Show $\mathbb{E}[\hat{\beta}] = \beta$ for OLS.

For OLS, we have

$$\hat{\beta} = (X^T X)^{-1} X^T y \qquad (1)$$

Our linear model say:

$$y = X\beta + \epsilon \qquad (2)$$

Substitute y in expression (1)

$$\hat{\beta} = (X^T X)^{-1} X^T (X\beta + \epsilon)$$

$$= (X^T X)^{-1} X^T X \beta + X^T \epsilon (X^T X)^{-1}$$

We notice that

$$(X^T X)^{-1} X^T X = I$$

$$\hat{\beta} = I\beta + (X^T X)^{-1} X^T \epsilon$$

Taking the expection:

$$\mathbb{E}[\hat{\beta}] = \mathbb{E}[\beta + (X^T X)^{-1} X^T \epsilon]$$

Expected value of 0, so it makes the whole expression vanish.

$$\mathbb{E}[\hat{\beta}] = \mathbb{E}[\underbrace{\beta}_{\text{Deterministic}} + 0]$$

$$\mathbb{E}[\hat{\beta}] = \beta$$