Hand-In Exercise: Admittance Controller

Name 1 (Username 1), Name 2 (Username 2), Name 3 (Username 3)

1 System Modeling

This section contains a derivation of the equations of motion for a robot arm consisting of the three first links of a UR5e robot. The model is derived using Lagrange–D'Alembert's Principle.

An illustration of the robot is shown in Figure 1 that includes the kinematic parameters specified in Table 1. The figure also show position vectors for the center of mass of each joint; the coordinates of the center of masses are specified in (1).

Insert a sketch of the 3-link robot including kinematic parameters and coordinate frames.

Figure 1: Illustration of considered robot, including kinematic parameters and reference frames.

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

Table 1: DH-parameters for the 3-link robot arm.

$$\boldsymbol{p}_{l_1}^1 = \begin{bmatrix} p_{l_1,x}^1 \\ p_{l_1,y}^1 \\ p_{l_1,z}^1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.02561 \\ 0.00193 \end{bmatrix}, \quad \boldsymbol{p}_{l_2}^2 = \begin{bmatrix} p_{l_2,x}^2 \\ p_{l_2,y}^2 \\ p_{l_2,z}^2 \end{bmatrix} = \begin{bmatrix} 0.2125 \\ 0 \\ 0.11336 \end{bmatrix}, \quad \boldsymbol{p}_{l_3}^3 = \begin{bmatrix} p_{l_3,x}^3 \\ p_{l_3,y}^3 \\ p_{l_3,z}^3 \end{bmatrix} = \begin{bmatrix} 0.15 \\ 0.0 \\ 0.0265 \end{bmatrix} \tag{1}$$

To derive a dynamical model of the robot, the mass and inertia tensor of each link are given in

$$I_1^1 = \begin{bmatrix} 0.0084 & 0 & 0 \\ 0 & 0.0064 & 0 \\ 0 & 0 & 0.0084 \end{bmatrix}, I_2^2 = \begin{bmatrix} 0.0078 & 0 & 0 \\ 0 & 0.21 & 0 \\ 0 & 0 & 0.21 \end{bmatrix}, I_3^3 = \begin{bmatrix} 0.0016 & 0 & 0 \\ 0 & 0.0462 & 0 \\ 0 & 0 & 0.0462 \end{bmatrix} \tag{2}$$

Finally, the masses of the links are $m_1 = 3.761$ kg, $m_2 = 8.058$ kg, $m_3 = 2.846$ kg.

1.1 Robot Kinematics

Derive the kinematics of the robot symbolically.

The kinematics of the robot are given by the homogeneous transformations

$$A_1^0 = \text{Insert},\ A_2^1 = \text{Insert},\ A_3^2 = \text{Insert} \tag{3}$$

where $c_i = \cos(\theta_i)$ and $s_i = \sin(\theta_i)$ in addition to the transformations

$$T_2^0 = \text{Insert},\ T_3^0 = \text{Insert} \tag{4}$$

where $c_{ij} = \cos(\theta_i + \theta_j)$ and $s_{ij} = \sin(\theta_i + \theta_j)$.

1.2 Center of Mass of Links

Derive symbolic expressions for the center of mass of each link.

The center of mass for each link have the following coordinates

$$p^0_{c1} = \text{Insert} \tag{5}$$
$$p^0_{c2} = \text{Insert} \tag{6}$$
$$p^0_{c3} = \text{Insert} \tag{7}$$

where $c_{ij} = \cos(\theta_i + \theta_j)$ and $s_{ij} = \sin(\theta_i + \theta_j)$.

1.3 Link Velocities

Derive symbolic expressions for Jacobians of each link.

The rotational part of the three link Jacobians are

$$J^{l_1}_O = \text{Insert},\ J^{l_2}_O = \text{Insert},\ J^{l_3}_O = \text{Insert} \tag{8}$$

1.3.1 Link Translational Velocities

Derive symbolic expressions for Jacobians of each link. ONLY for link 1 and link 2

The translational part of the link Jacobians are

$$J^{l_1}_P = \text{Insert} \tag{9}$$
$$J^{l_2}_P = \text{Insert} \tag{10}$$

1.4 Inertia-Tensors of Links

Derive symbolic expressions for inertia tensors in Base frame

The inertia tensors in Base frame are

$$I_{l_1}(\boldsymbol{q}) = \text{Insert} \tag{11}$$
$$I_{l_2}(\boldsymbol{q}) = \text{Insert} \tag{12}$$
$$I_{l_3}(\boldsymbol{q}) = \text{Insert} \tag{13}$$

1.5 Potential Energy

Derive symbolic expressions for the potential energy

The potential energy of the robot is

$$E_{\text{pot}}(\boldsymbol{q}) = \text{Insert} \tag{14}$$

1.6 Kinetic Energy

Derive symbolic expressions for the kinetic energy. Write down the general expressions and compute the matrix $B(q)$ using some software

The kinetic energy of the robot is

$$E_{\text{kin}}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2} q^T B(q) q \tag{15}$$

where the mass matrix is given by

$$B(q) = \tag{16}$$

1.7 Robot Dynamics including External Forces

We use Lagrange–D'Alembert's Principle to setup a dynamical model of the robot from

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} = \boldsymbol{Q} \tag{17}$$

where Q is an n-dimensional vector of generalized forces and $\mathcal{L} = E_{\text{kin}} - E_{\text{pot}}$.

