MODULE $EWD840$

TLA+ specification of an algorithm for distributed termination detection on a ring, due to *Dijkstra*, published as *EWD* 840: Derivation of a termination detection algorithm for distributed computations (with *W.H.J.Feijen* and *A.J.M.* van *Gasteren*).

EXTENDS $Naturals$

CONSTANT N
ASSUME $NAssumption \triangleq N \in Nat \setminus \{0\}$

VARIABLES $active$, $color$, $tpos$, $tcolor$

$Node \triangleq 0 \,..\, N-1$
$Color \triangleq \{\text{“white”}, \text{“black”}\}$

$TypeOK \triangleq$
- $\wedge$ $active \in [Node \to \text{BOOLEAN}\ ]$ status of nodes (active or passive)
- $\wedge$ $color \in [Node \to Color]$ color of nodes
- $\wedge$ $tpos \in Node$ token position
- $\wedge$ $tcolor \in Color$ token color

Initially the token is black. The other variables may take any “type-correct” values.

$Init \triangleq$
- $\wedge\ active \in [Node \to \text{BOOLEAN}\ ]$
- $\wedge\ color \in [Node \to Color]$
- $\wedge\ tpos \in Node$
- $\wedge\ tcolor = \text{“black”}$

Node 0 may initiate a probe when it has the token and when either it is black or the token is black. It passes a white token to node $N-1$ and paints itself white.

$InitiateProbe \triangleq$
- $\wedge\ tpos = 0$
- $\wedge\ tcolor = \text{“black”} \vee color[0] = \text{“black”}$
- $\wedge\ tpos' = N-1$
- $\wedge\ tcolor' = \text{“white”}$
- $\wedge\ active' = active$
- $\wedge\ color' = [color \text{ EXCEPT } ![0] = \text{“white”}]$

A node i different from 0 that possesses the token may pass it to node $i-1$ under the following circumstances :
- node i is inactive or
- node i is colored black or
- the token is black.

Note that the last two conditions will result in an inconclusive round, since the token will be black. The token will be stained if node i is black, otherwise its color is unchanged. Node i will be made white.

$PassToken(i) \triangleq$

$\land\ tpos = i$
$\land\ \neg active[i] \lor color[i] = \text{“black”} \lor tcolor = \text{“black”}$
$\land\ tpos' = i - 1$
$\land\ tcolor = \text{IF } color[i] = \text{“black”} \text{ THEN “black” ELSE } tcolor$
$\land\ active' = active$
$\land\ color' = [color \text{ EXCEPT } ![i] = \text{“white”}]$

token passing actions controlled by the termination detection algorithm

$System \triangleq InitiateProbe \lor \exists\, i \in Node \setminus \{0\} : PassToken(i)$