MARTIN BROCCA - CEIA - PEIA

1) 10 MONEDAS - 2 FALSAS (8 JUSTAS)

$P(\text{CARA}) = \underline{0,5}$ para una moneda justa
$P(\text{CARA}) = \underline{0,5}$ para una moneda falsa

Calcular:

$P(F / 11 \text{ CARAS})$

$P(J / 11 \text{ CARAS})$

$P(\text{JUSTA}) = \frac{8}{10} = 0,8$ $\qquad$ $P(\text{FALSA}) = \frac{2}{10} = 0,2$

Probabilidad de obtener 11 CARAS:

CASO MONEDA JUSTA: $P(11 \text{ CARAS} | J) = (0,5)^{11} = \frac{1}{2048}$

CASO MONEDA FALSA: $(P_{11\text{caras}} | F) = (0,5)^{11} = 1/2048$

$P(11 \text{CARAS} | J)\, P(J) + P(11 \text{CARAS} | F) . P(F) = P(11 \text{ CARAS}) \Rightarrow$

$P(11 \text{ CARAS}) = \frac{1}{2048} \times 0,8 + \frac{1}{2048} \times 02 = \frac{1}{2048} (0,8 + 0,2) = \frac{1}{2048}$

APLICAMOS BAYES:

$$P(J/11\text{ CARAS}) = \frac{P(11\text{ CARAS}|\text{JUSTA}) \cdot P(J)}{P(11\text{ CARAS})} = \frac{\frac{1}{2048} \cdot 0{,}8}{1/2048} = \boxed{0{,}8}$$

$$P(F/11\text{ CARAS}) = \frac{P(11\text{ CARAS}/F) \cdot P(F)}{P(11\text{ CARAS})} = \frac{1/2048 \cdot 0{,}2}{1/2048} = \boxed{0{,}2}$$

Resultado:

$P(J/11\text{ CARAS}) = 0{,}8 = 80\%$

$P(F/11\text{ CARAS}) = 0{,}2 = 20\%$

Nota: En el ejercicio la P de la moneda falsa es igual a la verdadera y no aporta información para distinguirla.

EJERCICIO 2: $f_{x,y} = \begin{cases} Ky & 1x^2 \leq y \leq 10x \\ 0 & \text{en otro caso.} \end{cases}$

a) Determinar el valor de k

Para que sea función de densidad de probabilidad

$\Rightarrow$

$$\iint f_{x,y}(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^{10}\int_{x^2}^{10x} ky\,dy\,dx = \int_0^{10} k\left[\frac{y^2}{2}\right]_{x^2}^{10x} dx$$

$$= \int_0^{10} k\left(\frac{(10x)^2}{2} - \frac{(x^2)^2}{2}\right)dx = \int_0^{10} k\left(\frac{100x^2}{2} - \frac{x^4}{2}\right)dx$$

$$= \int_0^{10} k\left(50x^2 - \frac{x^4}{2}\right) dx = k\int_0^{10} 10x^2 - \frac{x^4}{2}\,dx$$

$$= k\left[\frac{50x^3}{3} - \frac{x^5}{2.5}\right]_0^{10} = k\left[\frac{50x^3}{3} - \frac{x^5}{10}\right]_0^{10} =$$

$$= k\left[\frac{50 \cdot 10^3}{3} - \frac{10^{\cancel{5}4}}{\cancel{10}}\right] = k\left[\frac{50.000}{3} - 10.000\right] = \left[\frac{20.000}{3}\right]k = 1$$

$$\Rightarrow \boxed{k = \frac{3}{20.000}}$$

2b) Densidad Marginal:

$$f_Y(y) = \int f_{x,y}(x,y)\,dx$$

Límites de integración:

$$x^2 \leq y \leq 10x \Rightarrow \quad x^2 \leq y \Rightarrow x \leq \sqrt{y}$$
$$y \leq 10x \Rightarrow x \geq y/10$$

$$\Rightarrow f_Y(y) = \int_{y/10}^{\sqrt{y}} k y\,dx \quad , \quad y \text{ como } k = 3/20{,}000$$

$$= \int_{y/10}^{\sqrt{y} = y^{1/2}} \frac{3}{20000} y\,dx = \frac{3}{20000} y \left(y^{1/2} - \frac{y}{10}\right)$$

$$\boxed{f_Y(y) = \begin{cases} \frac{3}{20.000}\left(y^{3/2} - \frac{y^2}{10}\right) & \text{para } \emptyset \leq y \leq 100 \\ 0 & \text{en otro caso.} \end{cases}}$$

