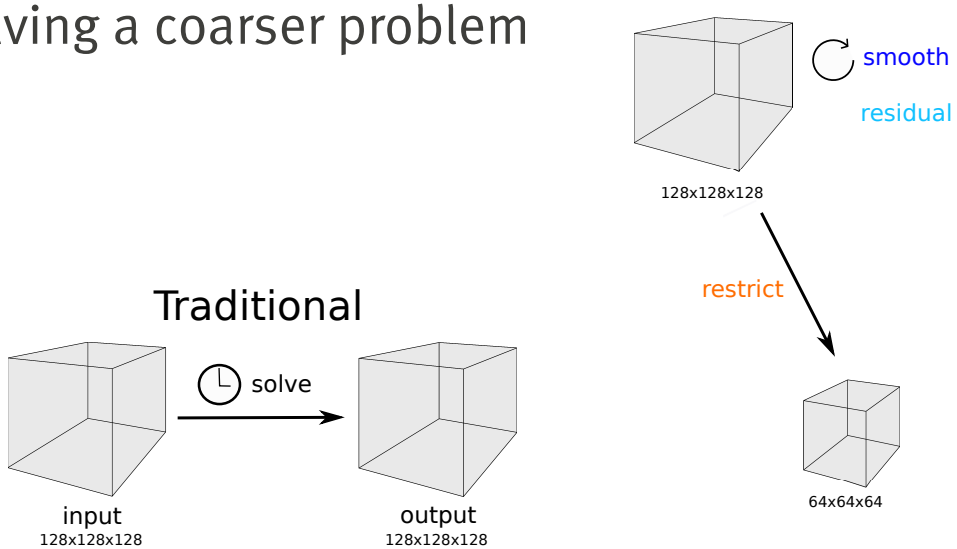

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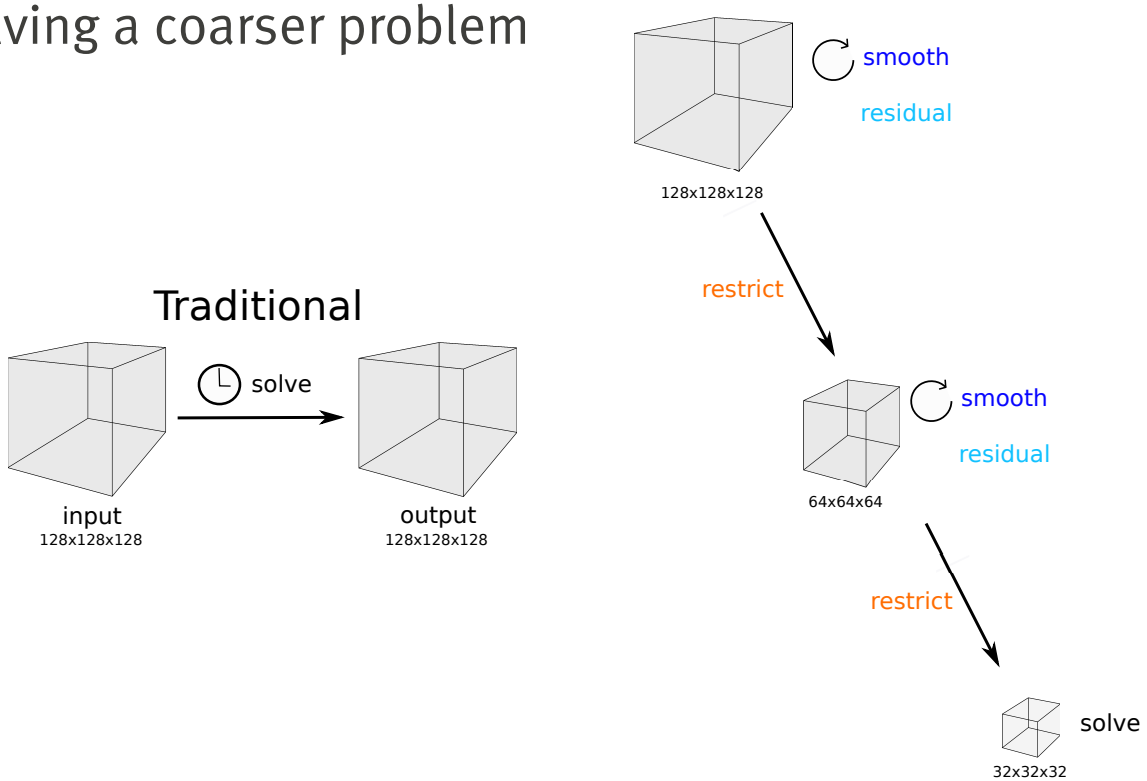

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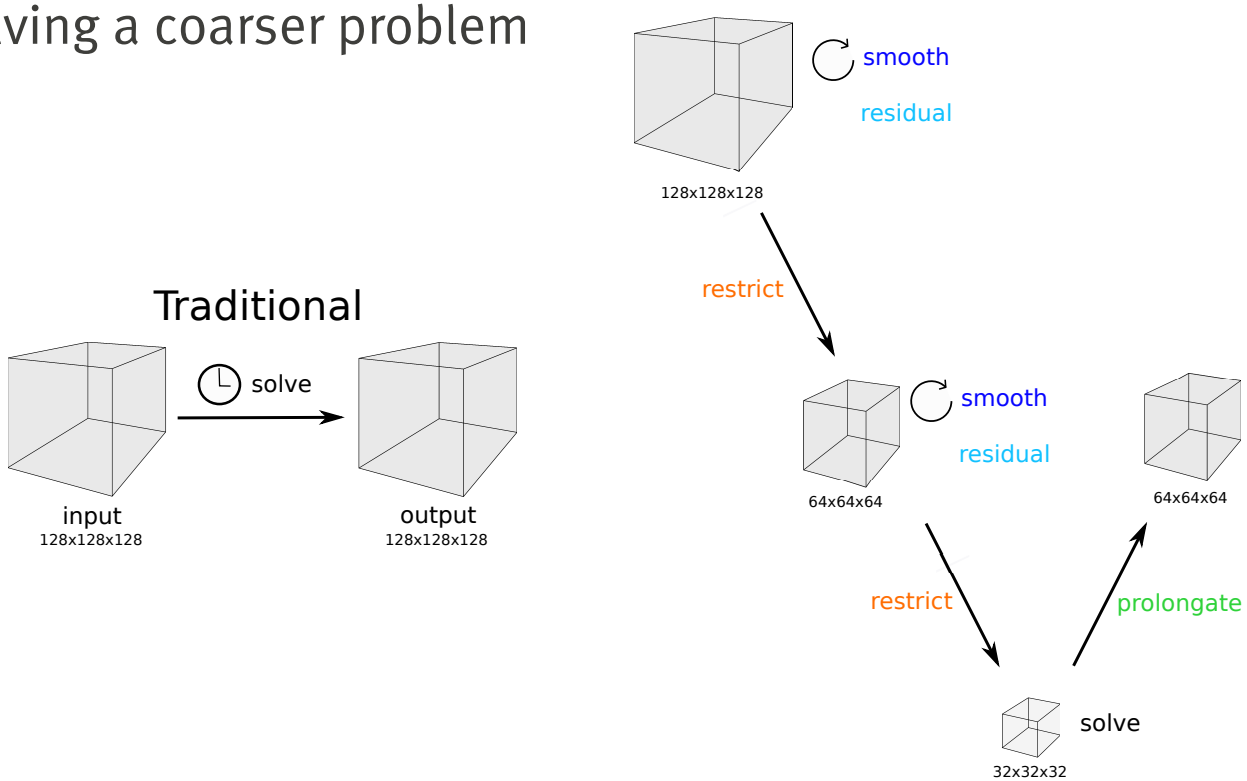

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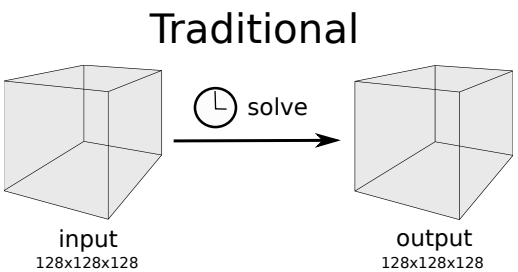
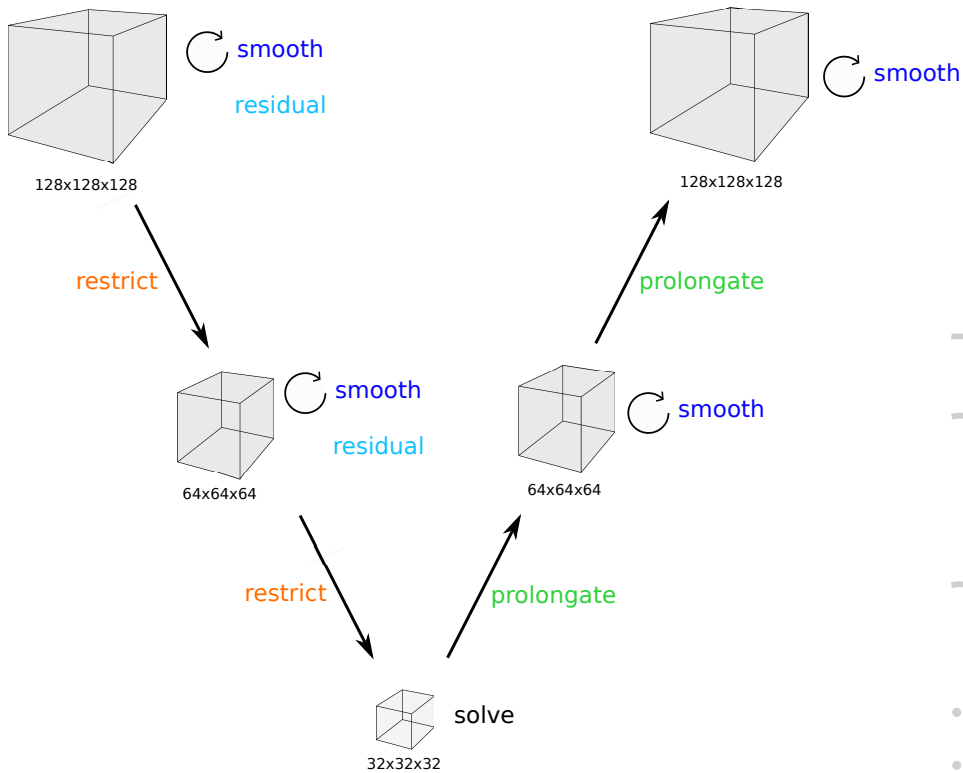
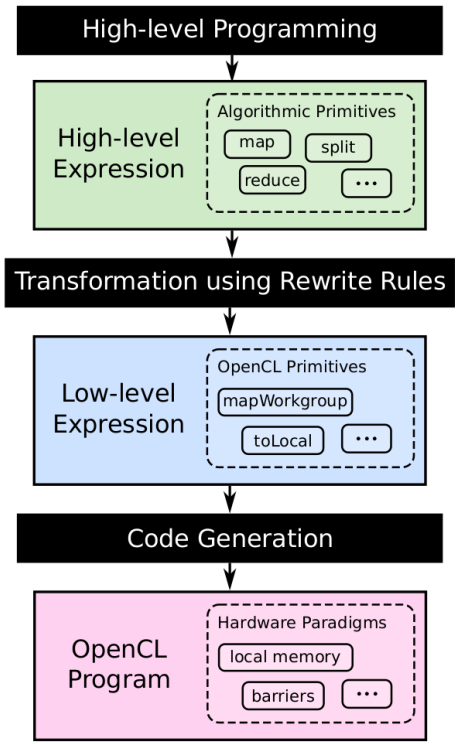




Lift

Lift

- Generating high-performance programs from functional programming
- Optimizations encoded in rewrite-rules



Lift

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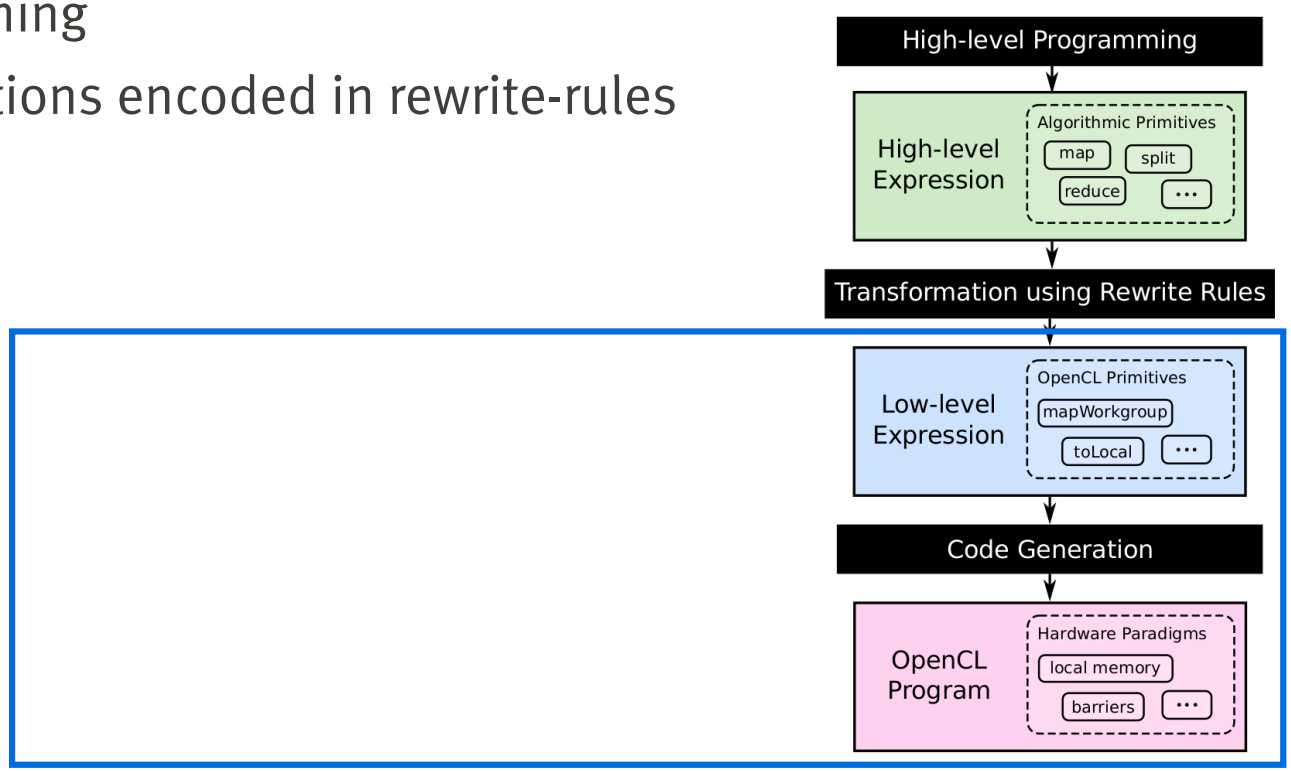
This Thesis:



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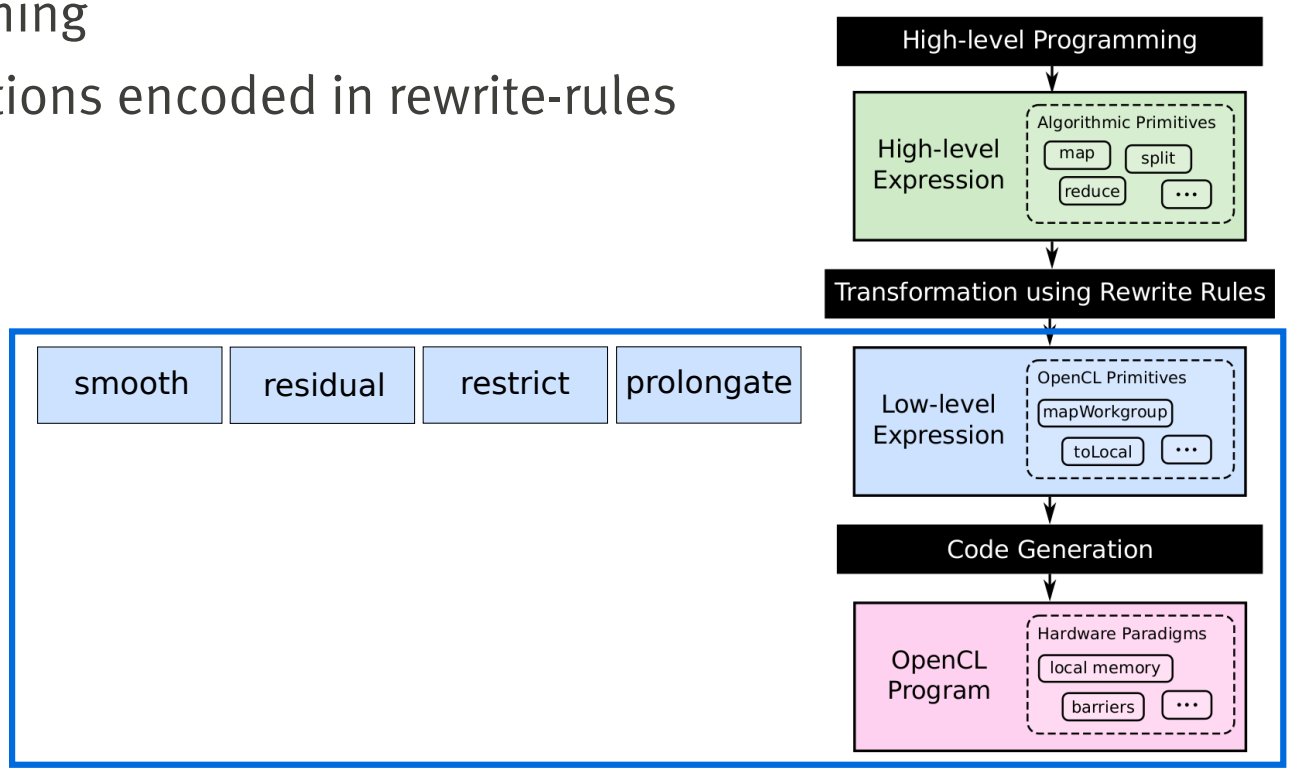

Lift

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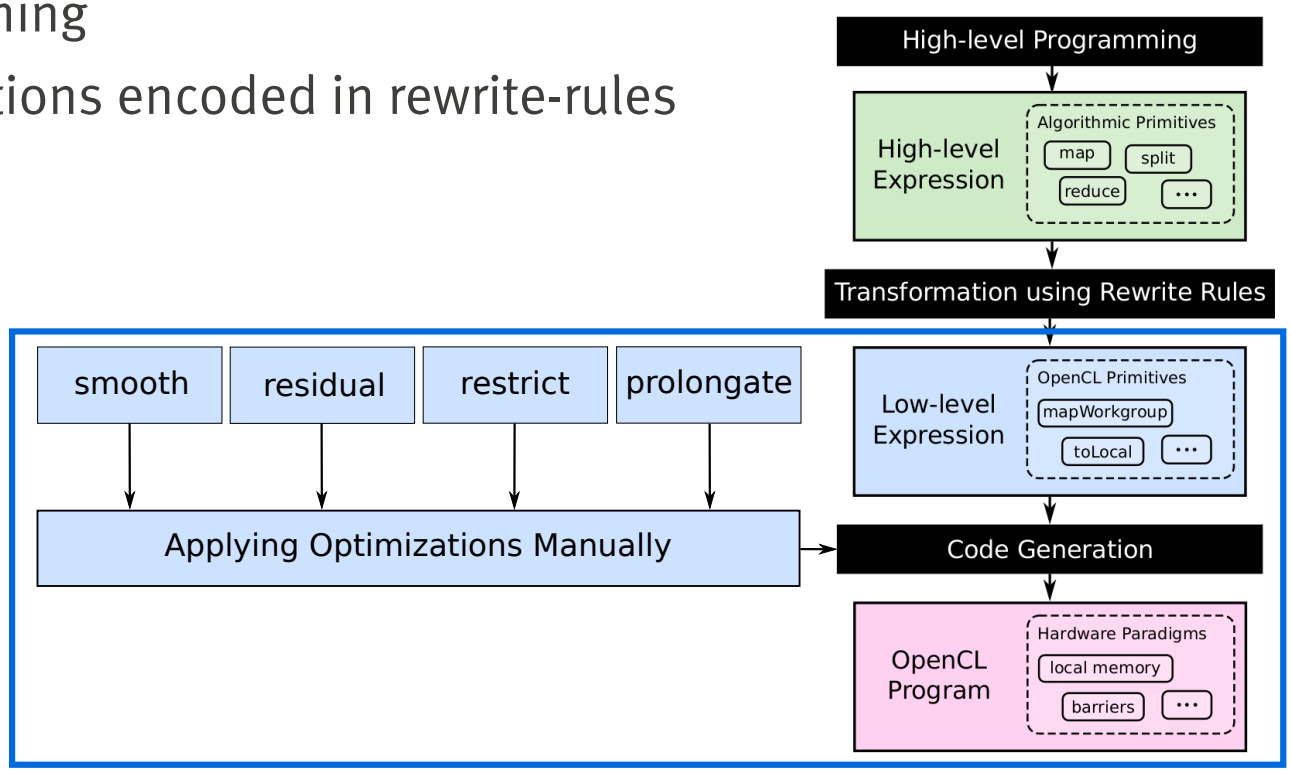

