# Relational Event Models 2.0

Mixed effect additive REMs - Tutorial Section: Sheet

Boschi, Martina

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# Mixed Effect Additive REMs: Application to Alien Species Invasions

# 0. Preparatory Steps: Introducing Alien Species Invasions

#### 0.1. Installing libraries

In the following section, we will install the necessary packages for this tutorial. The provided code ensures that the packages are installed only if they are not already present on your system.

```
if (!require("mgcv", quietly = TRUE)) {
  # If not installed, install it
  install.packages("mgcv")
  # Load the package
 library("mgcv")
} else {
  if (!require("splines", quietly = TRUE)) {
    install.packages("splines")
   library("splines")
    if (!require("ggplot2", quietly = TRUE)) {
      install.packages("ggplot2")
      library("ggplot2")
   } else {
      if (!require("tidyverse", quietly = TRUE)) {
        install.packages("tidyverse")
        library("tidyverse")
      } else {
        if (!require("RColorBrewer", quietly = TRUE)){
          install.packages("RColorBrewer")
        } else {
          if (!require("mgcViz", quietly = TRUE)){
            install.packages("mgcViz")
          } else {
            library("mgcv")
            library("splines")
            library("ggplot2")
            library("tidyverse")
            library("RColorBrewer")
            library("mgcViz")
          }
       }
```

```
}
}
}
```

During the tutorial, we will refer to the following color palettes to improve the visualization of the results.

## 0.2. Introducing alien species invasions



# Demo 1:

D1.1: Load the input data "input00.RData".

```
load(file="01-Data/01-Inputs/input00.RData")
```

D1.2: Inspect the original data contained in the object FR. Have a look at the columns: "LifeForm", "Taxon", "Region", "FirstRecord", "Source".

```
## LifeForm Taxon Region FirstRecord
## 1 Algae Acanthophora muscoides Turkey 1986
## 2 Algae Acanthophora nayadiformis Cyprus 1997
```

```
## 3
        Algae Acanthophora navadiformis
                                                                    1970
                                                     Turkey
## 4
                  Acanthophora spicifera Hawaiian Islands
                                                                    1952
        Algae
## 5
        Algae
                  Acetabularia caliculus
                                                     Israel
                                                                    1943
##
        Algae
                  Acetabularia caliculus
                                                                    1957
  6
                                                      Spain
##
                        Source
## 1
          Cinar et al. (2005)
## 2
                        DAISIE
## 3
          Cinar et al. (2005)
## 4 Carlton & Eldrege (2009)
## 5
                        DAISIE
## 6
                        DAISIE
```

D1.3: Interpret this data as a relational event network. Define sender set, receiver set, and time-window.

D1.4: Inspect the object native. A species' native range (NR) is the collection of areas where it is indigenous. How would you relate this concept to that of risk set?

```
native[sample(1:nrow(native), 10),c("species", "region")]
```

```
##
                          species
                                          region
## 996157
             Aonidomytilus albus
                                            Cuba
## 1033011
                Vespula vulgaris
                                           China
## 1009074
                    Hylastes ater
                                        Denmark
## 1010145
                Lilioceris lilii
                                        Slovakia
## 1021700
            Pheidole megacephala
                                   South Africa
## 998489
               Bombus terrestris
                                           Corse
## 1009158 Bostrichus ligniperda
                                          Russia
## 995569
              Anastrepha obliqua
                                      Nicaragua
## 1001341
              Chilo suppressalis
                                           India
## 1001809
                 Cinara cupressi United States
```

D1.5: Inspect the object first\_records. Describe this relational event network and identify why it can be considered a subset of the previous one.

```
##
         year
                   lf
                                      species
                                                      region
## 9337
         1880 Insects
                        Adelges nordmannianae
                                                 Switzerland
## 15937 1881 Insects
                         Pheidole megacephala
                                                      France
## 16225 1884 Insects
                                  Pineus pini
                                                 New Zealand
## 12792 1886 Insects
                             Eulachnus rileyi United States
## 14019 1887 Insects
                           Linepithema humile
                                                    Portugal
## 11797 1889 Insects Ctenarytaina eucalypti
                                                 New Zealand
```