2c) Densidad condicional $f_{X|Y}(x|y)$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

Como $f_{XY} = \begin{cases} ky & 1x^2 \leq y \leq 10x \\ 0 & \text{en otro caso} \end{cases}$

sustituyo para $x^2 \leq y \leq 10x$

$$f_{X|Y}(x|y) = \frac{ky}{f_Y(y)}, \quad k = 3/20.000 \Rightarrow$$

$$= \frac{\frac{3}{20.000}\, y}{\frac{3}{20000}\left(y^{2/3} - \frac{y^2}{10}\right)} = \frac{y}{y^{3/2} - \frac{y^2}{10}} = \frac{\cancel{y}}{\cancel{y}\left(y^{1/2} - \frac{y}{10}\right)}$$

$$\boxed{\begin{cases} = \dfrac{1}{\left(\sqrt{y} - \frac{y}{10}\right)} & \text{con } x \in \frac{y}{10} \leq x \leq \sqrt{y} \\ 0 & \text{en otro caso} \end{cases}}$$

TP1

March 26, 2025

1 CEIA - PEIA - TP1 - Martin Brocca

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

```
Matplotlib is building the font cache; this may take a moment.
```

[5]:
```
df_ventas = pd.read_excel('./Datos_primer_TP_20Co2025_i0601.xlsx')
df_ventas
```

[5]:

	Fecha	Ventas
0	2021-01-01	13015.353281
1	2021-01-02	12694.725807
2	2021-01-03	18188.522984
3	2021-01-04	18422.939904
4	2021-01-05	16412.068524
…	…	…
1090	2023-12-27	21176.251780
1091	2023-12-28	18188.456739
1092	2023-12-29	17505.530420
1093	2023-12-30	18015.741430
1094	2023-12-31	13799.255368

[1095 rows x 2 columns]

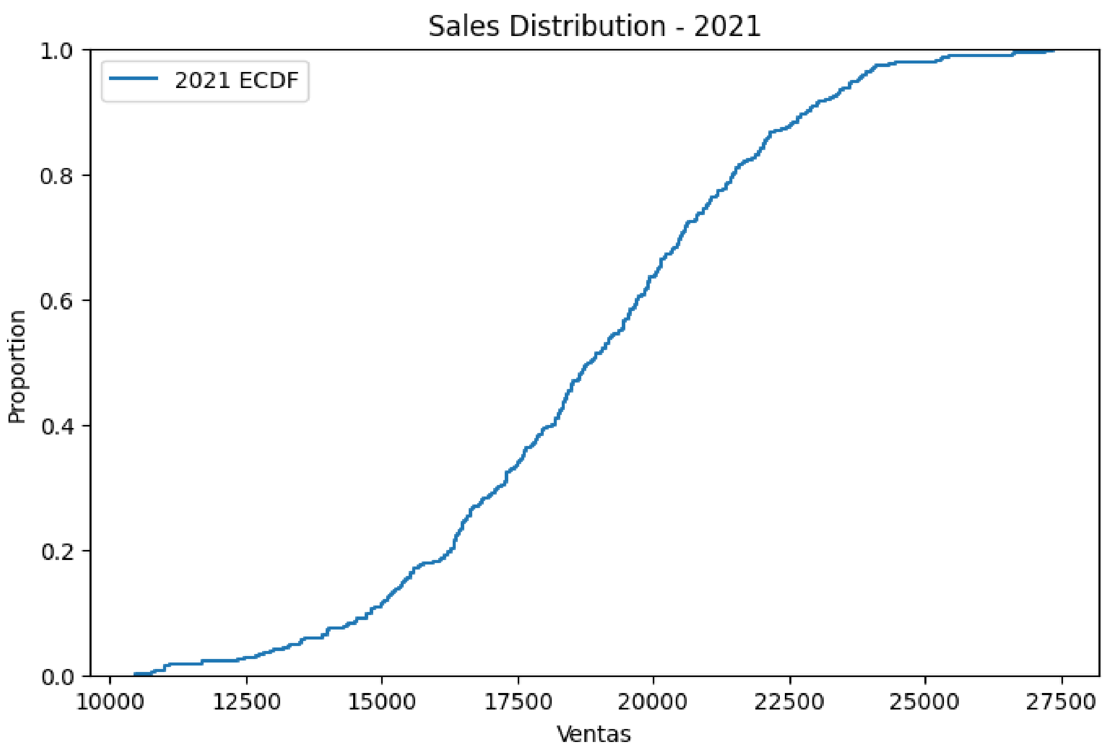
[6]:
```
ventas = df_ventas.Ventas
len(ventas)
```