Lift

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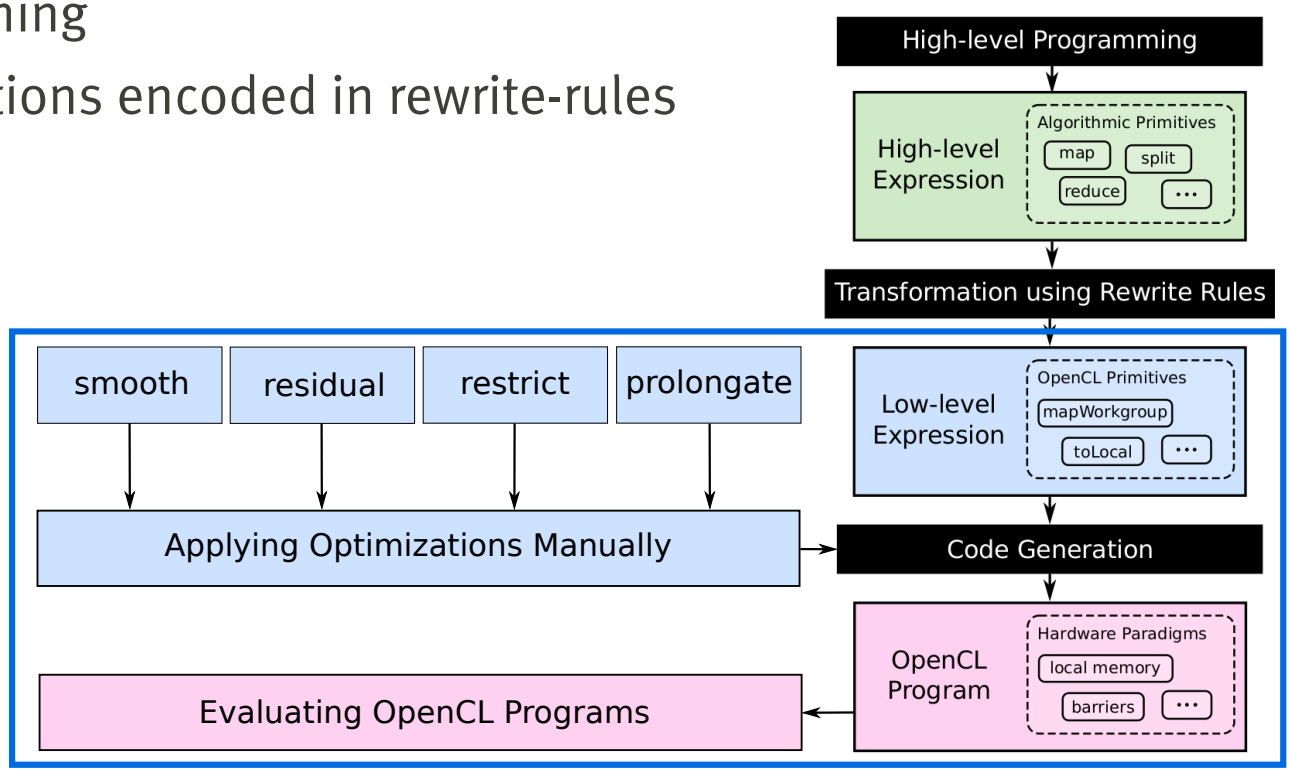

Designing Lift Expressions

- Identifying the general structure of the operation

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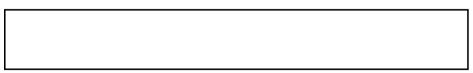
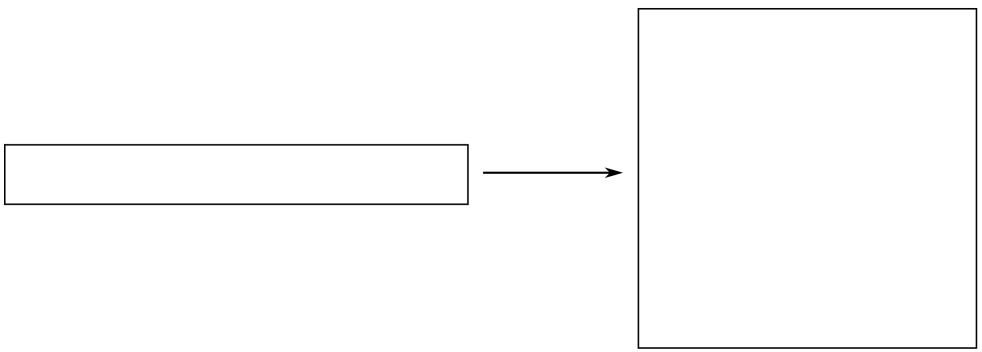
- Identifying the general structure of the operation
- Expressing the operation in 1 dimension



Designing Lift Expressions

- Identifying the general structure of the operation
- Expressing the operation in 1 dimension
- Scaling the operation to 2 and 3 dimensions



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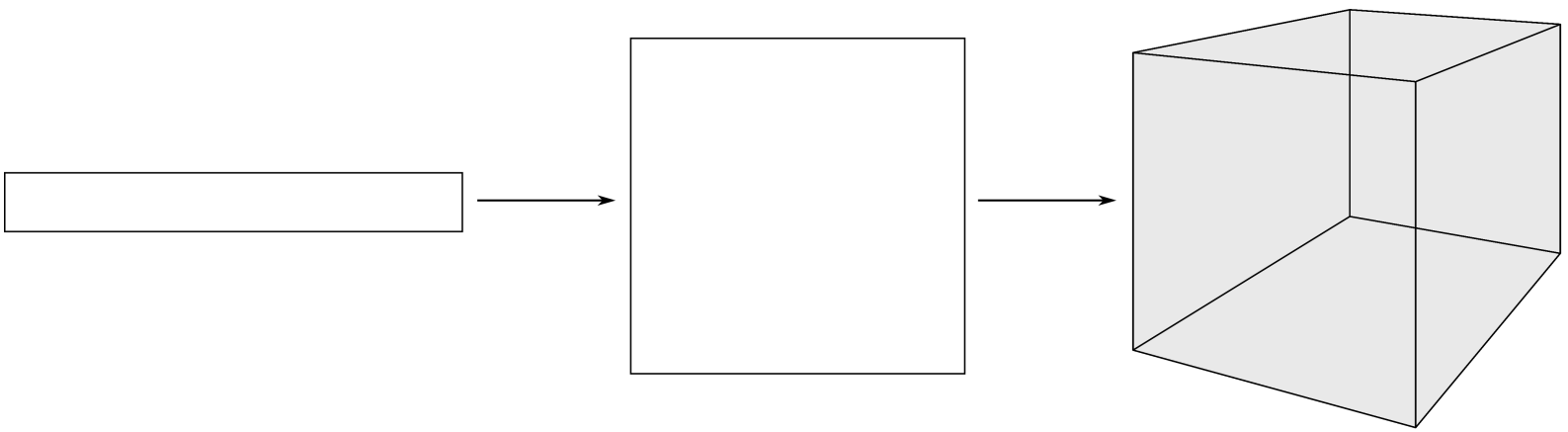
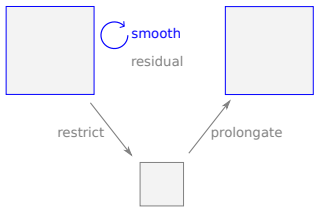
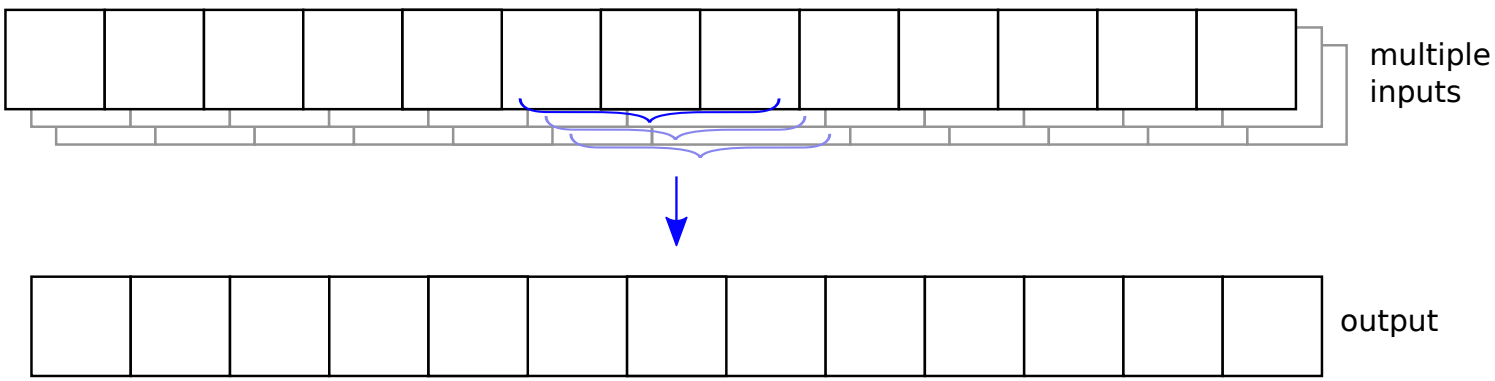


Smooth & Residual





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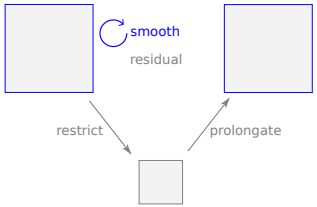
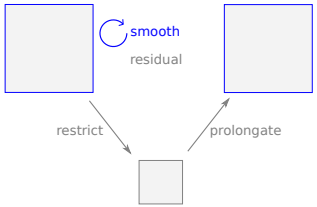
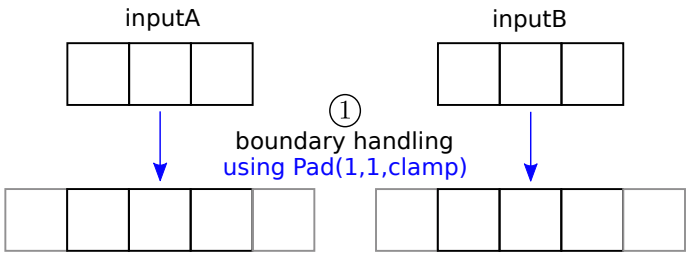


A,
B

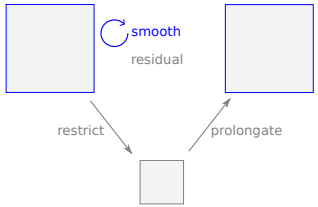
Smooth & Residual

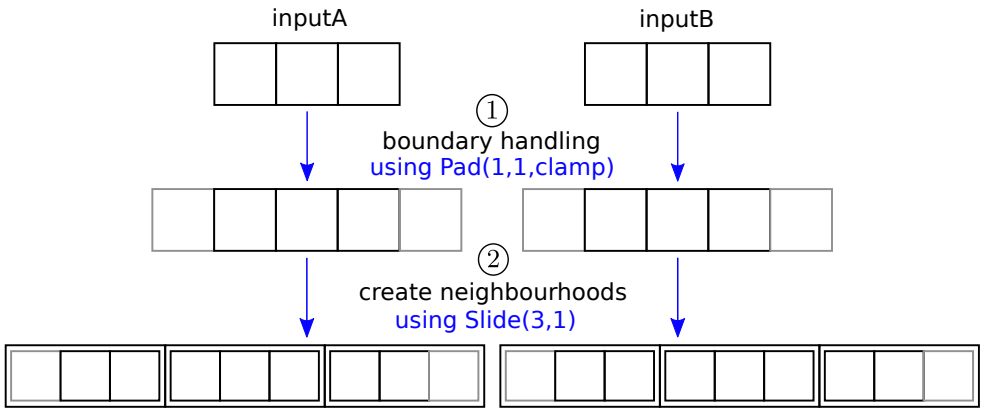





```
Pad(1,1,clamp) $ A,
Pad(1,1,clamp) $ B
```

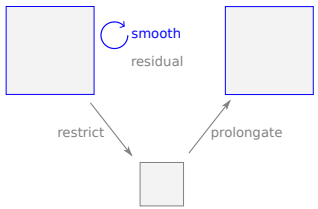
Smooth & Residual

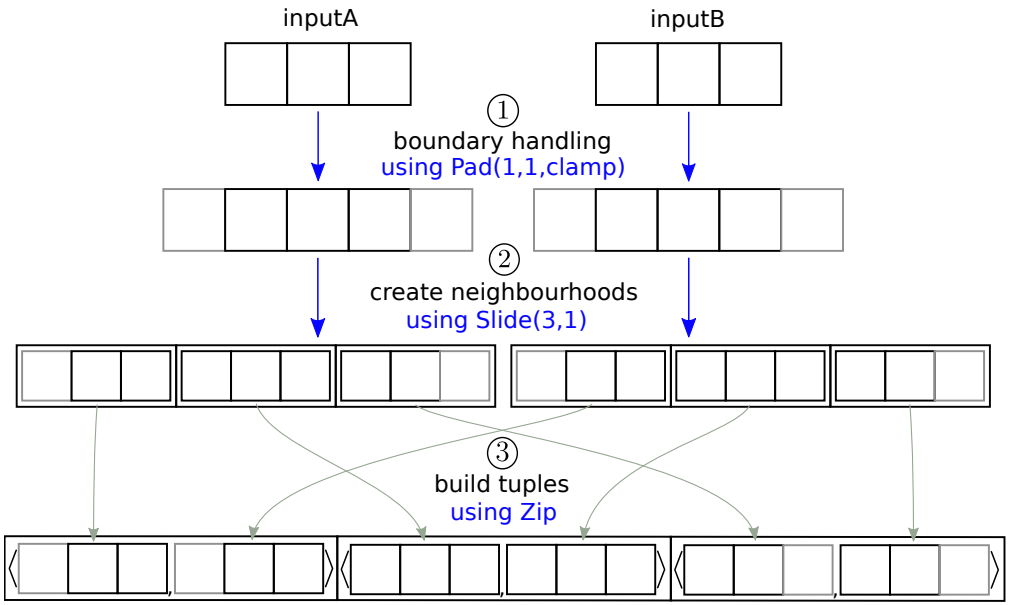





```
Slide(3,1) o Pad(1,1,clamp) $ A,
Slide(3,1) o Pad(1,1,clamp) $ B
```

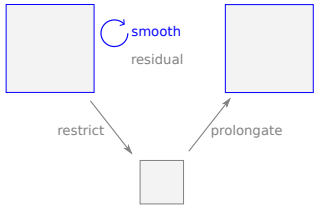
Smooth & Residual

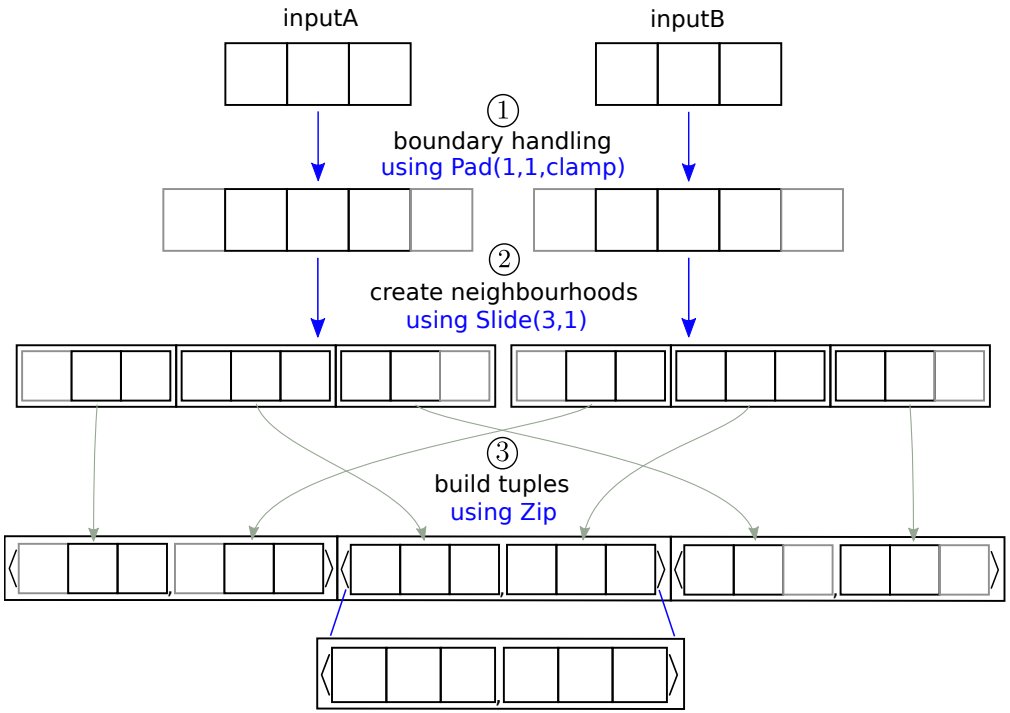





```
Zip(
     Slide(3,1) o Pad(1,1,clamp) $ A,
     Slide(3,1) o Pad(1,1,clamp) $ B )
```

