We now take the expectation of the same element y_i:

$$\mathbb{E}[y_i] = \mathbb{E}\left[\sum_j x_{ij}\beta_j + \epsilon_i\right]$$

Using lnearity to split:

$$\mathbb{E}[y_i] = \mathbb{E}\left[\sum_j x_{ij}\beta_j\right] + \mathbb{E}[\epsilon_i]$$

We know from our assumption that ϵ has zero mean.

And $x_{ij}\beta_j$ is deterministic (no randomness)

This means:

$$\mathbb{E}\left[\sum_j x_{ij}\beta_j\right] = \sum_j x_{ij}\beta_j$$

Therefor

$$\mathbb{E}[y_i] = \sum_j x_{ij}\beta_j$$

Using matrix notation since $x_{ij}\beta_j$ is the dot product of the i-th row of X with β.

$$x_{ij}\beta_j = X_{i,*}\beta \implies \underline{\mathbb{E}[y_i] = \Sigma_{ij}\beta_j = X_{i,*}\beta.}$$