D1.6: Read the commented function invaded.regions that allows computing which regions the species invaded before the current year. Which type of covariates does this function allow to compute: exogenous, endogenous, or global?

```
invaded.regions <- function(sp.n, r.n, y, native, first_records){

# Convert input arguments to numeric type if not already

sp.n <- as.numeric(sp.n)

r.n <- as.numeric(r.n)

y <- as.numeric(y)</pre>
```

```
# Get unique combinations of species number and region number from native data
t <- unique(as.vector(subset(native, sp.num == sp.n, r.num)))

# Get region numbers from first records data where species number matches
pr <- as.vector(subset(first_records, sp.num == sp.n, r.num))

# If the invasion is both present in first records data and in native range
# consider the former as actual piece of information
t <- setdiff(t, pr)

# Find indices of first records occurring before end date for the species
set.sp <- which(first_records$p.num == sp.n & first_records$year < y)

# Combine regions in native range with regions in first records before date
t <- na.omit(c(t, first_records$r.num[set.sp]))

# Do not consider the involved region
inv <- unlist(setdiff(t, r.n))

# Return invaded regions
return(inv)
}</pre>
```

#### 0.3. Building the Case-Control Dataset

#### Demo 2:

D2.1: Consider the smaller relational event network descripted by the data in first\_records. Begin by defining the sender set and storing it in spec, followed by defining the receiver set and storing it in reg.lf.

```
# Define sender set - species
spec <- unique(first_records$species)
(s <- length(spec))</pre>
```

```
## [1] 114
```

```
# Define receiver set - regions
reg.lf <- unique(first_records$region)
(r <- length(reg.lf))</pre>
```

```
## [1] 159
```

The dynamic network of FRs consists of two sets of nodes: the set of species and the set of regions. Additionally, in the dataset, we refer to a numerical formulation of species and regions, which correspond to the set of unique observed species in the data and the set of regions reported in the matrix of geographical distances, respectively.

```
reg <- colnames(data_distance)</pre>
```