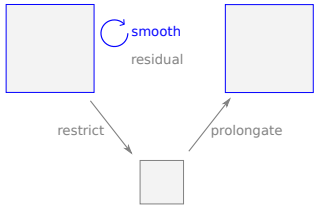
Smooth & Residual

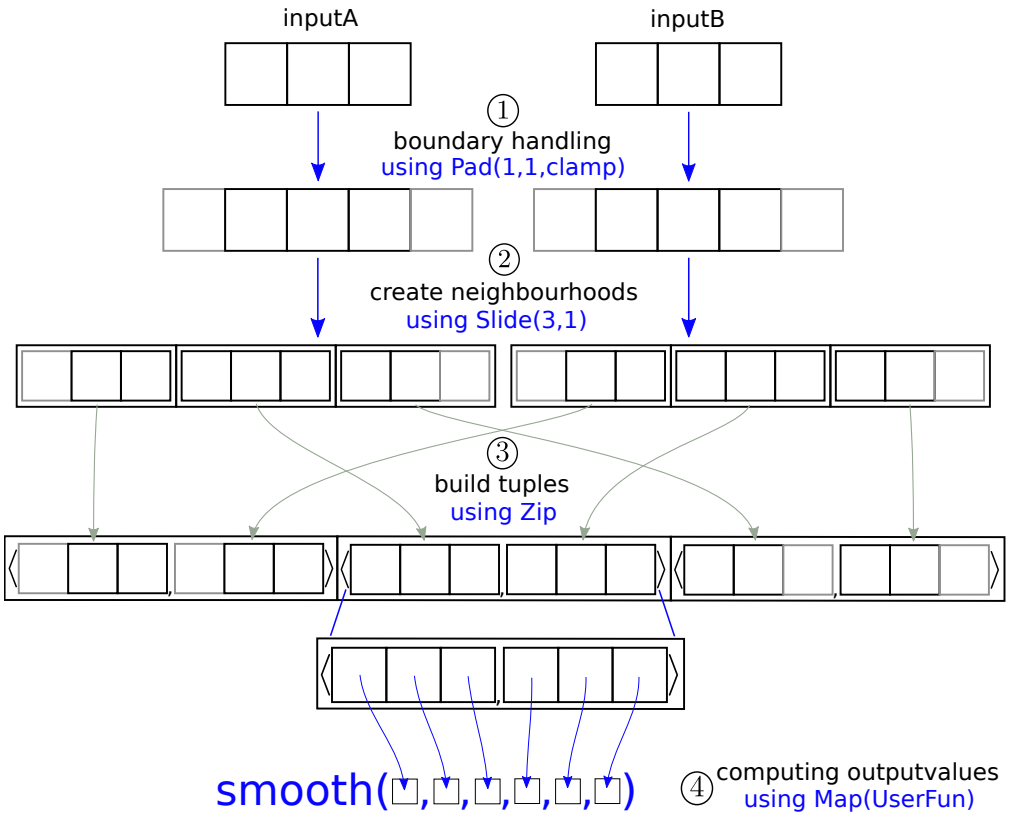





```
MapGlb( compute ) o
Zip(
     Slide(3,1) o Pad(1,1,clamp) $ A,
     Slide(3,1) o Pad(1,1,clamp) $ B )
```

Smooth & Residual






```
compute(Tuple t) = {
      a1 = tuple<0>[0]
      a2 = tuple<0>[1]
      a3 = tuple<0>[2]
      b1 = tuple<1>[0]
      b2 = tuple<1>[1]
      b3 = tuple<1>[2]

      smooth(a1, a2, a3, b1, b2, b3)
}


MapGlb( compute ) o
Zip(
      Slide(3,1) o Pad(1,1,clamp) $ A,
      Slide(3,1) o Pad(1,1,clamp) $ B )
```

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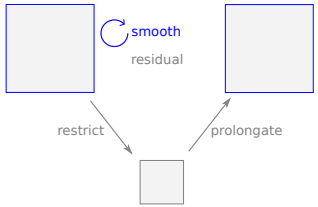

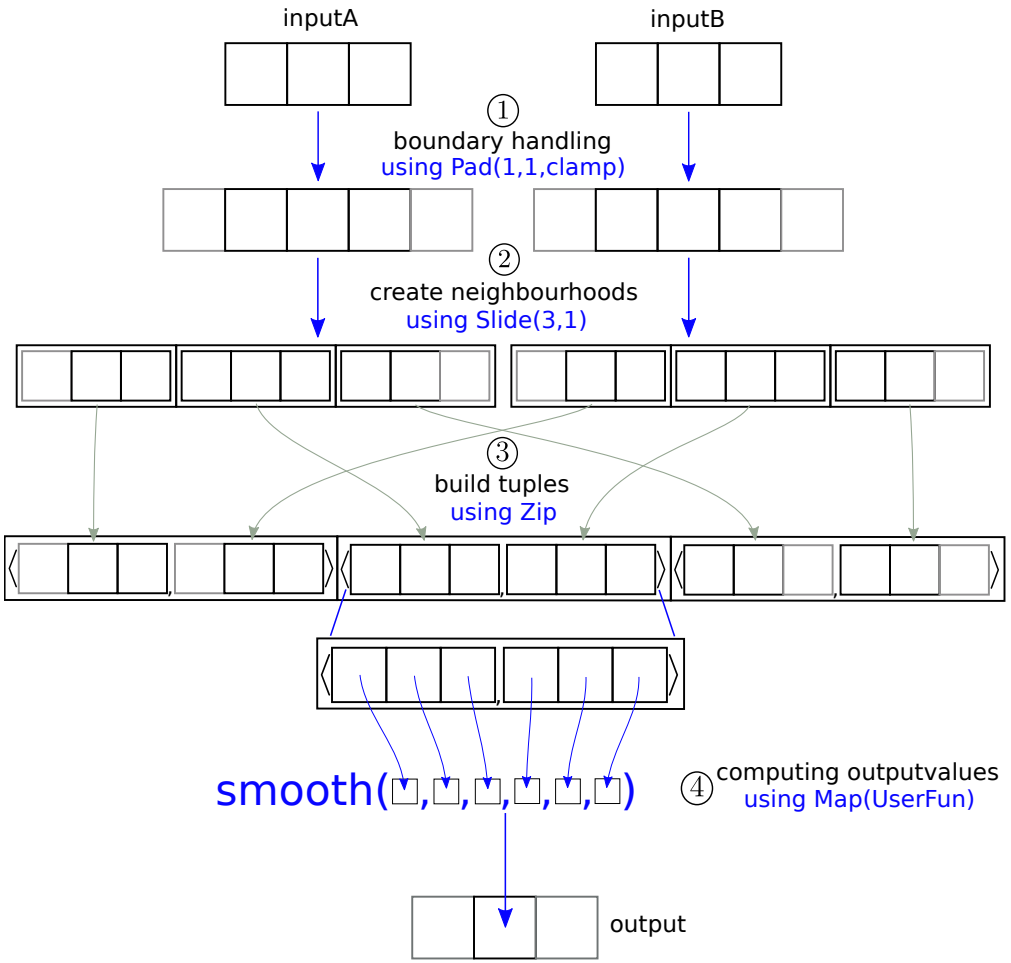


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      b1 = tuple<1>[0]
      b2 = tuple<1>[1]
      b3 = tuple<1>[2]

      smooth(a1, a2, a3, b1, b2, b3)
}


MapGlb( compute ) o
Zip(
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      Slide(3,1) o Pad(1,1,clamp) $ B )
```

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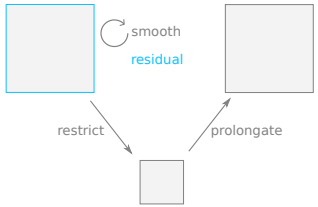

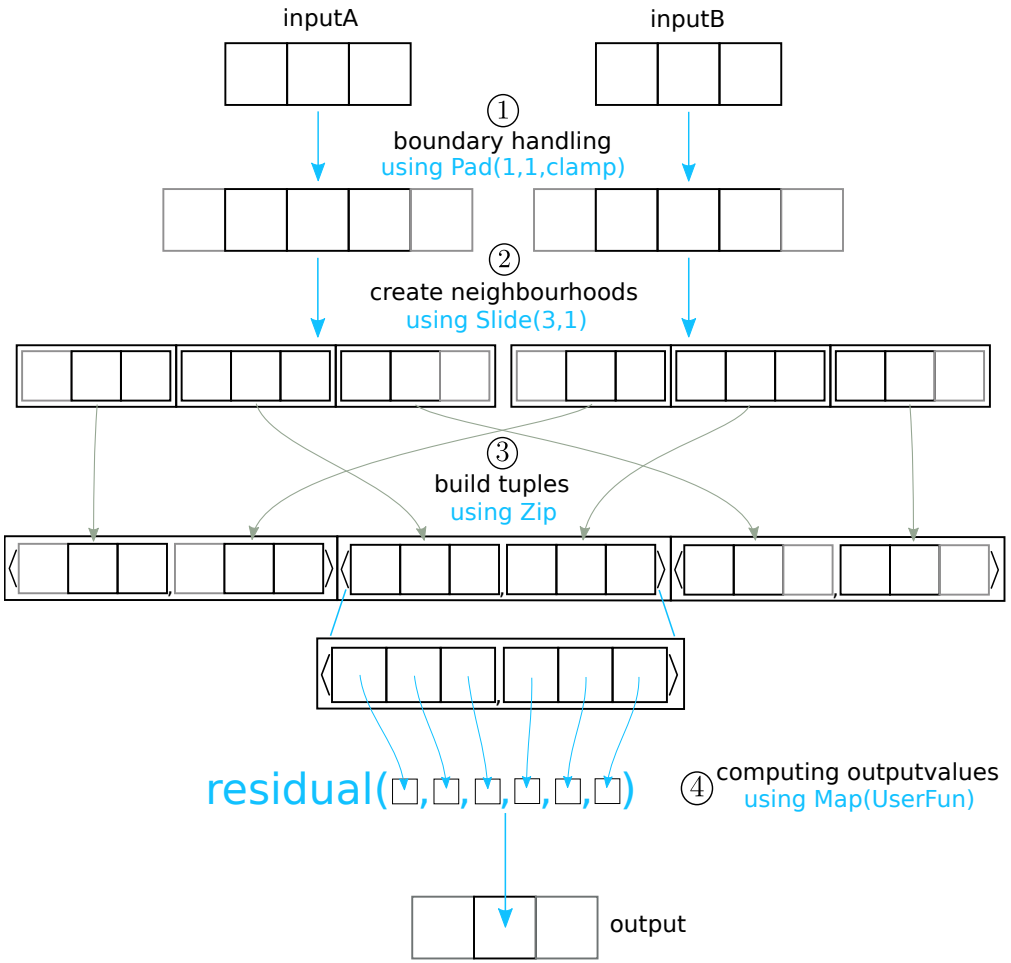


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      a3 = tuple<0>[2]
      b1 = tuple<1>[0]
      b2 = tuple<1>[1]
      b3 = tuple<1>[2]

      residual(a1, a2, a3, b1, b2, b3)
}


MapGlb( compute ) o
Zip(
      Slide(3,1) o Pad(1,1,clamp) $ A,
      Slide(3,1) o Pad(1,1,clamp) $ B )
```
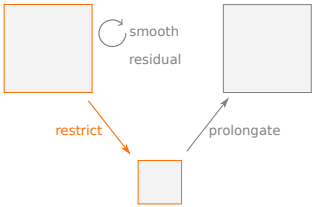
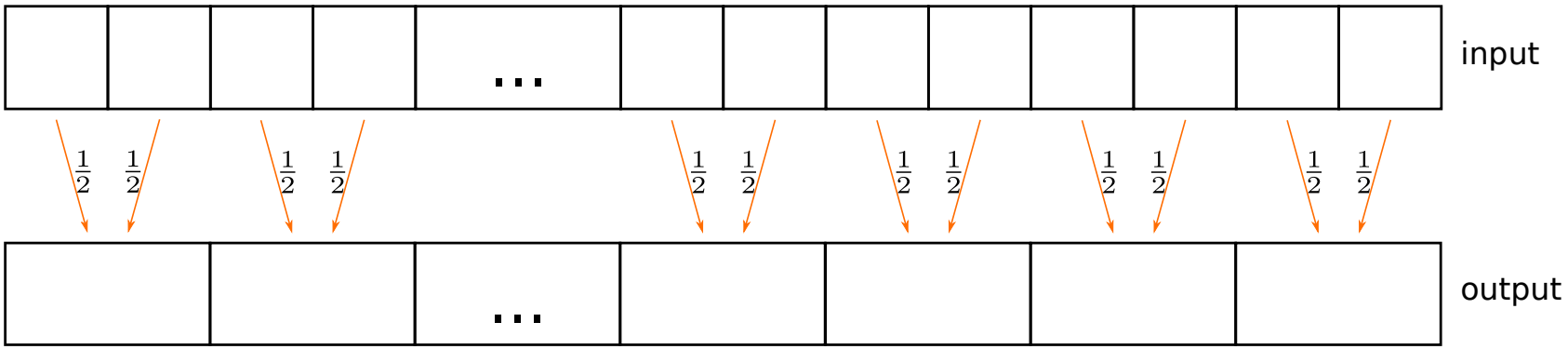
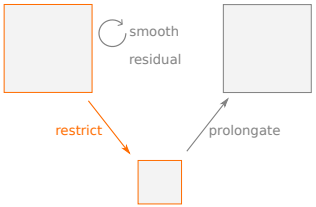
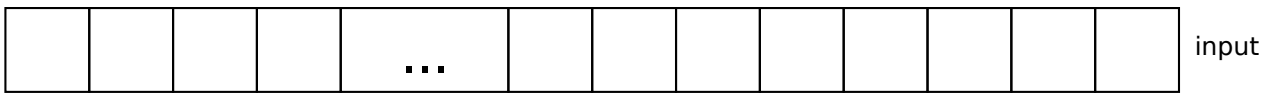

Restriction 1D





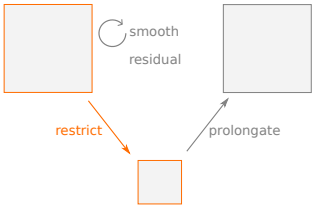
Restriction 1D

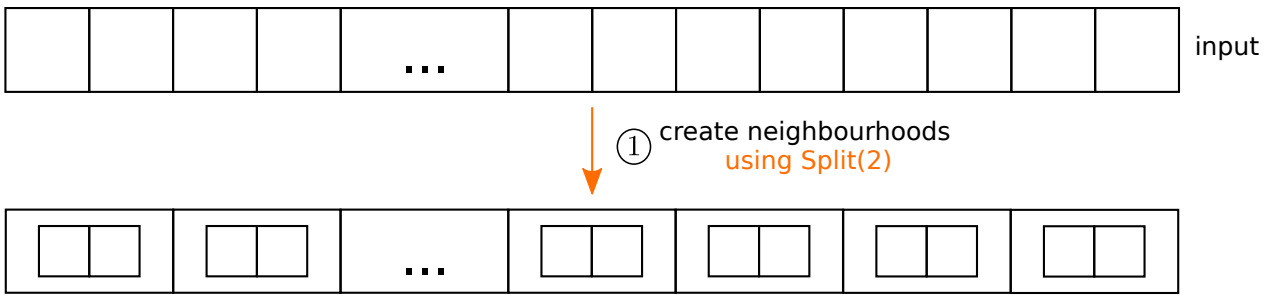




input

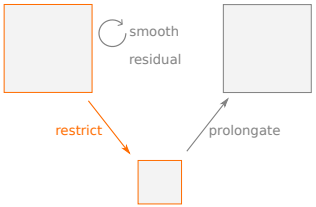
Restriction 1D

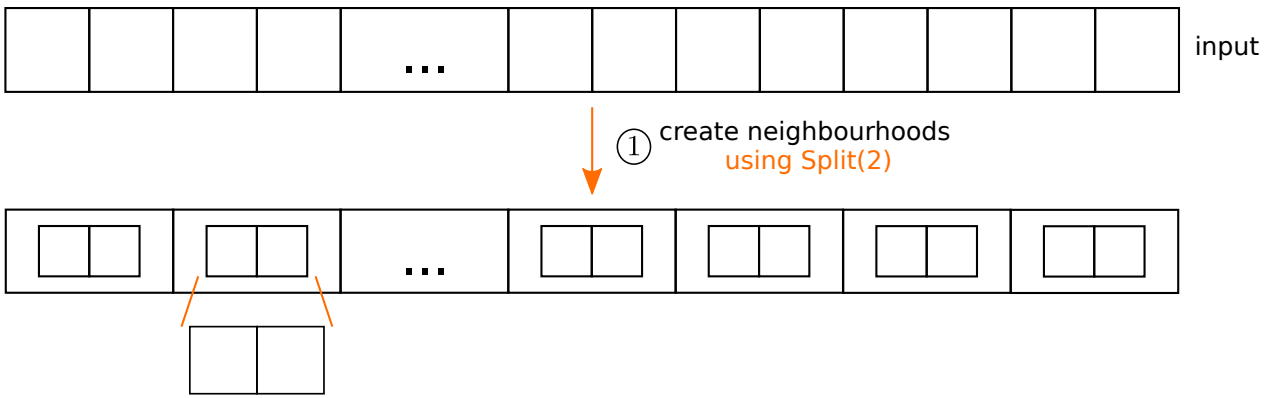





```
Split(2) $ input
```

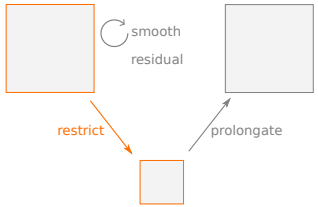
Restriction 1D

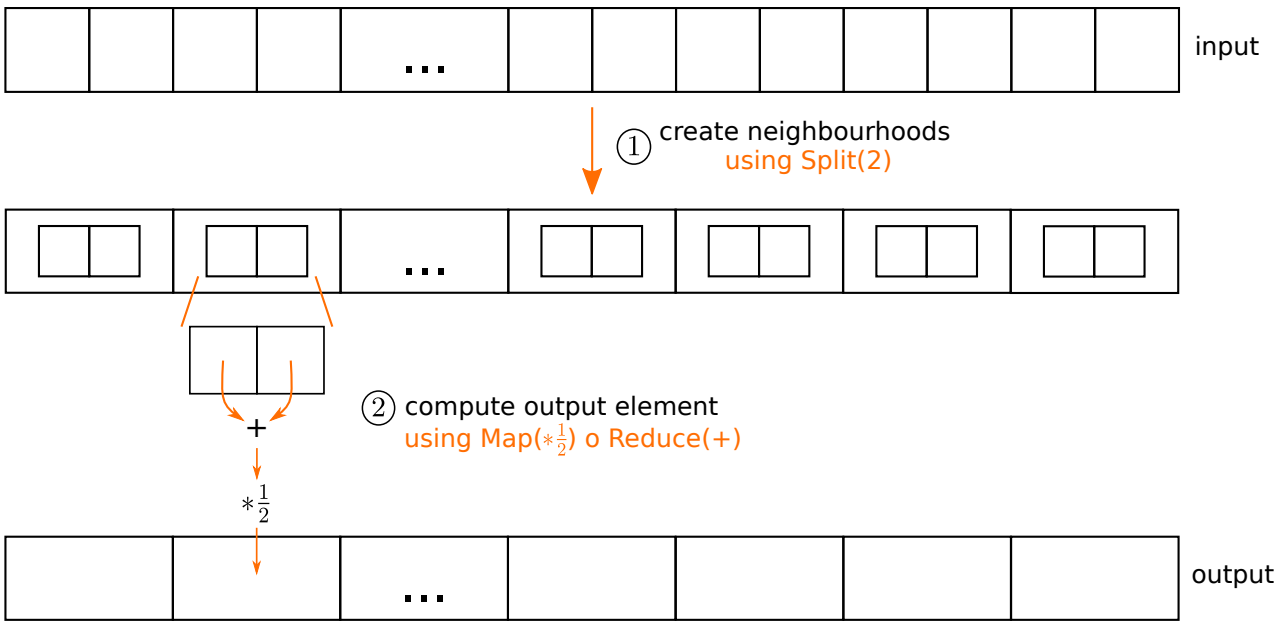





```
MapGlb(

) o
Split(2) $ input
```

Restriction 1D

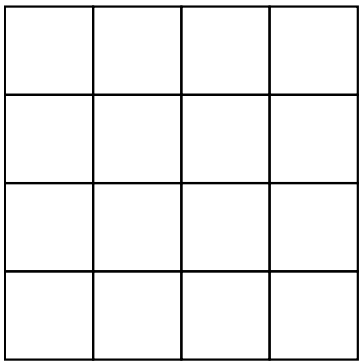


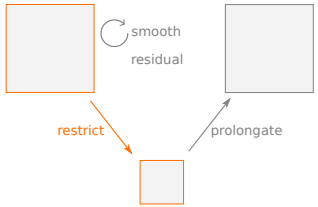



```
MapGlb(
  Map(∗1/2) o
  Reduce(+)
) o
Split(2) $ input
```

