ELEC 442 HW2

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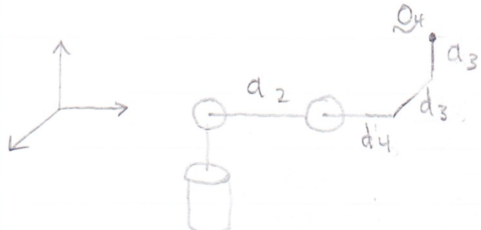
HW2 Documentation

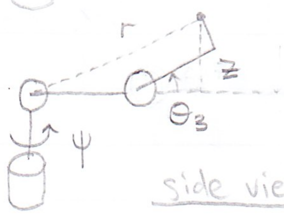
We know our desired translational vector $\underset{\sim}{o}_6 - \underset{\sim}{o}_0$

We find our desired wrist centre because we know our desired $\mathbb{k}_6$

$$\underset{\sim}{o}_4 = \underset{\sim}{o}_6 - d_6 \mathbb{k}_6 \quad , \text{ where } d_6 = 60.00 \text{ mm}$$

Now we've reduced the manipulator to only an arm:





side view

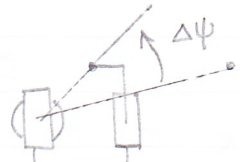
To determine the effects of $\theta_1, \theta_2, \theta_3$ on translation, look at the arm in cylindrical coordinates.

We see that r varies only with θ_3, because of the linkage geometry

- Only θ_3 can change r
- θ_1 can only change ψ, not z or r
- θ_2 changes z
- because of a_3, θ_2 and θ_3 change ψ slightly:

top view

home position
$\theta_2 = \theta_3 = 0$

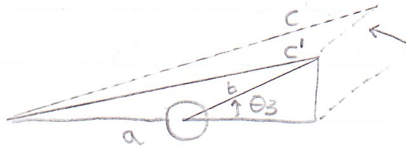


$\theta_2 = 0, \theta_3 = 90°$

we see that by changing θ_3, we've caused a $\Delta\psi$

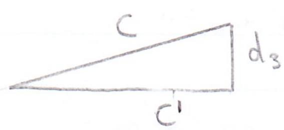
Finding θ_3

Since only θ_3 can change r, we will solve it first



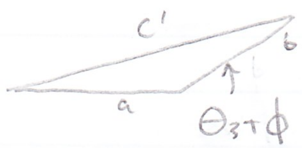
No matter what $|\underset{\sim}{O}_4 - \underset{\sim}{O}_0|$ is, these relationships hold because of the offset

top view



given our desired c, find c'

$$c' = \sqrt{c^2 - d_3^2}$$



$$b = \sqrt{d_4^2 + a_3^2}$$

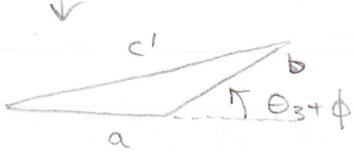
and d_4 and a_3 cause an offset ϕ

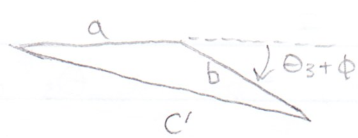
$$\phi = \tan^{-1}\left(\frac{a_3}{d_4}\right)$$

And this is just a Kahan P4 problem, so

$$\theta_3 = \text{Kahan P4}(a, b, c') - \phi$$

We also see that we'll have two solutions for θ_3, the "elbow up" and "elbow down".



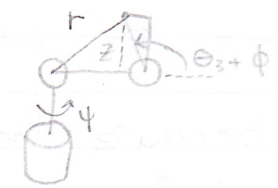


So, we have two solutions for θ_3 to give us the correct magnitude of our translation vector $\underset{\sim}{O}_4 - \underset{\sim}{O}_0$