D2.2: Review the commented function creating\_case\_control\_dataset, which utilizes information from the relational event network, noting that the relational event is non-recurrent.

```
reg.lf.num <- match(reg.lf, reg)</pre>
s <- length(spec)
r <- length(reg.lf)
## POSSIBLE (S,R) INTERACTIONS ####
# Initialize a vector to keep track of the risk set size over time
at.risk <- NULL
# Create a matrix to indicate possible (species, region) interactions
alien.occ <- matrix(0, nrow = s, ncol = r)</pre>
rownames(alien.occ) <- spec</pre>
colnames(alien.occ) <- reg.lf</pre>
for (n.sp in 1:s){
  # Identify native regions for each species
  nat.id <- unique(native$r.num[native$sp.num == n.sp])</pre>
  # Identify regions where the species is not native
  possible.to <- setdiff(reg.lf.num,nat.id)</pre>
  # Mark these regions as possible invasion sites for the species
  alien.occ[n.sp,reg[possible.to]] <- 1</pre>
}
## COLLECTING INFORMATION ####
dat.gam <- data.frame(matrix(NA, nrow=nrow(first_records),ncol=6))</pre>
colnames(dat.gam) <- c("y", "year",</pre>
                        "sp1", "r1",
                        "sp2", "r2")
# The response is fixed and equal to 1
dat.gam[,1] <- rep(1, nrow(first_records))</pre>
set.seed(seed)
# For each FR:
for (i in 1:nrow(first_records)){
  ### INFORMATION CONCERNING THE EVENT ####
  # year of the invasion event
  dat.gam[i,2] <- year <- first_records[i,"year"]</pre>
  # invading species
  dat.gam[i,3] <- s.ev <- first_records[i, "species"]</pre>
  # invaded country
  dat.gam[i,4] <- r.ev <- first_records[i,"region"]</pre>
  ### POSSIBLE EVENTS ####
  # Events occurred at the same time of the considered event
  # are removed from the risk set
  sub_stp <- first_records[first_records$year==year,</pre>
                            c("species", "region")]
  ni <- nrow(sub_stp)</pre>
  for (j in 1:ni){
    # Mark these (species, region) pairs as not at risk
    alien.occ[sub_stp[j,1],sub_stp[j,2]] <- 0</pre>
  at.risk <- c(at.risk, sum(alien.occ==1))
  ### SAMPLING THE NON-EVENT ####
```

```
sr.nv<-sample(which(alien.occ!=0),1)
# species non-event
dat.gam[i,5] <- s.nv <- spec[(sr.nv-1)%%s+1]
# region non-event
dat.gam[i,6] <- r.nv <- reg.lf[(sr.nv-1)%/%s+1]
}
return(dat.gam)
}</pre>
```

```
D2.3: Use the function creating case control dataset with first records and generate the case-
control dataset. Additionally, determine the numerical codification of species and regions.
dat.gam <- creating_case_control_dataset(first_records,</pre>
                                           spec, reg.lf, reg)
dat.gam$sp1.num <- match(dat.gam$sp1, spec)</pre>
dat.gam$r1.num <- match(dat.gam$r1, reg)</pre>
dat.gam$sp2.num <- match(dat.gam$sp2, spec)</pre>
dat.gam$r2.num <- match(dat.gam$r2, reg)</pre>
head(dat.gam[c("year", "sp1", "r1", "sp2", "r2")])
##
     year
                                              r1
                              sp1
                                                                         sp2
## 1 1880
           Adelges nordmannianae
                                     Switzerland
                                                       Epiphyas postvittana
## 2 1881
            Pheidole megacephala
                                          France
                                                         Chilo suppressalis
## 3 1884
                      Pineus pini
                                     New Zealand Coccinella septempunctata
## 4 1886
                 Eulachnus rileyi United States
                                                        Agrilus planipennis
## 5 1887
              Linepithema humile
                                        Portugal
                                                         Tremex fuscicornis
## 6 1889 Ctenarytaina eucalypti New Zealand
                                                        Xyleborus glabratus
##
                    r2
## 1
            Venezuela
## 2
                 Benin
## 3
               Gambia
## 4
                 Benin
## 5
        Guinea-Bissau
## 6 Hawaiian Islands
head(dat.gam[c("year", "sp1.num", "r1.num", "sp2.num", "r2.num")])
     year sp1.num r1.num sp2.num r2.num
## 1 1880
                                       77
                       90
                                22
                 1
## 2 1881
                 2
                       11
                                34
                                      195
## 3 1884
                 3
                                66
                                      208
                       31
## 4 1886
                 4
                       37
                               106
                                      195
## 5 1887
                 5
                       25
                              103
                                      211
## 6 1889
                 6
                       31
                               108
                                        3
```

# 1. Time-Varying Effect of Trade on Species Invasions



# 1.1. Trade: Covariate Computation

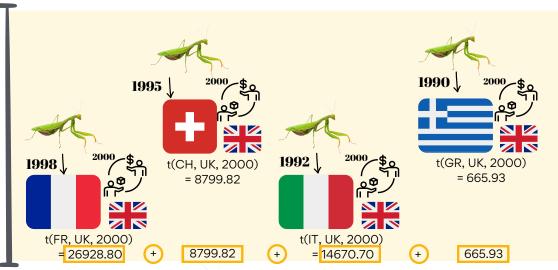
# Demo 3:

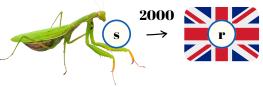
D3.1: Load the input data "input01.RData".

load("01-Data/01-Inputs/input01.RData")