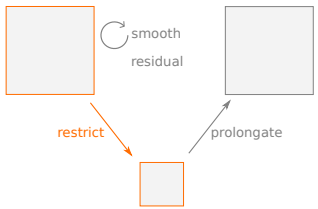
Restriction 2D

input



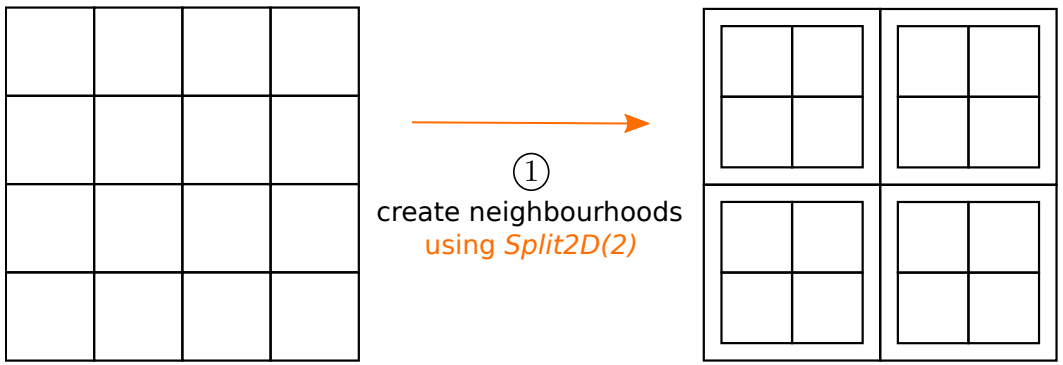


input



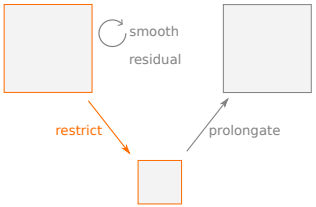
Restriction 2D

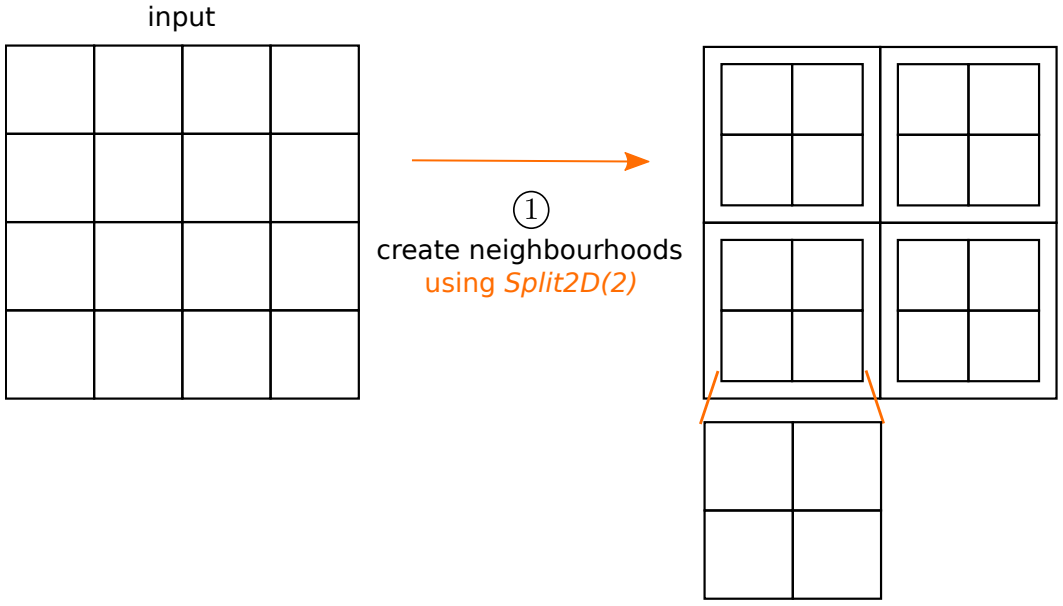
input



Split2D(2) $ input

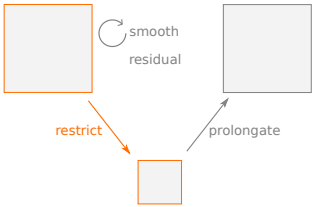
Restriction 2D

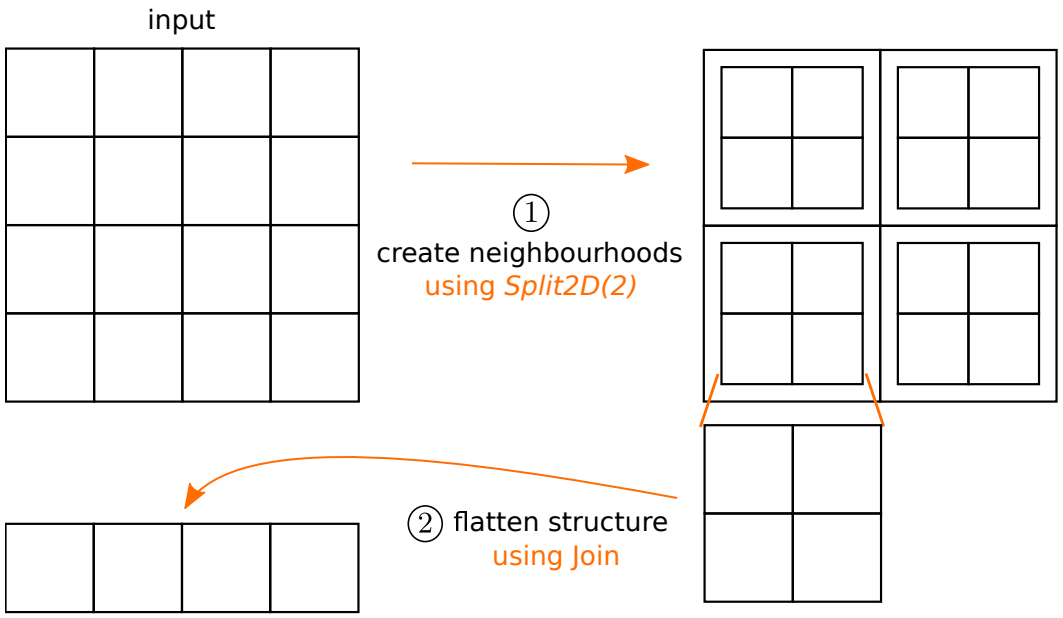




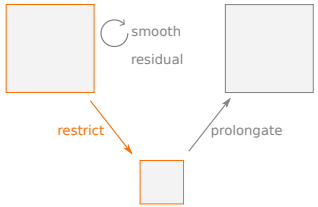

```
MapGlb(
  MapGlb(

)) o
Split2D(2) $ input
```

Restriction 2D

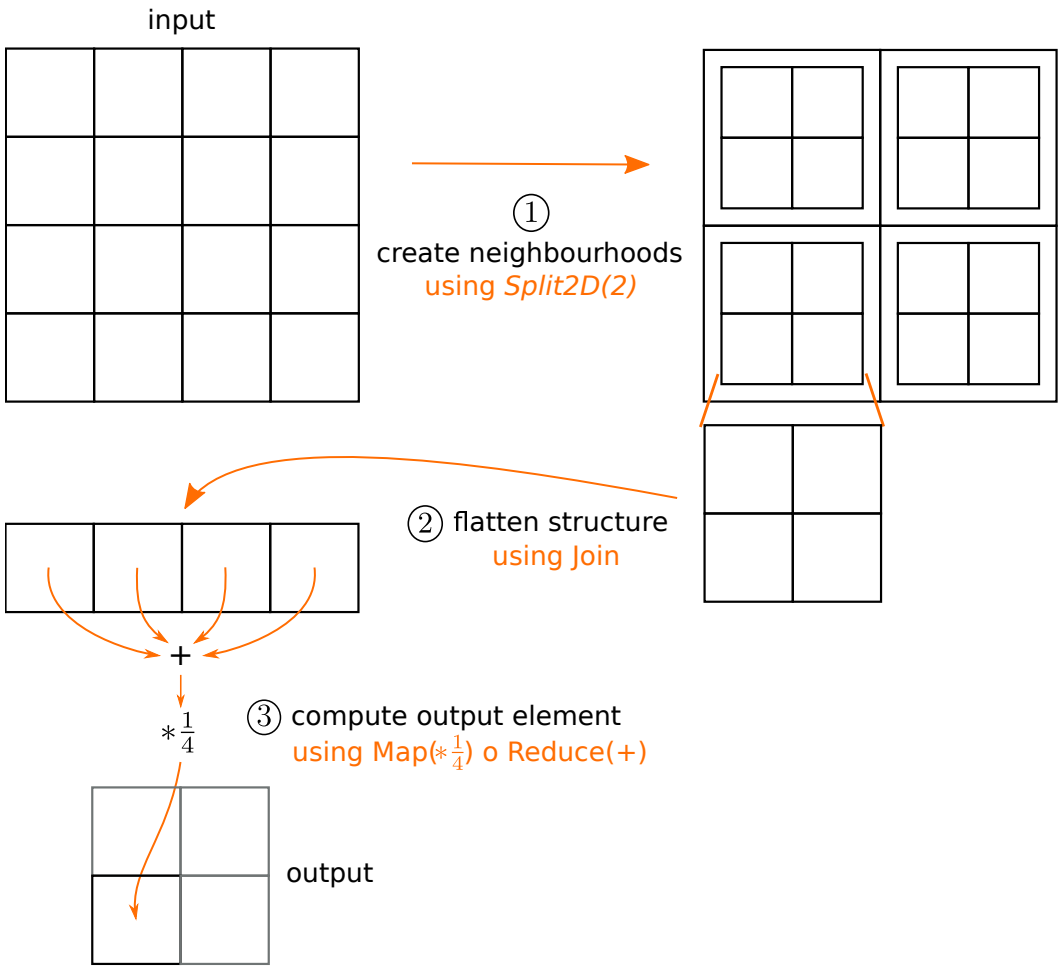





```
MapGlb(
  MapGlb(


    Join
)) o
Split2D(2) $ input
```

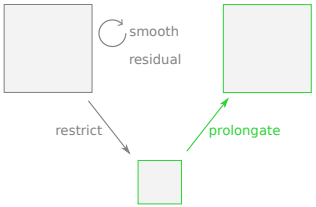


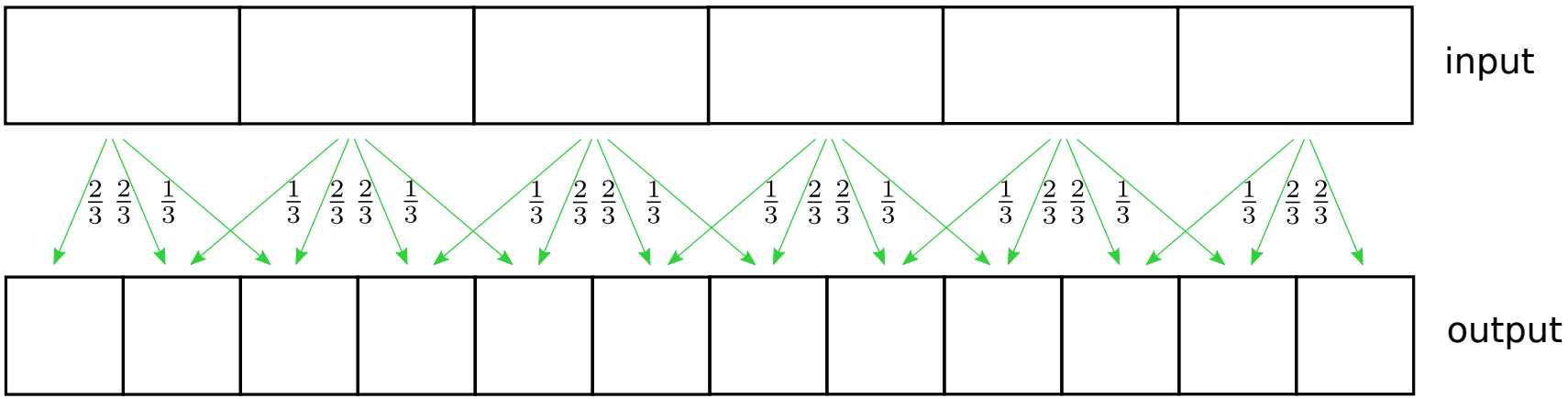
Restriction 2D



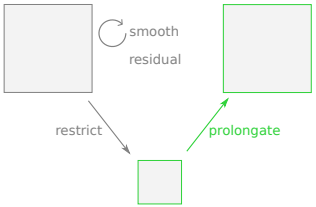

```
MapGlb(
  MapGlb(
    Map(*1/4) o
    Reduce(+) o
    Join
)) o
Split2D(2) $ input
```

Prolongation



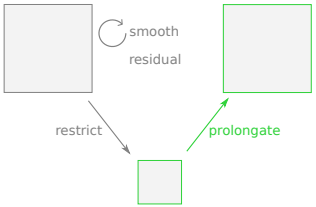


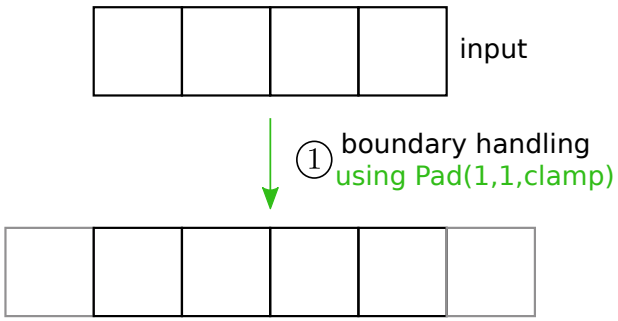
Prolongation



input

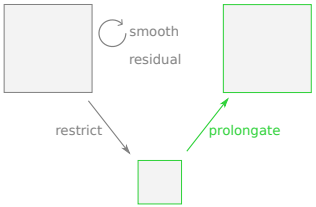
Prolongation

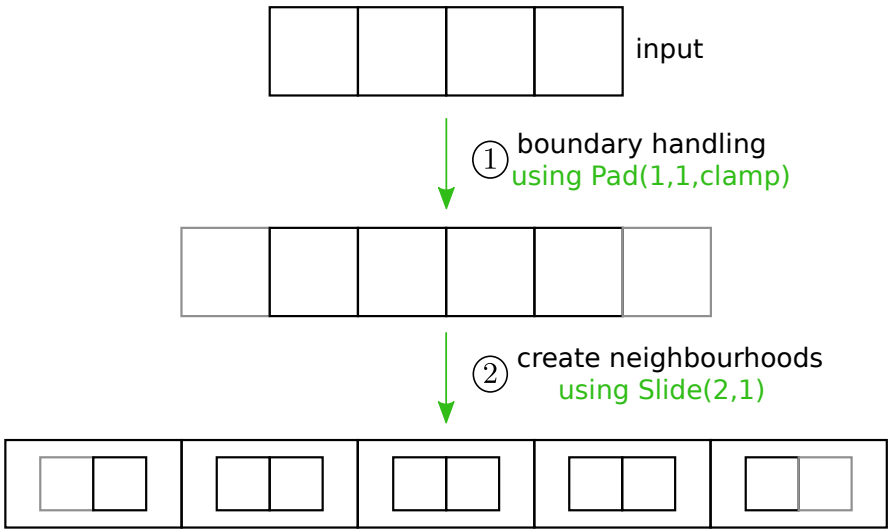





```
Pad(1,1,clamp) $ input
```

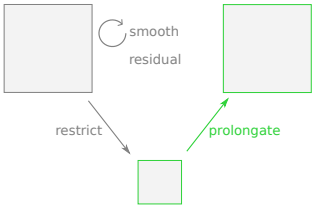
Prolongation

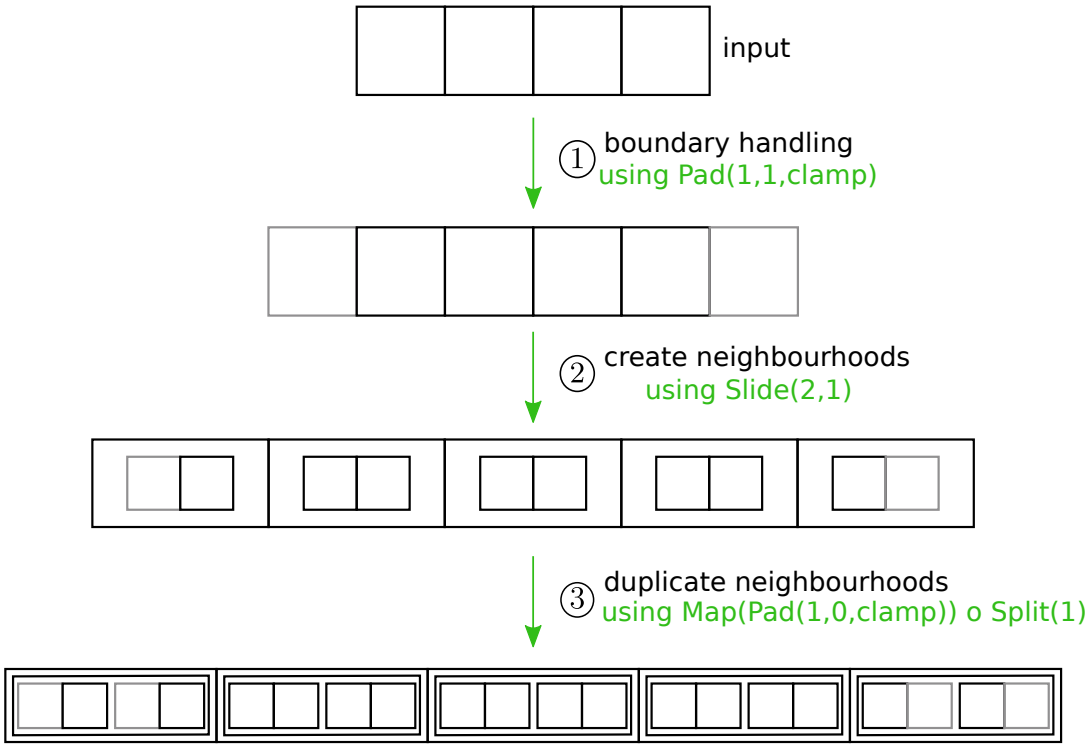





```
Slide(2,1) o
Pad(1,1,clamp) $ input
```

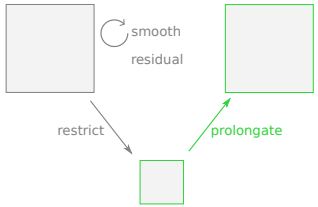
Prolongation

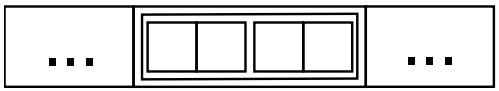





```
Map(Pad(1,0,clamp)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $ input
```

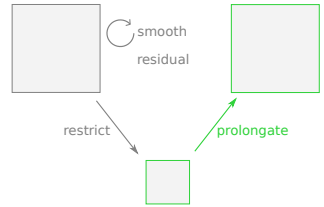
Prolongation

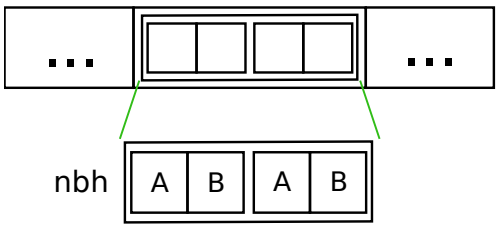





```
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $ input
```

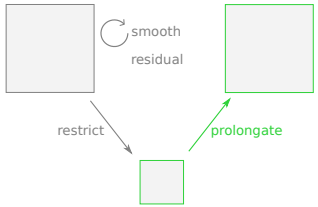
Prolongation



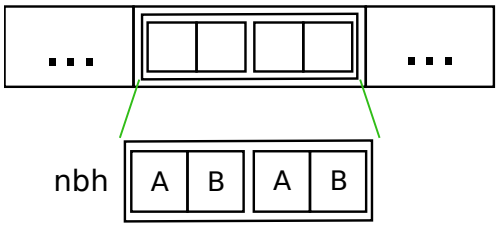



```
Map(nbh =>

        nbh,



) o
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $ input
```