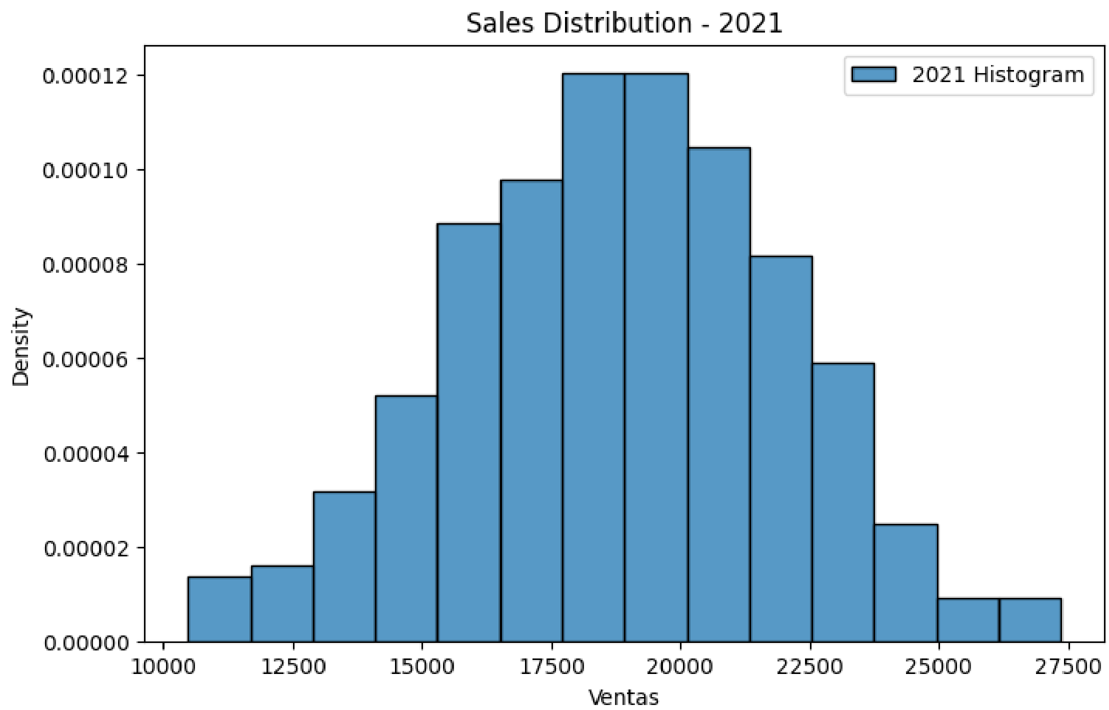
[6]: 1095

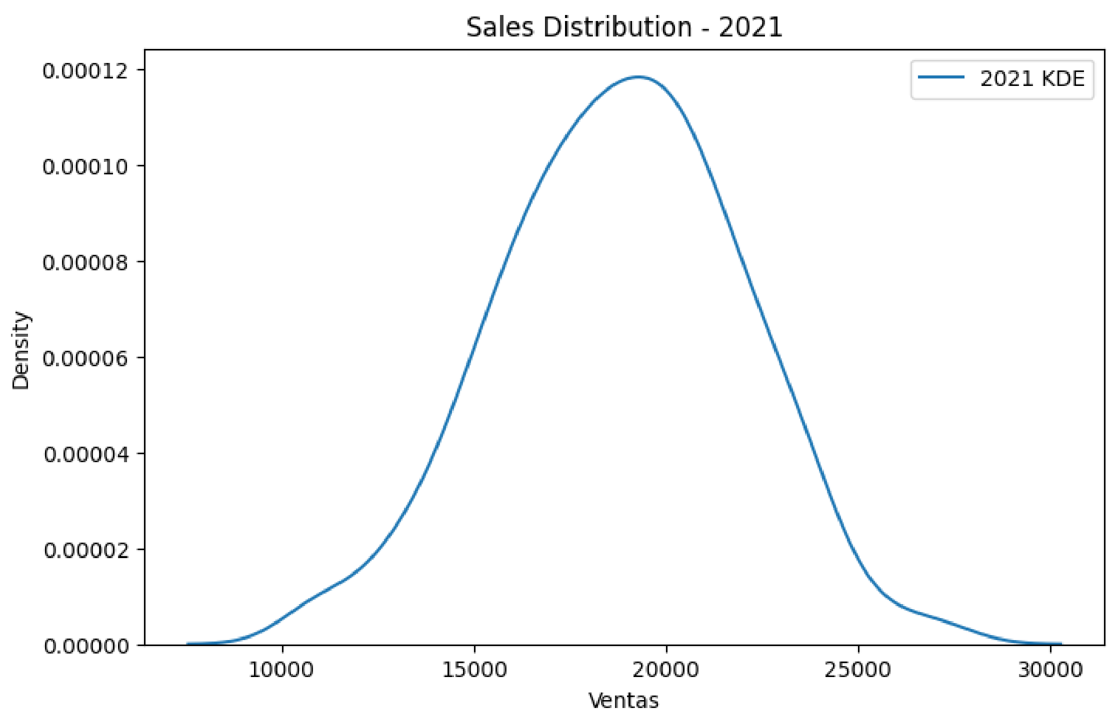
1.1 Plot the distributions per year