D3.2: Write a function log\_trade that, given a relational event (and all the relevant information required), computes the annual trade between the involved region and the previously invaded regions by the species. Apply the function to the events and the sampled non-events, and compute the difference. Refer to the figure in the following slide to understand how the covariate is computed.







```
t <- which(data_trade$transfer < 0)
data_trade$transfer[t] <- 0</pre>
trade.funct <- function(inv, r.n, y, reg, data_trade){</pre>
  # Convert input arguments to numeric type if not already
  r.n <- as.numeric(r.n)</pre>
  y <- as.numeric(y)
  # Check if there are already invaded countries
  if(length(inv)!=0){
    # Find rows that involve invaded countries as sending trade
    u <- which(data_trade$FromRegion %in% reg[inv])</pre>
    # Find rows that involve region of interest as receiving trade
    v <- which(data_trade$ToRegion == reg[r.n])</pre>
    # Find the intersection of the two sets
    w <- intersect(u,v)
    x <- data_trade[w,]</pre>
    # Consider the trade instances occurred before or at the time of interest
    x \leftarrow x[x\$year \leftarrow y,]
    trade_value <- NULL</pre>
    # If there are rows in the filtered dataset
    if(nrow(x)>0){
      # For each invaded country, the maximum year is recorded
      o <- aggregate(x$year, list(x$FromRegion), FUN=max)</pre>
      # For each of them, the corresponding transfer is stored
      for (o.i in 1:nrow(o)){
        trade_value <- c(trade_value,</pre>
                           x$transfer[x$FromRegion==o[o.i,1] &
                                         x$year==o[o.i,2]])}
```

```
}
  } else {
  # If there are not already invaded countries, trade is set equal to 0
   trade_value <- 0
  # Compute the log-transformed sum of trade values (with an added constant 1)
  log_trade.value <- ifelse(length(trade_value)>0,
                             log(sum(trade value, na.rm =T)+1),0)
  # Return the computed log-transformed trade value
  return(log_trade.value)
}
log_trade <- function(sp.n, r.n, y, native, first_records, reg, data_trade){</pre>
  inv <- invaded.regions(sp.n = sp.n,</pre>
                          r.n = r.n,
                          y = y,
                          native = native,
                          first_records = first_records)
  log_trade.value <- ifelse(r.n==match("USACanada", reg),</pre>
                             mean(trade.funct(inv = inv,
                                               r.n = match("United States", reg),
                                               y = y,
                                              reg = reg,
                                               data_trade = data_trade),
                                  trade.funct(inv = inv,
                                              r.n = match("Canada", reg),
                                               y = y,
                                              reg = reg,
                                               data_trade = data_trade)),
                             trade.funct(inv = inv,
                                         r.n = r.n,
                                         y = y,
                                         reg = reg,
                                         data_trade = data_trade))
  return(log_trade.value)
dat.gam$tr1 <- apply(dat.gam[,c("sp1.num", "r1.num", "year")], 1,</pre>
                             function(x) log_trade(sp.n = x[1],
                                                    r.n = x[2],
                                                    y = x[3],
                                                    native = native,
                                                    first_records =
                                                      first_records,
                                                    reg = reg,
                                                    data_trade = data_trade))
dat.gam$tr2 <- apply(dat.gam[,c("sp2.num", "r2.num", "year")], 1,</pre>
                             function(x) log_trade(x[1], x[2], x[3],
                                                    native = native,
                                                       first_records =
```

```
first_records,
reg = reg,
data_trade = data_trade))
dat.gam$tr = dat.gam$tr1 - dat.gam$tr2
```

D3.3: Inspect the nature of the covariate. Is it a monadic or dyadic covariate?

D3.4: Is it an endogenous or exogenous covariate?

#### 1.2. Effect of Trade as Spline function of Time

 $\overline{D4.0}$  Choose the basis-dimension q.

q = 10

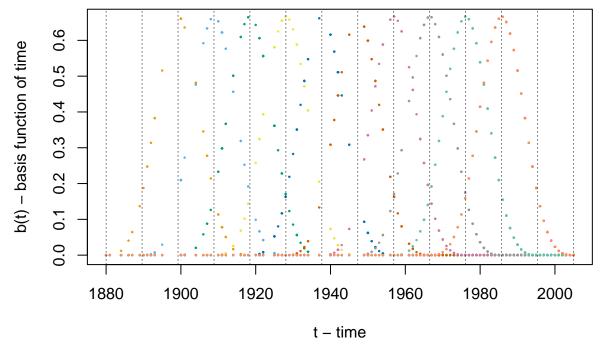
#### D4.1 Select the spline basis functions types

```
bspline \leftarrow function(x, k, i, m = 2) {
  # ith B-spline basis function of order m at the values in x
  # given knot locations in k
  if (m == -1) {
    # Base case of the recursion:
    # when m is -1, we are at the lowest order basis function
    res \leftarrow as.numeric(x < k[i + 1] & x >= k[i])
    # Returns 1 if x is within the interval [k[i], k[i+1])
 } else {
    # Recursive case:
    # B-spline basis function from lower order basis functions
    # Calculate the first term's coefficient
    z0 \leftarrow (x - k[i]) / (k[i + m + 1] - k[i])
    # Calculate the second term's coefficient
    z1 \leftarrow (k[i + m + 2] - x) / (k[i + m + 2] - k[i + 1])
    # Recursive calls to the lower order basis functions
    res <-z0 * bspline(x, k, i, m - 1) + z1 * bspline(x, k, i + 1, m - 1)
  }
  return(res) # Return the evaluated B-spline basis function
}
```

