1

CEIA - PROBABILIDAD y ESTADÍSTICA - TP 2

MARTIN BROCCA

1) $x = \{0,1,2,3\}$

X	0	1	2	3
p	$\frac{3\theta}{3}$	$\frac{6\theta}{3}$	$\frac{1-3\theta}{3}$	$\frac{2(1-3\theta)}{3}$

$n_0 = x = 0 =$ 2 veces

$n_1 = x = 1 =$ 1 vez

$n_2 = x = 2 =$ 4 veces

$n_3 = x = 3 =$ 3 veces

$$L(\theta) = f_{x_0}(x_0 . \theta)^{n_0} . f_{x_1}(x_1 . \theta)^{n_1} . f_{x_2}(x_2 . \theta)^{n_2} . f_{x_3}(x_3 . \theta)^{n_3}$$

$$= \left(\frac{\cancel{3}\theta}{\cancel{3}}\right)^2 . \left(\frac{\cancel{6}2\theta}{\cancel{3}}\right)^1 . \left(\frac{1-3\theta}{3}\right)^4 . \left(\frac{2 \cdot (1-3\theta)}{3}\right)^3$$

$$L(\theta) = \underbrace{\theta^2 . 2\theta}_{} . \left(\frac{1-3\theta}{3}\right)^4 . \left(\frac{2(1-3\theta)}{3}\right)^3$$

$$= 2\theta^3 . \left(\frac{1}{3}\right)^4 (1-3\theta)^4 . \left(\frac{2}{3}\right)^3 . (1-3\theta)^3$$

$$= 2 . \frac{1}{81} . \frac{8}{27} . \theta^3 . (1-3\theta)^4 . (1-3\theta)^3$$

$$= \frac{16}{2187} . \theta^3 (1-3\theta)^3 . (1-3\theta)^4$$

$$\boxed{= \frac{16}{2187} . \theta^3 (1-3\theta)^7}$$

//

MAXIMIZAMOS $L(\theta)$ aplicando $\ln$.

$$\ell(\theta) = \ln\left(\frac{16}{2187} \cdot \theta^3 \cdot (1-3\theta)^7\right)$$

$$= \ln\left(\frac{16}{2187}\right) + \ln(\theta^3) + \ln\left((1-3\theta)^7\right)$$

$$= \ln(16) - \ln(2187) + \ln(\theta^3) + \ln\left((1-3\theta)^7\right)$$

$$= \ln(16) - \ln(2187) + 3\ln(\theta) + 7\left(\ln(1-3\theta)\right)$$

Aplico derivada con respecto a θ

$$\Rightarrow \frac{d\ell(\theta)}{d\theta} = d\ln(16) - d\left(\ln(2187)\right) + d\left(3\ln(\theta)\right) + d\left(7\ln(1-3\theta)\right)$$

$$= 0 - 0 + \frac{3}{\theta} + \frac{7.(-3)}{(1-3\theta)}$$

$$= \frac{3}{\theta} - \frac{21}{1-3\theta} = 0$$

$$\Rightarrow \frac{3}{\theta} = \frac{21}{1-3\theta} = 3(1-3\theta) = 21\theta$$

$$\Rightarrow 3 - 9\theta = 21\theta \Rightarrow 3 = 30\theta$$

$$\Rightarrow \theta = \frac{3}{30} \Rightarrow \boxed{\theta = \frac{1}{10}}$$

Ejercicio 2:

x	y
0	0
6	71
11	91
16	219
23	540

$y = a + bx + cx^2$

$$S = \sum_{i=1}^{n} \left(y_i - (a + bx + cx^2)\right)^2$$

derivando: $\left\{ \frac{de_i^2}{da} = 2e_i \cdot \frac{de_i}{da} \right\}$

$$\frac{dS}{da} = \frac{d}{da} \sum \left(y_i - (a + bx_1 + cx_2^2)\right)^2$$

$$= \sum \frac{d}{da} \left(\underbrace{y_i - (a + bx + cx^2)}_{e_i}\right)^2$$

$$\Rightarrow \frac{de_i}{da} = \frac{d}{da}\left(y_i - (a + bx + cx^2)\right) = \frac{d}{da}\left(\underset{0}{y_i} \underset{-1}{-a} \underset{0}{-bx} \underset{0}{-cx^2}\right)$$

$$\therefore \frac{dS}{da} = \sum 2\left(y_i - (a + bx + cx^2)\right) \cdot (-1) = \sum -2\left(y_i - a - bx - cx^2\right)$$

$$\boxed{= -2 \sum \left(y_i - a - bx_i - cx_i^2\right)}$$

iguelo a $\emptyset$

$$\frac{dS}{da} = -2 \sum \left(y_i - a - bx_i - cx_i^2\right) = 0$$

$$= \sum y_i - \sum a - \sum bx_i - \sum cx_i^2 = 0$$

$$= \sum y_i - \underbrace{\sum a}_{a \cdot n} - b\sum x_i - c\sum x_i^2 = 0$$

$$= \boxed{\sum y_i - a \cdot n - b \sum x_i - c \sum x_i^2 = 0} \quad (1)$$

$$\frac{dS}{db} = -2\sum x_i\left(y_i - a - bx_i - cx_i^2\right) = 0$$

$$\Rightarrow \boxed{\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3} \quad ②$$

$$\frac{dS}{dc} = -2\sum x_i^2\left(y_i - a - bx_i - cx_i^2\right) = 0$$

$$\Rightarrow \boxed{\sum x_i^2 y = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4} \quad ③$$

