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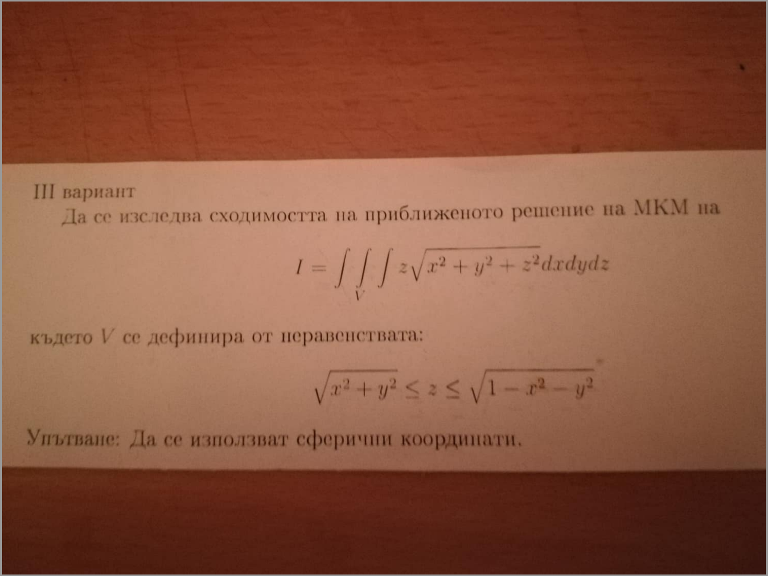
The Problem



III вариант

Да се изследва сходимостта на приближеното решение на МКМ на

$$I = \iiint\limits_V z\sqrt{x^2+y^2+z^2}\,dxdydz$$

където V се дефинира от неравенствата:

$$\sqrt{x^2+y^2} \le z \le \sqrt{1-x^2-y^2}$$

Упътване: Да се използват сферични координати.

Figure 1: problem

Solution

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^1 r^4 cos(\theta) sin(\theta) dr d\theta d\psi = \pi/10$$

Solution

$I_1 = \int_0^{2\pi} d\psi$
$I_2 = \int_0^{\pi/4} cos(\theta)sin(\theta)d\theta$
$I_3 = \int_0^{1} r^4 dr$
$I = I_1 I_2 I_3 = \pi/10$

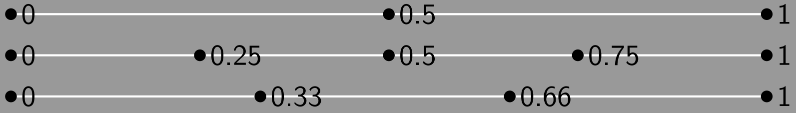
Basic monte carlo

- $I = \int_a^b f(x)dx$ - Integral
- n - size; $\Delta = b - a$
- $x_1, .., x_n, x_i \in [a, b]$
- $y_i = \frac{f(x_i)\Delta}{n}$
- $I = \sum_{i=1}^{n} y_i$
- error - $O(N^{-1/2})$

Vand der Corput

- corput(n, b), n-size, b-base
- The algorithm splits 1D axis to n intervals by base b

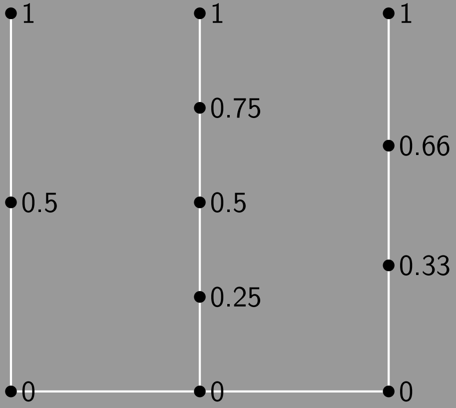
Vand der Corput



Halton

- $S_b(n) := \frac{\pi_b(d_0)}{b} + \frac{\pi_b(d_1)}{b^2} + ... + \frac{\pi_b(d_j)}{b^{j+1}}$, π_b - permutations
- $x_n = (S_{b1}(n), ..., S_{bs}(n)), n = 0, 1, ...$
- Halton returns a matrix nxb

