Show: $Var(y_i) = \sigma^2$

Using the same assumptions

Have our regression model (linear).

$$y = X\beta + \epsilon$$

As illustrated earlier:

$$y_i = \sum_j x_{ij} \beta_j + \epsilon_i,$$

$\sum x_{ij} \beta_j$ is deterministic term, which result in 0 variance. Therefor the $var(y_i)$ is only influenced by ϵ. ϵ is independent of the predictor.

$$Var(y_i) = var(\overbrace{\sum x_{ij} \beta_j}^{\to 0}) + Var(\epsilon_i)$$

$$var(y_i) = var(\epsilon_i)$$

From assumption $\epsilon \sim N(0, \sigma^2)$

$$Var(\epsilon) = \sigma^2$$

$$\underline{var(y_i) = \sigma^2}$$