$$[\theta_{3U}] \quad [\theta_{3D}]$$

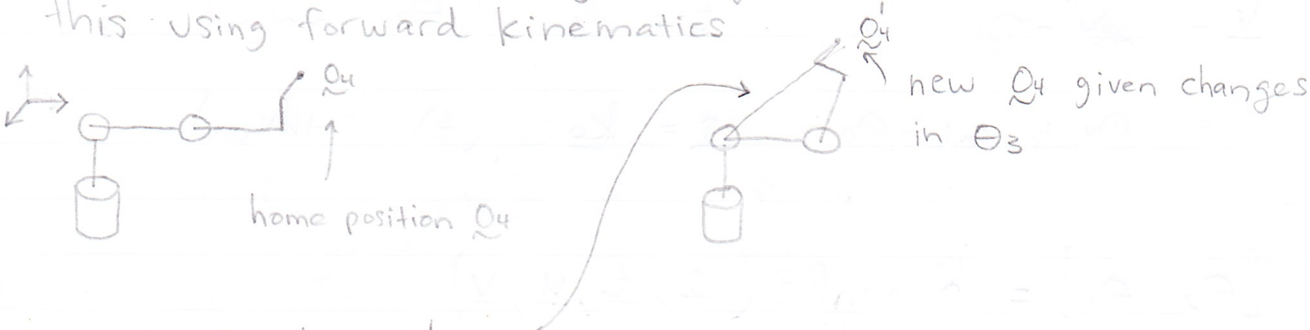
Finding θ_1, θ_2

Back to the cylindrical diagram:



we've adjusted θ_3 to get the correct $|\underset{\sim}{o}_4 - \underset{\sim}{o}_0|$, which is r.
However, the z and ψ now need to be adjusted

We now assume that θ_1, θ_2 didn't change, and solve for a $\underset{\sim}{o}_4'$, given only changes in θ_3. We solve for this using forward kinematics



we solve for this $\underset{\sim}{o}_4' - \underset{\sim}{o}_0$

So, now we have $\underset{\sim}{o}_4' - \underset{\sim}{o}_0$ that needs to be rotated by $\underline{k}_1$ and $\underline{k}_0$, the θ_2 and θ_1 translations.

We know from before that θ_1 can only change ψ, and θ_2 can change both ψ and z.
We can set this up as a kahan P3 problem

$\underset{\sim}{o}_4' - \underset{\sim}{o}_0$ will be our $\underline{v}$. $\underset{\sim}{o}_{4\_desired} - \underset{\sim}{o}_0$ will be our $\underline{u}$

$\underline{v}$ rotates about $\underline{t}$, and our $\underline{t}$ will be $\underline{k}_1$, so that $\underline{v}$ rotates about $\underline{k}_1$ by θ_2 degrees.

$\underline{u}$ rotates about $\underline{s}$, and our $\underline{s}$ will be $\underline{k}_0$, so that $\underline{u}$ rotates about $\underline{k}_0$ by θ_1 degrees.

$\underset{\sim}{o}_{4\text{-desired}} - \underset{\sim}{o}_0$ and $\underset{\sim}{o}_4' - \underset{\sim}{o}_0$ will rotate and meet at some arbitrary vector, but the angles of their rotations their rotations will be our joint parameters θ_2 and θ_1

The reason we rotate by θ_2 first is because the rotation will change ψ and z, not just z.

If we rotated by θ_1 first, we'd set a ψ. Then, the rotation by θ_2 would change the ψ, as seen at the bottom of page 1.

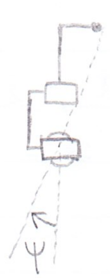
$\underline{v} = \underset{\sim}{o}_4' - \underset{\sim}{o}_0$ $\qquad$ $\underline{t} = \mathbb{k}_1$ (θ_2 rotation)

$\underline{u} = \underset{\sim}{o}_{4\text{-desired}} - \underset{\sim}{o}_0$ $\qquad$ $\underline{s} = \mathbb{k}_0$ (θ_1 rotation)

$$[\theta_2, \theta_1] = \text{KahanP3}(\underline{s}, \underline{t}, \underline{u}, \underline{v})$$

We'll have two sets of solutions:

left arm:



$[\theta_{1L}, \theta_{2L}]$

right arm:

$[\theta_{1R}, \theta_{2R}]$

Solving the Wrist:

We've solved for the translation, set by the "arm" of the manipulator. Because of the two elbow solutions for Θ_3, and the two arm solution sets for Θ_1 and Θ_2, we have four translational solution sets:

$$\begin{bmatrix} \Theta_{3U} \\ \Theta_{1L} \\ \Theta_{2L} \end{bmatrix} \quad \begin{bmatrix} \Theta_{3U} \\ \Theta_{1R} \\ \Theta_{2R} \end{bmatrix} \quad \begin{bmatrix} \Theta_{3D} \\ \Theta_{1L} \\ \Theta_{2L} \end{bmatrix} \quad \begin{bmatrix} \Theta_{3D} \\ \Theta_{1R} \\ \Theta_{2R} \end{bmatrix}$$

We now run these sets through forward kinematics to find $\underline{C_3}$. We can then compare $\underline{C_3}$ against our desired $\underline{C_6}$ to find the wrist parameters Θ_4, Θ_5, and Θ_6

Solving The Wrist

Through forward kinematics, we found our current $\underline{C_3}$.
We need to rotate $\underline{C_3}$ to get to $\underline{C_d}$

$\underline{k_3}$ rotates to get a $\underline{k_d}$

We see that the last joint, θ_6 rotating about k_5, doesn't change our $\underline{k_d}$. So, $\underline{k_5} = \underline{k_6}$

So, we can see that only θ_4 and θ_5 will change $\underline{k_3}$ to get our desired $\underline{k_6}$

We have two vectors, $\underline{k_3}$ and $\underline{k_6}$, and two angles, θ_4 and θ_5, relating them, and we're trying to solve for those angles. We can set this up as a KahanP3.

$$e^{\theta_4 \underline{k_3} \times} \underline{k_d} = e^{\theta_5 \underline{k_4} \times} \underline{k_3}$$

but $\underline{k_4} = -j_3$, and we already know $-j_3$

$$e^{\theta_4 \underline{k_3} \times} \underline{k_d} = e^{\theta_5 (-j_3) \times} \underline{k_3}$$

$\underline{s} = \underline{k_3}$ $\qquad$ $\underline{t} = -j_3$

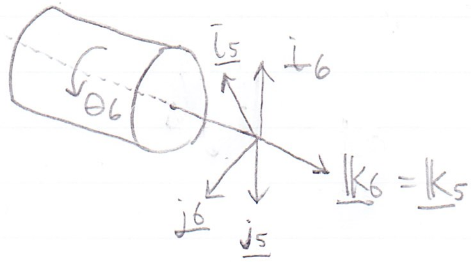
$\underline{u} = \underline{k_6}$ $\qquad$ $\underline{v} = \underline{k_3}$

$$[\theta_4, \theta_5] = \text{KahanP3}(\underline{s}, \underline{t}, \underline{u}, \underline{v})$$

This will again yield two solution sets:

$$[\theta_{4A}, \theta_{5A}], \quad [\theta_{4B}, \theta_{5B}]$$

Now that $\underline{k_6}$ has been properly oriented by changing θ_4 and θ_5, we have



We now have to rotate $\underline{j_5}$ about $\underline{k_6}$ by θ_6 degrees to get our desired $\underline{j_6}$, which was an input.

When we know two vectors, and we want to find the angle between them, we can set it up as a Kahan P2
θ_6 rotates about $\underline{k_5}$, but $\underline{k_5} = \underline{k_6}$

$$e^{\theta_6 \underline{k_6} \times} \underline{j_5} = \underline{j_6}$$

$\underline{s} = \underline{k_6}$, $\underline{u} = \underline{j_5}$, $\underline{w} = \underline{j_6}$

$\theta_6 = \text{KahanP2}(\underline{s}, \underline{u}, \underline{w})$ This will have 2 solutions

$$\left.\begin{array}{ll} \theta_3 & \text{2 sets} \\ \theta_1, \theta_2 & \text{2 sets} \\ \theta_4, \theta_5 & \text{2 sets} \\ \theta_6 & \text{2 sets} \end{array}\right\} \text{This gives us up to 16 solution sets for a given input.}$$

However, when we apply our joint constraints:
$-160° < \theta_1 < 160°$, $-225° < \theta_2 < 45°$, $-135° < \theta_3 < 135°$,
$-110° < \theta_4 < 170°$, $-100° < \theta_5 < 100°$, $-266° < \theta_6 < 266°$

We find we get up to 12 solutions for any given input.
This finding was based on testing the check values given in the HW description.