Workshop Notation: Extending the Relational Event Model

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1 Introduction to Relational (Hyper) Event Models

Symbol	**Description**
$V = \{1, 2, \ldots, p\}$	Set of entities in the system
$e = (s, r, t)$	Relational event
$s \subset V$	Sender of event e
$S \subseteq V$	Senders of hyperevent / Actors of a meeting e
$r \subset V$	Receiver of event e
$R \subseteq V$	Receivers of hyperevent e
$t \in \mathbb{R}^+$	Time of event
$E = \{e_1, \ldots, e_i, \ldots, e_n\}$	Event sequence
n	Number of events in the event sequence
i	Index of a generic event in the event sequence
$\{N_{sr}(t)\}_{s \subset V, r \subset V, t \in \mathbb{R}^+}$	Longitudinal network / multivariate counting process
$N_{sr}(t)$	Counting process evaluated for dyad (s, r) at time t
$\Lambda_{sr}(t)$	Cumulative intensity function
$M_{sr}(t)$	Martingale residual
$\lambda_{sr}(t)$	Hazard / instantaneous rate / intensity function
$W_{sr}(t)$	Risk Indicator Function
$\lambda_0(t)$	Baseline Function
$f_{sr}(t)$	(Log-Hazard) Contribution Function
$\Delta T_{sr}(t)$	Waiting Time for dyad (s, r);

2 Relational (Hyper) Event Statistics

Symbol	Description
$\boldsymbol{x}_{sr}(t)$	Vector of **generic statistics** for dyad (s, r) at time t;
$\boldsymbol{z}_{sr}(t)$	Vector of **generic attributes** for dyad (s, r) at time t;
name_statistics(s, r, t)	**particular** statistic / attribute
	named name_statistics for dyad (s, r) at time t;
att(s, s', t)	N. of previous events from s to s' before t (also prior(s, r, t));
`repetition`	N. of previous events from s to r before t;
`reciproc.`	N. of previous events from r to s before t;
`outDegSource`	N. of previous events from s to s' before t for all possible s';
`inDegTarget`	N. of previous events from s' to r before t for all possible s';
`transitive`	N. of previous events from s to s' times the ones from s' to r before t
	for all possible s' (also `trans.`);
`cycl.`	N. of previous events from s' to s times the ones from r to s' before t
	for all possible s';
`shared rec.`(s,r,t)	N. of previous events from s to s' times the ones from r to s' before t
	for all possible s';
prior(s, s', t)	N. of previous events from s to s' before t;
recent(s, s', t)	N. of previous events from s to s' before t
	with larger weight to recent events;
$\mathcal{R}_t$	Risk set;
$\boldsymbol{z}_S(t)$	Vector of **generic attributes** for undirected hyperevent S at time t;
repetition(S, t)	N. of previous events with (exactly) S;
hy.deg(S, t)	N. of previous events involving S;
sub.rep$^{(k)}(S, t)$	Average N. of previous joint events involving S',
	for all possible subsets $S' \subset S$ of size k;
$\binom{S}{k}$	Set of all possible subsets of S of size k;
$\binom{\lvert S\rvert}{k}$	$\lvert S\rvert$ choose k;
closure(S, t)	Average N. of two-paths via a third node for any pair of nodes in S
avg.z(S, t)	Average attribute value for hyperevent,
	computed as mean of attribute values of individuals;
diff.z(S, t)	Average absolute difference among attribute values
	of pairs of individuals, for all possible pairs in the hyperevent;
	computed as mean of attribute values of individuals;
$\mathbb{E}_M(\cdot)$	Expectation according to model M;

3 Relational (Hyper) Event Model: Inference

Symbol	Description
$\mathbb{W} = \{\mathcal{W}_t\}_{t\in\mathbb{R}^+}$	Filtration arising from events and exogenous information
$\Delta N_{sr}(t_i)$	Difference between counts of events between t_{i-1} and t_i
$\ell(\boldsymbol{\beta})$	Log-Likelihood Function
$\mathcal{L}^P(\boldsymbol{\beta})$	Partial Likelihood Function
$\ell^P(\boldsymbol{\beta})$	Partial Log-Likelihood Function
$\mathcal{SR}_t$	Sampled Risk set evaluated at time t
m	number of sampled non-events
$\mathbb{F} = \{\mathcal{F}_t\}_{t\in\mathbb{R}^+}$	Filtration integrated with sampling information
$\pi_t(\mathcal{SR}\|(s,r))$	Probability of sampling the risk set given the observed dyad
y_i	Response of (Conditional) Logistic Regression for observation i
π_i	Success Probability of (Conditional) Logistic Regression for observation i
$\lambda_{sr,\mathcal{SR}}(t)$	Intensity of the sampling counting process
$\mathcal{L}^S(\boldsymbol{\beta})$	Sampled Partial Likelihood Function
$\ell^S(\boldsymbol{\beta})$	Sampled Partial Log-Likelihood Function
$\Delta\boldsymbol{x}_i$	Difference between the covariate evaluated for the event and the non-event