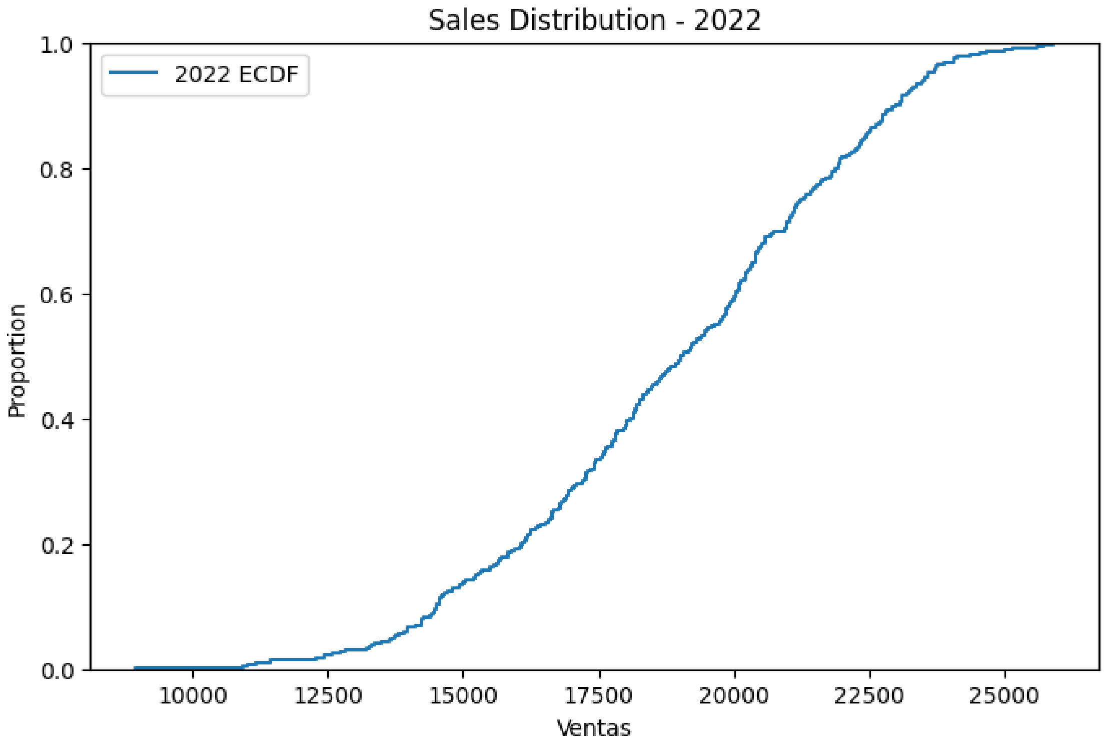
[14]:
```
years = [2021, 2022, 2023]

for year in years:
    df_year = df_ventas[df_ventas['Fecha'].dt.year == year]
```