Use software to derive the robot dynamics and provide the results in the box below.

The dynamics of the 3 link robot can be written as

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + J^T(q) h_e \tag{18}$$

where h_e is an external wrench applied to the end-effector, the generalized coordinate q, $B(q)$ is given in (16), and the remaining parameter matrices are

$$q = \text{Insert expression.} \tag{19}$$
$$C(q, \dot{q}) = \text{Insert expression.} \tag{20}$$
$$\boldsymbol{g}(q) = \text{Insert expression.} \tag{21}$$
$$J(q) = \text{Insert expression.} \tag{22}$$
$$\tag{23}$$

1.8 Verification of Model

Simulate the robot arm with input $\boldsymbol{Q} = (\tau_1, \tau_2, \tau_3) = -D\dot{\boldsymbol{q}}$, where D is a diagonal matrix. Use initial condition $\boldsymbol{q} = (\theta_1, \theta_2, \theta_3) = (1, \pi/3, \pi/3)$.

Insert simulation results, i.e. graphs of q, $\dot{q}$ and τ.

Figure 2: Insert text.

2 Admittance Control

Design admittance controllers in operational space with the orientational part expressed a) with quaternion and b) with Euler angles.

2.1 Control Law

This section presents the admittance controller, where the translational part of the admittance controller is given by

$$M_p \Delta \ddot{p}_{cd} + D_p \Delta \dot{p}_{cd} + K_p \Delta p_{cd} = f \tag{24}$$

where M_p, D_p, K_p are 3×3 matrices, $f \in \mathbb{R}^3$ is force given in Base frame, and $\Delta p_{cd} = p_c - p_d$.

The rotational part of the admittance controller can be expressed in different ways depending on the representation use for orientation. Two representations are

$$M_o \Delta\dot{\omega}^d_{cd} + D_o \Delta\omega^d_{cd} + K'_o \epsilon^d_{cd} = \mu^d \tag{25}$$

$$M_o \Delta\ddot{\phi}_{cd} + D_o \Delta\dot{\phi}_{cd} + K_o \Delta\phi_{cd} = T^T(\phi_c)\mu \tag{26}$$

where M_o, D_o, K_o are 3×3 matrices, $\mu^d \in \mathbb{R}^3$ is the torque applied to the end-effector given in desired frame, $\Delta\omega_{cd} = \omega_c - \omega_d$, $\epsilon^d_{cd} = \eta_d \epsilon_c - \eta_c \epsilon_d - S(\epsilon_c)\epsilon_d$, and $\Delta\phi_{cd} = \phi_c - \phi_d$. The rotational stiffness matrix is

$$K'_o = 2E^T(\eta_{cd}, \epsilon^d_{cd})K_o$$

where K_o is the stiffness in Euler angle representation and

$$E(\eta, \epsilon) = \eta I - S(\epsilon)$$

2.2 Gain Selection

Write how the gains should be selected to obtain a critically damped system.

2.3 Implementation

Provide details on the implementation of the controller.

The moment in desired frame is computed from the wrench h_e as

$$\mu^d = \textit{Insert} \tag{27}$$

where h_e is an external wrench applied to the end-effector.

2.3.1 Quaternion-Based Controller

The quaternion $(\eta_{cd}, \epsilon^d_{cd})$ is obtained from integration of ω^d_{cd} as

$$\textit{Insert} \tag{28}$$

The compliant frame p_c is obtained from the quaternion $(\eta_{cd}, \epsilon^d_{cd})$ as

$$\textit{Insert} \tag{29}$$

2.3.2 Euler Angle-Based Controller

The Euler angle $\Delta\phi_{cd}$ is obtained from integration of $\Delta\dot{\phi}_{cd}$ as

$$\textit{Insert} \tag{30}$$

The compliant frame p_c is obtained from the Euler angle $\Delta\phi_{cd}$ as

$$\textit{Insert} \tag{31}$$

3 Simulation

Insert simulation results for the admittance controlled robot. The admittance controller should have a desired motion of your choice. In addition, no external force should be applied for the first five seconds, then a force $f = (1, 2, 3)$ N should be added for five seconds, then no external force for five seconds, then a torque $\mu = (1, 0.5, 1)$ Nm for five seconds and lastly no external force for five seconds. You should include figures that documents the simulation including applied wrench, desired motion, actual motion.