Halton

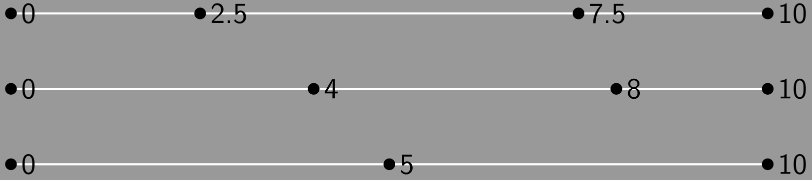


Quasi monte carlo

- Same algorithm as basic monte carlo
- uses quasi random numbers

Symmetric monte carlo

- Same algorithm as basic monte carlo
- $f(x_i) = \frac{f(x_i)+f(b-a-x-i)}{2}$
- no result in $x_i = \frac{b-a}{2}$



First Result

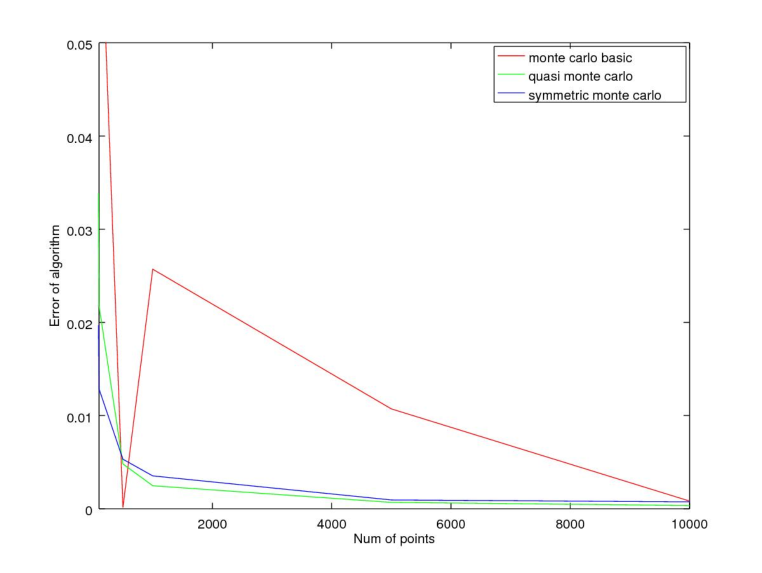