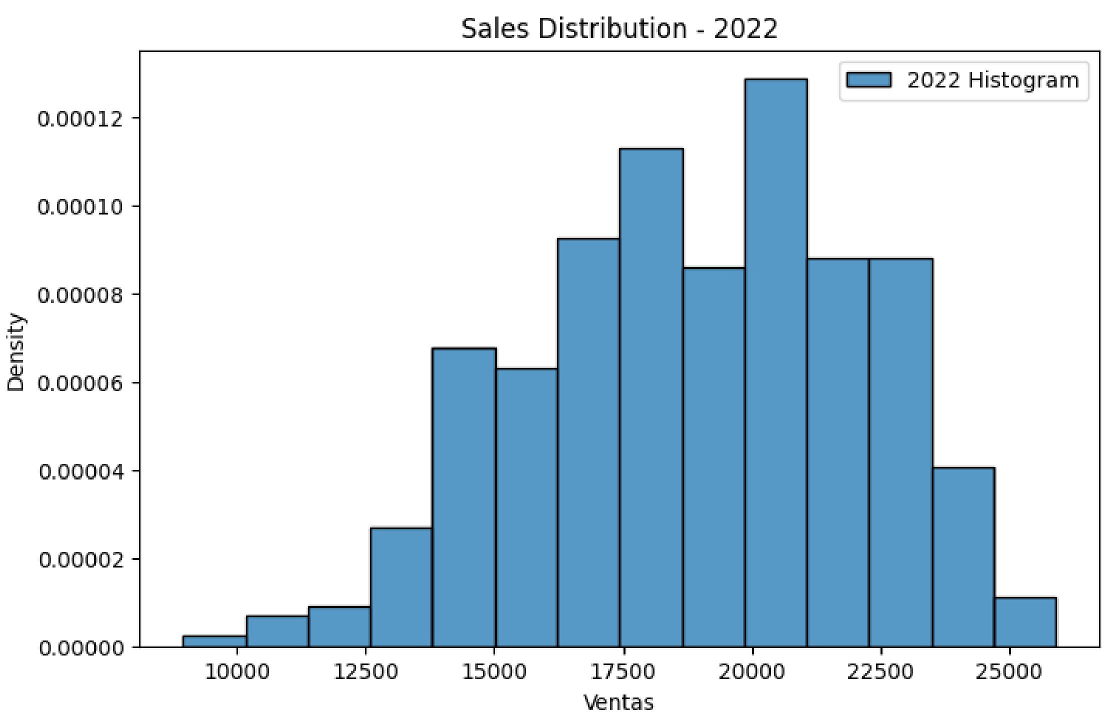
```python
ventas = df_year['Ventas']

plt.figure(figsize=(8, 5))
sns.ecdfplot(ventas, label=f'{year} ECDF')
plt.legend()
plt.title(f'Sales Distribution - {year}')
plt.show()

plt.figure(figsize=(8, 5))
sns.histplot(ventas, stat='density', label=f'{year} Histogram')
plt.legend()
plt.title(f'Sales Distribution - {year}')
plt.show()

plt.figure(figsize=(8, 5))
sns.kdeplot(ventas, label=f'{year} KDE')
plt.legend()
plt.title(f'Sales Distribution - {year}')
plt.show()
```