#### D4.2 Given range of the variable, select the basis evaluation points.

#### D4.3 Evaluate the q basis functions of time $b_l(t)$ .

```
m = 2
basis = matrix(0, nrow = length(dat.gam$year), ncol = q)
for (j in 1:q) {
  basis[, j] = bspline(dat.gam$year, k = knots, i = j)
}
```



#### 1.3. Determine Trade's Contribution

• To the log-hazard of an interaction}

$$f_{sr} = \dots + \sum_{j=1}^{q} \theta_l b_j(t) t r_{sr}(t) + \dots$$

• To the sampled likelihood function}

$$PL(\theta) = \prod_{i=1}^{n} \frac{e^{\left[\sum_{j=1}^{q} \theta_{j} b_{j}(t)\right] \Delta t r_{i}}}{1 + e^{\left[\sum_{j=1}^{q} \theta_{j} b_{j}(t)\right] \Delta t r_{i}}}$$

#### 1.4. Estimate the Coefficients of the Spline

Demo 5:

D5.1 :Let x.ev and x.nv be  $n \times 1$  vectors of covariate evaluated for events & non-events. Then: x = x.ev - x.nv;

```
x.ev <- dat.gam$tr1
x.nv <- dat.gam$tr2
x <- x.ev - x.nv</pre>
```

D5.2 Let stp be  $n \times 1$  vector of event-times.

```
stp <- dat.gam$year</pre>
```

D5.3 Fit the model incorporating a time-varying effect for x.

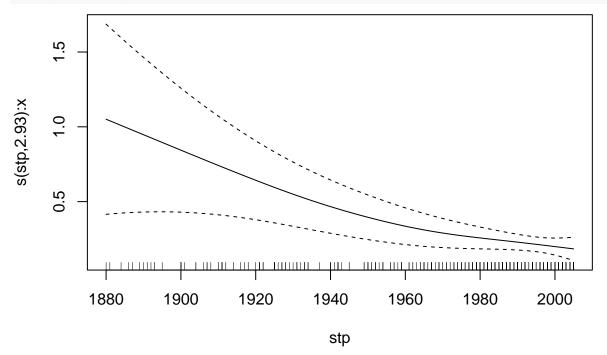
REMARK: default spline's type for gam in mgcv consists of thin plate regression spline.

```
gam_tr.only <- gam(y ~ s(stp, by=x) - 1,
    family="binomial"(link = 'logit'),
    method="REML", data=dat.gam)</pre>
```

# 1.5. Interpreting Trade's Effect

# Demo 6

plot(gam\_tr.only)