Calcular los valores para reemplazar en las ecuaciones.

	x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
	0	0	0	0	0	0	0
	6	71	36	216	1296	426	2556
	11	91	121	1331	14641	1001	11011
	16	219	256	4096	65536	3504	56064
	23	540	529	12167	279841	12420	285660
$\Sigma =$	56	921	942	17810	361314	17351	355291

Sistema de ecuaciones:

1. $an + b\sum x_i + c\sum x_i^2 = \sum y_i$

2. $a\sum x_i + b\sum x_i^2 + c\sum x_i^3 = \sum x_i y_i$

3. $a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4 = \sum x_i^2 y_i$

Trabajamos con 1 y 2:

$1 \times \sum x_i = \quad an\sum x_i + b\sum x_i \sum x_i + c\sum x_i^2 \sum x_i = \sum y_i \sum x_i$

$2 \times n = - \quad an\sum x_i + bn\sum x_i^2 + c\sum x_i^3 . n = n . \sum x_i y_i$

$$\boxed{\sum y_i \sum x_i - n\sum x_i y_i = 0 + b\left(\sum x_i \sum x_i - n\sum x_i^2\right) + c\left(\sum x_i^2 \sum x_i - \sum x_i^3 n\right) =}$$

ecuación 4

1 y 3:

$1 \times \sum x_i^2 \quad an\sum x_i^2 + b\sum x_i \sum x_i^2 + c\sum x_i^2 \sum x_i^2 = \sum y_i \sum x_i^2$

$3 \times n: \quad an\sum x_i^2 + bn\sum x_i^3 + cn\sum x_i^4 = n\sum x_i^2 y_i$

$$\boxed{\sum y_i \sum x_i^2 - n\sum x_i^2 y_i = 0 + b\left(\sum x_i \sum x_i^2 - n\sum x_i^3\right) + c\left(\sum x_i^2 \sum x_i^2 - n\sum x_i^4\right)}$$

ecuación 5

Resolver b y c de las ecuaciones 4 y 5

3 $\Sigma y_i \Sigma x - n \Sigma x_i y_i = b(\Sigma x_i \Sigma x_i - n \Sigma x_i^2) + c(\Sigma x_i^2 \Sigma x_i - \Sigma x_i^3 \cdot n)$

4 $\Sigma y \Sigma x_i^2 - n \Sigma x^2 y = b(\Sigma x_i \Sigma x_i^2 - n \Sigma x_i^3) + c(\Sigma x_i^2 \Sigma x_i^2 - n \Sigma x_i^4)$

de 3 $b = \dfrac{\Sigma y_i \Sigma x - n \Sigma x_i y_i - c(\Sigma x_i^2 \Sigma x_i - \Sigma x_i^3 n)}{\Sigma x_i \Sigma x_i - n \Sigma x_i^2}$

Reemplazo en 4

$$\Sigma y \Sigma x_i^2 - n \Sigma x^2 y = \frac{(\Sigma y_i \Sigma x - n \Sigma xy)}{\Sigma x_i \Sigma x_i - n \Sigma x_i^2}(\Sigma x_i \Sigma x_i^2 - n \Sigma x_i^3) - c(\Sigma x_i^2 \Sigma x_i - \Sigma x_i^3 n) \cdot \frac{(\Sigma x_i^2 \Sigma x_i - \Sigma x_i^3 n)}{\Sigma x_i \Sigma x_i - n \Sigma x_i^2} + c(\Sigma x_i^2 \Sigma x_i^2 - n \Sigma x_i^4)$$

despejo c y resuelvo $\boxed{c = 1,1884}$

Resolviendo b en 4 $\Rightarrow \boxed{b = -5,0549}$

USANDO b y c en 1 $\Rightarrow \boxed{a = 16,9262}$

$$\boxed{Y = 16,93 - 5,05x + 1,19x^2}$$

Ejercicio 3:

Nº de clientes: $n=7$
Cliente en mora: $x=1$
Priori para % Morosidad: $p \sim \beta(2,2)$

encontrar: posteriori, media, varianza

$\Rightarrow$ Distribución a priori: $p \sim Beta(\alpha=2, \beta=2)$
Verosimilitud: $Binomial(x=1, n=7)$

Distribución posterior: como $L(\theta)$ es Binomial $\Rightarrow$ Beta

$$p/datos \sim Beta(\alpha+x, \beta+n-x)$$

$$\Rightarrow p/datos \sim Beta(2+1, 2+6) = \boxed{Beta(3,8)}$$

$$E(p) = \frac{\alpha}{\alpha+\beta} = \frac{3}{3+8} = \frac{3}{11} \approx \boxed{0{,}273}$$

$$Var(p) = \frac{\alpha\beta}{(\alpha+\beta)^2 \cdot (\alpha+\beta+1)} = \frac{3 \cdot 8}{(3+8)^2(3+8+1)} = \frac{24}{121 \cdot 12} = \frac{24}{1452} \approx \boxed{0{,}0165}$$