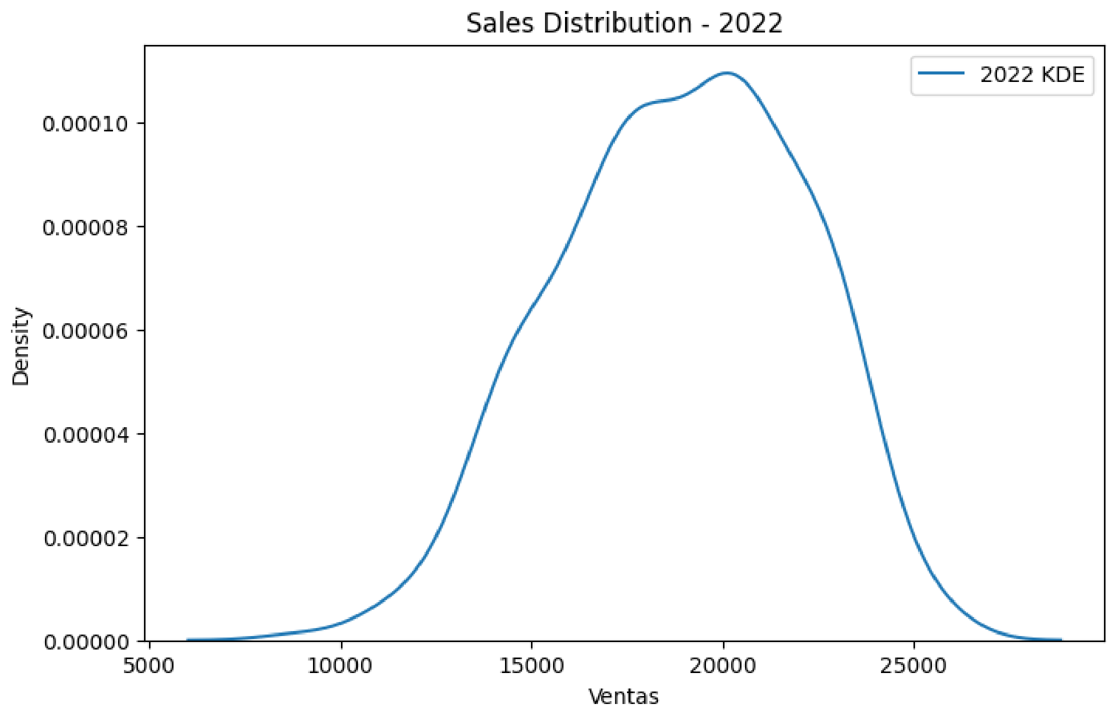



Sales Distribution - 2021
2021 Histogram
Density
Ventas
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
10000
12500
15000
17500
20000
22500
25000
27500