Prolongation

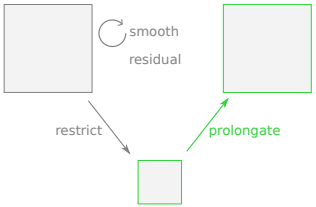


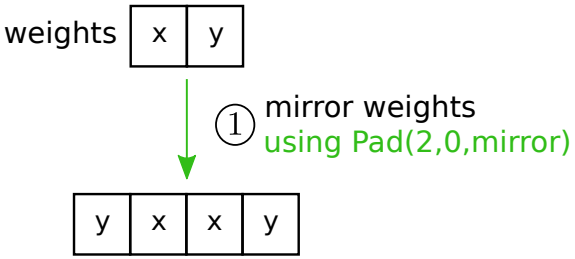
weights | x | y |

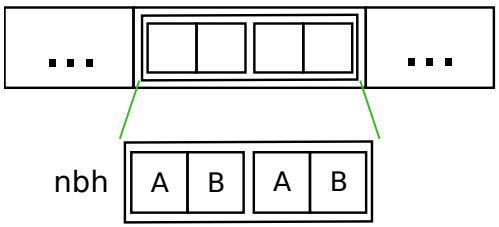



```
Map(nbh =>

        nbh,

        weights

) o
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $ input
```

Prolongation

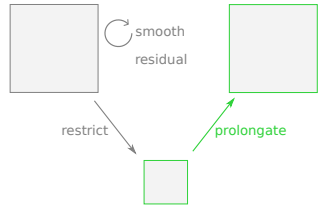


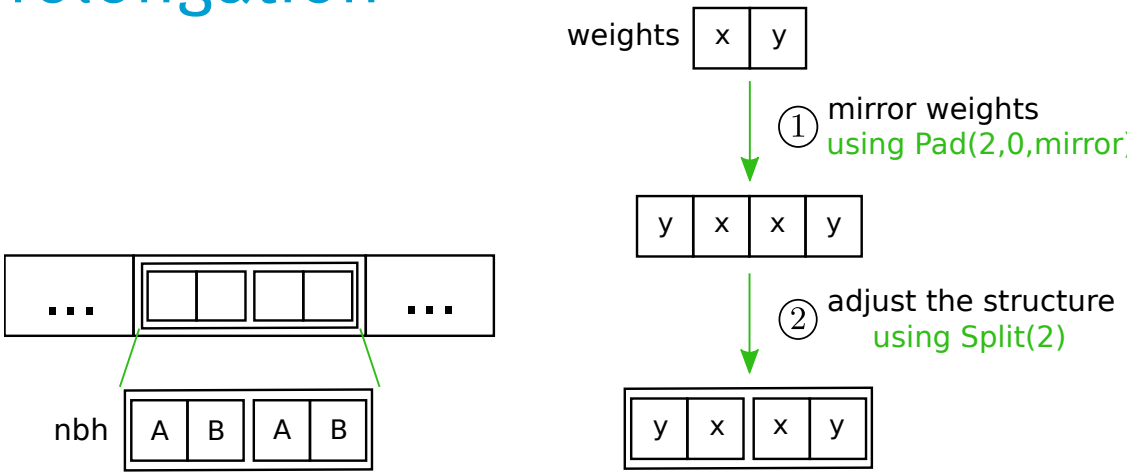





```
Map(nbh =>

        nbh,
                    Pad(2,0,Mirror) $
        weights

) o
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $ input
```

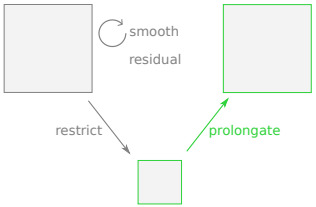
Prolongation

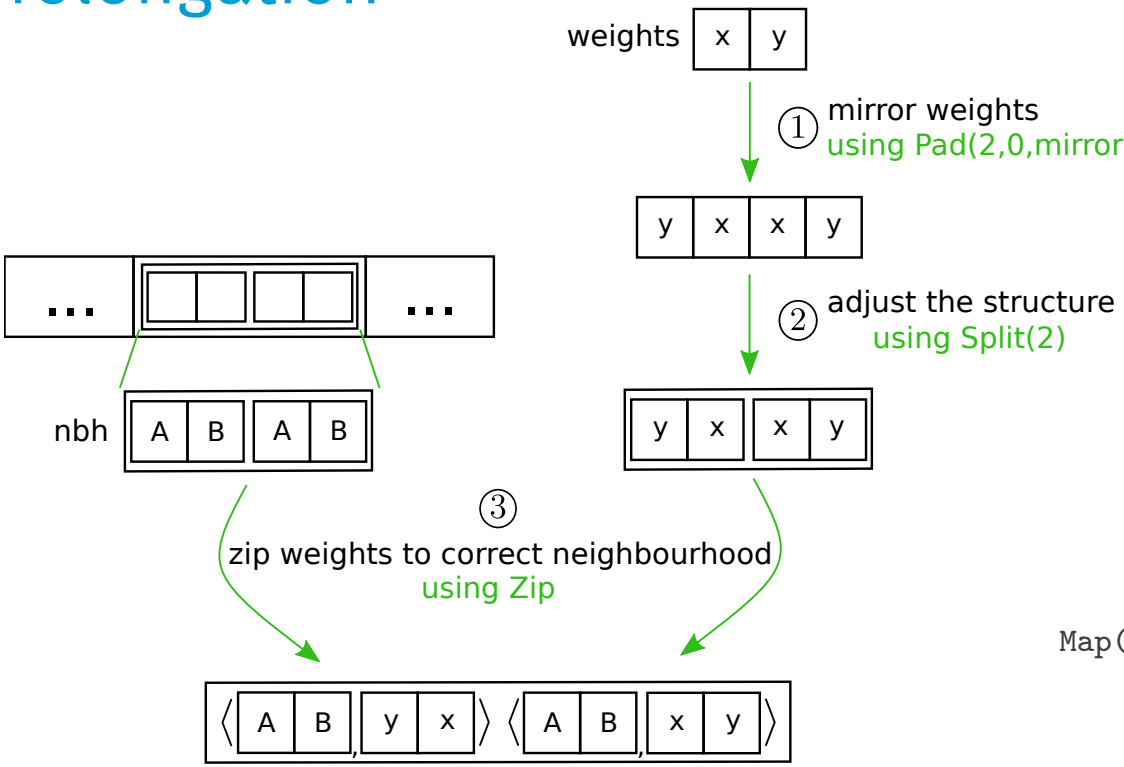





```
Map(nbh =>

        nbh,
        Split(2) o  Pad(2,0,Mirror) $
        weights

) o
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $  input
```

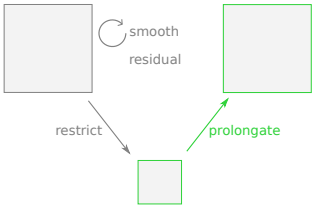
Prolongation

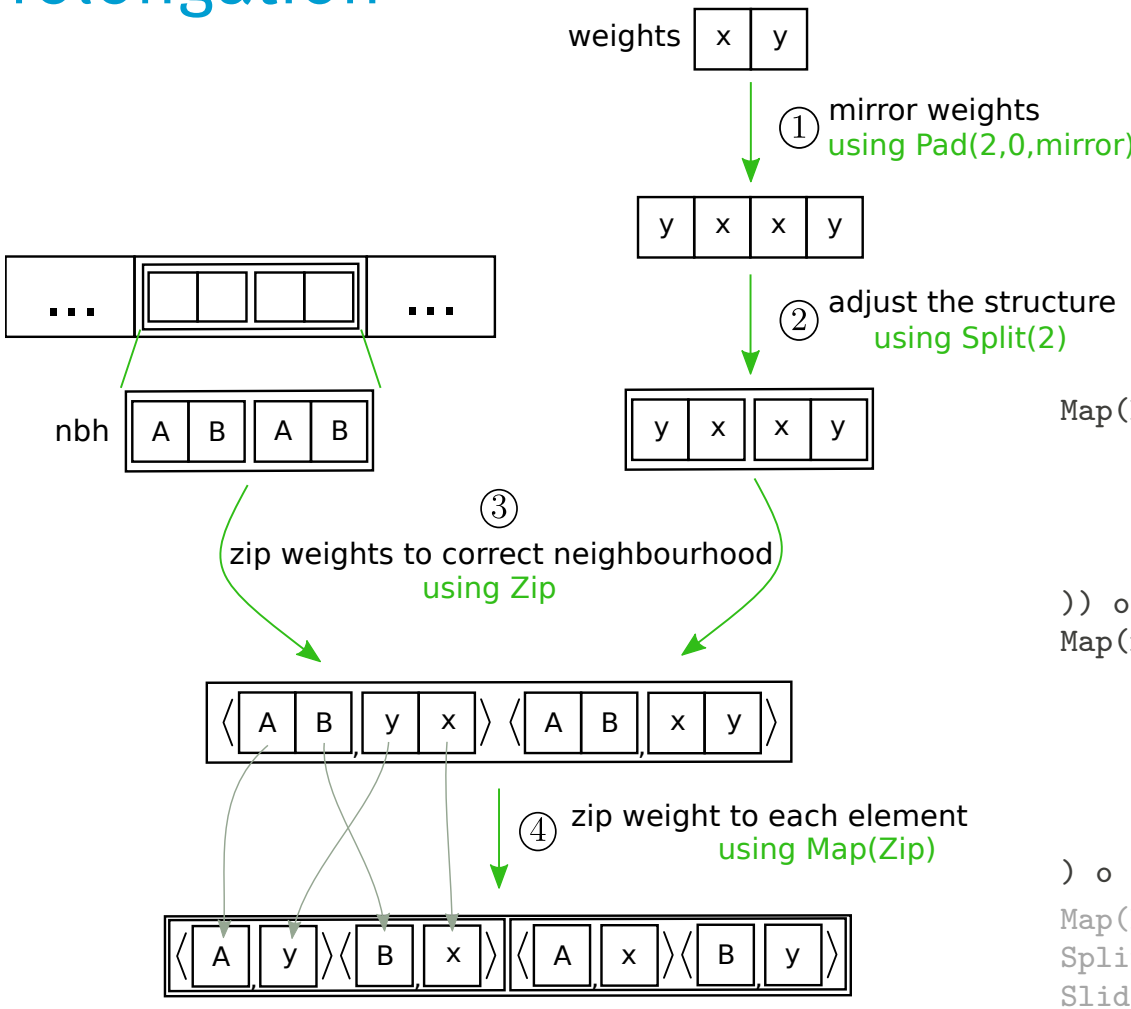





```
Map(nbh =>
    Zip(
        nbh,
        Split(2) o Pad(2,0,Mirror) $
        weights
    )
) o
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $ input
```

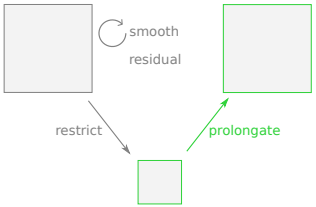
Prolongation

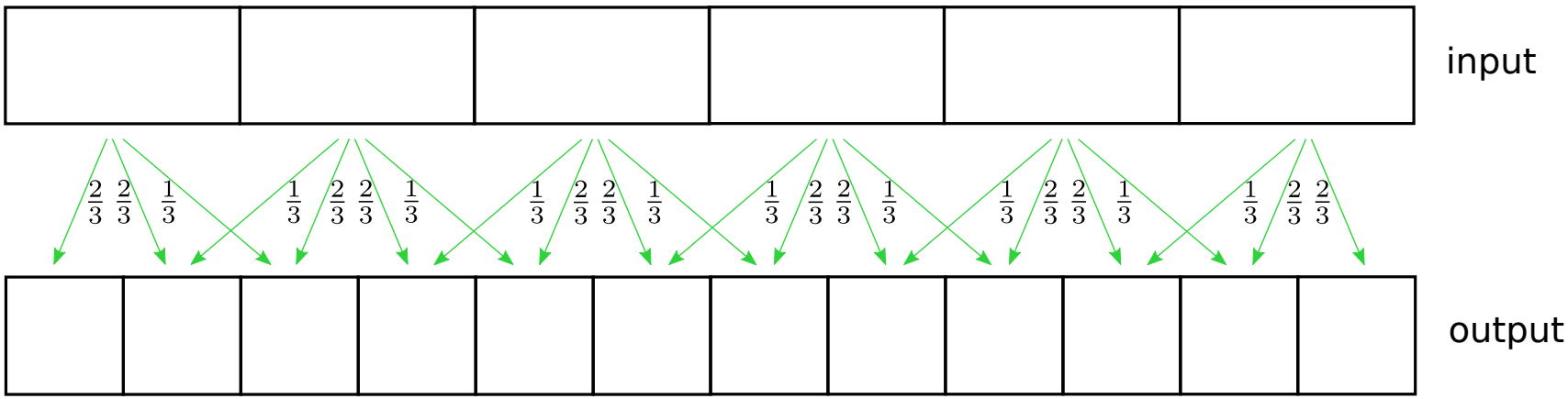




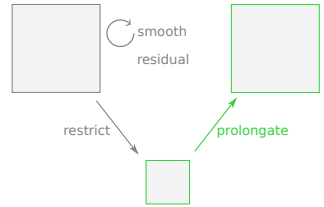

```
Map(Map(tuple =>
     Zip(
          tuple<0>,
          tuple<1>
     )
)) o
Map(nbh =>
     Zip(
          nbh,
          Split(2) o  Pad(2,0,Mirror) $
          weights
     )
) o
Map(Pad(1,0,wrap)) o
Split(1) o
Slide(2,1) o
Pad(1,1,clamp) $  input
```

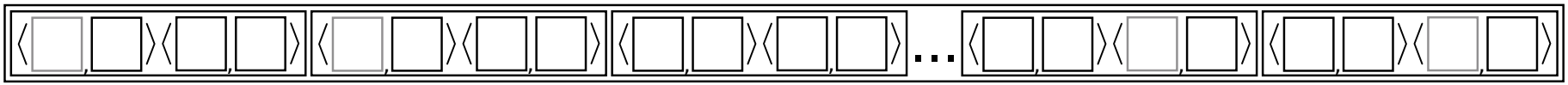
Prolongation





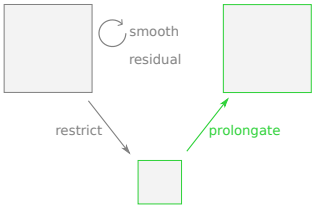
Prolongation

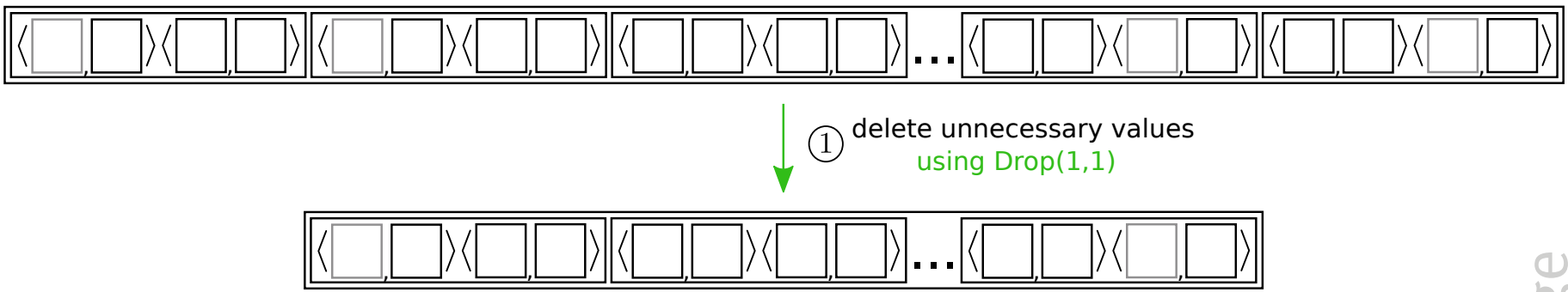




```
Map(Map(tuple =>
     Zip(
          tuple<0>,
          tuple<1>)
     )
)) o
[...]
```

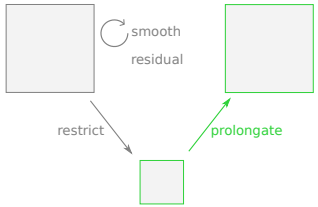
Prolongation

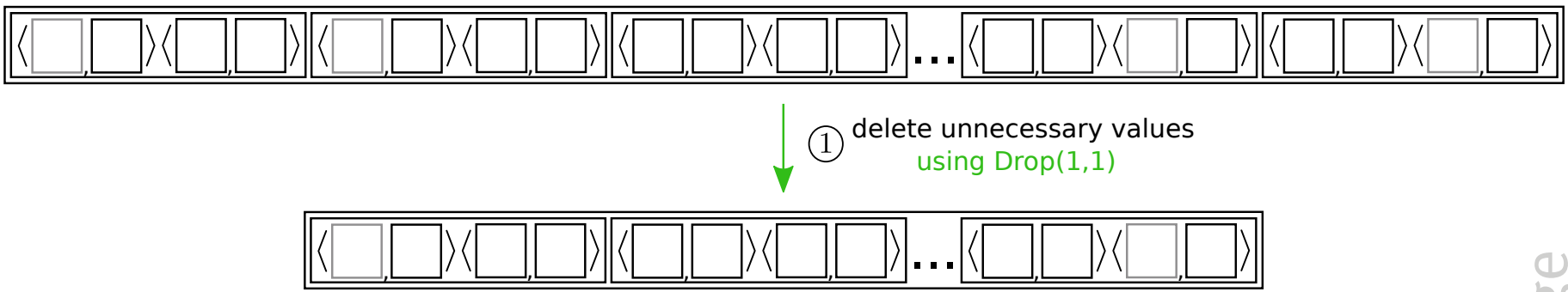





```
Drop(1,1) o
Map(Map(tuple =>
     Zip(
          tuple<0>,
          tuple<1>)
     )
)) o
[...]
```

Prolongation





- `Drop(l,r)` was introduced in this thesis
- Extends LIFT to be capable of expressing prolongation

```
Drop(1,1) o
Map(Map(tuple =>
     Zip(
          tuple<0>,
          tuple<1>)
     )
)) o
[...]
```

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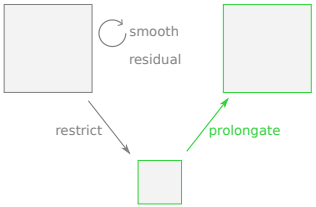
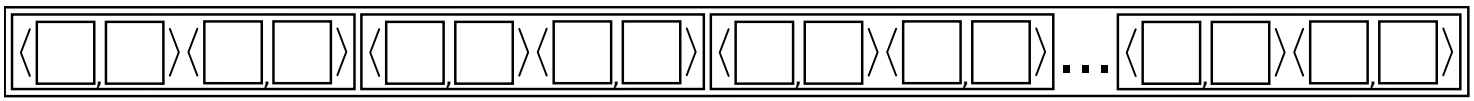
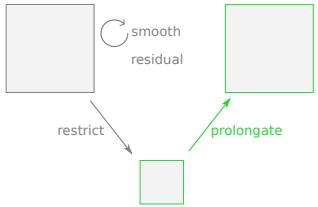
Prolongation