4 Mixed Effect Additive Relational Event Models

Symbol	Description
$\boldsymbol{\beta}(t)$	Time-varying effect vector
$f(\boldsymbol{x}_{sr}(t))$	Non-linear effect of $x_{sr}(t)$
$\boldsymbol{\gamma}$	Random effect vector
$b(\cdot)$	Basis function
j	Index of a generic basis $b_j(\cdot)$
q	Spline dimension
$\boldsymbol{\theta}$	Basis coefficient vector
$\boldsymbol{x}_{sr}^{(1)}$	Vector of covariates with linear fixed effect
$\boldsymbol{x}_{sr}^{(2)}$	Vector of covariates with time-varying effect
	Speciically, Bike example: distance from public transport
$\boldsymbol{x}_{sr}^{(3)}$	Vector of covariates with non-linear effect
	Speciically, Bike example: distance between bike stations
$\boldsymbol{z}_{sr}$	Vector of covariates with random effect
	Bike example: start station indicator
σ^2	variance of a single random effect

5 Goodness of Fit

Symbol	Description
$G[\hat{\boldsymbol{\beta}}, t]$	Observed cumulative martingale residual process
$\hat{J}$	Variance-covariance matrix of process G
$\mathcal{I}[\hat{\boldsymbol{\beta}}]$	Observed information matrix
S	penalty matrix
T	test statistic

6 Advanced Relational Hyper Event Models

Symbol	**Description**
V_1	Sender set in a two-mode relational graph;
V_2	Receiver set in a two-mode relational graph;
V_t	Set of potential nodes at time t
$V_{*,t}$	Set of nodes of type $*$ at time t
$V_{2,t}(s)$	Set of receiver at time t available for sender s;
$\overline{\lambda}_{s,\lvert R\rvert}(t)$	baseline intensity in directed RHEM ;
$\mathcal{L}^P(\boldsymbol{\beta}, t)$	Partial likelihood
$\lambda_{sR}^{(\text{dyadic})}(t)$ / $\lambda_{sR}^{(\text{rhem})}(t)$	intensity process
repetition(s, R, t)	weigthed sum of previous $s \to R$ events before t
hy.deg$^{(in)}(R', t)$	N. (or weighted count) of previous events with receiver set $\subset R'$
r.sub.rep$^{(k)}(s, R, t)$	Average N. (or weighted count) of previous joint events with receivers set $\subset R$
hy.deg(s, R', t)	N. (or weighted count) of previous events from s to receiver set $\subset R'$
s.r.sub.rep$^{(k)}(s, R, t)$	Average N. (or weighted count) of previous joint events with receivers set $\subset R$
interact.receivers$^{(k)}(s, R, t)$	Interaction among receivers
reciprocation(s, R, t)	N. (or weighted count) of previous events from $r \in R$ to s over $\lvert R\rvert$
deg$^{(out)}(s', t)$	N. (or weighted count) of previous events from s' (to whoever)
gen.recip(s, R, t)	N. (or weighted count) of previous events from $r \in R$ (to whoever) over $\lvert R\rvert$
trans.closure(s, R, t)	N. (or weighted count) of previous triangles $s \to a$ and $a \to r \in R$ for possible a (over $\lvert R\rvert$)
cyclic.closure(s, R, t)	N. (or weighted count) of previous 2-path $a \to s$ and $r \in R \to a$ for possible a (over $\lvert R\rvert$)
shared.sender(s, R, t)	N. (or weighted count) of previous 2-path$a \to s$ and $a \to r \in R$ for possible a (over $\lvert R\rvert$)
shared.receiver(s, R, t)	N. (or weighted count) of previous 2-path $s \to a$ and $r \in R \to a$ for possible a (over $\lvert R\rvert$)
$e_i = (S_i, t_i, x_i, y_i)/e_i = (S_i, t_i, y_i)$	Relational event with outcome y_i and covariates x_i;
hy.deg(S', t, E) / hy.deg$(S', t, G[E, t])$	Same as hy.deg(S', t)
sub.rep$^{(k)}(S, t, E)$	Same as sub.rep$^{(k)}(S, t)$
performance(S', t)	Total prior success;
prior.success$^{(k)}(S, t)$	Prior share success of order k;
$e_i = ([S_i, l_i], t_i)$	Labeled (l_i) relational hyperevent;
$e_i = (r(t_i), S_i, R_i, t_i)$	Publication of paper $r(t_i)$ at time r_i by authors S_i citing papers R_i;
$r(t)$	paper published at time t;
cocite(R', t)	N. (or weighted count) of previous publication events with common references R';
cocite rep$^{(k)}(s, R, t)$	Average N. (or weighted count) of publication joint events including set of references of size k subset of R;
cite paper and its refs(R, t)	
cite coauthor(S, R, t)	
coauth(s, s', t)	
author(s', r, t)	
auth cite repet(S, R, t)	
auth cite recipr(S, R, t)	