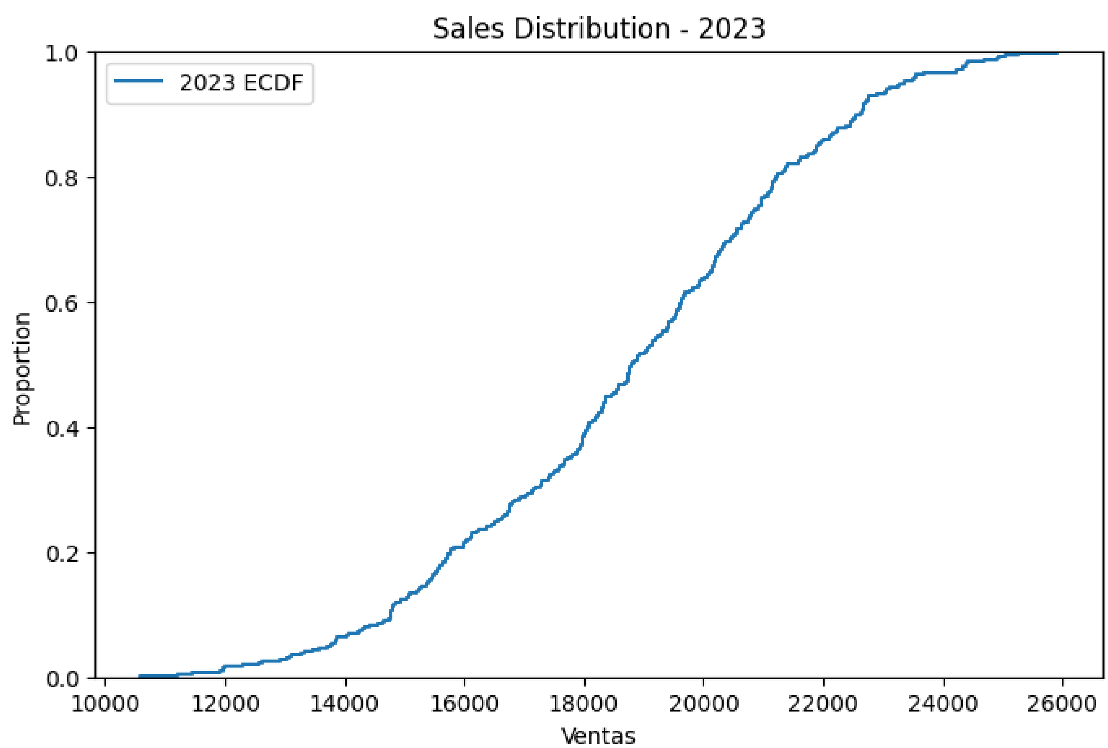

Sales Distribution - 2021
2021 KDE
Density
Ventas
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
10000
15000
20000
25000
30000


Sales Distribution - 2022
2022 ECDF
Proportion
0.0
0.2
0.4
0.6
0.8
1.0
10000
12500
15000
17500
20000
22500
25000
Ventas