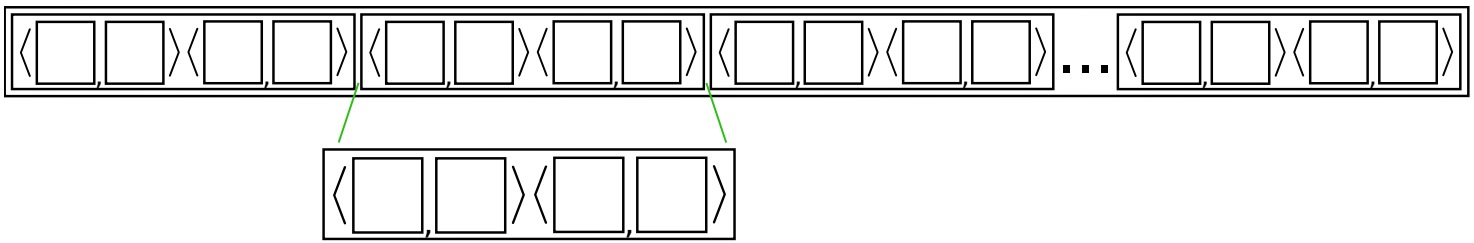




```
Drop(1,1) o
[...]
```

Prolongation

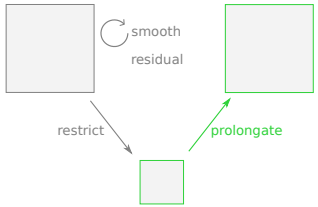


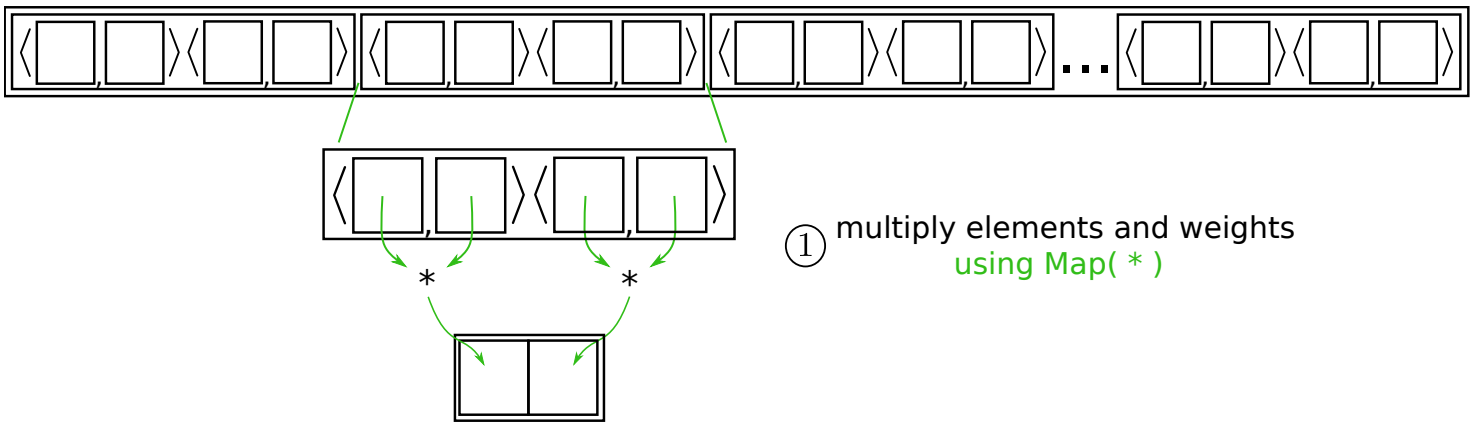


```
MapGlb(

) o
Drop(1,1) o
[...]
```

Prolongation



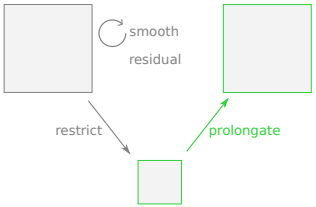


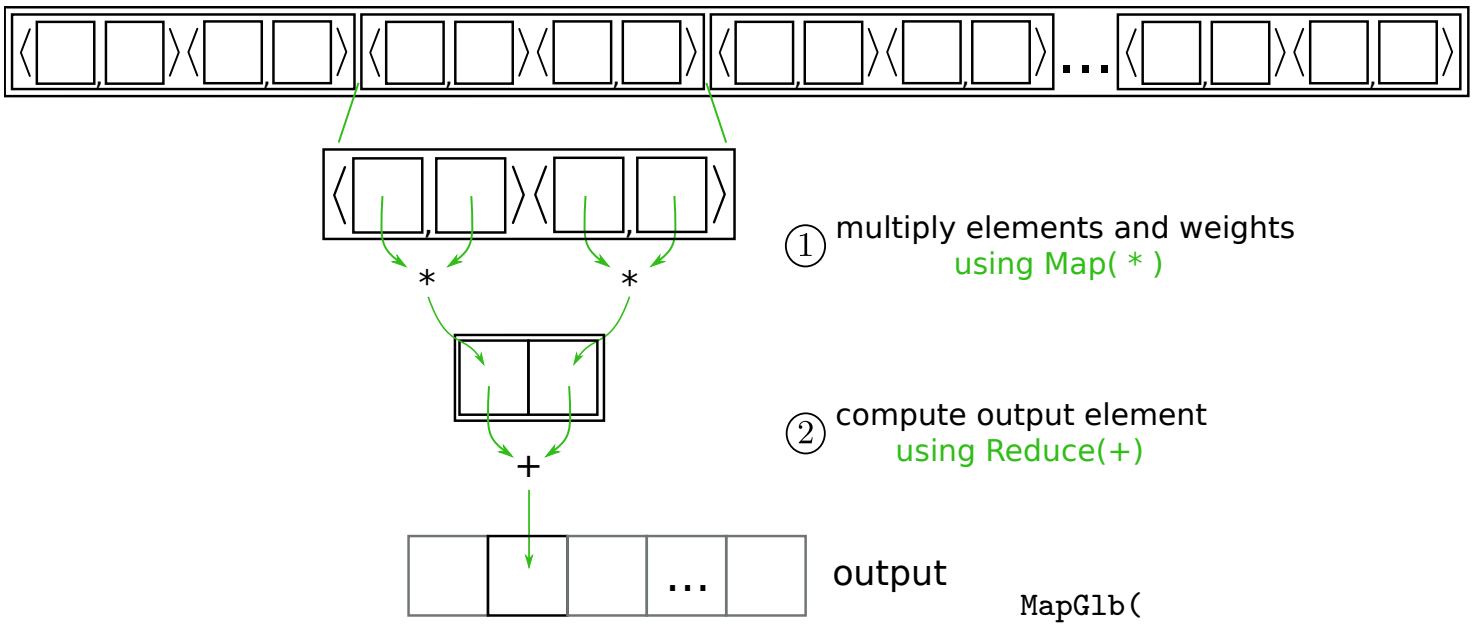
① multiply elements and weights
using Map(*)

```
MapGlb(

    MapSeq(
        tuple<0> * tuple<1>
    )
) o
Drop(1,1) o
[...]
```

Prolongation





① multiply elements and weights
using Map(*)

② compute output element
using Reduce(+)

output

```
MapGlb(
    Reduce(+) o
    MapSeq(
        tuple<0> * tuple<1>
    )
) o
Drop(1,1) o
[...]
```

Evaluation

- Lift does not support building entire programs yet
- All operations expressed in Lift in multiple dimensions

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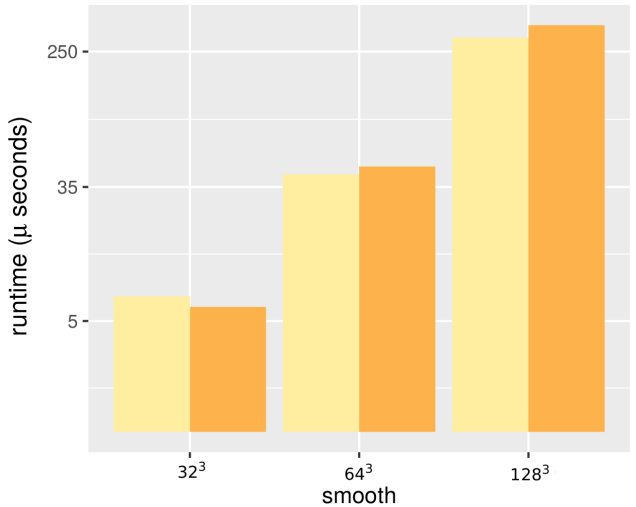
Workflow

- Used GPU: Nvidia GeForce GTX 1080
- All parameters auto-tuned with the Auto Tuning Framework (ATF)
- Tuning time: 1 hour per framework per operation per input size

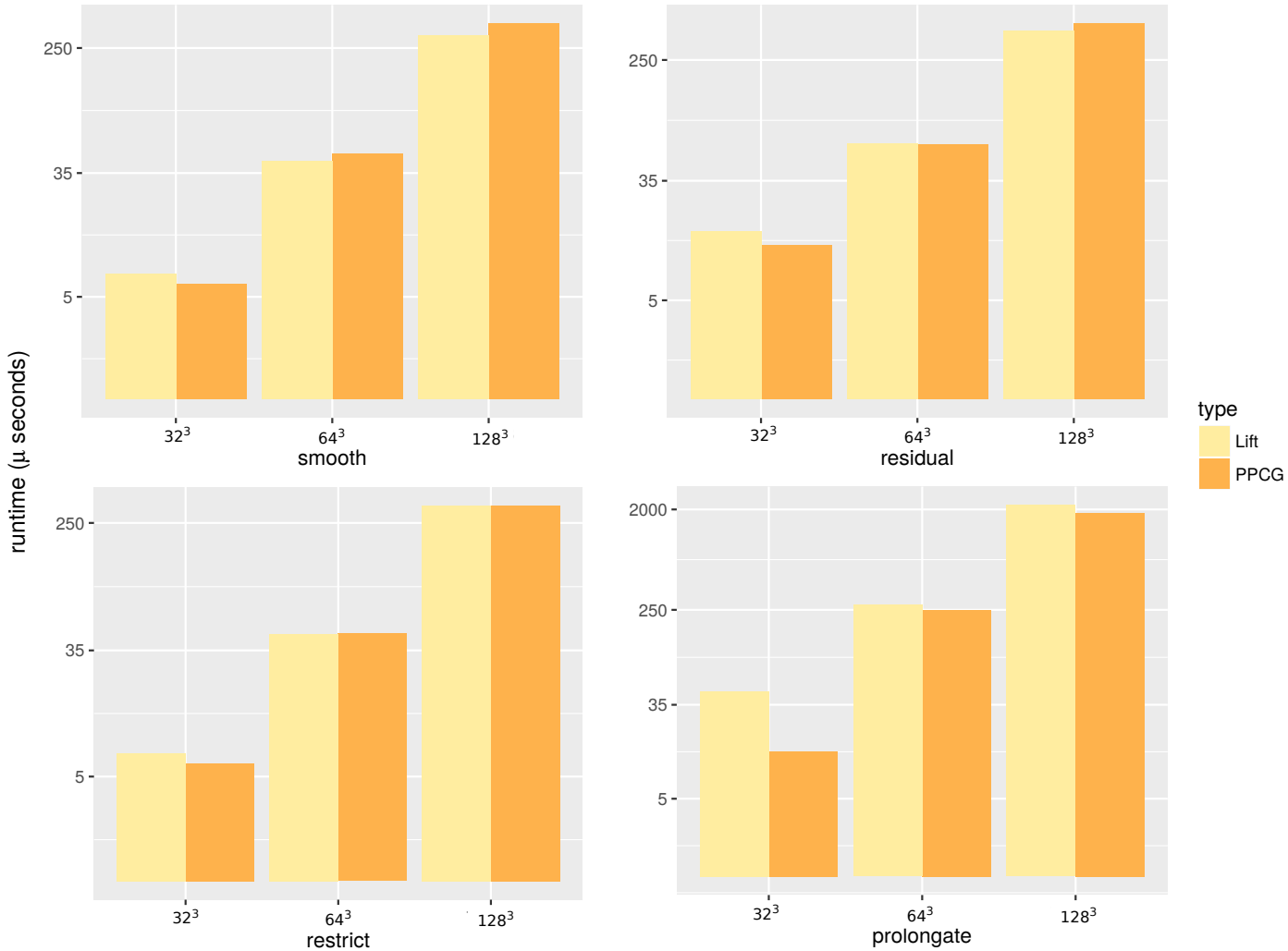
Runtime Comparison



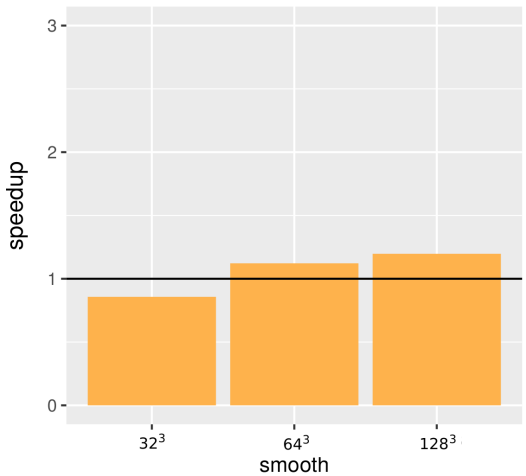
- x: output elements (32^3, 64^3, 128^3)
- y: runtime in μs (lower is better)

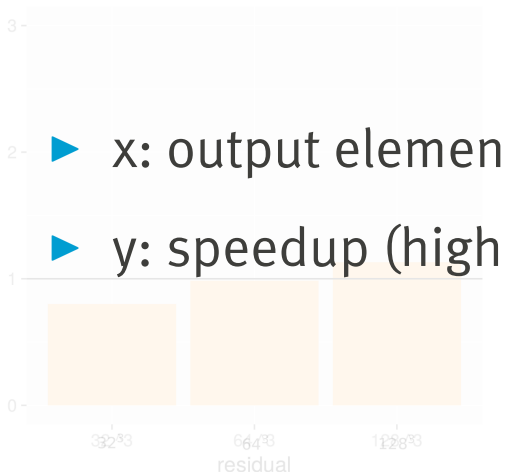
LIFT PPCG

Runtime Comparison



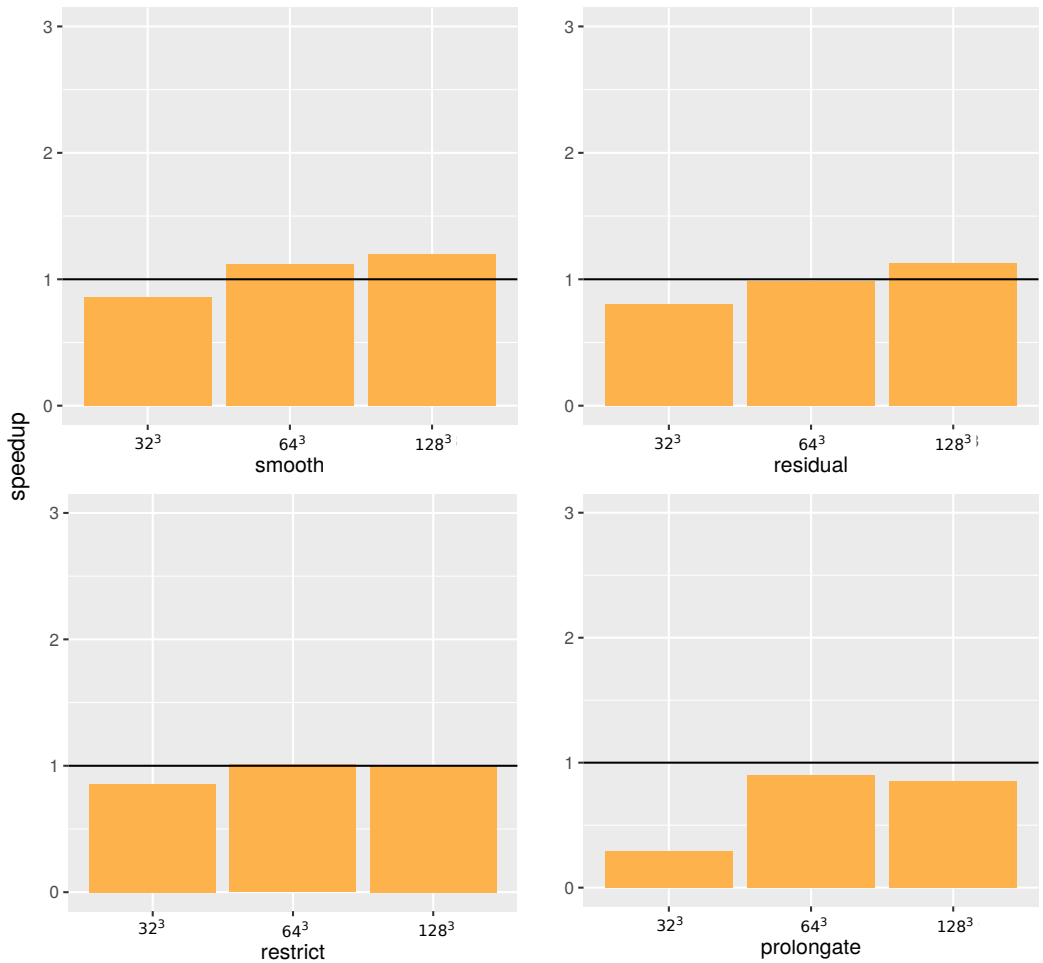
Speedup Comparison





- x: output elements (32^3, 64^3, 128^3)
- y: speedup (higher is better)

Speedup Comparison



GMG Program

In a GMG solver the operations vary in number of execution, so further evaluation was necessary:

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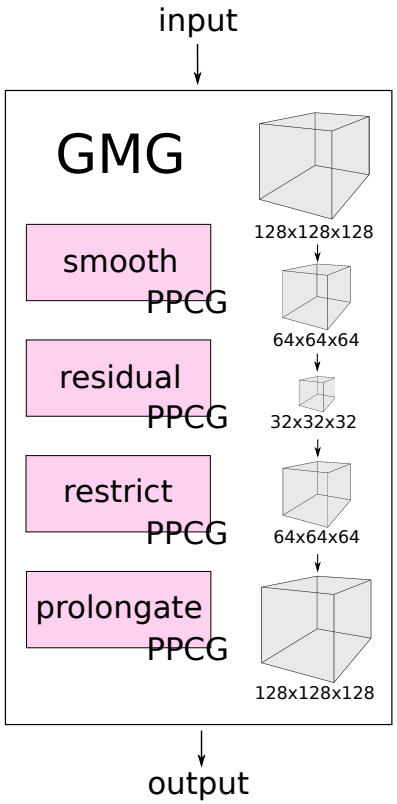

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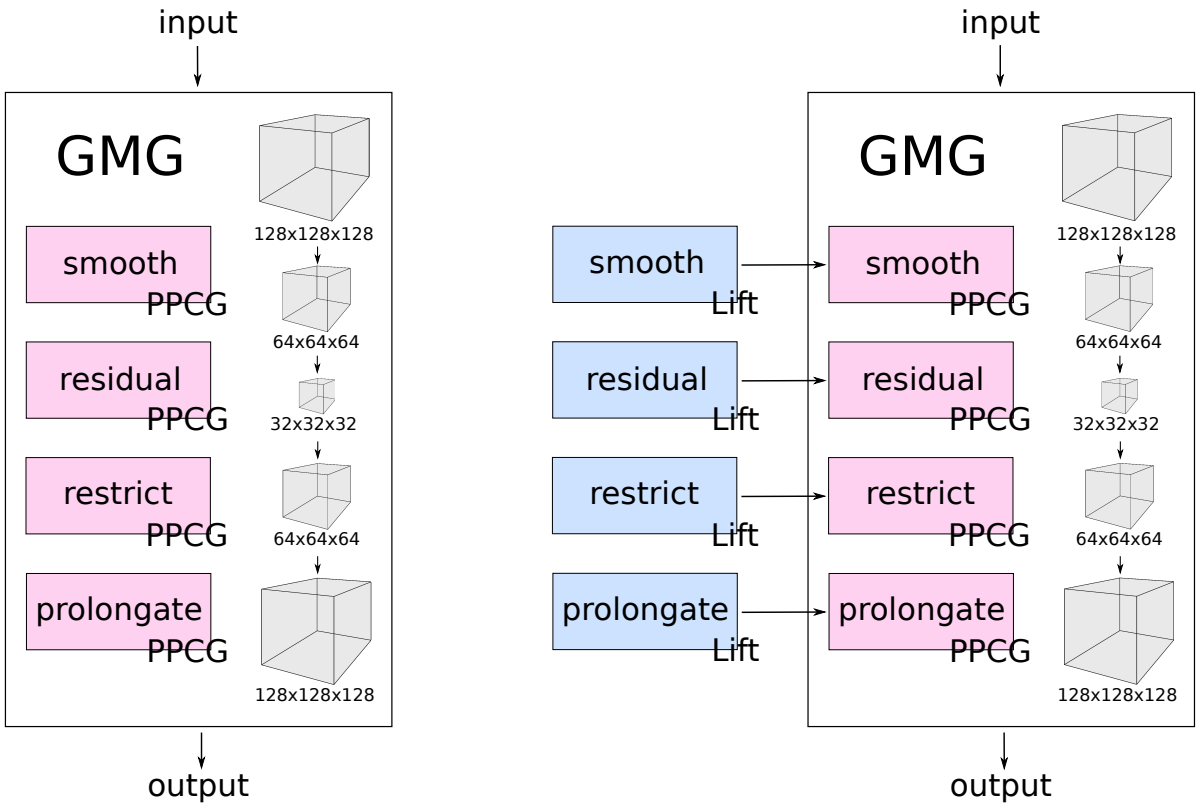