Figure 2: first result

Second Result

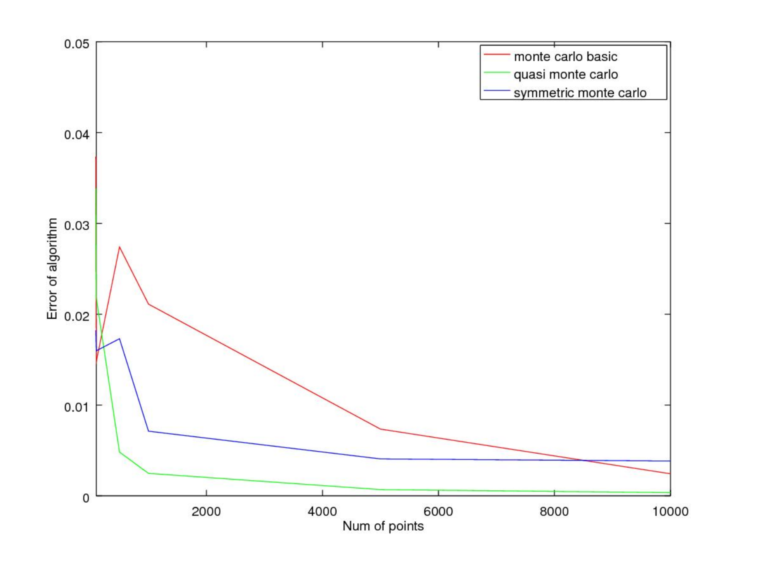


Figure 3: second result

Third Result

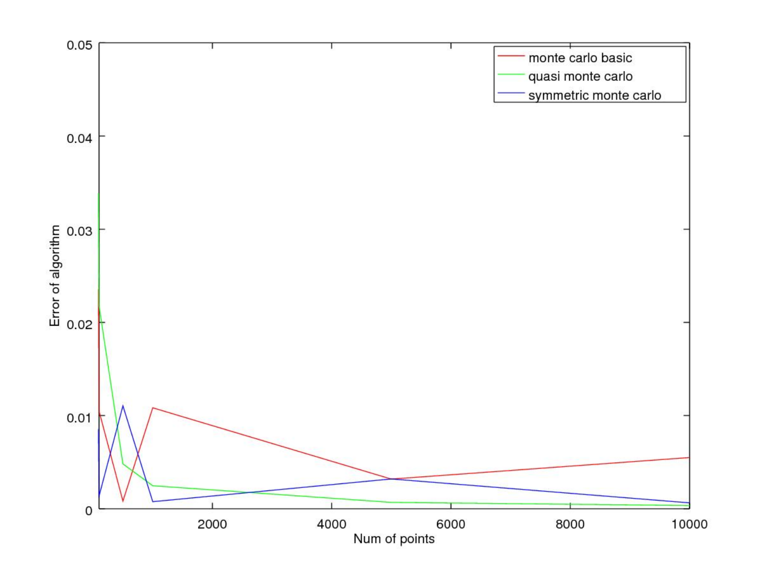


Figure 4: 3rd result

Fourth Result

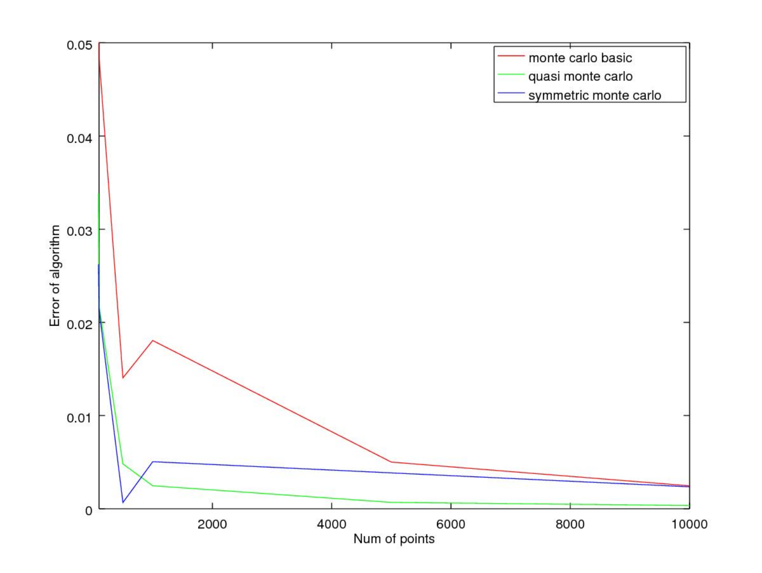


Figure 5: 4th result

After 100 iterations

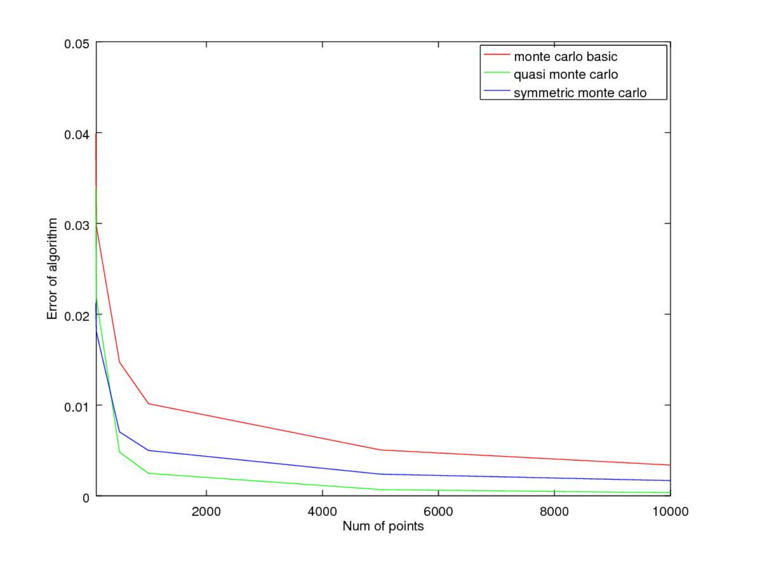


Figure 6: 100 iterations