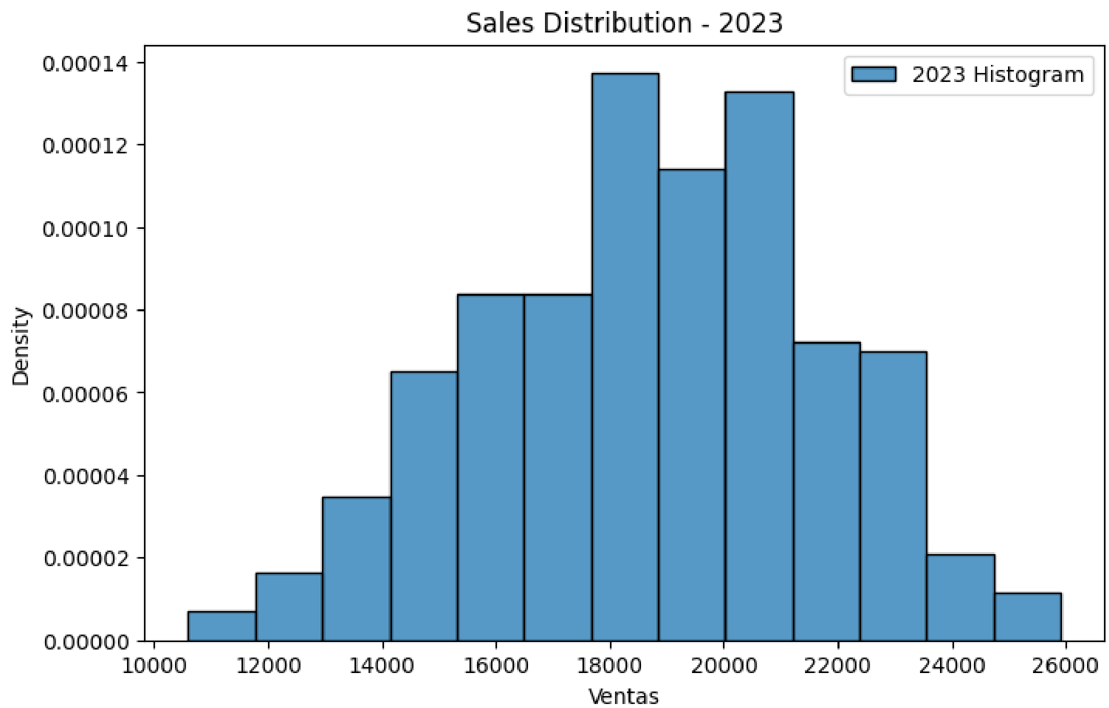

Sales Distribution - 2022
2022 Histogram
Density
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
10000
12500
15000
17500
20000
22500
25000
Ventas

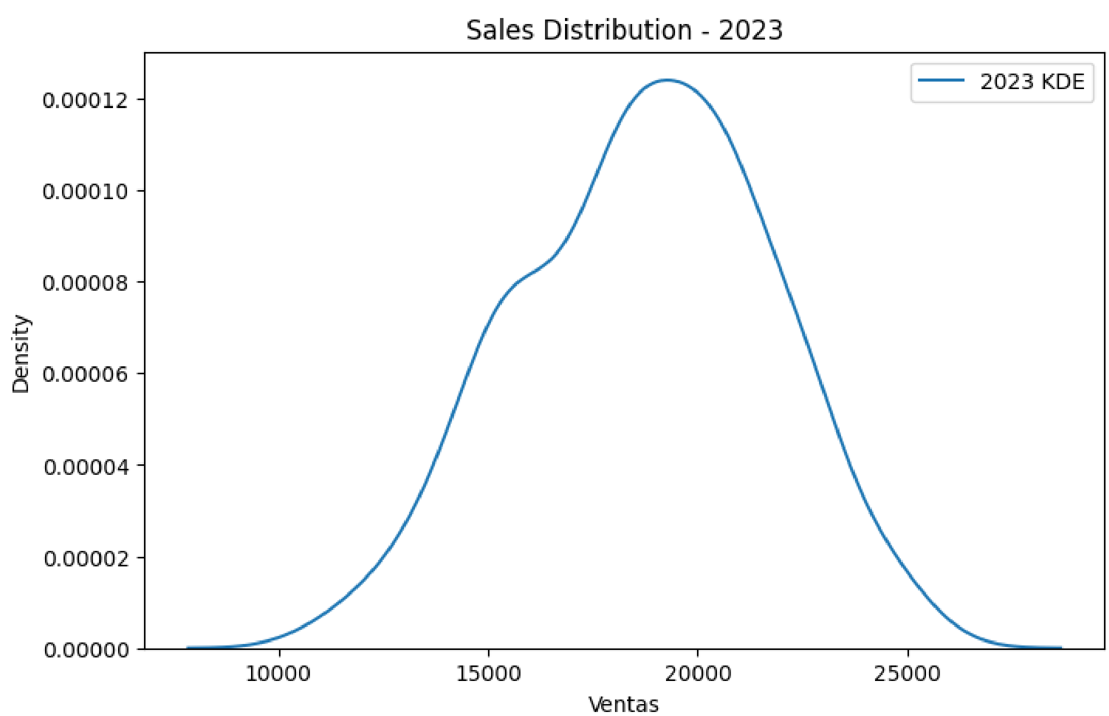


Sales Distribution - 2022
2022 KDE
Density
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
5000
10000
15000
20000
25000
Ventas



Sales Distribution - 2023
2023 ECDF
Proportion
0.0
0.2
0.4
0.6
0.8
1.0
10000
12000
14000
16000
18000
20000
22000
24000
26000
Ventas


Sales Distribution - 2023
2023 Histogram
Density
Ventas
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
0.00014
10000
12000
14000
16000
18000
20000
22000
24000
26000


Sales Distribution - 2023
2023 KDE
Density
Ventas
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
10000
15000
20000
25000