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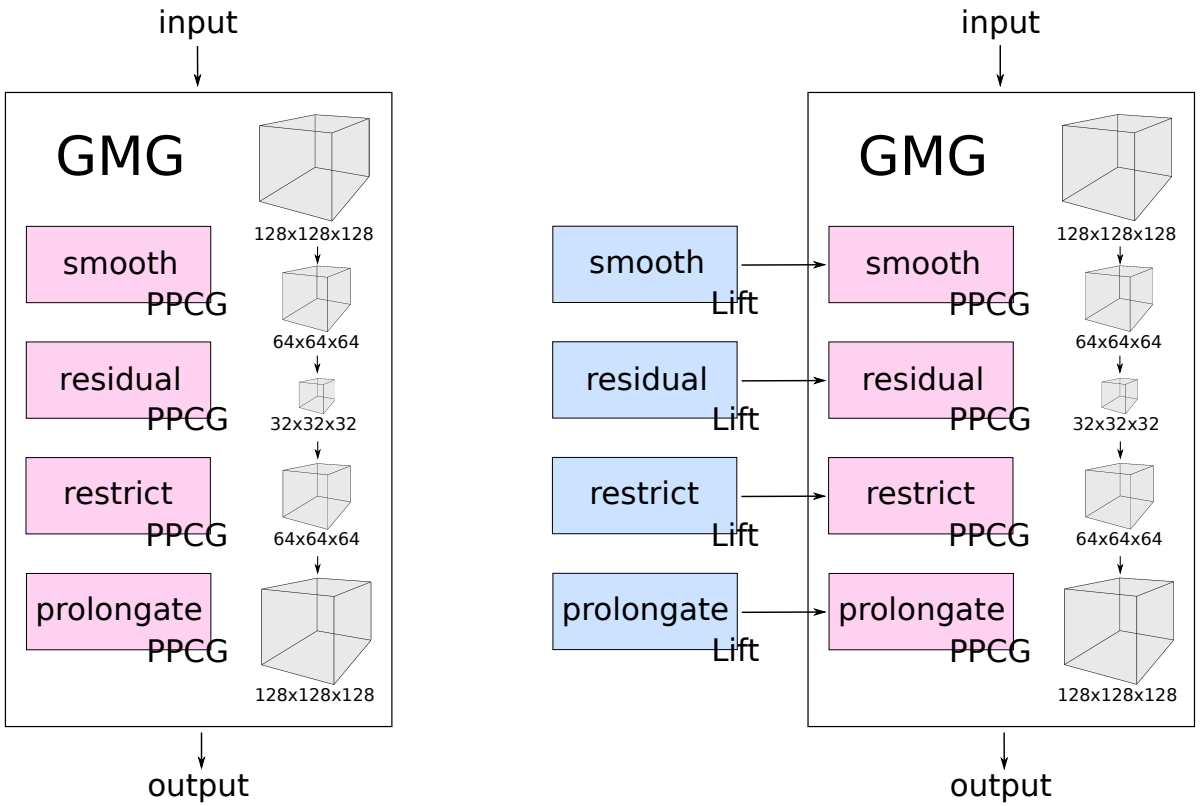

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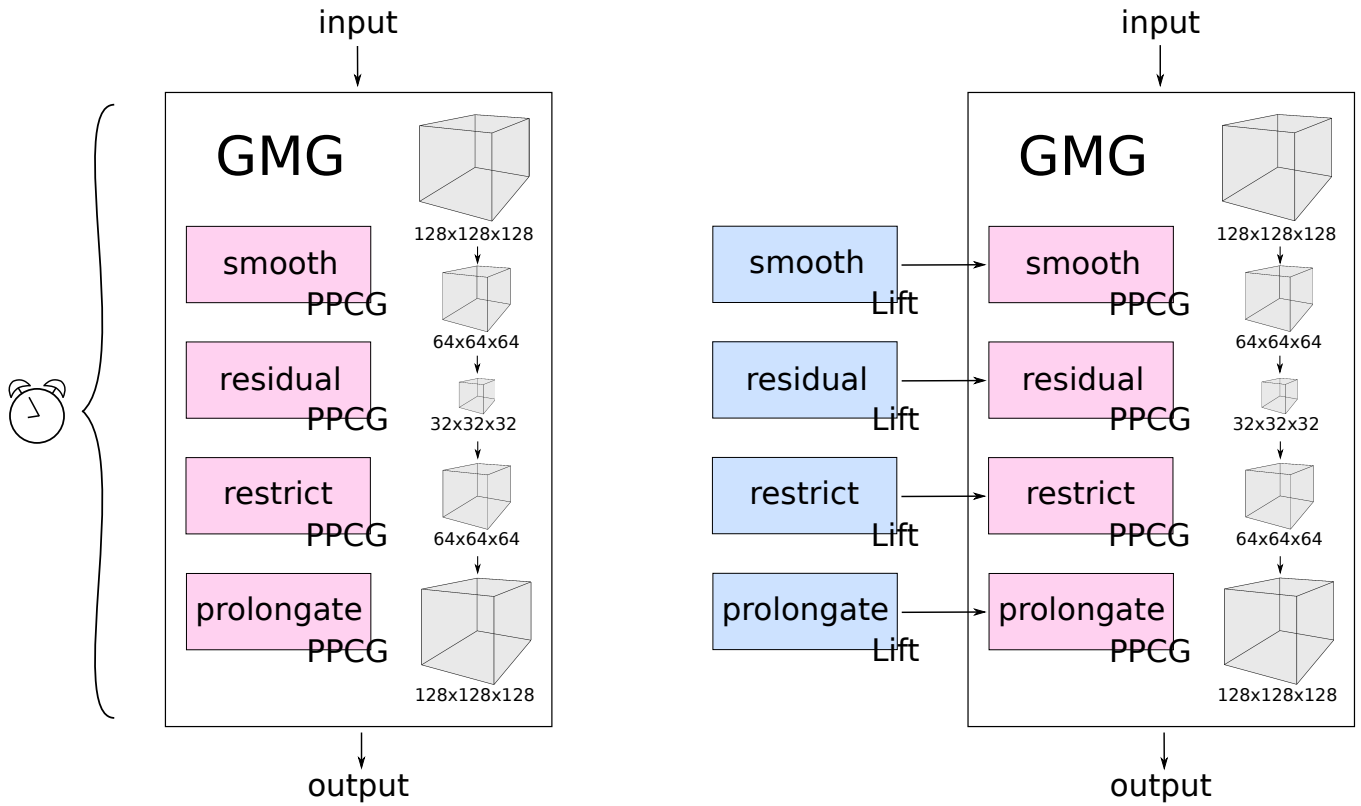

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GMG Program Runtime Comparison

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- Smooth executed 24 times

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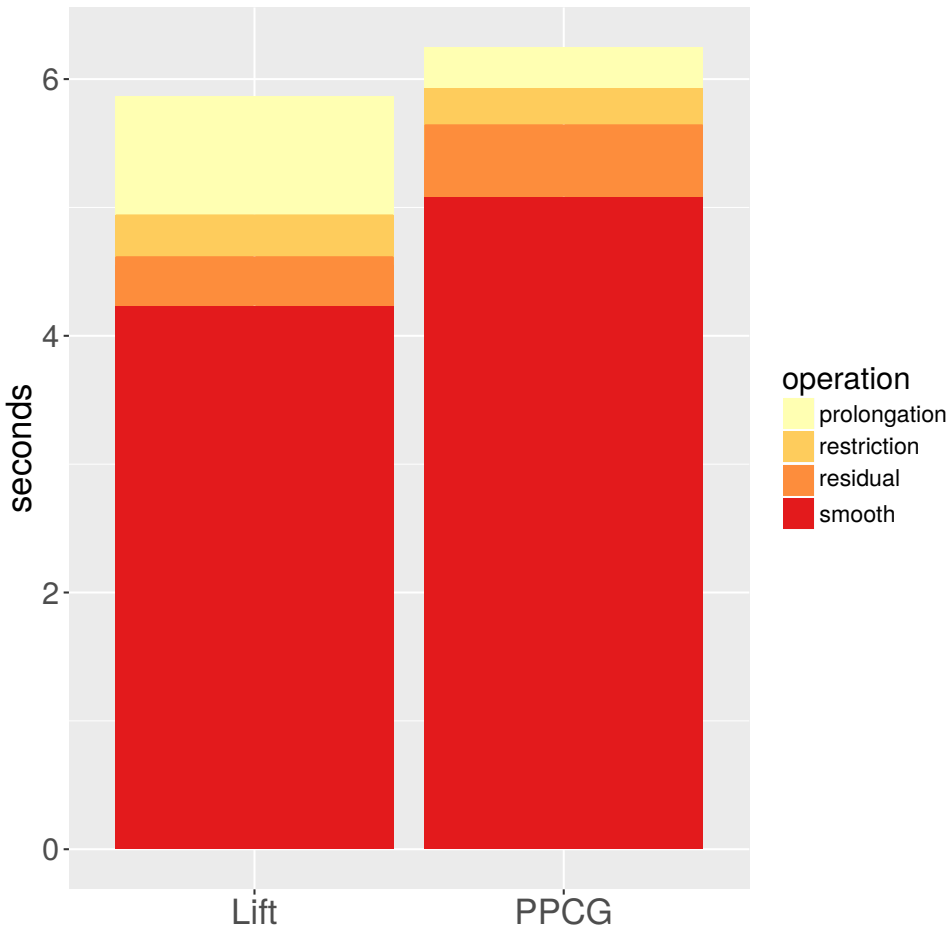

GMG Program Runtime Comparison

- Residual, restrict, prolongate each executed 2 times
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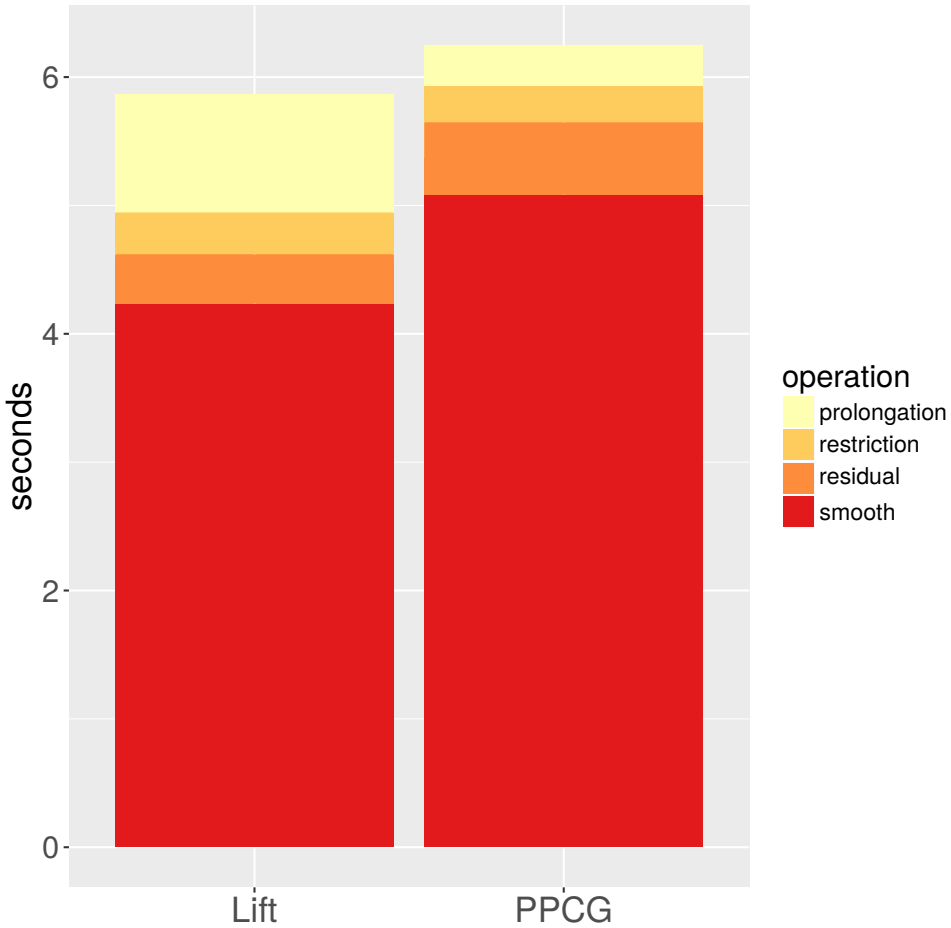
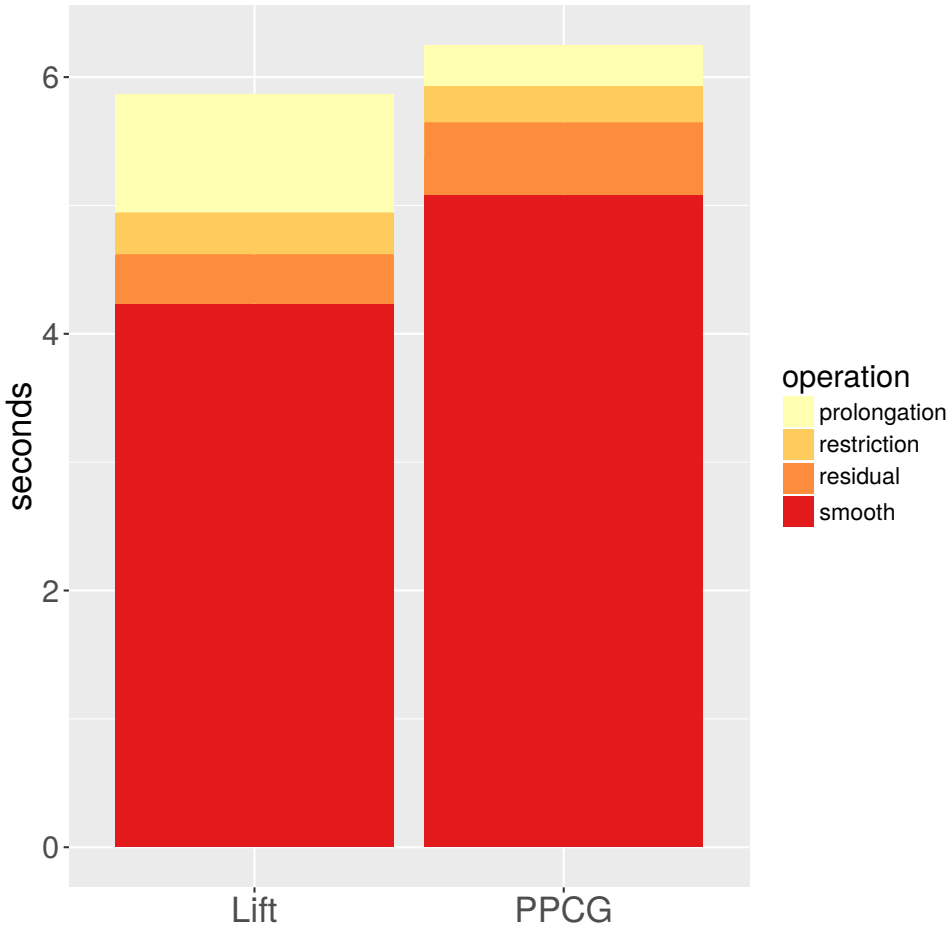
- Small improvement in smooth has large impact on overall runtime



GMG Program Runtime Comparison

- Residual, restrict, prolongate each executed 2 times
- Smooth executed 24 times

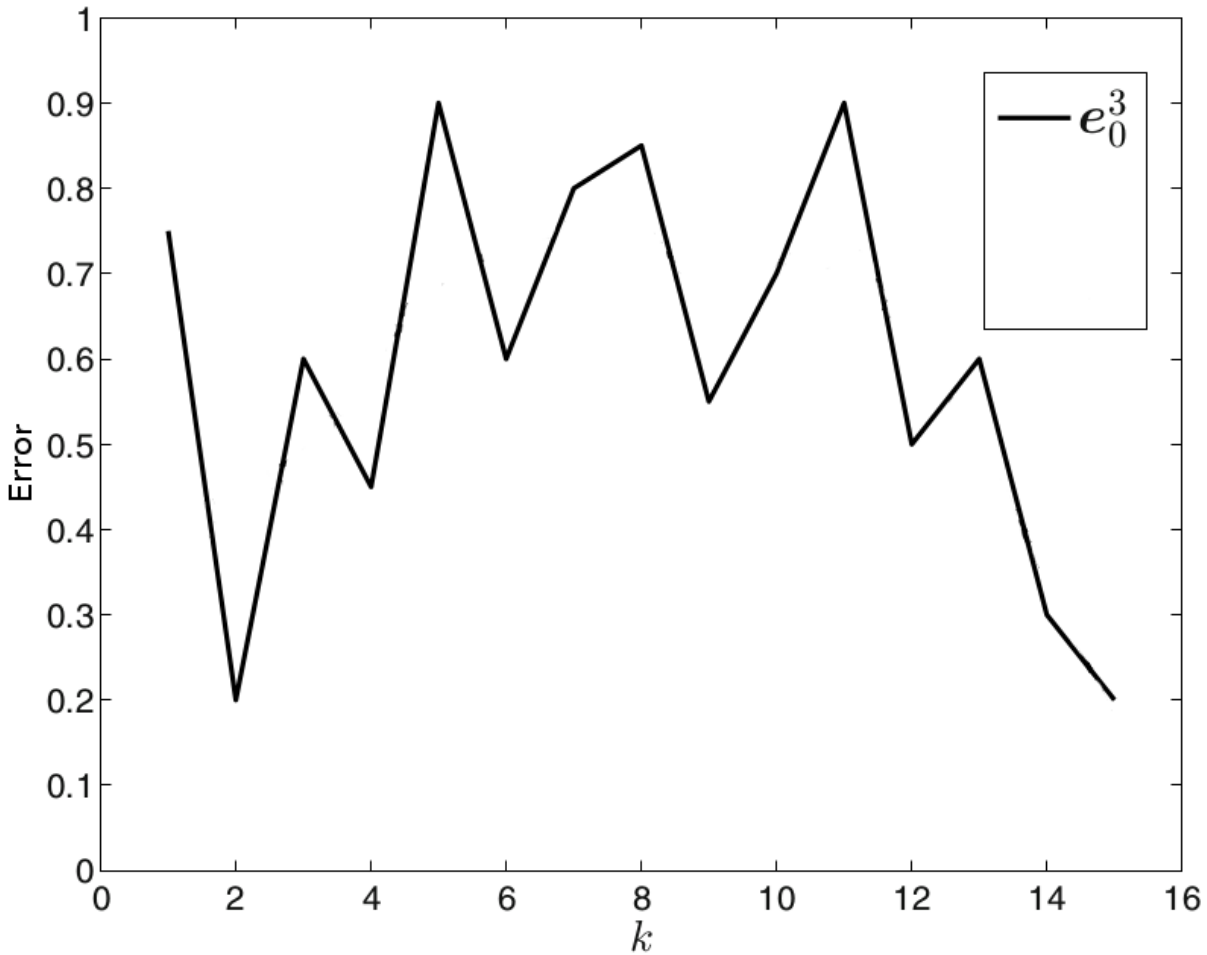
- Small improvement in smooth has large impact on overall runtime
- Experiments with optimizations for iterative kernels in Lift