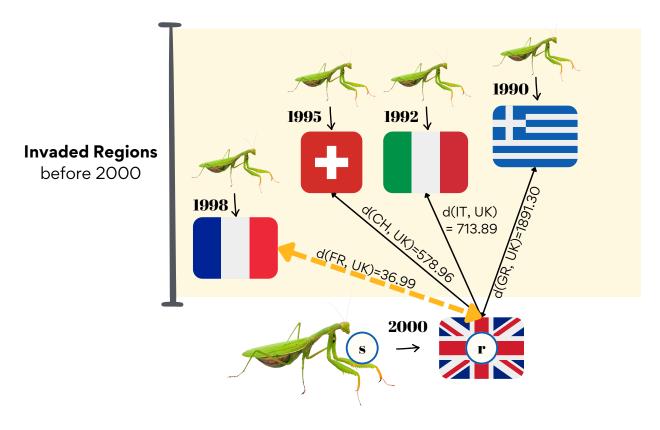


# 2. Non-Linear Effect of Distance on Species Invasions



2.1. Distance: Covariate Computation

Supplementary 2



#### 2.2. Effect: Spline function of Distance

#### Supplementary 3

#### 2.3. Determine Distance's Contribution

• To the log-hazard of an interaction}

$$f_{sr} = ... + \sum_{j=1}^{q} \theta_{j} b_{j} [d_{sr}(t)] + ...$$

• To the sampled likelihood function}

$$PL(\theta) = \prod_{i=1}^{n} \frac{e^{\left[\sum_{j=1}^{q} \theta_{j} b_{j} [d_{s_{i}r_{i}}(t_{i})] - b_{j} [d_{s_{i}^{*}r_{i}^{*}}(t_{i})]\right]}}{1 + e^{\left[\sum_{j=1}^{q} \theta_{j} b_{j} [d_{s_{i}r_{i}}(t_{i})] - b_{j} [d_{s_{i}^{*}r_{i}^{*}}(t_{i})]\right]}}$$

#### 2.4. Estimate the Coefficients of the Spline

#### Exercise 1:

E1.1: Load the input data "input02.RData".

E1.2: Let x.ev and x.nv be  $n \times 1$  vectors of covariate evaluated for events & non-events.

E1.2: Let unit be  $n \times 1$  unit vector.

E1.3: Define X, combining x.ev and x.nv and I combining unit and -unit.

E1.4 Fit the model incorporating a non-linear effect.

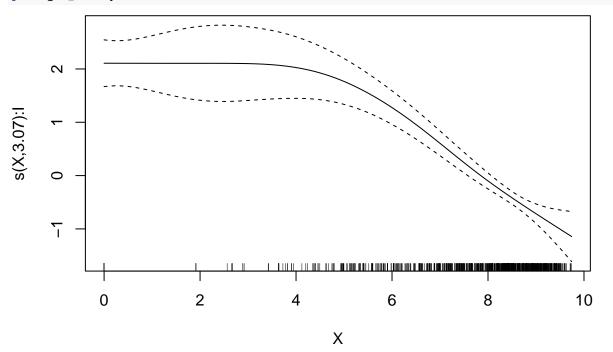
REMARK: default spline's type for gam in mgcv consists of thin plate regression spline.

# 2.5. Interpreting Distance's Effect

# Demo 7

load(file="01-Data/02-Gam-Fits/gam\_d.only.RData")

plot(gam\_d.only)



# 3. Species Invasiveness Random Effect



#### 3.1. Species Invasiveness: Covariate Computation

## Demo 8:

D8.1: Load the input data "input03.RData".

load("01-Data/01-Inputs/input03.RData")

D8.2: Define the covariate:  $z_s(t)$ : involvement of species s in first record occurring at t. Create a  $n \times 2$  matrix sp containing in the fist column spec levels related to species involved in the events and in the second those involved in the non-events.

```
sp1 <- dat.gam$sp1
sp2 <- dat.gam$sp2
sp <- factor(c(sp1,sp2))
dim(sp) <- c(length(sp1),2)</pre>
```

D7.3: Inspect the nature of the covariate. Is it a monadic or dyadic covariate? It is just reporting the label of species involved s, so the covariate  $z_s(t)$  is monadic.

## 3.2. Random Effects as Smooths

$$\gamma' z_s(t) = \sum_{s' \in \mathcal{S}} \gamma_{s'} 1\{s = s'\}$$

#### 3.3. Species Invasiveness' Contribution

• To the log-hazard of an interaction}

$$f_{sr} = \dots + \sum_{s' \in S} \gamma_{s'} 1\{s = s'\} + \dots$$

• To the sampled likelihood function}

$$PL(\gamma) = \prod_{i=1}^{n} \frac{e^{\gamma_{s_i} - \gamma_{s_i^*}}}{1 + e^{\gamma_{s_i} - \gamma_{s_i^*}}} - \frac{1}{2\sigma_{\rm sp}^2} \gamma^T \gamma$$

#### 3.4. Estimate the Coefficients of the Spline

#### Exercise 3:

E3.1: Let unit be  $n \times 1$  unit vector. Define I combining unit and -unit.

E3.2 Fit the model incorporating a random effect.