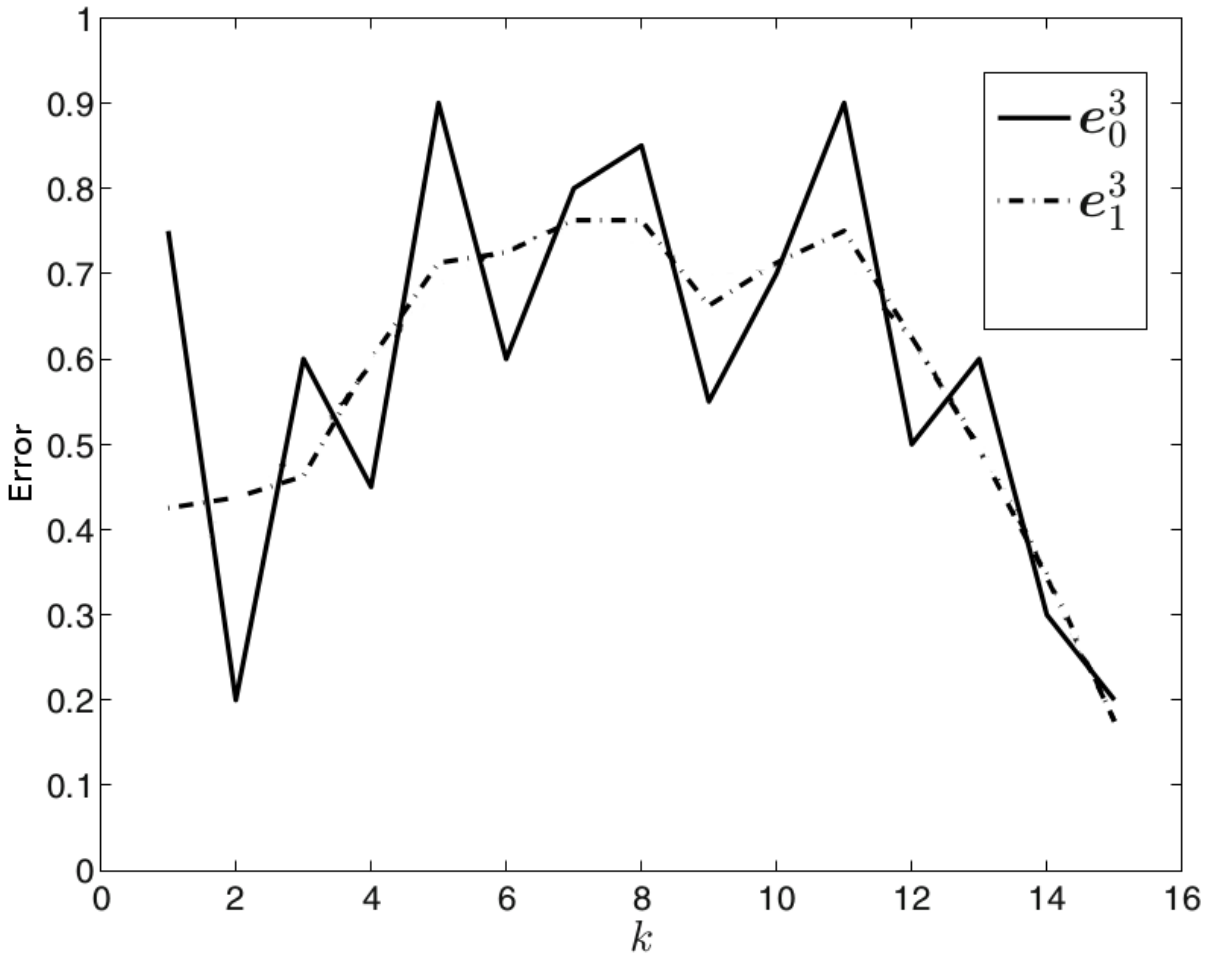
Questions?

Backup Slides

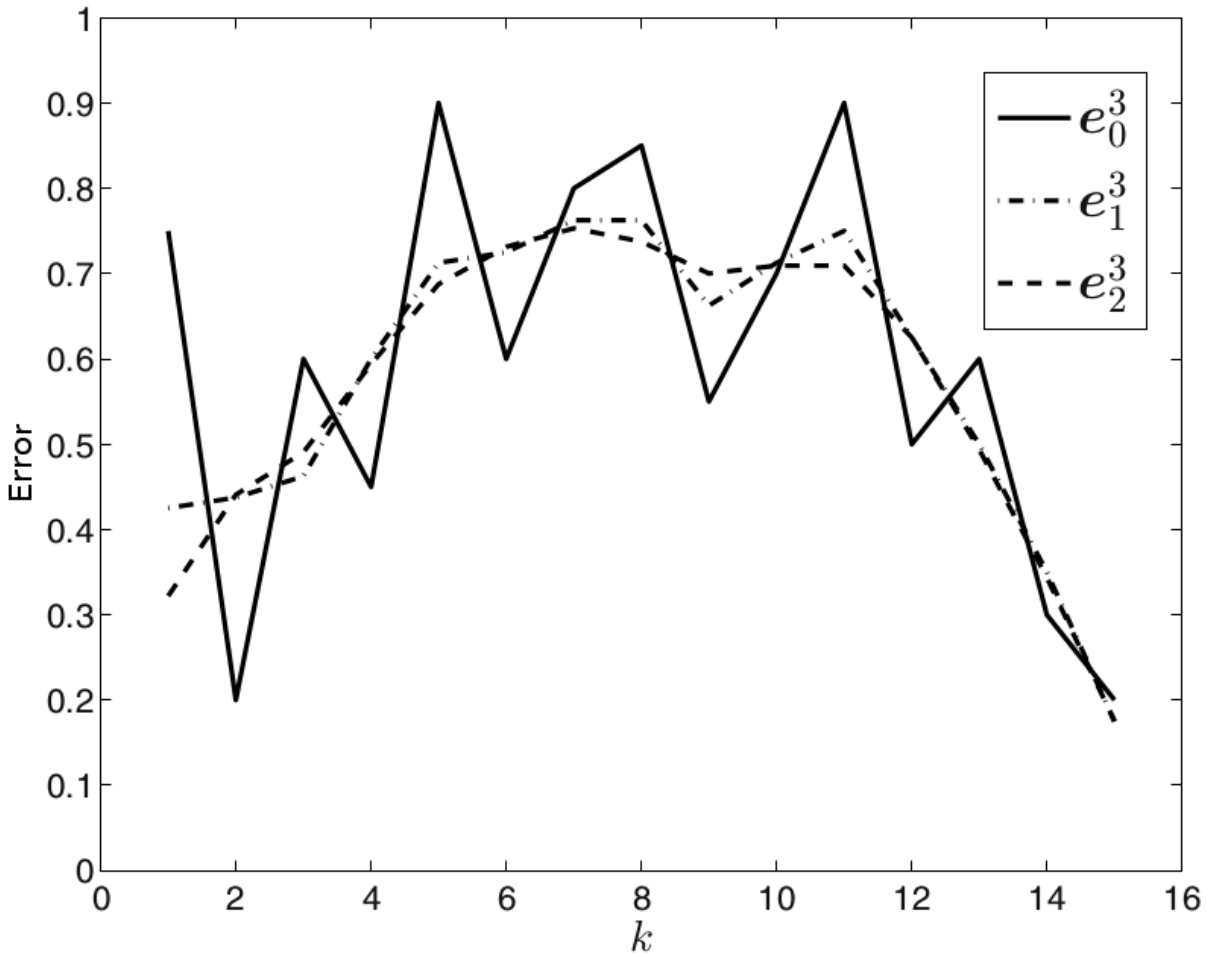
Error Smoothing



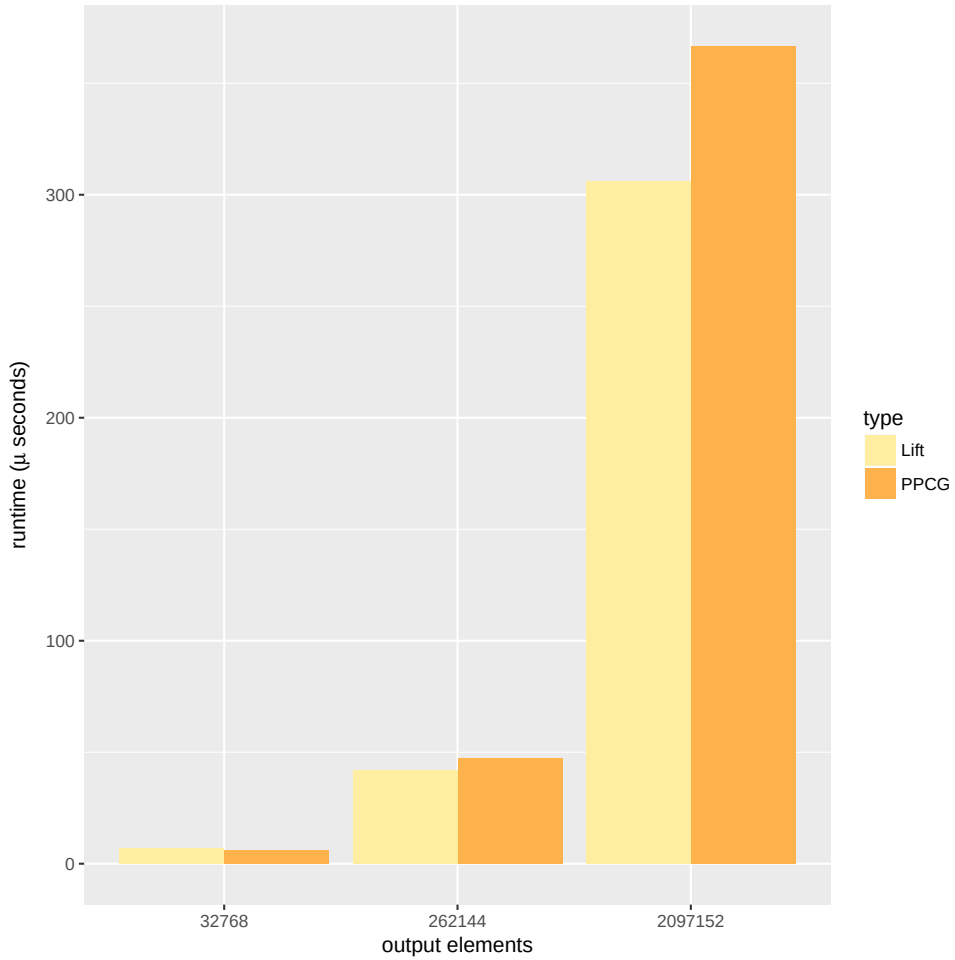
Error Smoothing



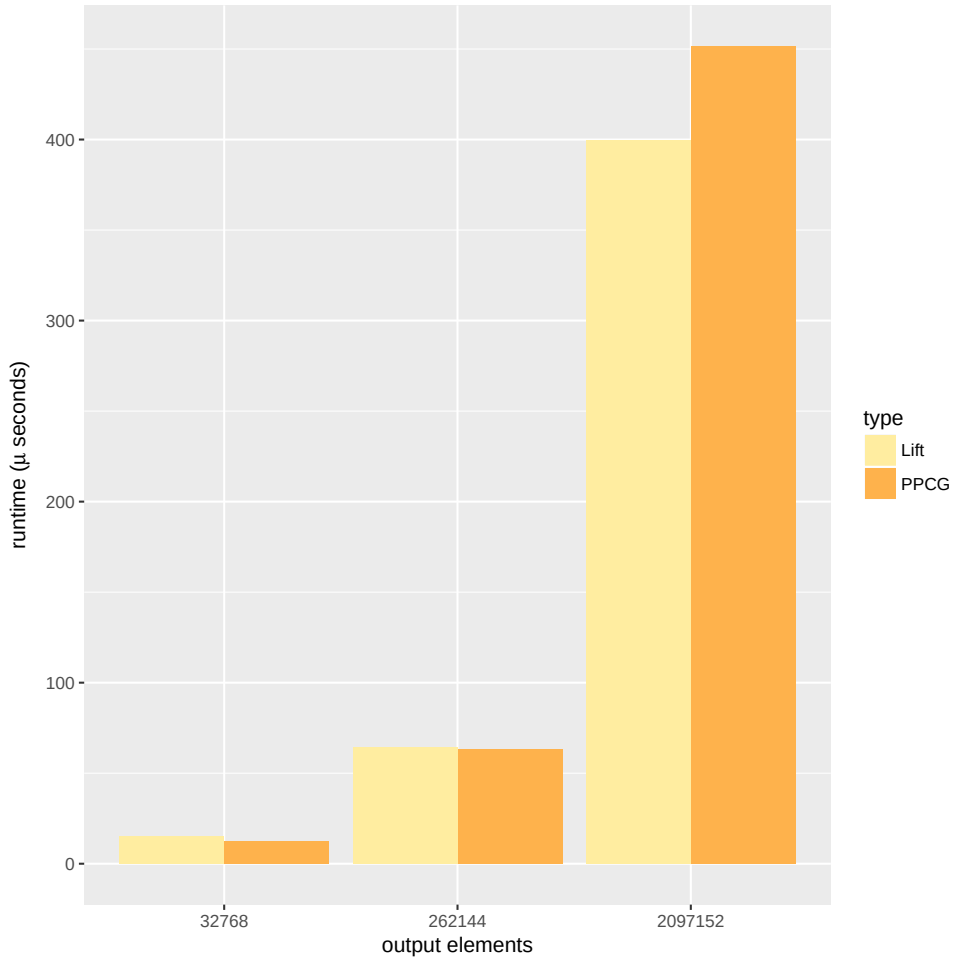
Error Smoothing



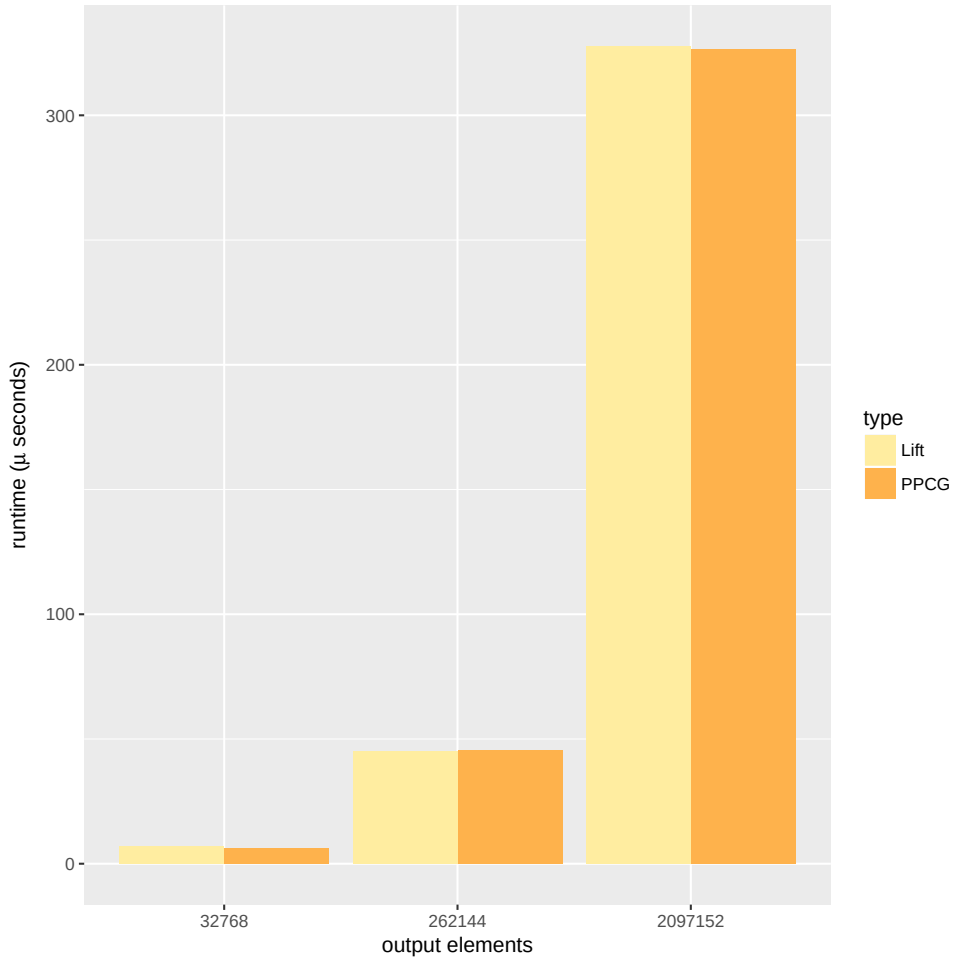
Smooth



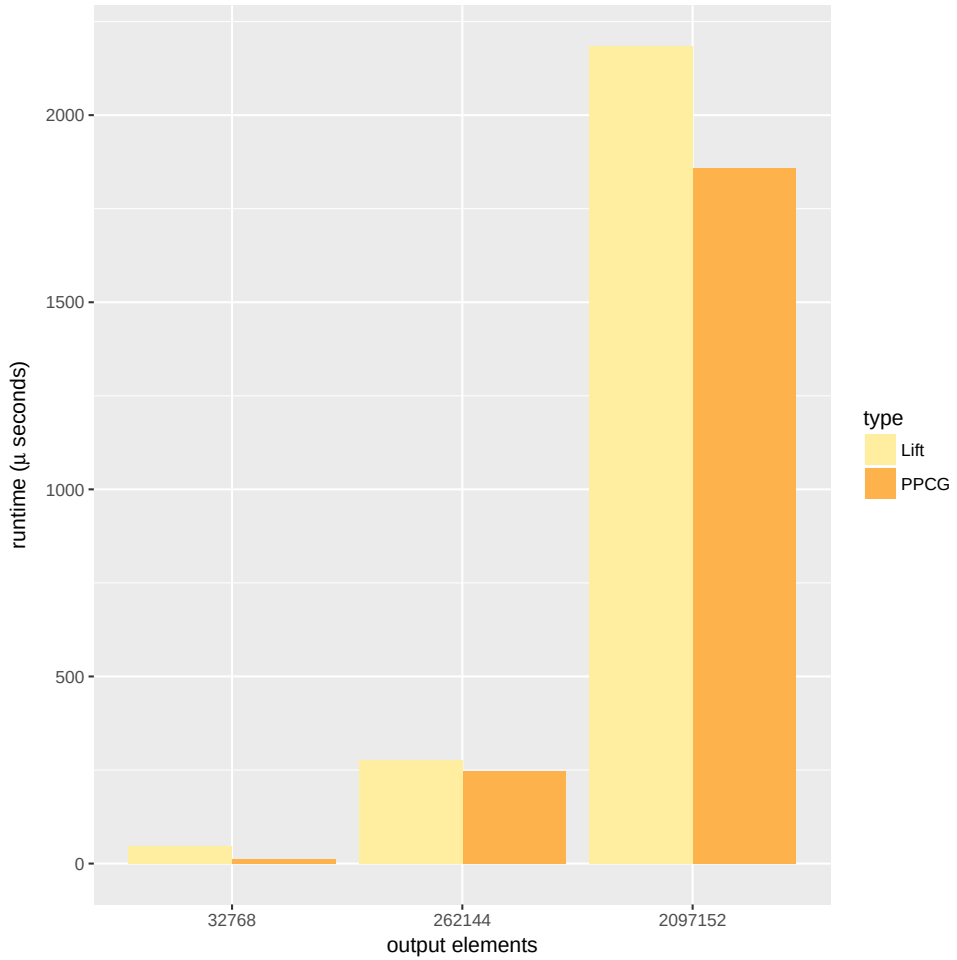
Residual



Restrict



Interpolate



Comparison