REMARK: default spline's type for gam in mgcv consists of thin plate regression spline. Instead, we need to specify that we aim to fit a random effect.

#### 3.5. Interpreting Species Invasiveness Random Effects

#### Demo 8:

```
load(file="01-Data/02-Gam-Fits/gam sp.only.RData")
re.species <- coefficients(gam_sp.only)
names(re.species) <- levels(sp)</pre>
sort(re.species, decreasing = TRUE)[1:5]
## Frankliniella occidentalis
                                         Aphis spiraecola
##
                      2.411066
                                                  1.848464
##
              Cinara cupressi
                                     Phenacoccus manihoti
##
                      1.789825
                                                  1.749139
##
               Apis mellifera
                      1.571748
##
sort(re.species)[1:5]
    Xylosandrus compactus
                              Solenopsis richteri
                                                      Scolytus schevyrewi
##
               -1.5031237
                                       -1.2676923
                                                               -1.0597829
                              Agrilus planipennis
## Archips fuscocupreanus
               -1.0562797
                                       -0.9503508
```

# 4. Fit a model with all the components included

#### Exercise 4:

```
E4.1: Load the input data "input04.RData".
```

```
load("01-Data/01-Inputs/input04.RData")
```

```
unit <- rep(1, nrow(dat.gam))
stp = dat.gam$year
X = cbind(dat.gam$d1,dat.gam$d2)
I = cbind(unit,-unit)</pre>
```

```
gam_complete <- gam(y ~ dt +</pre>
                      s(stp, by=tr) +
                      s(X, by=I) +
                      s(sp, by=I, bs="re") - 1,
    family="binomial"(link = 'logit'),
    method="REML", data=dat.gam)
load(file="01-Data/02-Gam-Fits/gam_dt.only.RData")
load(file="01-Data/02-Gam-Fits/gam_tr.only.RData")
load(file="01-Data/02-Gam-Fits/gam_d.only.RData")
load(file="01-Data/02-Gam-Fits/gam_sp.only.RData")
AIC(gam_dt.only)
Model Selection
## [1] 742.7089
AIC(gam_tr.only)
## [1] 630.9921
AIC(gam_d.only)
## [1] 550.0876
AIC(gam_sp.only)
## [1] 642.3556
AIC(gam_complete)
## [1] 436.6027
sort(re.species, decreasing = TRUE)[1:5]
Changes in random effect predictions
## Frankliniella occidentalis
                                         Aphis spiraecola
##
                     2.411066
                                                 1.848464
##
                                     Phenacoccus manihoti
              Cinara cupressi
##
                     1.789825
                                                 1.749139
##
               Apis mellifera
##
                     1.571748
sort(re.species)[1:5]
    Xylosandrus compactus
                             Solenopsis richteri
                                                     Scolytus schevyrewi
##
                                                               -1.0597829
##
               -1.5031237
                                       -1.2676923
## Archips fuscocupreanus
                             Agrilus planipennis
##
               -1.0562797
                                       -0.9503508
re.species_complete <- coefficients(gam_complete)[21:length(coefficients(gam_complete))]
names(re.species_complete) <- levels(sp)</pre>
sort(re.species_complete, decreasing = TRUE)[1:5]
```

```
##
       Bactrocera invadens
                              Phenacoccus manihoti Maconellicoccus hirsutus
##
                 1.2972472
                                          1.1558752
                                                                   0.9404922
                             Wasmannia auropunctata
##
           Aphis spiraecola
##
                 0.8913869
                                          0.8754983
sort(re.species_complete)[1:5]
                                     Curculio conicus
##
      Phloeomyzus passerinii
                                                          Xylosandrus compactus
##
                 -1.0033272
                                           -0.9886417
                                                                     -0.8874932
## Xylosandrus crassiusculus
                                Bostrichus ligniperda
                                           -0.7592472
                 -0.8056282
sort(re.species)[1:3]
## Xylosandrus compactus
                          Solenopsis richteri
                                                Scolytus schevyrewi
                                                          -1.059783
              -1.503124
                                    -1.267692
sort(re.species_complete)[1:3]
## Phloeomyzus passerinii
                               Curculio conicus Xylosandrus compactus
              -1.0033272
                                     -0.9886417
                                                            -0.8874932
##
```